Lecture 11
Image restoration and reconstruction II

1. Periodic noise
2. Periodic noise reduction by frequency domain filter
3. Introduction to degradation and filtering technique
   - Linear, position-invariant degradation
   - Estimation of degradation function
   - Inverse filter
   - Wiener filter
   - Constrained least squares filtering

Periodic noise

- Noise that appears periodically, typically from electronic or electromechanical interference during image acquisition.
- Example:

$$r(x,y) = A\sin(2\pi u_0(x+B_x)/M + 2\pi v_0(y+B_y)/N)$$

$$g(x,y) = f(x,y) + r(x,y)$$

- Periodic noise can be reduced significantly via frequency domain filtering

- The DFT of the r(x,y) is

$$R(u,v) = i\frac{A}{2}[(e^{2\pi i u_0 B_x/M})\delta(u+u_0, v+v_0) - (e^{2\pi i v_0 B_y/N})\delta(u-u_0, v-v_0)]$$

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Periodic Noise Reduction by Frequency Domain Filtering

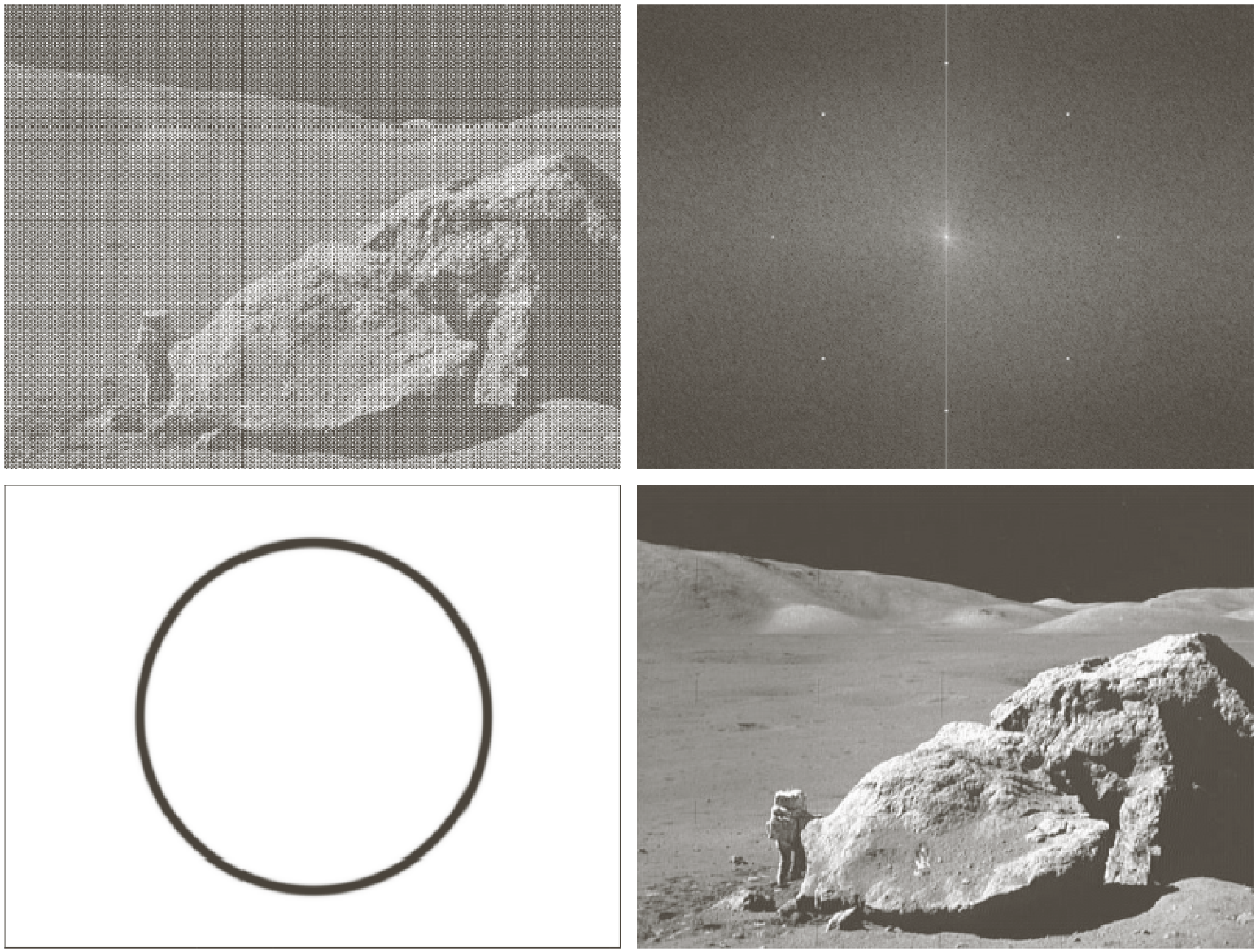
- When the bandwidth of noise is known, bandreject filters can be used for filtering the noise

- Bandreject filters $H_{BR}(u, v)$ filtering that reject certain bandwidth
  - Idea, Butterworth, Gaussian

- Bandpass filters

$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a b
c d

FIGURE 5.16
(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

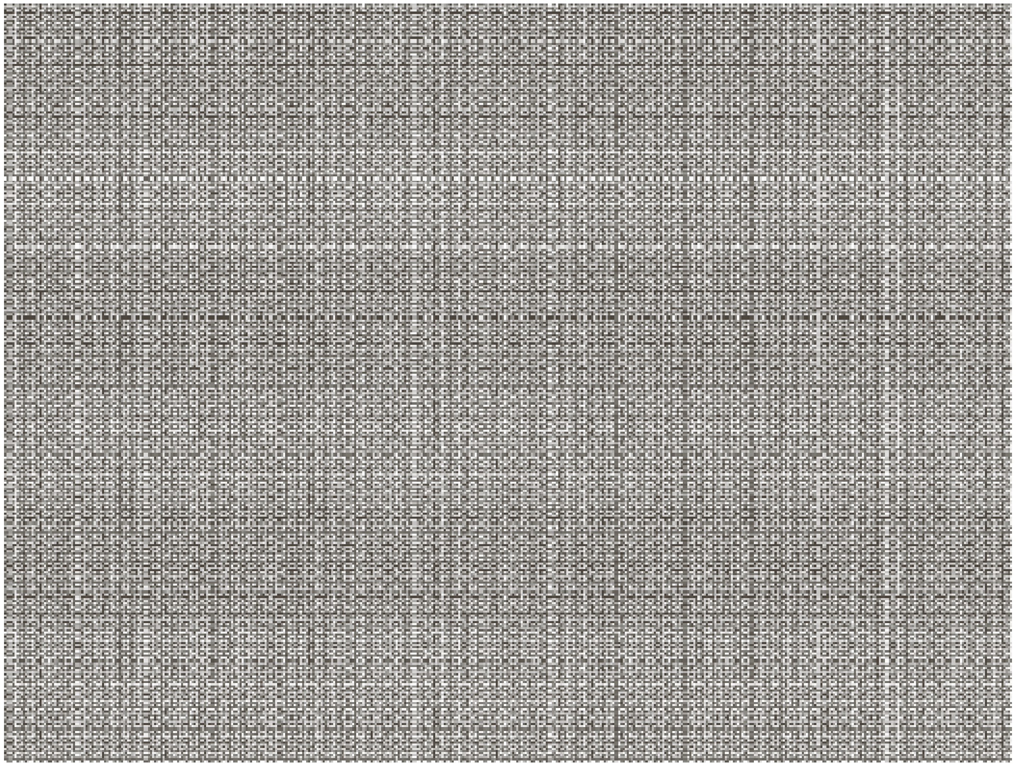


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

Obtained by Bandpass filters

$H_{BP}(u, v) = 1- H_{BR}(u, v)$

The inverse FT

Notch Filters

- Reject (or pass) frequencies in a predefined neighborhood about the center of he frequency rectangle. Zero-phase-shift filters must be symmetric about the origin.
- The Notch reject and pass filter can be represented as

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

where $H_k(u, v)$, $H_{-k}(u, v)$ are the highpass filters whose centers are at (u_k, v_k) and (u_k, v_k), respectively. These centers are specified w.r.p.t the center of the frequency rectangle $(M/2, N/2)$

$$H_{NR}(u,v) = \prod_{k=1}^{Q} [\frac{1}{1+[D_{0k} / D_k(u,v)]^{2n}}][\frac{1}{1+[D_{0k} / D_{-k}(u,v)]^{2n}}]$$

$$D_k(u,v) = [(u - M / 2 - u_k)^2 + (v - N / 2 - v_k)^2]^{1/2}$$

$$D_{-k}(u,v) = [(u - M / 2 + u_k)^2 + (v - N / 2 + v_k)^2]^{1/2}$$

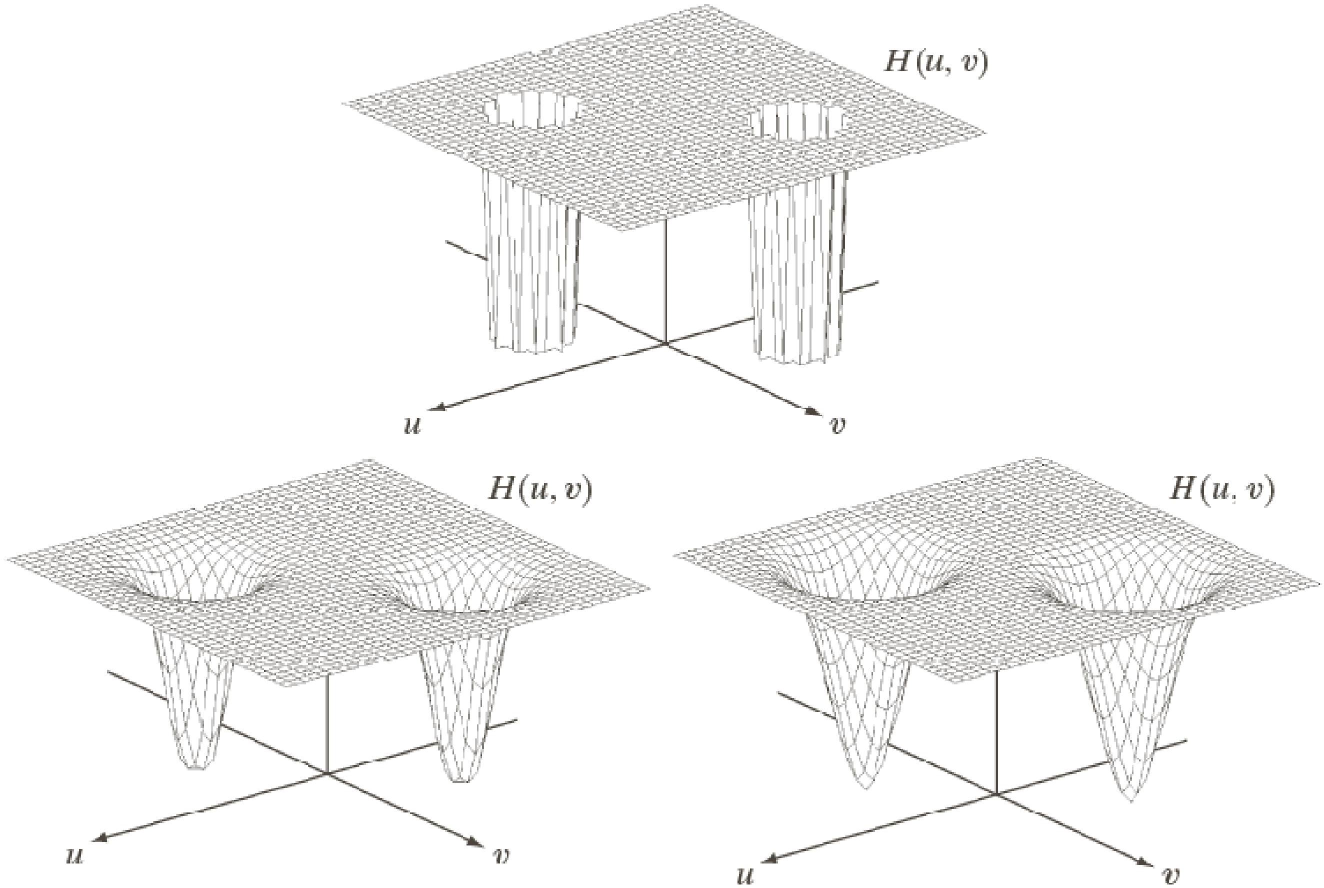
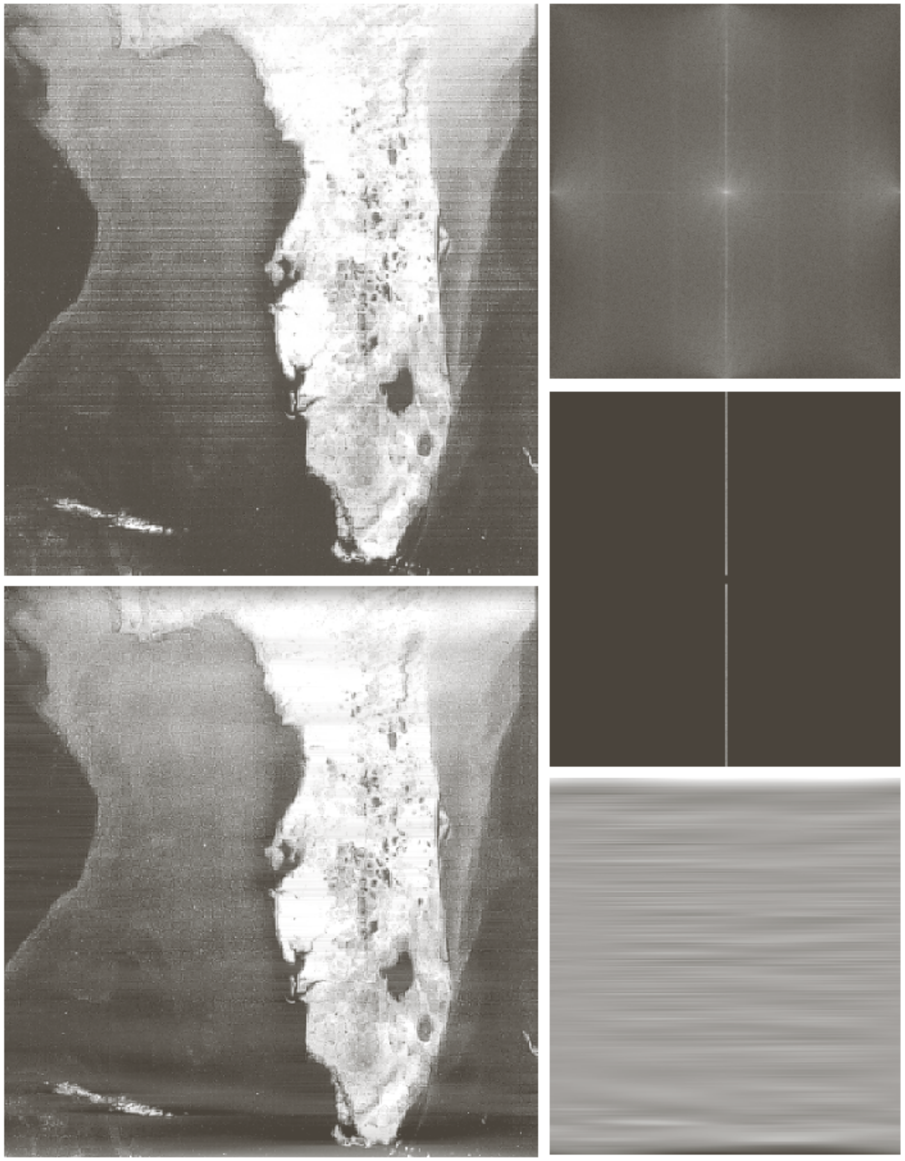


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



a	b
	c
e	d

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Linear, position-invariant degradation

- The image with degradation and noise:

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

- Assume $\eta(x, y) = 0$
- H is linear if

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

- Additivity:

$$H[f_1(x, y) + f_2(x, y)] = H[f_1(x, y)] + H[f_2(x, y)]$$

- Homogenerity:

$$H[af_1(x, y)] = aH[f_1(x, y)]$$

- Position invariant

$$g(x, y) = H[f(x, y)]$$

$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$$

Impulse response

- The image with degradation and noise:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha\, d\beta$$

$$g(x,y)=H[f(x,y)]$$

$$=H[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha\, d\beta]$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\delta(x-\alpha,y-\beta)]d\alpha\, d\beta$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H[\delta(x-\alpha,y-\beta)]d\alpha\, d\beta$$

- Impulse response of H (point spread)

$$h(x,\alpha,y,\beta)=H[\delta(x-\alpha,y-\beta)]$$

- Superposition integral of the first kind

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)d\alpha\, d\beta$$

Convolution

- If H is position invariant

$$H[\delta(x-\alpha, y-\beta)] = h(x-\alpha, y-\beta)$$

- The convolution integral

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha\, d\beta$$

- With noise

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha\, d\beta + \eta(x,y)$$

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Summary

- A linear spatially-invariant degradation system with additive noise can be modelled in spatial domain as the convolution of the degradation (point spread) function with an image, followed by the addition of the noise

  Or equivalently in frequency domain: the product of the FTs of the image and degradation, followed by the addition of Ft of the noise

- Solution is available for linear spatially-invariant degradation model.

a b

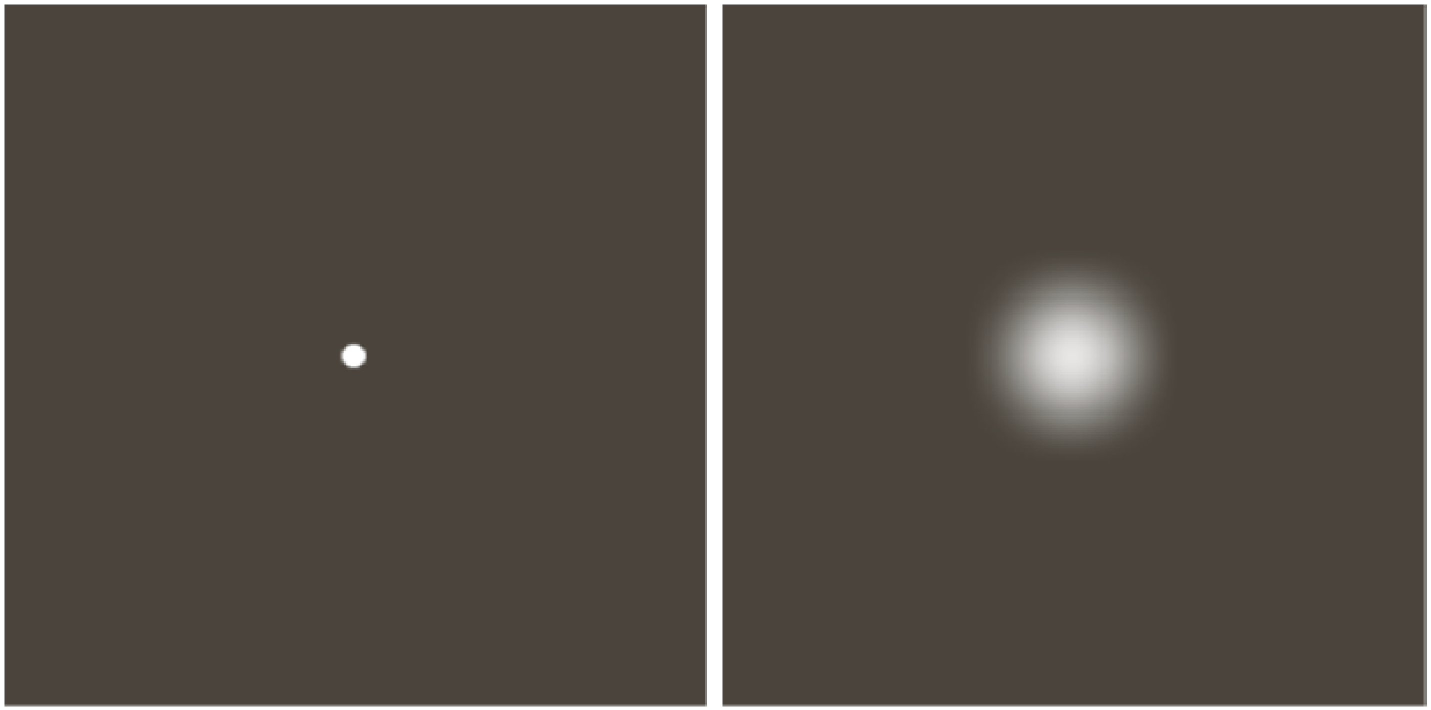


FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Inverse filter

- Restoration image degraded by a degradation function H, where H is given or obtained by estimation

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Minimum Mean Square Error (Wiener) Filtering

- Incorporate both the degradation function and statistical characterization of the noise into restoration process.
- Find the estimate image which minimize the error

$$e^2 = E\{(f - \hat{f})^2\}$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v) |H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

$$= \frac{1}{H(u,v)} \left[\frac{H(u,v)^2}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

$$S_\eta(u,v) = |N(u,v)|^2$$

$$S_f(u,v) = |F(u,v)|^2$$

$$\hat{F}(u,v) = \frac{1}{H(u,v)} \left[\frac{H(u,v)^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Constrained Least Square Filtering

- When H and the mean and variance of the noise are known.
- To minimize $C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x,y)]^2$

  Subject to $\| g - H\hat{f} \|^2 = \| \eta \|^2$

  Then $$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2}\right] G(u,v)$$

- Where $P(u,v)$ is the FT of

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

a	b	c
d	e	f
g	h	i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.