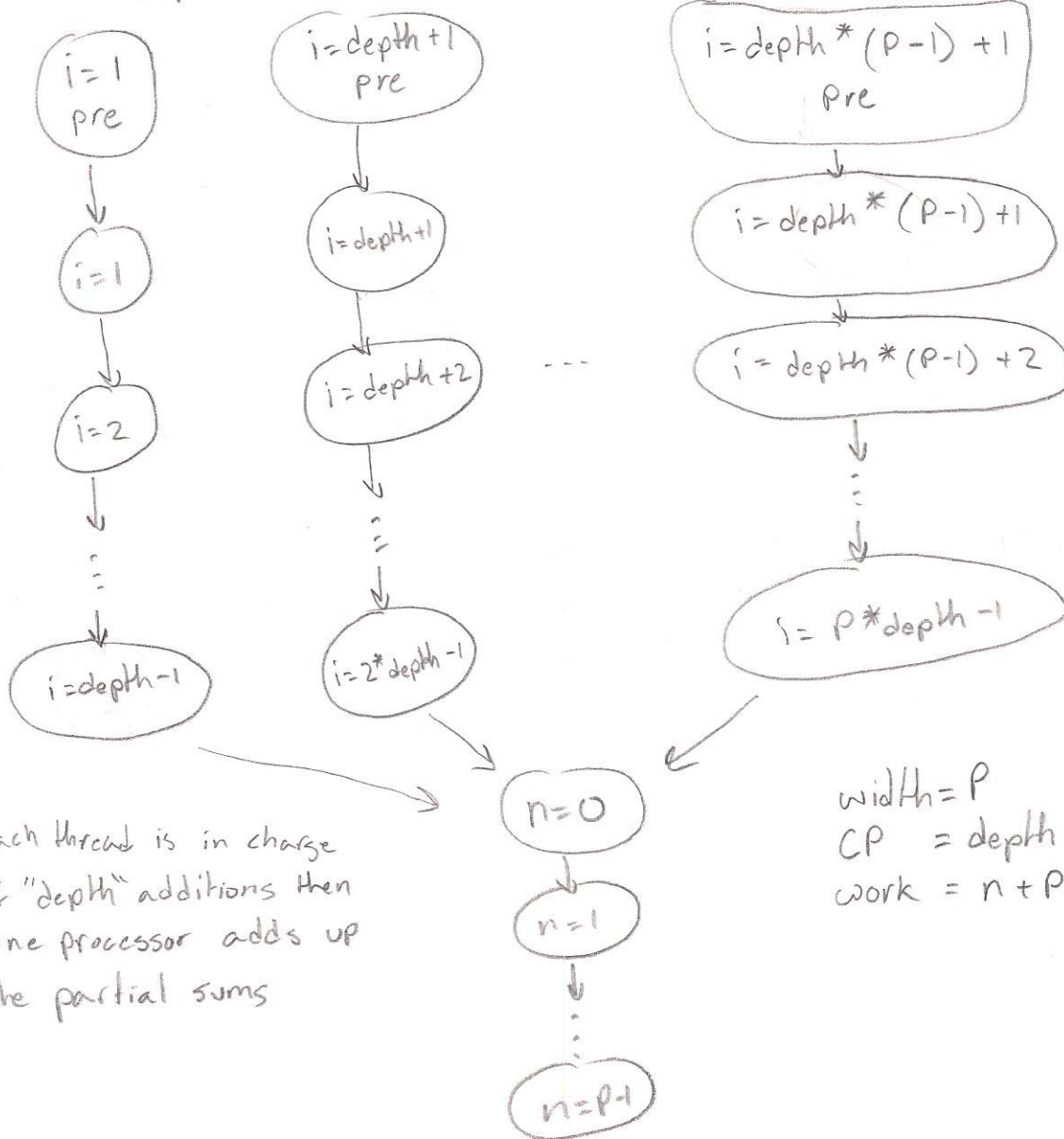


All tasks are dependent on variable "result"

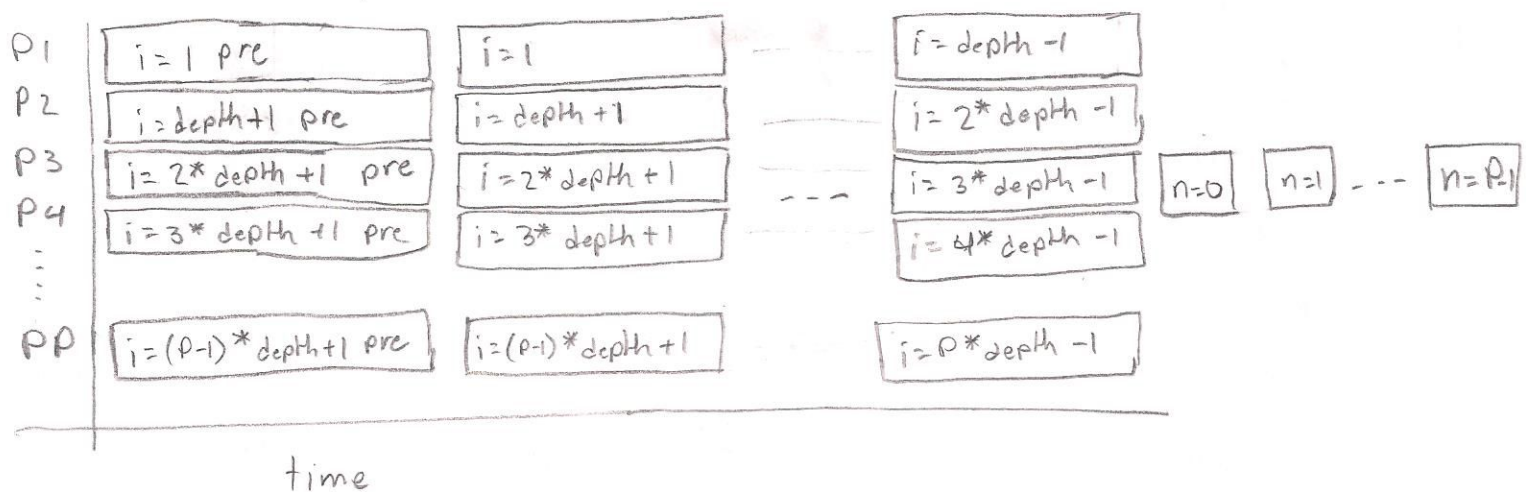
width = 1  
CP = n  
work = n

depth = floor(n/P) ← this is the number of additions each processor will do



each thread is in charge of "depth" additions then one processor adds up the partial sums

width = P  
CP = depth + P  
work = n + P



Each thread is in charge of "depth" number of sums. Then one thread (P3) is in charge of adding up the partial sums. The pre task for each thread is the initial set of the result variable

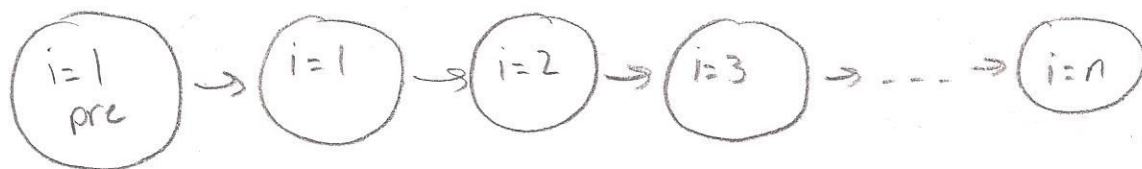
3.2) `reduce<int, max>` - Yes. It would find the max element in each section of the array then compare the max for each section against each other to find the whole max

`reduce<std::string, concat>` - yes, He also would concat each section then concat each section together, it would maintain proper order

`reduce<float, sum>` - yes, code is templeted, and int is similar to float & would work the same way

`reduce<float, max>` - yes, similar to `reduce<int, max>`

4)



all tasks are dependent on the  $i-1$  iteration. This is because to calculate  $pr[i]$ , you need  $pr[i-1]$

width = 1  
CP =  $n+1$   
work =  $n+1$

### Parallel version

Similar to the reduce function, the array will be split into smaller chunks and prefix sum will be computed then the last sum of each chunk is added into the next chunk

Pass 1	ar	pr	ar	pr	ar	pr
	0	0	6	0	12	0
	1	0	7	6	13	12
	2	1	8	13	14	25
	3	3	9	21	15	39
	4	6	10	30	16	54
	5	10	11	40	17	70
		15		51		87
	thread 1 offset = 15		thread 2 offset = 51		thread 3 offset = 87	

do partial prefix sum for each chunk of the data in this case 3 chunks. the offset is the largest partial sum of that chunk. This is step 1

Pass 2	pr	fin	pr	fin	pr	fin
	0	0	0	15	0	66
	0	0	6	21	12	78
	1	1	13	28	25	91
	3	3	21	36	39	105
	6	6	30	45	54	120
	10	10	40	55	70	136
	15	15	51	66	87	153
	error = 0		error = 15		error = 51 + 15 = 66	

Now add all the offsets from the previous chunks to each member of the partial sum the sum of the offsets is the error. This summation is independent and can be threaded similar to step 1. This is step 2 and final step

## 5) mergesort super highlevel pseudocode

mergesort(arr)

if  $\text{len}(\text{arr}) \leq 1$ :  
return arr

left = left half of arr  
right = right half of arr

left = mergesort(left)  
right = mergesort(right)  
return merge(left, right)

merge(left, right)

result = empty array

while left is not empty  
and right is not empty:

if  $\text{left}[0] \leq \text{right}[0]$ :  
result += left[0]  
rem left[0]

else:  
result += right[0]  
rem right[0]

if left is not empty  
result += left

if right is not empty  
result += right

return result

an array of size 1 is sorted

split into 2 parts

sort both left + right parts  
recursively

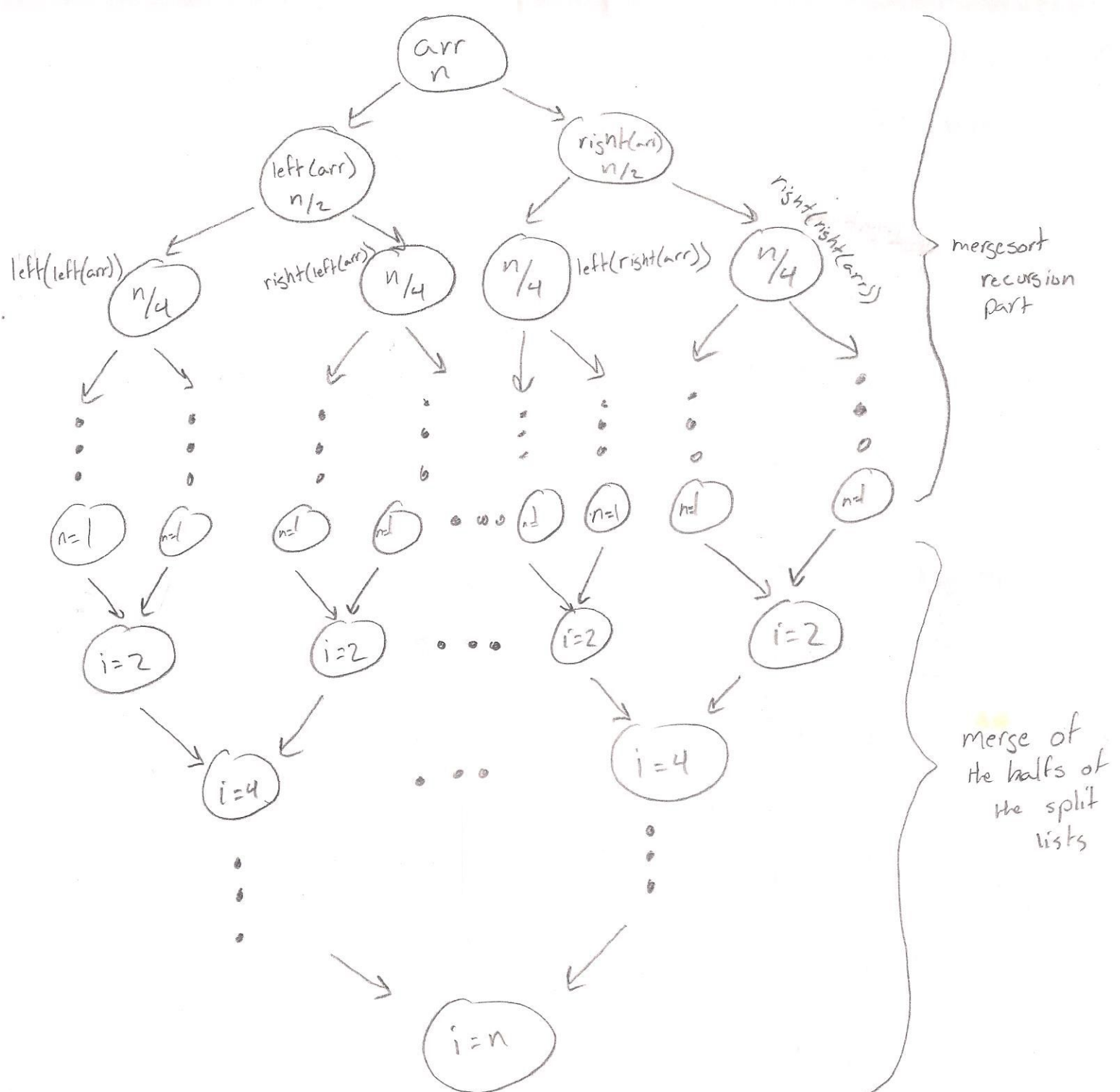
merge the two parts in order

create empty result array

go through both left + right sides and  
place the smaller of the first element into  
the result array

place the rest of the elements from  
either list into the result array



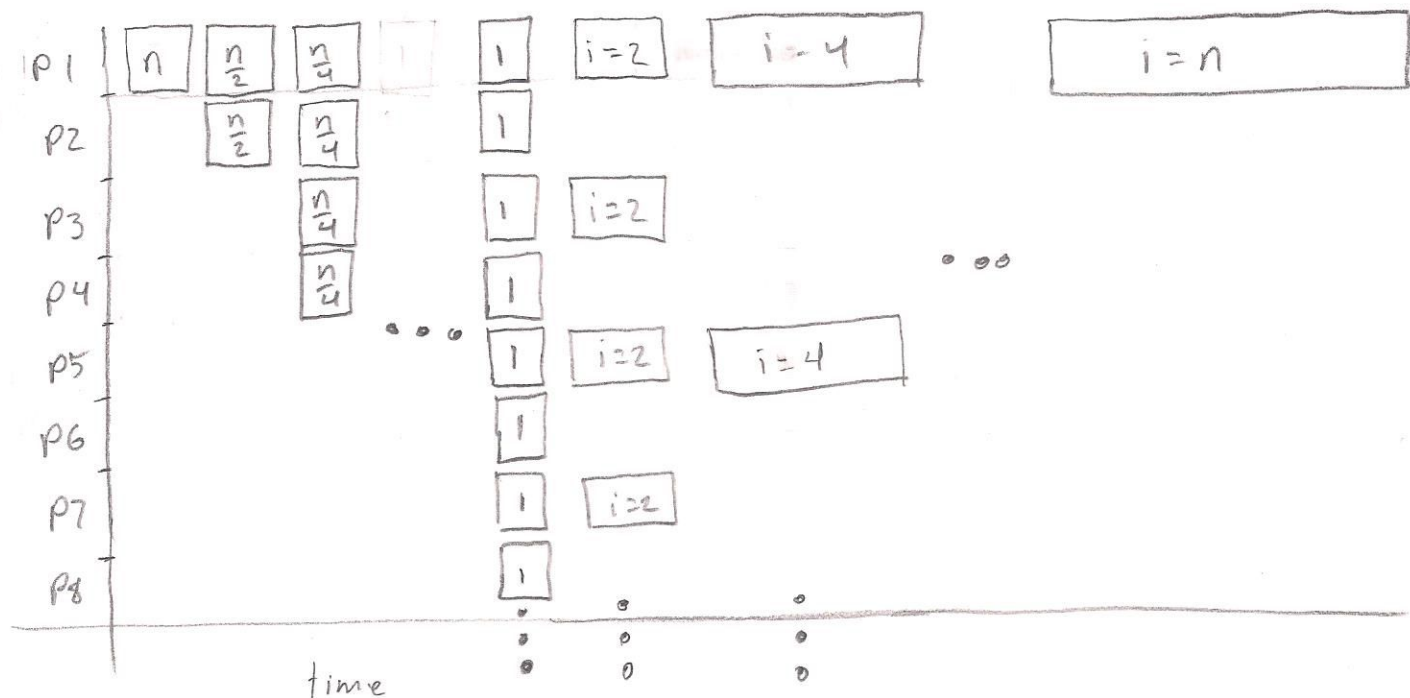


The first half of the dependency tree is the recursive part of mergesort() function. It splits the array into 2 subarrays. Then these subarrays are sorted and merged in the second half of the dependency graph. The merge part of the dependency graph takes more time to compute.

$$\text{work} = 3n - 2$$

$$\text{CP} = 2 \log_2(n) + 1$$

$$\text{width} = 1$$



Diamond shape? If we don't have any penalty for spawning threads, then each recursive call would spawn 2 threads: one for each half of its current thread. There is very little "work" happening other than the creation of the threads. That is why the bars here are thin. When the merger of two subarrays happens, one thread will handle it but because it has to go through 2 subarrays, the time taken to sort will double as the recursion stack collapses to the end condition ' $i=n$ '.

To make the code more parallel, I would remove some of the initial recursion so there is little dependency at the beginning of the run. For example with a P of 4, the first 2 "layers" of the Gantt chart / dependency graph will be gone. Then the processor mergesorts the 4 halves of the array. At the end a thread sorts the 2 halves that are left.