Image Restoration

Image Restoration by Inverse Filtering

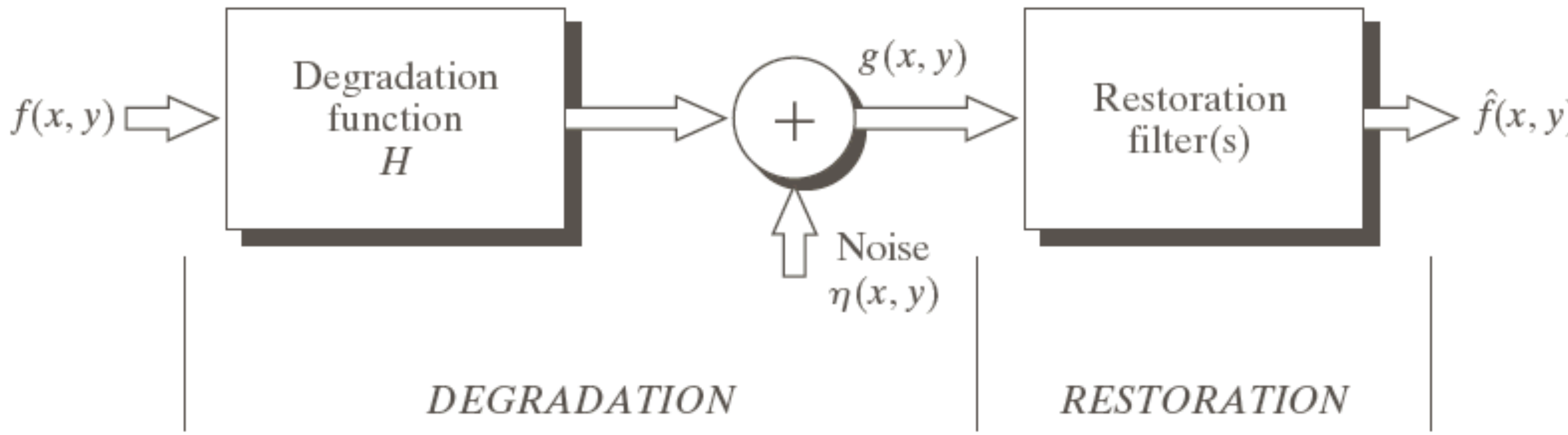

FIGURE 5.1 A model of the image degradation/restoration process.

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Image Restoration by Inverse Filtering

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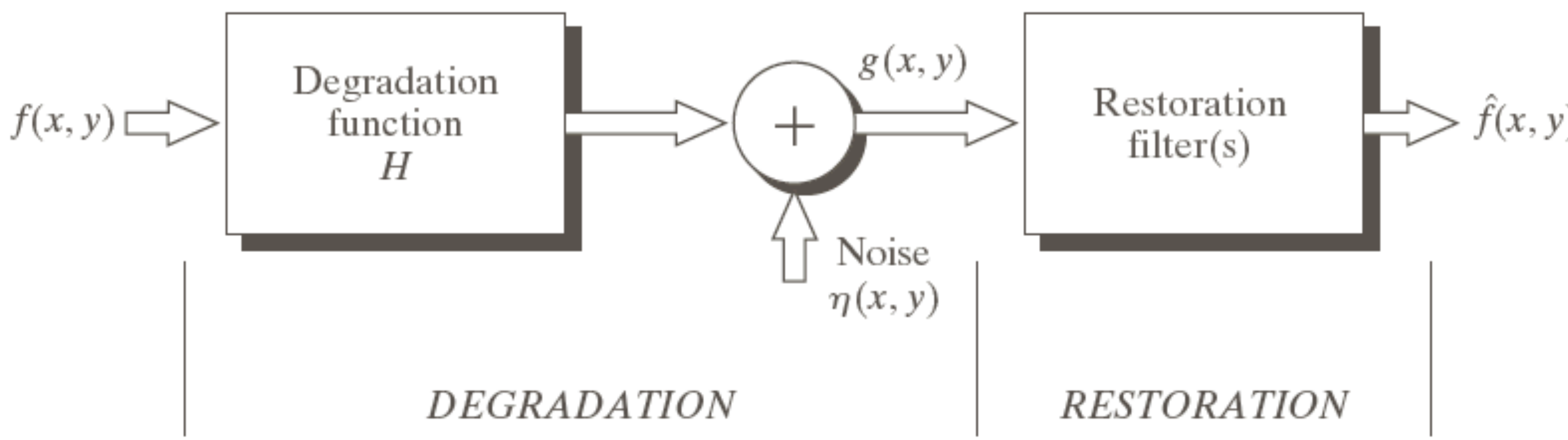

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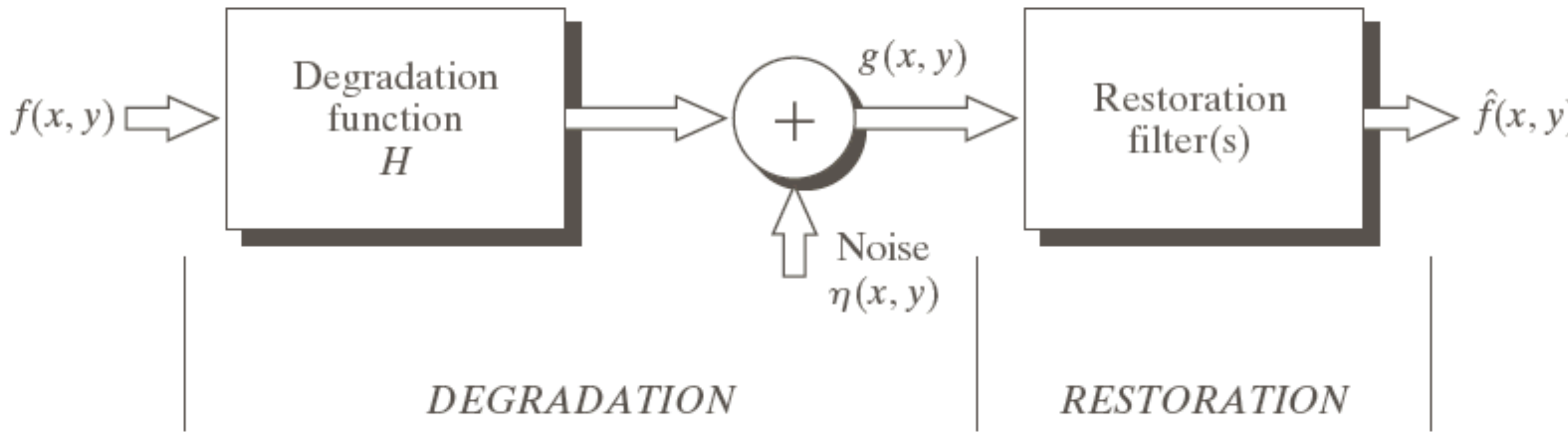

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$$\hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

Inverse Filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Bad news:

- Even when H(u,v) is known, there is always unknown noise
- Often H(u,v) has values close to zero

Example: Inverse Filtering

Atmospheric turbulence effect

$$H(u,v) = \exp\left\{-k\left[(u - M/2)^2 + (v - N/2)^2\right]^{5/6}\right\}$$

Example: Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$\frac{G(u,v)}{H(u,v)}$$

Wiener Filtering = Mean Squared Error Filtering

- Incorporates both:
  - Degradation function
  - Statistical characteristics of noise
- Assumption: noise and the image are uncorrelated
- Optimizes the filter so that MSE is minimized

$$e = \sum_{x} \sum_{y} (f(x, y) - \hat{f}(x, y))^2$$

Wiener Filter — Derivation

unknown original

after Wiener filtering

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

Wiener Filter — Derivation

unknown original　　　after Wiener filtering

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

$$= \sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2$$

Parseval's Theorem

Wiener Filter — Derivation

$$
\begin{aligned}
e &= MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2 \\
&= \sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v) - [F(u,v)H(u,v) + N(u,v)]W(u,v))|^2
\end{aligned}
$$

Unknown original

Corrupted original

Wiener filter

Wiener Filter — Derivation

independent signals

$$
\begin{aligned}
e &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2 \\
&= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2
\end{aligned}
$$

Wiener Filter — Derivation

$$
\begin{aligned}
e &= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\
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&= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2
\end{aligned}
$$

$$
\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W(u,v)
$$

Wiener Filter — Derivation

$$\frac{\partial}{\partial z}(zz^*) = 2z^*$$

$$\begin{aligned} e &= \textstyle\sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2 \\ &= \textstyle\sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2 \\ &= \textstyle\sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2 \end{aligned}$$

$$\frac{\partial e}{\partial W(u,v)} = |F|^2[2(1 - W^*H^*)(-H)] + |N|^2[2W^*]$$

Wiener Filter — Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

$$\Downarrow$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

Wiener Filter — Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

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inverse filter

Wiener Filter -- Approximation

$$W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{1}{\mathrm{SNR}}}$$

Signal-to-noise ratio

Example: Wiener Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Example: Wiener Filtering

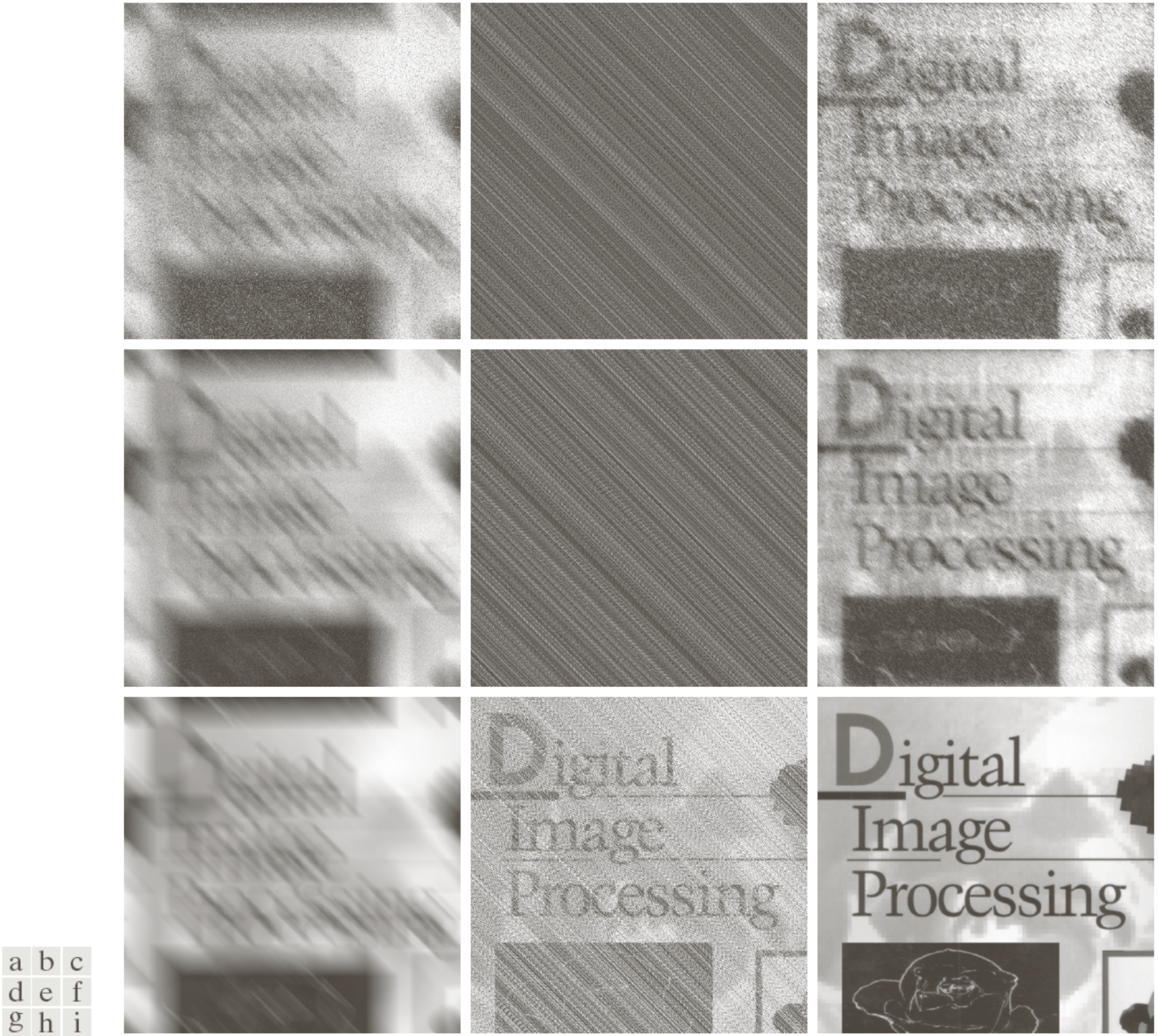

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.

Next Class

- Image reconstruction from projections (Textbook 5.11)
- Radon Transform (Textbook 5.11.3)