Overview and Descriptive Statistics

Populations, Samples and Variables

Collections of facts, or data

Population: all objects that are interested for a particular project. Eg. census.

Sample: a subset of population

Variable: a particular characteristic, measured

Pictorial and Tabular Methods in Descriptive Statistics

Two subject areas of descriptive statistics:

visual vs numerical.

Examples of familiar visual techniques:

frequency tables,

tally sheets,

histograms,

pie charts,

bar graphs,

scatter diagrams

Notation

*sample size: t*he number of observations in a sample, n

eg, $n = 4$, for the sample of universities {Stanford, Iowa State, Wyoming, Rochester}

and also for the sample of pH measurements

{ 6.3, 6.2, 5.9, 6.5}.

Given a data set of n observations on a variable x, the individual observations will be denoted by $x_1, x_2, x_3, \ldots, x_n$.

Usually, x_1 will be the first observation gathered, x_2 the second, and so on. The *i*th observation in the data set will be denoted by x_i.

Dotplots

Each observation is one dot above a horizontal scale.

When more than once observations have the same value, the dots are stacked vertically.

Good for small data set

data set with a few distinct values

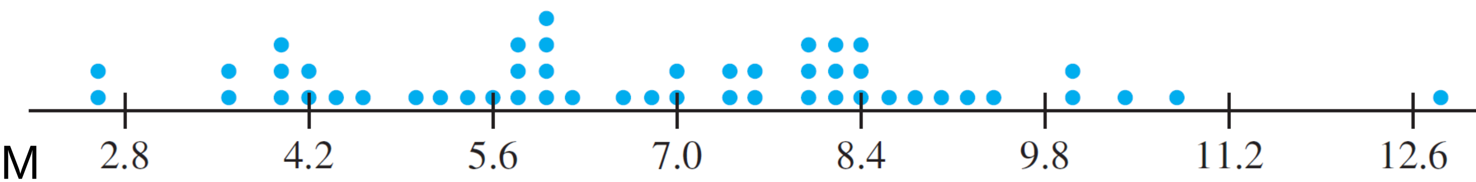
A dotplot gives information about location, spread, extremes, and gaps.

Example

Higher education as a % of state and local tax revenue for the fiscal year 2006–2007 for all states (AL first, WY last):

10.8	6.9	8.0	8.8	7.3	3.6	4.1	6.0	4.4	8.3
8.1	8.0	5.9	5.9	7.6	8.9	8.5	8.1	4.2	5.7
4.0	6.7	5.8	9.9	5.6	5.8	9.3	6.2	2.5	4.5
12.8	3.5	10.0	9.1	5.0	8.1	5.3	3.9	4.0	8.0
7.4	7.5	8.4	8.3	2.6	5.1	6.0	7.0	6.5	10.3

NM



VM
NH

Histograms

Discrete variable: finite or countable possible. Often obtained by counting to determine the value of a variable, say the # of trucks passing through a checkpoint in a hour.

Continuous variable: possible values consist of an entire interval on the number line. Often obtained by taking measurements.

Histograms for discrete and continuous variables are constructed slightly different.

Histogram Concepts for Discrete Variable

The **frequency** of an *x* value is the number of times that value occurs in the data set.

The **relative frequency** of a value is the fraction or proportion of times the value occurs:

$$\text{relative frequency of a value} = \frac{\text{number of times the value occurs}}{\text{number of observations in the data set}}$$

Example: a data set with 200 observations on *x* = the number of courses a college student is taking this term. If 70 of these *x* values are 3, then

frequency of the *x* value 3: 70

relative frequency of the *x* value 3: 70/200=0.35 or 35%

The relative frequencies of all values should sum to 1 (allowing rounding errors)

Histograms for Discrete Data

Constructing a Histogram for Discrete Data

First, determine the frequency and relative frequency of each x value. Then mark possible x values on a horizontal scale. Above each value, draw a rectangle whose height is the relative frequency (or alternatively, the frequency) of that value.

Example 9

cont'd

Table 1.1 is a frequency distribution for the number of hits per baseball team per game for all nine-inning games that were played between 1989 and 1993.

Hits/Game	Number of Games	Relative Frequency	Hits/Game	Number of Games	Relative Frequency
0	20	.0010	14	569	.0294
1	72	.0037	15	393	.0203
2	209	.0108	16	253	.0131
3	527	.0272	17	171	.0088
4	1048	.0541	18	97	.0050
5	1457	.0752	19	53	.0027
6	1988	.1026	20	31	.0016
7	2256	.1164	21	19	.0010
8	2403	.1240	22	13	.0007
9	2256	.1164	23	5	.0003
10	1967	.1015	24	1	.0001
11	1509	.0779	25	0	.0000
12	1230	.0635	26	1	.0001
13	834	.0430	27	1	.0001
				19,383	1.0005

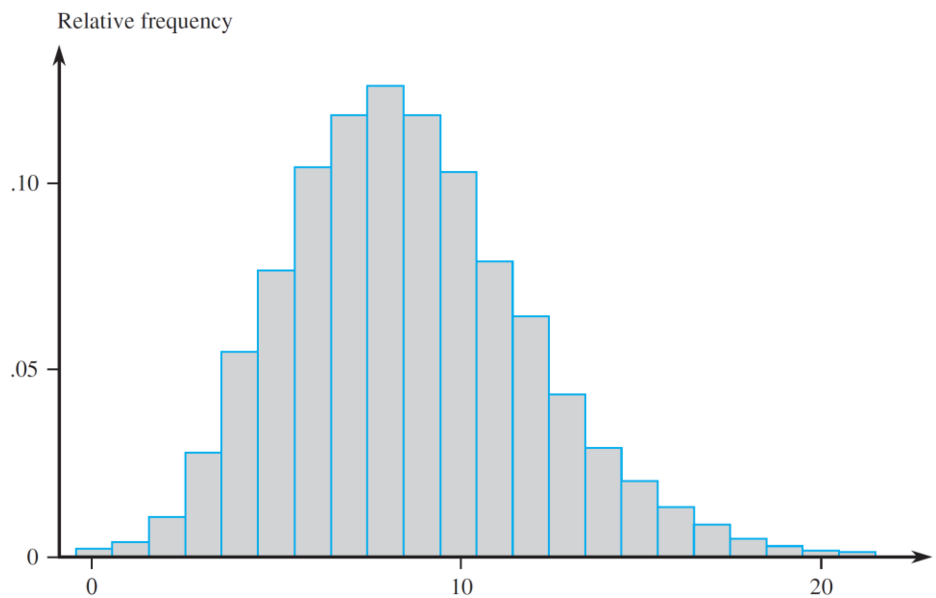
Frequency Distribution for Hits in Nine-Inning Games

Table 1.1

Example 9

cont'd

The corresponding histogram in Figure 1.7 rises rather smoothly to a single peak and then declines. The histogram extends a bit more on the right (toward large values) than it does on the left—a slight "positive skew."



Histogram of number of hits per nine-inning game

Figure 1.7

Example 9

cont'd

Either from the tabulated information or from the histogram itself, we can determine the following:

$$\begin{array}{l} \text{proportion of games with} \\ \text{at most two hits} \end{array} = \begin{array}{c} \text{relative} \\ \text{frequency} \\ \text{for } x = 0 \end{array} + \begin{array}{c} \text{relative} \\ \text{frequency} \\ \text{for } x = 1 \end{array} + \begin{array}{c} \text{relative} \\ \text{frequency} \\ \text{for } x = 2 \end{array}$$

$$= .0010 + .0037 + .0108$$

$$= .0155$$

Similarly,

$$\begin{array}{l} \text{proportion of games with} \\ \text{between 5 and 10 hits (inclusive)} \end{array} = .0752 + .1026 + \cdots + .1015$$

$$= .6361$$

That is, roughly 64% of all these games resulted in between 5 and 10 (inclusive) hits.

Histogram for Continuous varaibles

Constructing a Histogram for Continuous Data: Equal Class Widths

Determine the frequency and relative frequency for each class. Mark the class boundaries on a horizontal measurement axis. Above each class interval, draw a rectangle whose height is the corresponding relative frequency (or frequency).

Constructing a Histogram for Continuous Data: Unequal Class Widths

After determining frequencies and relative frequencies, calculate the height of each rectangle using the formula

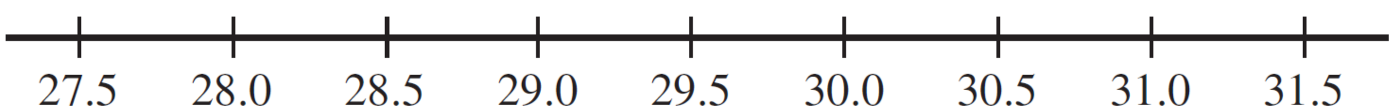
$$\text{rectangle height} = \frac{\text{relative frequency of the class}}{\text{class width}}$$

The resulting rectangle heights are usually called *densities,* and the vertical scale is the **density scale.** This prescription will also work when class widths are equal.

Histograms

Constructing a histogram for continuous data (measurements) entails subdividing the measurement axis into a suitable number of **class intervals** or **classes,** such that each observation is contained in exactly one class.

Suppose, for example, that we have 50 observations on x = fuel efficiency of an automobile (mpg), the smallest of which is 27.8 and the largest of which is 31.4. Then we could use the class boundaries 27.5, 28.0, 28.5, . . . , and 31.5 as shown here:



Histograms: Dealing with boundaries

One potential difficulty is that occasionally an observation lies on a class boundary so therefore does not fall in exactly one interval, for example, 29.0.

One way to deal with this problem is to use boundaries like 27.55, 28.05, . . . , 31.55. Adding a hundredths digit to the class boundaries prevents observations from falling on the resulting boundaries.

Another approach is to place an observation on a boundary in the interval to the *right* of the boundary. This is how Minitab constructs a histogram.

Histogram Shapes

A **Unimodal** histogram: single peak

A **bimodal** histogram: two different peaks

Bimodality can occur when the data set consists of observations on two quite different kinds of objects. Eg. a data set of driving times between San Luis Obispo, and Monterey, CA, two routes: inland (≈ 2.5 hr) and coast (3.5–4 hr). However,

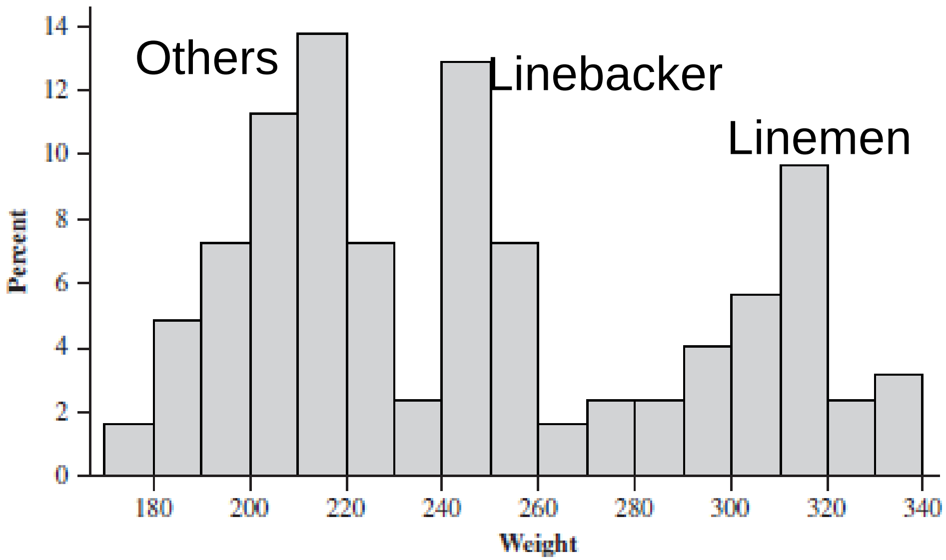
A data set of college student heights may NOT result in a bimodal histogram, male around 69”, female around 64”.

A **multimodal** histogram: multiple peaks

The number of peaks may depend on the choice of class intervals, particularly with a small number of observations. The more classes, the more peaks will manifest itself.

Example 12

Figure 1.11(a) shows a Minitab histogram of the weights (lb) of the 124 players listed on the rosters of the San Francisco 49ers and the New England Patriots (teams the author would like to see meet in the Super Bowl) as of Nov. 20, 2009.



NFL player weights Histogram

Figure 1.11(a)

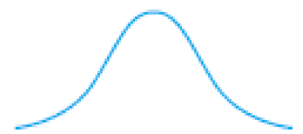
Histogram Shapes: symmetric or skewed

A histogram is **symmetric** if the left half is a mirror image of the right half. A unimodal histogram is **positively skewed** if the right or upper tail is stretched out compared with the left or lower tail and **negatively skewed** if the stretching is to the left.

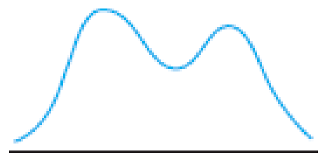
Example 12

cont'd

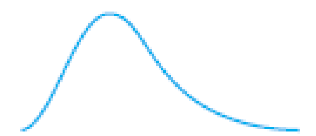
Figure 1.12 shows "smoothed" histograms, obtained by superimposing a smooth curve on the rectangles, that illustrate the various possibilities.



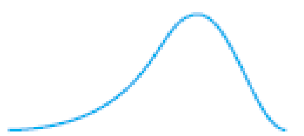
(a) symmetric unimodal



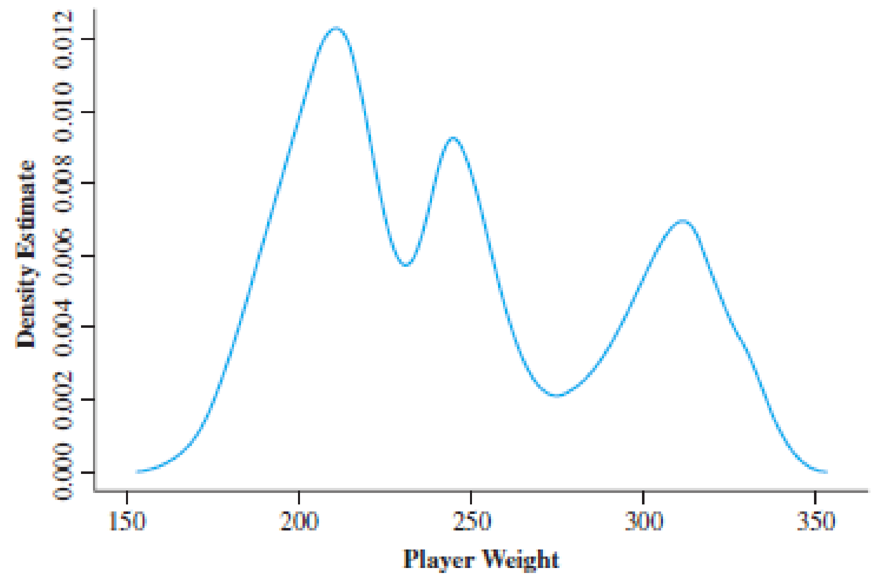
(b) bimodal



(c) Positively skewed



(d) negatively skewed



(e) NFL player weights Smoothed histogram

Smoothed histograms

Figure 1.12

Qualitative Data

Both a frequency distribution and a histogram can be constructed when the data set is *qualitative* (categorical) in nature.

Ordered: eg, freshmen, sophomores, juniors, seniors, graduate students

Non-ordered: eg, Catholic, Jewish, Protestant, Muslim and the like.

The intervals should have equal width.

Example 13

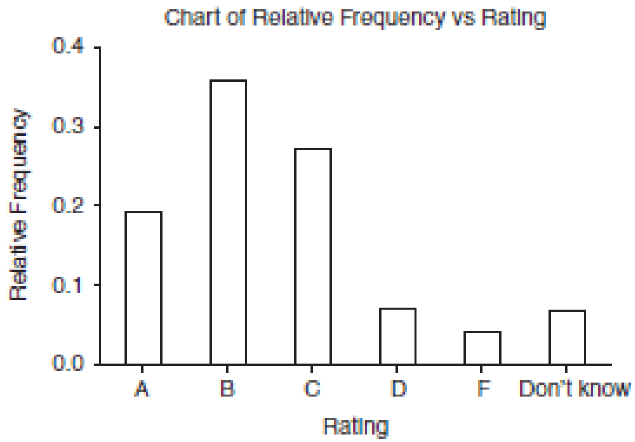
cont'd

How do you rate your school?

Rating	Frequency	Relative Frequency
A	478	.191
B	893	.357
C	680	.272
D	178	.071
F	100	.040
Don't know	172	.069
	2501	1.000

Frequency Distribution for the School Rating Data

Table 1.2



Histogram of the school rating data from Minitab

Figure 1.13

More than half the respondents gave an A or B rating, and only slightly more than 10% gave a D or F rating.

Multivariate Data

Multivariate data is generally rather difficult to describe visually. Several methods for doing so appear later in the book, notably scatter plots for bivariate numerical data.