INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Chemical Engineering

End-semester (Autumn) Examination 2022-2023

Subject: Advanced Mathematical Techniques in Chemical Engineering (CH61015)

Remarks:

1. This question paper contains two parts: **Part A** and **Part B**. Attempt both parts.
2. Unless otherwise stated, usual mathematical notations apply.
3. Time = 3 h; maximum marks = 100; total number of printed pages = 2.

Part A: Linear algebra

1. For the following set of simultaneous equations, verify whether the system has only real solutions for t, θ, $x_1(0), x_2(0) \in \mathbb{R}$. θ may be treated as a parameter independent of x_i and t. You may solve only for x_1 and deduce the conclusions from there.

$$\frac{dx_1}{dt} = (cos\theta)x_1 - (sin\theta)x_2 \tag{1}$$

$$\frac{dx_2}{dt} = (sin\theta)x_1 + (cos\theta)x_2 \tag{2}$$

. . . 20 marks

2. Determine the dimension and basis for the range space of the following set of equations using Fredholm's alternative theorem.

$$ix_1 + 2x_2 - 3ix_3 = 2 \tag{3}$$

$$5ix_1 + 10x_2 - 15ix_3 = 9 \tag{4}$$

$$2ix_1 + 4x_2 - 6ix_3 = 5 \tag{5}$$

. . . 15 marks

3. The function $H_n : \mathbb{R} \to \mathbb{R}$ defined as

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2}) \tag{6}$$

yields a set of Hermite polynomials. Sketch the first three polynomials and verify if the polynomials form an orthogonal set in $[\,-1\,,\,1\,]$ by considering $n = 0, 1, 2$. You must test all inner products.

. . . 15 marks

Part B: Differential equations

4. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} \tag{7}$$

At $t = 0$, $u = f(r, \theta)$. At $r = 1$, $u = 0$. Use the suitable physical boundary conditions on the rest of the boundaries.

...10 marks

5. Solve completely:

$$\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) \tag{8}$$

At $r = 1$, $\frac{\partial u}{\partial r} + 3u = 0$ and at $t = 0$, $u = 1$.
Use the suitable physical boundary conditions on the rest of the boundaries.

...10 marks

6. Solve completely:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{1}{sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2}\frac{1}{sin^2\theta}\frac{\partial^2 u}{\partial \phi^2} = 0 \tag{9}$$

The boundary condition at $r = 1$, $u = f(\theta, \phi)$. Use the suitable physical boundary conditions on rest of the boundaries.

...10 marks

7. Solve completely using Green's function method:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t \tag{10}$$

At $t = 0$, $u = 1$. At $x = 0$, $\frac{\partial u}{\partial x} = 0$. At $x = 1$, $u = 2$.

...20 marks

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INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester Examination 2022-23

Date of Examination: 22.11.22 Session: (FN/AN) AN Duration: 3 hrs. Full Marks:50

Subject No.: CH61011

Subject: Advanced Fluid Dynamics

Department/Center/School: Chemical Engineering

Specific charts, graph paper, log book etc., required: Nil

Special Instructions (if any): Make rational assumptions wherever necessary. No doubts will be clarified by the paper setters during examination time

Answer all components of a questions and all questions of each Part together.

PART A

1. A painter (it could be considered either a person making a coating on a wall/surface or an artist trying to create a picture on a canvas) has kept his/her shallow tray (rectangle shaped) containing the paint in such a way that the left edge of the tray is exposed to intense sunlight and the right edge falls under the shade of a tree. This creates a significant difference in temperature between the two edges and a temperature gradient across the tray. The tray contains paint to its brim. The hight of the tray could be considered as '*h*' and the length be considered as '*L*'. It is understood that this temperature difference is causing a steady macro-Marangoni type transport of paint in the tray. Based on this context, answer the following:
   (a) Draw a clear velocity profile of the transport inside the tray. Mark your coordinates clearly (3 marks)
   (b) Neglecting edge and end effects, derive an equation for velocity profile of the transport, by starting from a fundamental fluid dynamics equation. While deriving this, it may be assumed that (not mandatory) the surface tension gradient across the paint surface is linear (12 marks).
   (c) For L=10 cm, h=1 mm, temperature gradient = 40°C, viscosity of paint = 5 cP, estimate the surface velocity (10 marks).

   [A few helpful points that could be paid attention to: (a) If you do not mark your coordinate axes while drawing the velocity profile in part (a), 50% marks will be deducted; (b) If the derivation in part (b), is wrong, part (c) will not be checked at all during evaluation and straight away zero marks will be awarded for part (c). While doing derivation in part (b), if you do not want to assume a linear gradient in surface tension across the surface (as given in the question), you should clearly mention your own assumption, without which 50% marks from part (b) will be deducted even if the derivation is correct. If some data is missing in part (c), make assumptions of those values that are physically feasible. This means the assumed values should be within maximum one order of magnitude of the realistic value. If this is not followed or your assumptions over shoot one order of magnitude, then even if the calculated value for surface velocity is "mathematically" correct, 50% of marks from part (c) will be deducted.]

PART B

2. (a) What is Reynold's Decomposition of Turbulence? (2)

   (b) What is Boundary Layer Momentum Thickness? (2)

   (c) Distinguish with suitable graph between Stationary and Homogeneous Turbulence, including their physical significance. (2)

   (d) How can one obtain the Similarity Parameter in the Blasius Solution of Boundary Layer? (3)

   (e) What is the physical significance of Reynolds Number within a boundary layer? (2)

   (f) Discuss if the edge of the boundary layer is a Stream line or not. (2)

3. Discuss how additional stresses get augmented in case of a Turbulent flow. What is Eddy Diffusivity? What is Prandalt Mixing Length? What is the utility of Prandalt Mixing Length? (8+1+2+1=12)

End of Question Paper. All the Best