# Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B,\ \ B \rightarrow C, A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g.: on RHS: $\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow CD\}$ can be simplified to

      $$\{A \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$$
    - E.g.: on LHS: $\{\mathrm{A} \rightarrow B,\ \ B \rightarrow C,\ \ AC \rightarrow D\}$ can be simplified to

      $$\{\mathrm{A} \rightarrow B,\ \ B \rightarrow C,\ \ A \rightarrow D\}$$
- Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

# Extraneous Attributes

- Consider a set $F$ of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in $F$.
  - Attribute A is **extraneous** in $\alpha$ if $A \in \alpha$ and $F$ logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute $A$ is **extraneous** in $\beta$ if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies $F$.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - $B$ is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping $B$ from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - $C$ is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting $C$

# Testing if an Attribute is Extraneous

- Consider a set *F* of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in *F*.
- To test if attribute A $\in \alpha$ is extraneous in $\alpha$
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in *F*
  2. check that $(\{\alpha\} - A)^+$ contains $\beta$; if it does, *A* is extraneous in $\alpha$
- To test if attribute *A* $\in \beta$ is extraneous in $\beta$
  1. compute $\alpha^+$ using only the dependencies in
     $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  2. check that $\alpha^+$ contains *A;* if it does, *A* is extraneous in $\beta$

# Canonical Cover

- A **canonical cover** for $F$ is a set of dependencies $F_c$ such that
  - $F$ logically implies all dependencies in $F_c$, and
  - $F_c$ logically implies all dependencies in $F$, and
  - No functional dependency in $F_c$ contains an extraneous attribute, and
  - Each left side of functional dependency in $F_c$ is unique.
- To compute a canonical cover for $F$:

  **repeat**

  Use the union rule to replace any dependencies in $F$

  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$

  Find a functional dependency $\alpha \rightarrow \beta$ with an extraneous attribute either in $\alpha$ or in $\beta$

  /* Note: test for extraneous attributes done using $F_c$, not F*/

  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$

  **until** $F$ does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

# Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- $A$ is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- $C$ is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of $A$ in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$