Discrete and Continuous Data

Discrete data can only take on certain individual values.

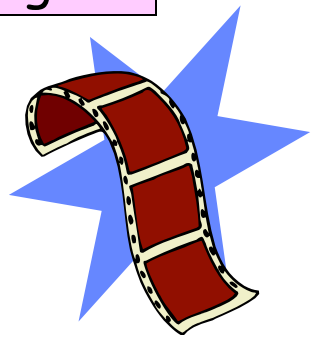
Continuous data can take on any value in a certain range.

Example 1

Number of pages in a book is a **discrete variable.**

Example 2

Length of a film is a **continuous variable.**



Example 3

Shoe size is a **Discrete variable.** E.g. $5, 5\frac{1}{2}, 6, 6\frac{1}{2}$ etc. Not in between.

Example 4

Temperature is a **continuous variable.**

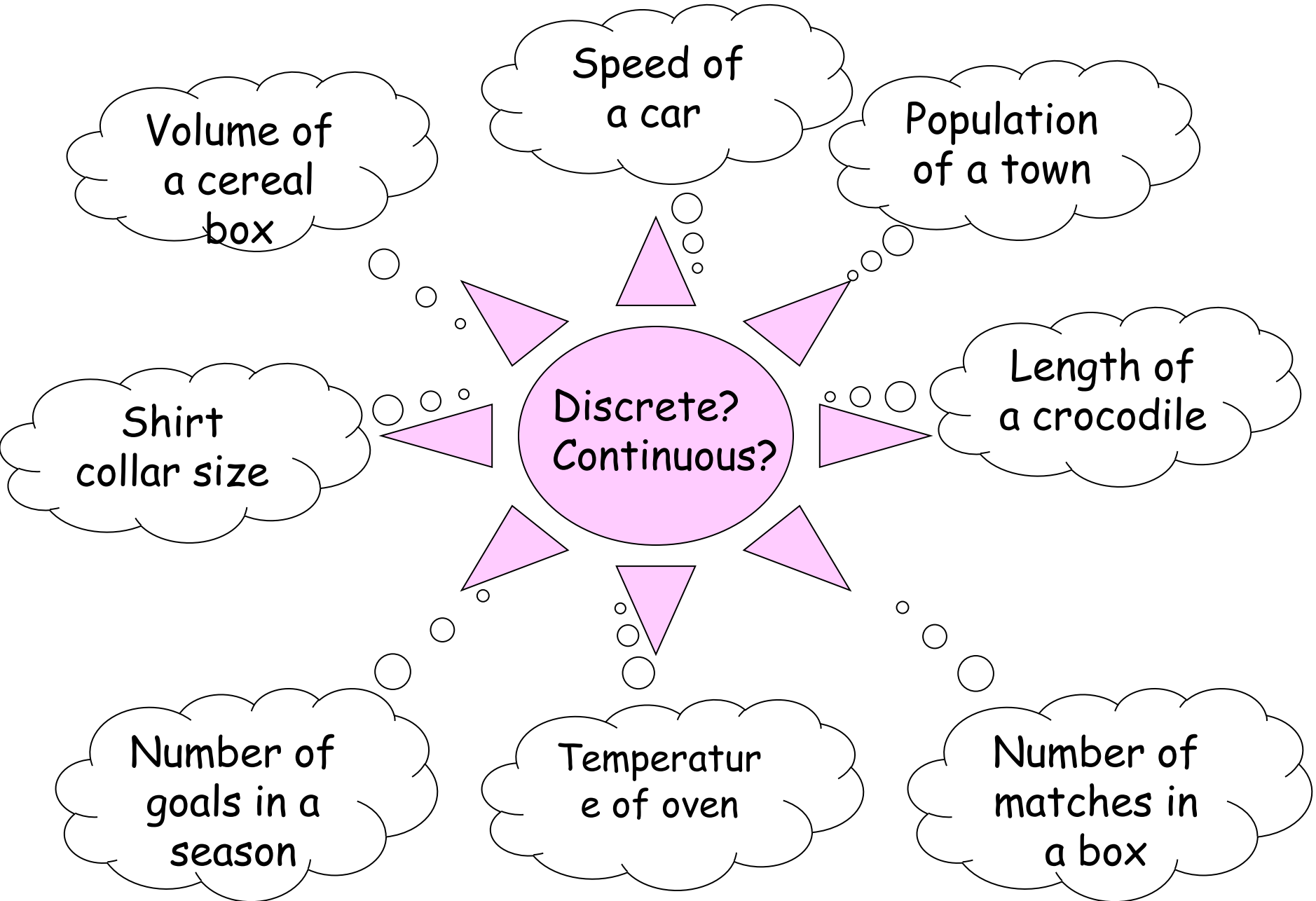
Example 5

Number of people in a race is a **discrete variable.**

Example 6

Time taken to run a race is a **continuous variable**

Group the following as either discrete or continuous data.



Discrete

Population of a town

Number of matches in a box

Shirt collar size

Number of goals in a season

Continuous

Volume of a cereal box

Top speed of a car

Length of a crocodile

Temperatur e of oven

Discrete Data

A discrete space is one in which all possible outcomes can be clearly identified and counted:

- Die Roll: S = {1,2,3,4,5,6}
- Number of Boys among 4 children: S = {0, 1, 2, 3, 4}
- Number of baskets made on two free throws: S = {0, 1, 2}

Data that can has a clearly **finite** number of values is known as discrete data.

A non-discrete space, which is called called a continuous interval/space, is one in which the outcomes are too numerous to identify every possible outcome:

- Height of Human Beings: S = [0.00 inches, 100.00 inches] → It is impractical to specify every possible height from 0.00 to 100.00.
- Muscle gain over 1 year of weight training: [0 pounds, 100+ pounds]

As with disjoint/non-disjoint, independent/non-independent, we will see that there are different probability formulas that are used depending on whether the sample space is discrete or not.

A basketball player shoots three free throws. He typically makes a basket exactly 60% of the time.

The random variable *X* is the number of baskets successfully made.

What is the probability that the player successfully makes at least* two baskets?

Value of *X*	0	1	2	3
Probability	0.064	0.288	0.432	0.216

Answer:

$P(X \geq 2)$ = $P(X=2$ or $X=3)$

= (0.432+0.216) = 0.648

Through careful reading of the question, we see that the event we are interested in is "2 or more free throws". The possible values of X making up this event are {2, 3}.

Key Point: The sample space discussed in this example contains discrete data.

* **"At least": A concept that has a way of showing up very often in real life (and on exams).**

Continuous random variables

Unlike a discrete variable, a variable that is continuous includes all possible values within an **interval** (range).

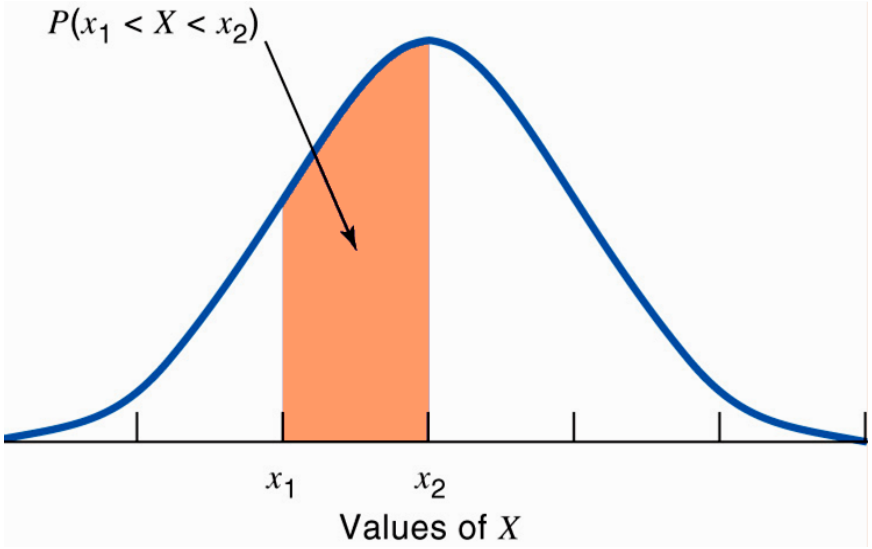
Example: There is an infinity of numbers between 0 and 1 (e.g., 0.001, 0.4, 0.0063876).

Previously, we've learned how to assign probabilities to events in a discrete sample space. But how do we assign probabilities to a continuous sample space which, rather than having a discrete number of outcomes, has an infinite number of outcomes?

Answer: We use **density curves** and compute the probabilities for **intervals**.

Determining probabilities for *continuous* random variables

The probability of any event in which the outcomes are continuous (as opposed to discrete) is the area under the density curve for the values of X that make up the event.



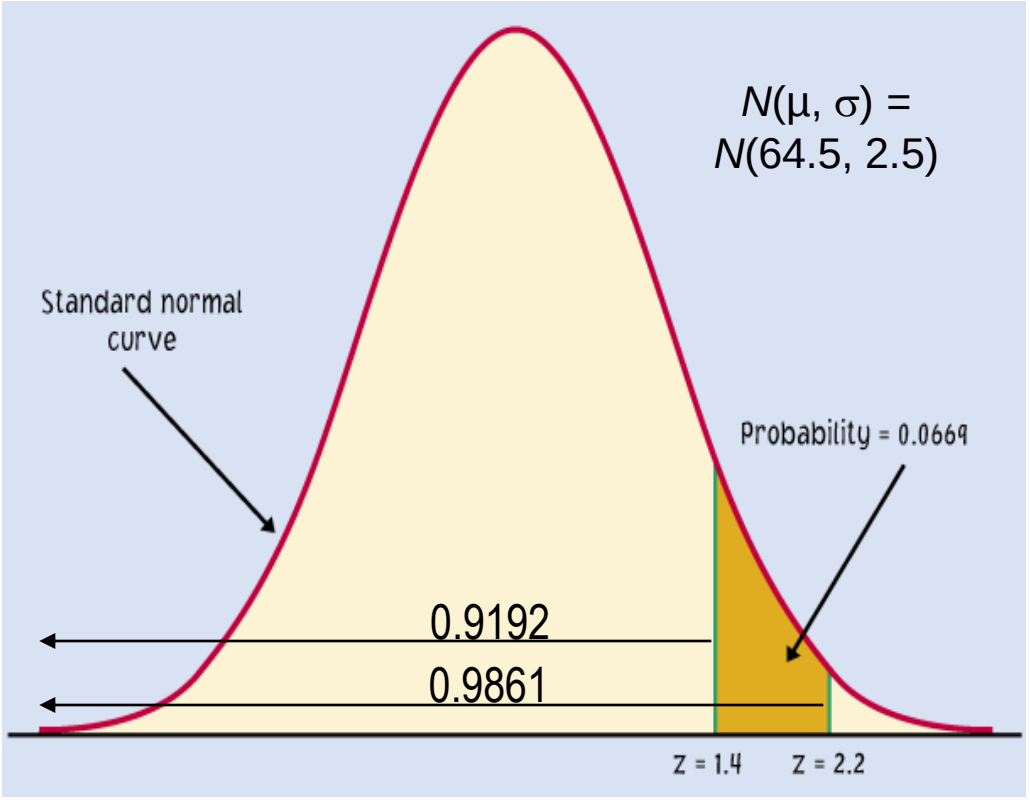
Example: The height of a sample of women has a distribution of approximately N(64.5, 2.5). What is the probability, if we pick one woman at random, that her height will be between 68 and 70 inches. I.e. ***P*(68 < *X* < 70)**?

$$z = \frac{(x - \mu)}{\sigma}$$

As we've done before, we calculate the *z*-scores for 68 and 70 and determine the area between them.

For x = 68", $z = \frac{(68 - 64.5)}{2.5} = 1.4$

For x = 70", $z = \frac{(70 - 64.5)}{2.5} = 2.2$

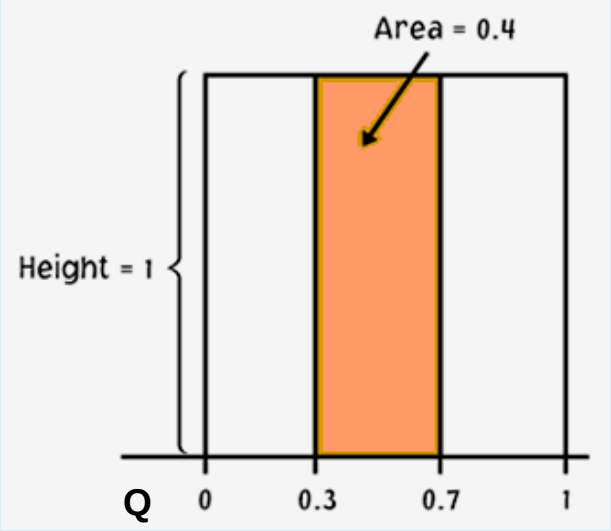


The area under the curve for the interval [68" to 70"] is 0.9861 − 0.9192 = 0.0669.

I.e. The probability that a randomly chosen woman falls into this range is 0.0669.

Probabilities of continuous random variables contd

- **Recall**: In order to determine probabilities of continuous data, we calculate the area under the density curve.
- Don't forget that density curves come in all kinds of shapes!
- Here is an example using a uniform density curve:



Shown here is a density curve for some variable *'Q'*.

What is the probability of randomly encountering a value for *Q* that falls between 0.3 and 0.7?

Answer: W*hen trying to determine the probability for a continuous interval, we look for the area under the density curve.*

In this case: $P(0.3 \leq Q \leq 0.7) = (0.7 - 0.3)*1 = 0.4$

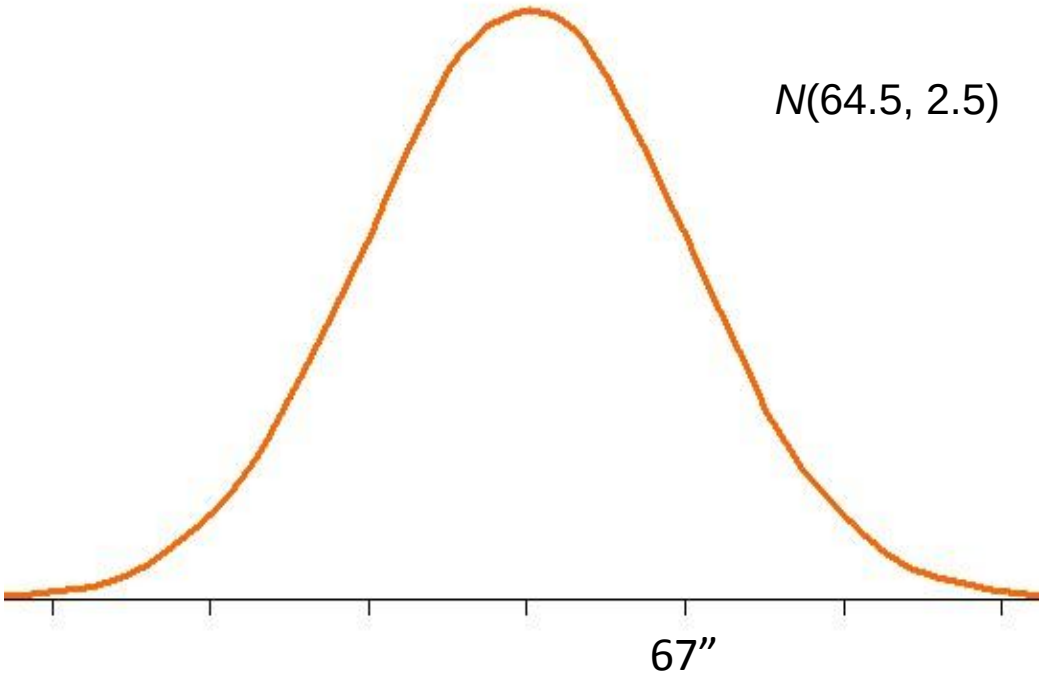
Only intervals can have a non-zero probability

The probability of a single event is ***meaningless*** for a continuous random variable. In fact, the calculated value is 0.

Only intervals can have a non-zero probability.

Example: The height of a sample of women has a distribution of approximately N(64.5, 2.5). What is the probability, if we pick one woman at random, that her height will be 67 inches?

Answer: We can NOT say! Recall that with continuous variables, it is not possible to determine the probability of a single value. We can only determine the probability of a *range* of values.



Calculating the mean of a random variable

- Mean of a discrete random variable
- Mean of a continuous random variable

Mean of a discrete random variable

For a discrete random variable X with the probability distribution shown here,

Value of X	x_1	x_2	x_3	$\cdots$	x_k
Probability	p_1	p_2	p_3	$\cdots$	p_k

the mean μ of X is found by multiplying each possible value of X by its probability, and then adding the products.

$$\mu_X = x_1p_1 + x_2p_2 + \cdots + x_kp_k$$
$$= \sum x_ip_i$$

A 60% shooting basketball player shoots three free throws. The random variable X is the number of baskets successfully made.

Value of X	**0**	**1**	**2**	**3**
Probability	**0.064**	**0.288**	**0.432**	**0.216**

The mean μ of X is

μ = (0*0.064) + (1*0.288) + (2*0.432) + (3*0.216)

= 1.8

In other words, out of 3 throws, in the long run, this player would make 1.8 baskets.

1.8 Baskets?!

- This number represents the average number of baskets the thrower would make out of many many 3-throw attempts.
  - Obviously on any invidivual attempt at 3 throws, the thrower could only make 0, 1, 2, or 3 baskets.

- This is similar to the idea that the average American household has 1.2 children. While no household has this particular number, this is the average number when looking at all households.

- There is a special name given to the mean value of a random variable (whether discrete or continuous). It is called the **expected value**.

Expected Value

The mean of a random variable *X* is called the expected value of *X*. Be aware that the method of calculating the mean of a random variable changes depending on whether the variable in question is *discrete* or *continuous*.

Calculating expected value: The mean of a random variable *X* is a weighted average of the possible values of *X*.

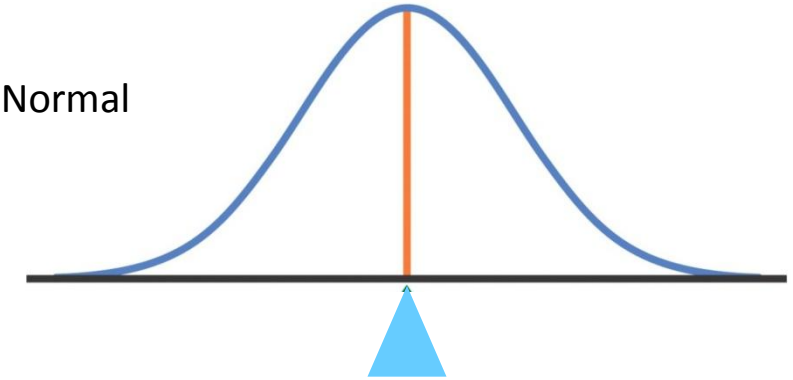
Weighted: We take each value of X and multiply it by its probability. We then add all those values together.

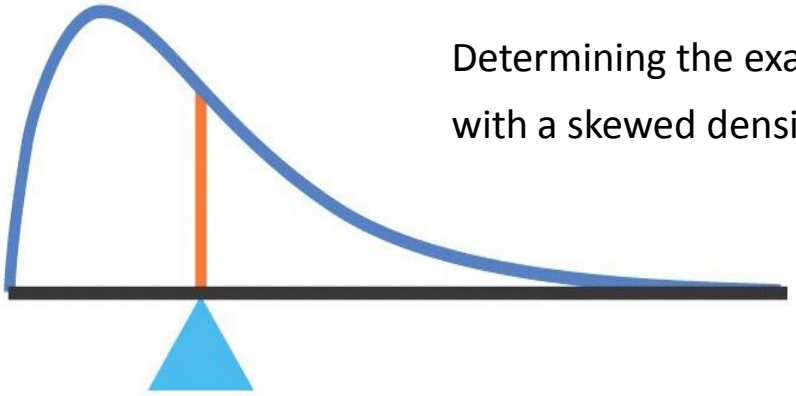
Mean of a continuous random variable

Now let's turn our attention to determining the mean (expected value) of a continuous random variable.

As we did with probabilities, when working with continuous data, we rely on the density curve to make various determinations.

With symmetric curves, such as the Normal curve, the mean lies at the center.





Determining the exact mean of a distribution with a skewed density curve is more complex.

Standard Deviation of a random variable

- As with expected value (i.e. mean) of a random variable, we can also calculate the standard deviation of a random variable.
- Calculation: Once again, we need to carefully take into account the probability of each outcome.

Standard Deviation of a discrete random variable

Value of X	x_1	x_2	x_3	$\cdots$	x_k
Probability	p_1	p_2	p_3	$\cdots$	p_k

For a discrete random variable X

with the probability distribution shown

and mean μ_X, the sd of X is found by multiplying each deviation by its probability and then adding all the products.

This formula shows variance, taking the square root would give us SD:

$$\sigma_X^2 = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_k - \mu_X)^2 p_k$$
$$= \sum (x_i - \mu_X)^2 p_i$$

A basketball player shoots three free throws. The random variable X is the number of baskets successfully made.

Value of X	**0**	**1**	**2**	**3**
Probability	**0.064**	**0.288**	**0.432**	**0.216**

$\mu_X = 1.8$.

The variance σ^2 of X is

$\sigma^2 = 0.064*(0-1.8)^2 + 0.288*(1-1.8)^2 + 0.432*(2-1.8)^2 + 0.216*(3-1.8)^2$

$= 0.20736 + 0.18432 + 0.01728 + 0.31104$

$= 0.72$ → Take the square root to get SD: SD = 0.85

Standard Deviation of a continuous random variable

- This topic and formula is a bit more involved. We will not discuss it for now.
- However, don't ever forget that while we like to think about measures of center (e.g. the mean), when examining a dataset, we should never neglect to also consider the spread (e.g. standard deviation).

Example

- Linda is a sales associate at a large auto dealership. At her comission rate of 25% of gross profit on each vehicle she sells, Linda expects to earn $350 for each car sold and $400 for each truck or SUV sold. Linda motivates herself by using probability estimates of her sales. For a sunny Saturday in April, she estimates her car sales as follows:

Cars Sold	0	1	2	3
Prob	0.3	0.4	0.2	0.1

Trucks Sold	0	1	2	3
Prob	0.4	0.5	0.1	0

- What is Linda's expected income?
- Linda is given the option of switching to a truck-only department. Should she take that job?

What is Linda's expected income?

Cars Sold	0	1	2	3
Prob	0.3	0.4	0.2	0.1

Trucks Sold	0	1	2	3
Prob	0.4	0.5	0.1	0

Recall how with certain probability questions you have to ask yourself "disjoint or non-disjoint" or "independent or non-indepndent". Similarly, when calculating an expected value, the very first question you must ask yourself is "discrete or non-discrete" since the method of determining the mean is completely different for each.

Because the type of variable we are discussing (# cars/trucks sold) is *discrete*, we will use the formula for calculating the mean of discrete data that was described earlier:

$$\mu_X = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$$
$$= \sum x_i p_i$$

$\mu_{\text{cars sold}}$ = 0*0.3 + 1*0.4 + 2+0.2 + 3*0.1 = 1.1 cars

$\mu_{\text{trucks sold}}$ = 0*0.4 + 1*0.5 + 2*0.1 + 3*0 = 0.7 trucks

If she gets \$350 per car, and \$400 per truck, Linda would expect to make: \$350*1.1 + \$400*0.7 = <u>\$665</u>

Linda is given the option of switching to a truck-only department. Should she take that job?

Cars Sold	0	1	2	3
Prob	0.3	0.4	0.2	0.1

Trucks Sold	0	1	2	3
Prob	0.4	0.5	0.1	0

Answer: It might be tempting to compare expected value for each: 350*1.1 = $385 for cars v.s. 400*0.7 = $280 for trucks and infer that Linda should focus specifically on cars and just forget about trucks.

However, this would be a mistake. Can you see why?

Answer: If Linda focused only on trucks, she would not be spending any of her time on cars. As a result, she is likely to sell more than 0.7 trucks on a given day. However, we have no idea just how many more trucks she would sell. In other words, we do not have enough information to make a sound decision.

IMPORTANT: In fact, the expected value for income we calculated earlier is not enough. To do a proper evaluation of Linda's expected income and so on, we should not have neglected to also consider the spread (e.g. the standard deviation). However, we left out these calculations for the time being, as they are very tedious to do by hand. Statistical software would give us this information.