Unit - 2

Set Theory

UNIT 2

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* Basic concepts of Set Theory

Set is collection of objects of any sort.
Ex: collection of natural numbers,
Pair of shoes.

set of sets called as collection of sets, it shares common property.
Ex: set of lines,
set of tangents.

Any object belonging to set is called as Member or an element of that set.

A set is said to be well defined if it is possible to determine by means of certain rules.

capital letters A to Z are used for Notation of sets.

Lowercase letter a to z are used for denote elements of sets.

Ex: if $p \in A$
p belongs to A
p is in A
p is an element of set A

if $q \notin A$
q is not belongs to A

The negation of the statement, "q is in A", that is, $\neg(q \in A) \Leftrightarrow q \notin A$

If we have to mention the set s,
$s = \{1, 2, a\}$

Elements are separated by comma. set enclosed with curly brackets.

Where equality sign is understood to mean that S is the set $\{1, 2, a\}$. obviously,
$1 \in S \quad 2 \in S \quad a \in S$

This method is sufficient if set is finite.

If set is infinite then we are using Predicate method.

Ex:

$$S = \{x \mid x \text{ is odd no}\}$$

$$S_1 = \{x \mid x \text{ is odd no}\}$$

$$S_2 = \{x \mid x = a \text{ or } x = b\}$$

$\forall$ x is odd no.
$x = a$ or $x = b$ are predictions

p(x) be any predicate then we can mention it as,

$\{x \mid p(x)\} = A$

Ex : $\{1, 3, a\}$ can be define in predicit as,

$\{x \mid (x=1) \vee (x=3) \vee (x=a)\}$

* Finite set :

The set which contain finite number of sets called as finite set.

* Infinite set :

The set which contain infinite number of elements called as infinite set.

if $S = \{a, \{1, 2\}, P, \{q\}\}$

$\{q\}$ is element of S but q is not element of S but member of $\{q\}$.

* Inclusion and Equality of sets.

Definition :

Let A and B be any two sets. If an every element of A is an element of B, then A is called as subset of B. or A is said to be included in B, or B includes A.

Symbolically this relation is denoted by $A \subseteq B$, or equivalently by $B \supseteq A$. Alternatively,

$A \subseteq B \Leftrightarrow (x)(x \in A \rightarrow x \in B) \Leftrightarrow B \supseteq A$

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If we have given sets.

$A = \{1,2,3\}$, $B = \{1,2\}$, $c = \{1,3\}$, $D = \{3\}$

then,

$B \subseteq A$, $C \subseteq A$, $D \subseteq A$

$\{1,2\} \subseteq \{1,2,3\}$

$\{1,3\} \subseteq \{1,2,3\}$

$\{3\} \subseteq \{1,2,3\}$

* Important properties of set

① $A \subseteq A$ Reflexive

② $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow (A \subseteq c)$ Transitive

③ $(x)(x \in A \rightarrow x \in B) \wedge (x)(x \in B \rightarrow x \in C) \Rightarrow$
$(x)(x \in A \rightarrow x \in c)$

* Equal set :

Two sets A and B are said to be equal iff $A \subseteq B$ and $B \subseteq A$, or symbolically

$A = B \Leftrightarrow (A \subseteq B \wedge B \subseteq A)$

From the equivalance,

$$\boxed{(x)(x \in A \rightarrow x \in B) \wedge (x)(x \in B \rightarrow x \in c) \Rightarrow (x)(x \in A \rightarrow x \in c)}$$

$(x)((P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow P(x)) \rightleftarrows$
$x(p(x) \rightleftarrows Q(x))$

$\{1,2,4\} = \{1,2,2,4\}$
$\{1,4,2\} = \{1,2,4\}$

If $P = \{\{1,2\},4\}$ and $Q = \{1,2,4\}$, then
$P \neq Q$
$\{1\} \neq \{\{1\}\}$
$A = B \iff B = A$

The equality of set is reflexive symmetric and transitive

* Proper subset :

A set A is called as proper subset of set B if $A \subseteq B$ and $A \neq B$. Written as $A \subset B$, so that
$A \subset B \iff (A \subseteq B \wedge A \neq B)$

$A \subset B$ also called proper inclusion.

A proper inclusion is non reflexive and it is transitive, i.e,

$(A \subset B) \wedge (B \subset C) \Rightarrow (A \subset C)$

* Universal set :

A set is called as a universal set, if it includes every set under de discussion. A universal set will be denoted by 'E'.

Ex: if A is a set then $A \subseteq E$

$x \in E \quad E = \{x \mid P(x) \vee \neg P(x)\}$

* Null set :

A set which does not contain any element is called as Null or empty set. A null set is denoted by ϕ

$$\phi = \{x \mid P(x) \wedge \neg P(x)\}$$

$P(x)$ is any predicate.

* Power set :

If given set is A then null set ϕ and set A are subset of A and also any element a is $a \in A$

Definition :

For set A a collection or family of all subsets of A is called power set of A. It is Denoted by $P(A) =$ or 2^A

$$\therefore P(A) = 2^A = \{x \mid x \subseteq A\}$$

Power set of null set ϕ having only element ϕ

$$\therefore P(\phi) = \{\phi\}$$

If $S_1 = \{a\}$ is given set then,

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power set $p(s_1) = 2^p = \{\phi, 0\} = \{\phi, s_1\}$

If $s_2 = \{a, b\}$
$p(s_2) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$
$= \{\phi, \{a\}, \{b\}, s_2\}$

If $s_3 = \{a, b, c\}$ find $p(s_3)$
If we have given some ordering i.e., arbitary
if $s_2 = \{a, b\}$ then ordering of given set will be
$\phi = B_{00}$ $\{b\} = B_{01}$
$\{a\} = B_{10}$ $\{a, b\} = B_{11}$
Read from left
Let Reading from subscript i.e, the subscripts are
00, 01, 10, 11
The subscripts could be $2^n = 2^2 = 4$
Binary Representation of decimal no 0 to $2^n - 1$

If a set
$s_6 = \{a_1, a_2, \text{-----} a_6\}$ then
binary of it
$B_0, B_1, \text{-----} B_{2^6-1}$
if
$B_7 = B_{111} \Rightarrow B_{000111}$
$\Rightarrow \{a_4, a_5, a_6\}$
$B_{12} = B_{1100} \Rightarrow B_{001100}$
$\Rightarrow \{a_3, a_4\}$

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* Types of operations on sets

① Intersection :

If A and B are two sets then intersection could be $A \cap B$. i.e., Elements belongs to both sets.

$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$$

- $A \cap B = B \cap A$ A & B are commulative
- $A \cap A = A$
- $A \cap \phi =$

Intersection is,

$$x \in A \cap B \Leftrightarrow x \in \{x \mid (x \in A) \wedge (x \in B)\}$$

$$\Leftrightarrow (x \in A) \wedge (x \wedge B)$$

$$\Leftrightarrow (x \wedge B) \wedge (x \wedge A)$$

$$\Leftrightarrow x \in \{x \mid (x \in B) \wedge (x \in A)\}$$

$$\Leftrightarrow x \in B \cap A$$

$A \cap (B \cap C) \Leftrightarrow (A \cap B) \cap C$

prove Associative of sets intersection

② Disjoint :

Two sets A and B are called disjoint iff $A \cap B = \phi$, that is A and B have no element in common.

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③ Mutually Disjoint :

A collection of sets is called disjoint collection. If for every pair of set in the collection, the two sets are disjoint

The elements of disjoint collection are said to be mutually disjoint.

Ex: if $A = \{ \{1,2\}, \{3\} \}$
$B = \{ \{1\}, \{2,3\} \}$
$C = \{ \{1,2,3\} \}$ then show A,B,C are mutually disjoint

$A \cap B = \phi$, $B \cap C = \phi$, $C \cap A = \phi$

④ Union :

For any two sets A and B, the union of A and B, written as $A \cup B$, is the set of all elements which are members of the set A or the set B or both.

$A \cup B = \{ x \mid x \in A \vee x \in B \}$

$A \cup B = B \cup A$ commutative

$A \cup \phi = A$

$A \cup A = A$

$A \cup (B \cup C) = (A \cup B) \cup C$ Associative

*prove $A \cup A = A$

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$x \in A \cup A \Leftrightarrow x \in \{x \mid x \in A \vee x \in A\}$

$\Leftrightarrow x \in A \vee x \in A$

$\Leftrightarrow x \in A$

$\Leftrightarrow x \in \{x \mid x \in A\}$

$x \in A$

$\Leftrightarrow \underline{\underline{A}}$

Ex : find $S \cup Q$, $S \cap Q$

if $S = \{a, b, p, q\}$

$Q = \{a, p, t\}$

$S \cup Q = \{a, b, p, q, t\}$

$S \cap Q = \{a, p\}$

~~Ex~~ :

if $A_1 \cup A_2 \cup \quad A_n = \bigcup_{i=1}^{n} A_i$

Ex : if $A_1 = \{1, 2\}$

$A_2 = \{2, 3\}$

$A_3 = \{1, 2, 3, 6\}$ then

$\bigcup_{i=1}^{3} A_i$, $\bigcap_{i=1}^{3} A_i = ?$

$\bigcup_{i=1}^{3} A_i = \{1, 2, 3, 6\} = A_1 \cup A_2 \cup A_3$

$\bigcap_{i=1}^{3} A_i = \{2\} = A_1 \cap A_2 \cap A_3$

⑤ Relative complement :

Let A and B are two sets. The relative complement of B in A, written as A − B, is the set consisting of all elements of A which are not element of B that is,

$$A - B = \{x \mid x \in A \wedge x \in B\}$$
$$= \{x \mid x \in A \wedge \neg(x \in B)\}$$

Relative complement of B in A is also called difference of A and B

⑥ Absolute complement:

If E is universal set. for any set A, the relative complement of A with respect to E, i.e., E − A is called the absolute complement of A

The absolute complement of A also called as complement of A

It is denoted as '$\sim A$'.

$$\sim A = E - A$$
$$= \{x \mid x \in E \wedge x \notin A\}$$
$$= \{x \mid x \notin A\}$$
$$= \{x \mid \neg(x \in A)\}$$

- $\sim(\sim A) = \sim\sim A = A$

$\sim \phi = E$

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$\sim E = \phi$

$A \cap \sim A = \phi$

$A \cup \sim A = E$

Ex : $A = \{2, 5, 6\}$

$B = \{3, 4, 2\}$

$C = \{1, 3, 4\}$

Find $A-B$, $B-A$ and show that $A-B \neq B-A$

$A-B = \{5, 6\}$

From above $A-B \neq B-A$

$B-A = \{3, 4\}$

Find $A-C = A$

Ex:② show that $A-B = A \cap \sim B$

$A-B = x \in \{x \mid x \in A \wedge x \notin B\}$

$= x \in \{x \mid x \in A \wedge \neg E \neg B\}$

$= \underline{\underline{A \cap \sim B}}$

⑦ Symmetric difference [Boolean sum] :

If A and B are two sets then symmetric difference [Boolean sum] of A and B is $\underline{\underline{A+B}}$

$A+B = (A-B) \cup (B-A)$

which is exclusive disjunction

$A+B = B+A$ $\quad (A+B)+C = A+(B+C)$

$A+\phi = A$ $\quad A+A = \phi$

We shall prove $A + \phi = A$. For any x

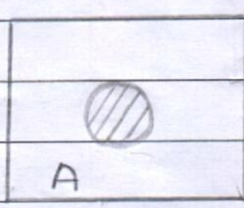
$$x \in A+\phi \Leftrightarrow x \in \{x \mid (x \in A \wedge x \notin \phi) \vee (x \in \phi \wedge x \notin A)\}$$
$$\Leftrightarrow (x \in A \wedge x \notin \phi) \vee (x \in \phi \wedge x \notin A)$$
$$\Leftrightarrow (x \in A) \vee F$$
$$\Leftrightarrow x \in A$$
$$\Leftrightarrow x \in \{x \mid x \in A\}$$
$$\Leftrightarrow x \in A$$

* Venn Diagram

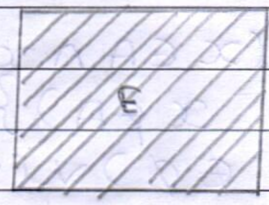
A Venn Diagram is a schematic representation of a set by a set of points.

The universal set E is represented by a set of points in rectangle. and a subset, say A, of E is represented by interior of circle or closed curve inside rectangle.

① Set A :

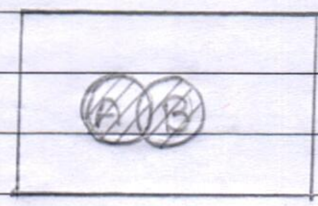


② Universal set E :



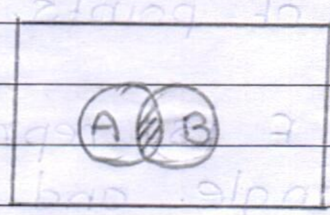
E

③ $A \cup B$



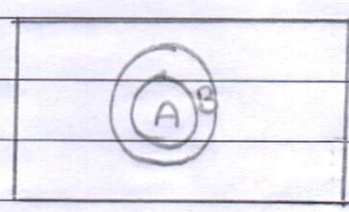
$A \cup B$

④ $A \cap B$:

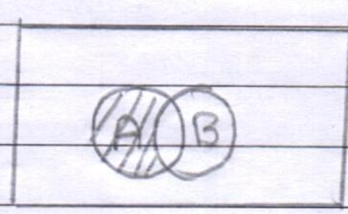


$A \cap B$

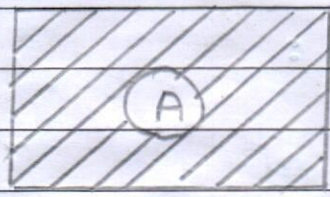
⑤ $A \subseteq B$ A is subset of B



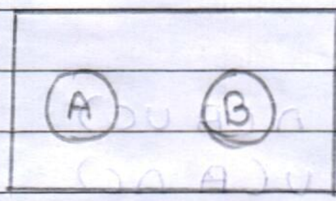
⑥ $A - B = A \cap \sim B$



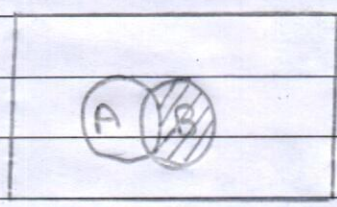
⑦ $\sim A$



⑧ $A \cap B = \phi$



⑨ $B - A$



$B - A = B \cap \sim A$

* Set Algebra

① Idempotent law's :

$A \cup A = A$
$A \cap A = A$

② Associative low's :

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

③ commulative law's :

$A \cup B = B \cup A$
$A \cap B = B \cap A$

④ Distributive law's :

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup \phi = A$
$A \cap \phi = \phi$
$A \cup E = E$
$A \cap E = A$
$A \cup \sim A = E$
$A \cap \sim A = \phi$

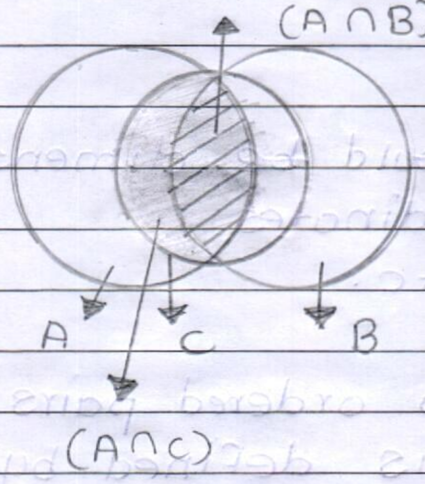
⑤ Absorption law's :

$A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$

⑥ De-Morgan's ~~theorem~~ law :

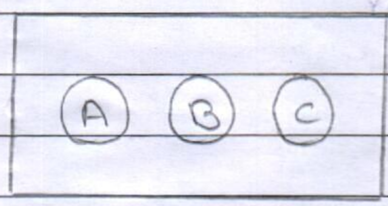
$\sim (A \cup B) = \sim A \cap \sim B$
$\sim (A \cap B) = \sim A \cup \sim B$
$\sim \phi = E$
$\sim E = \phi$
$\sim (\sim A) = A$

① $(A \cap B) \subset (A \cap C)$ but $(B \not\subset C)$

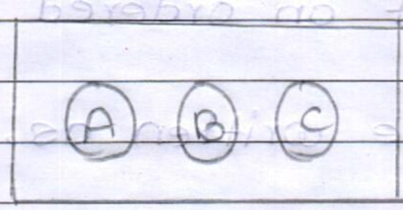


② $(A \cup B) \subseteq (A \cup C)$ but $(B \not\subseteq C)$.

③ $(A \cup B) = (A \cup C)$ but $B \neq C$



④ $(A \cap B) = (A \cap C)$ but $B \neq C$



* Ordered Pairs and n tuples

An ordered pair consists of two objects in a given fixed order. Means it not contain only two elements.

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The two object should be distinct denoted by $<x, y>$.

Ex: Two (objects) ~~should be~~ dimensional plane in cartesion co-ordinates
$<1,3>$, $<2,4>$ etc.

The equality of two ordered pairs $<x, y>$ and ~~$<y, z>$~~ $<u, v>$ is defined by,

$<x, y> = <u, v> \Leftrightarrow ((x = u) \cap (y = v))$
so
$<1,2> \neq <2,1>$ or
$<1,1> \neq <2,2>$

* ordered Triple :

An ordered triple is an ordered pair whose first member is itself an ordered pair.

ordered triple can be written as, $<<x, y>, z>$

If $<<x, y>, z>$ and $<<u, v>, \omega>$ are ordered triple then equality would be
$<x, y> = <u, v> \cap z = w$
but
$<x, y> = <u, v>$ equal to $<x = u \cap y = v>$
$\therefore ((x = u) \cap (y = v) \cap (z = \omega))$
From above definition we can write
$<<x, y> z> = <x, y, z>$

$\langle x, y, z \rangle \neq \langle y, z, x \rangle \neq \langle z, x, y \rangle$

* ordered quadruple :

It is defined as an ordered pair whose first member is an ordered triple.

It is written as, $\langle\langle x, y, z\rangle, u\rangle$ which actually is $\langle\langle\langle x, y\rangle, z\rangle, u\rangle$.

It is easy to show that two ordered quadruples $\langle\langle x, y, z\rangle, u\rangle$ and $\langle\langle p, q, r\rangle, s\rangle$ are equal.

$(x = p) \cap (y = q) \cap (z = r) \cap (u = s)$

* n tuple :

The ordered pair whose first member is ordered $(n-1)$ tuple we can write ordered n tuple as

$\langle\langle x_1, x_2, x_3 \text{------} x_{n-1}\rangle, x_n\rangle$

If two ordered n tuples then

$\langle\langle x_1, x_2, \text{----} x_{n-1}\rangle, x_n\rangle$ and

$\langle\langle u_1, u_2, \text{------} u_{n-1}\rangle, u_n\rangle$ then

$((x_1 = u_1) \cap (x_2 = y_2) \cap \text{----} \cap (x_n = u_n)$

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It can be written as,
$<x_1, x_2, ------ x_n>$

* Cartesion Products :

Let A and B be any two sets. The set of all ordered pairs such that the first member of the ordered pair is an element of A and the second member is an element of B is called cartesion products of A and B

A and B written as $A \times B$,
$A \times B = \{<x, y> | (x \in A) \wedge (y \in B)\}$

Cartesion set follows ordered set of given example.
If $A = \{\alpha, \beta\}$ and $B = \{1, 2, 3\}$ then what are ① $A \times B$ ② $B \times A$ ③ $A \times A$ ④ $B \times B$ and ⑤ $(A \times B) \wedge (B \times A)$?
→
① $A \times B = \{<\alpha, 1>, <\alpha, 2>, <\alpha, 3>,$
$<\beta, 1>, <\beta, 2>, <\beta, 3>\}$

② $B \times A = \{<1, \alpha>, <2, \alpha>, <3, \alpha>,$
$<1, \beta>, <2, \beta>, <3, \beta>\}$

③ $A \times A = \{<\alpha, \alpha>, <\alpha, \beta>, <\beta, \alpha>, <\beta, \beta>\}$

④ $B \times B = \{<1, 1>, <1, 2>, <1, 3>, <2, 1>, <2, 2>, <2, 3>$
$<3, 1>, <3, 2>, <3, 3>\}$

$(A \times B) \cap (B \times A) = \phi$

If we have given $A_1, A_2, \ldots A_n$ then cartesion product will be

$$\mathop{\times}_{i=1}^{n} A_i = A_1 \times A_2 \times \cdots \times A_n$$

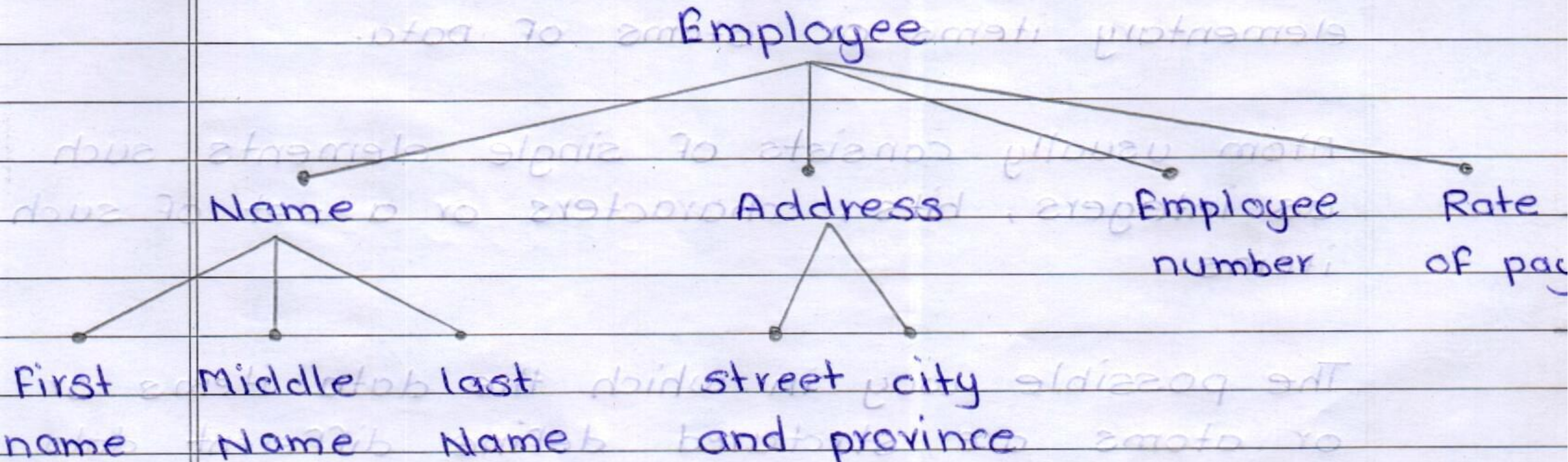
cartesion product of $A \times A = A^2$

$A \times A \times A = A^3$

* Representation of Discrete strctures

① Data strctures :

By using Data strcture we can reduce complexity of given programming language we can represent Payroll system. for Example in a payroll (data processing) application a tree-like representation of information for an employee



In FORTRAN it is not possible to write tree strcture but by using COBAL we can write the program into simpler Format.

By using simpler strcture we can write given program into subprograms.

Processes, modulas, concerned with operations performed on data strctures are frequently represented by subroutines or functions. and for the program implementation of any significant problem the organization of a program into suitable modul-as or subroutines is indispensible.

In organizing the solution to a problem, one is concerned with two classes concepts. The first class deals with particular data strcture. The second class of concepts with operations that must be performed on data strcture.

Data in a particular problem consist of a set of elementary items or atoms of Data.

Atom usually consists of single elements such as integers, bits, characters or a set of such items.

The possible way in which the data items or or atoms are strctured define different data strcture.

The data strctures can be classified into,
① frequently used
② Not frequently used

① frequently used

creation of data strcture at declaration of statement
① Destroyed
② Erased

change, add and delete of element in DS called update of data strcture

② Data can be classified as
① Batches
② Vectors
③ Plexes

① Batches : - understad
- fixed / variable
used in data processing problem

② vector : - fixed no of objects are there
- No deletion / No addition are performed

③ Plex : - variable no of elements
- deletion / addition can be done
- Two adjancy elements are called as linear and other called non-linear.

Linear list is used to instead of linear plex.

In plex if we have to add / delete element at ith location at such case we have to locate element at either i+1, or i-1 location

some plex are permit to add / delete element at one end of subclass called as stack.

① stack -

addition and deletion from same end

push - addition

pop - deletion

most accessible element - top

least accessible element - bottom

It used LIFO [Last in first out]

Ex. Pile of tray in cafeteria

② Queue -

Another excample of plex is queue push at one end and pop at opposite end (FIFO)

Ex. Row of students in any examination

② Storage strcture :

By using hardware and software, the way in which data strctures are represented in memory called storage strcture

We can avoid ① loss of efficiency
② solve problem from available tools and resources.

The memory can be ordered into sequences of words each word contain 8 to 64 bits.

* Computed address:

Method of giving disoriphre addressing called as computed address

* Link or pointer:

use the address to store memory somewhere in computer called links or pointer address

③ Sequential Allocation

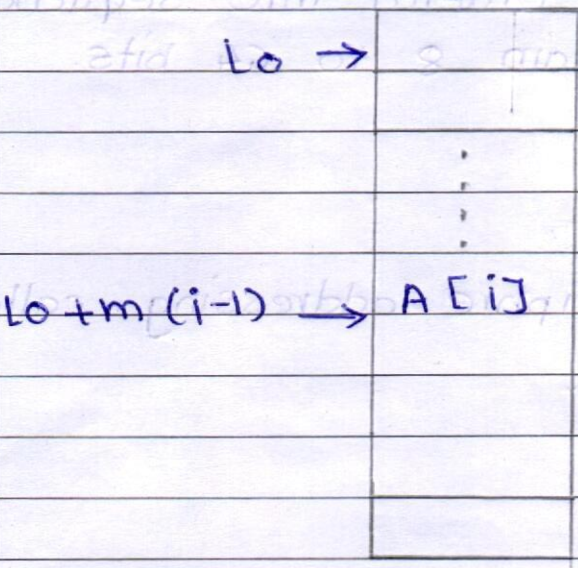
Integers, real numbers and character strings are called premitive data strcture because they are used for manipulate on other data strcture

* Vector:

locate elements in elements

contigous allocation of memory

If n memory elements are these then n consecutive words in memory



m words are allocated for each elements or node

Fig : Representation of vector

If we have given element A [i, j] then expression could be ,

Lo + (j-1) * 3 + (i-1)

Eg A [2, 3]

Lo + 7

} For 3 dimensional arrays with 3 row 2 column

For two dimensional array we can take it as 2 rows and 3 columns

If n rows and m columns then linearly they can be represented as

$$\boxed{L_0 + (j-1) * n + (i-1)}$$

Two dimensional array with 3 rows

A[1,1] A[2,1] A[3,1] for L_0

A[1,1] A[2,1] A[3,1] A[1,2] A[2,2] A[3,2] A[1,3] A[2,3] A[3,3]

If no rows and m columns then it can be linearly represented as A[2.2]

A[2,2] A[1,2] A[1,m] ------- A[n-1] A[n-2] A[n,m]

The address of matrix element A[i, j] is given by expression

$$\boxed{L_0 + (i-1) * m + (j-1)}$$