A functional dependency A->B in a relation holds if two tuples having same value of attribute A also have same value for attribute B.

STUDENT

STUD_NO	STUD_NAME	STUD_PHONE	STUD_STATE	STUD_COUNTRY	STUD_AGE
1	RAM	9716271721	Haryana	India	20
2	RAM	9898291281	Punjab	India	19
3	SUJIT	7898291981	Rajsthan	India	18
4	SURESH		Punjab	India	21

Table 1

STUD_NO->STUD_NAME, STUD_NO->STUD_PHONE hold

but

STUD_NAME->STUD_PHONE do not hold

Functional Dependency Set

Functional Dependency set or FD set of a relation is the set of all FDs present in the relation. For Example, FD set for relation STUDENT shown in table 1 is:

{ STUD_NO->STUD_NAME,

STUD_NO->STUD_PHONE,

STUD_NO->STUD_STATE,

STUD_NO->STUD_COUNTRY,

STUD_NO -> STUD_AGE,

STUD_STATE->STUD_COUNTRY }

Attribute Closure

Attribute closure of an attribute set can be defined as set of attributes which can be functionally determined from it.

Step-1: Add the attributes which are present on Left Hand Side in the original functional dependency.

Step-2: Add the attributes present on the Right Hand Side of the functional dependency.

Step-3: With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies. Repeat this process until all the possible attributes which can be derived are added in the closure.

$\{STUD\_NO\}^+ = \{STUD\_NO, STUD\_NAME, STUD\_PHONE, STUD\_STATE, STUD\_COUNTRY, STUD\_AGE\}$

$\{STUD\_STATE\}^+ = \{STUD\_STATE, STUD\_COUNTRY\}$

Consider the table student_details having (Roll_No, Name,Marks, Location) as the attributes and having two functional dependencies.

FD1: Roll_No -> Name, Marks

FD2: Name -> Marks, Location

We will calculate the closure of all the attributes present in the relation.

$\{Roll\_no\}^+ = \{Roll\_No, Marks, Name, Location\}$

$\{Name\}^+ = \{Name, Marks, Location\}$

$\{Marks\}^+ = \{Marks\}$

$\{Location\}^+ = \{Location\}$

Consider a relation R(A,B,C,D,E) having below mentioned functional dependencies.

FD1: A -> BC

FD2: C -> B

FD3: D -> E

FD4 : E -> D

We need to calculate the closure of attributes of the relation R.

$\{A\}^+ = \{A, B, C\}$

$\{B\}^+ = \{B\}$

$\{C\}^+ = \{B, C\}$

$\{D\}^+ = \{D, E\}$

$\{E\}^+ = \{E, D\}$

Find Candidate Keys and Super Keys using Attribute Closure

- If attribute closure of an attribute set contains all attributes of relation, the attribute set will be super key of the relation.
- If no subset of this attribute set can functionally determine all attributes of the relation, the set will be candidate key as well.

$\{STUD\_NO, STUD\_NAME\}^+$ = {STUD_NO, STUD_NAME, STUD_PHONE, STUD_STATE, STUD_COUNTRY, STUD_AGE}

$\{STUD\_NO\}^+$ = {STUD_NO, STUD_NAME, STUD_PHONE, STUD_STATE, STUD_COUNTRY, STUD_AGE}

{STUD_NO, STUD_NAME} will be super key but not candidate key because its subset $\{STUD\_NO\}^+$ is equal to all attributes of the relation. So, STUD_NO will be a candidate key.

Consider the relation R(A,B,C) with given functional dependencies :

FD1: A -> B

FD2: B -> C

Now, calculating the closure of the attributes as:

$\{A\}^+ = \{A, B, C\}$

$\{B\}^+ = \{B, C\}$

$\{C\}^+ = \{C\}$

Clearly, A is the candidate key as, its closure contains all the attributes present in the relation R.

Consider relation R(A, B, C, D, E) having the Functional dependencies :

FD1: A -> BC

FD2: C -> B

FD3: D -> E

FD4: E -> D

Now, calculating the closure of the attributes as:

$\{A\}^+ = \{A, B, C\}$

$\{B\}^+ = \{B\}$

$\{C\}^+ = \{C, B\}$

$\{D\}^+ = \{E, D\}$

$\{E\}^+ = \{E, D\}$

In this case, a single attribute is unable to determine all the attribute of the relation. Here, we need to combine two or more attributes to determine the candidate keys.

$\{A, D\}^+ = \{A, B, C, D, E\}$

$\{A, E\}^+ = \{A, B, C, D, E\}$

Hence, AD and AE are the two possible keys of the given relation R.

Prime Attributes

Attributes which are indispensable part of candidate keys. For example: A, D, E attributes are prime attributes in above example.

Non-Prime Attributes

Attributes other than prime attributes which does not take part in formation of candidate keys. For example, B and C attributes are non-prime attributes in above example.

Consider the relation R(A, B, C, D) with functional dependencies :

FD1: A -> BC

FD2: B -> C

FD3: D -> C

Find the candidate key.