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In [1]: import json
         import math
In [2]: # Load the document representatives
         with open("./input/documents.json", "r") as f:
             documents = json.load(f)
In [4]: # ---- Cosine Similarity ----
         def cosine_similarity(doc1, doc2):
             # union of all words
             words = set(doc1.keys()).union(set(doc2.keys()))
             # create vectors
             v1 = [doc1.get(w, 0) for w in words]
             v2 = [doc2.get(w, 0) for w in words]
             # dot product
             dot = sum(a*b for a, b in zip(v1, v2))
             # magnitudes
             mag1 = math.sqrt(sum(a*a for a in v1))
             mag2 = math.sqrt(sum(b*b for b in v2))
             if mag1 == 0 or mag2 == 0:
                 return 0.0
             return dot / (mag1 * mag2)
In [5]: # ---- Single-pass clustering ----
         def single_pass_clustering(documents, threshold=0.3):
             clusters = [] # list of (cluster_rep, [docs])
             for doc_name, doc_rep in documents.items():
                 if not clusters:
                     clusters.append((doc_rep, [doc_name]))
                     continue
                 placed = False
                 for i, (rep, docs) in enumerate(clusters):
                     sim = cosine_similarity(doc_rep, rep)
                     if sim >= threshold:
                         docs.append(doc_name)
                         # update cluster representative (average frequencies)
                         for w, f in doc_rep.items():
                             rep[w] = rep.get(w, 0) + f
                         clusters[i] = (rep, docs)
                         placed = True
                         break
                 if not placed:
                     clusters.append((doc_rep, [doc_name]))
             return clusters
In [22]: # ---- Run clustering ----
         clusters = single_pass_clustering(documents, threshold=0.75)
```