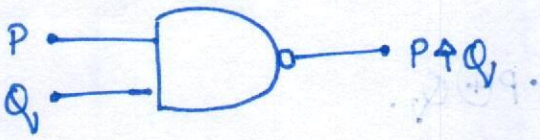
DMS

Q1.] Write other connectives in detail.

⇒ 1 NAND

P	Q	$P \wedge Q$	$\overline{P \wedge Q} \Leftrightarrow P \uparrow Q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

- Symbol



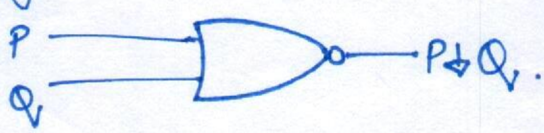
- It follows commutative law,
  i.e $P \uparrow Q = Q \uparrow P$

- It does not follow associative law,
  i.e $P \uparrow (Q \uparrow R) \neq (P \uparrow Q) \uparrow R$.

2.] NOR = OR + NOT

P	Q	$P \vee Q$	$\overline{P \vee Q} \Leftrightarrow P \downarrow Q$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

- Symbol



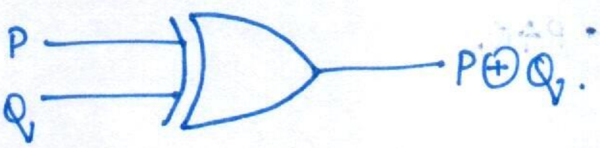
- It follows commutative law,
  i.e $P \downarrow Q = Q \downarrow P$.

- It also follows associative law,
  i.e $P \downarrow (Q \downarrow R) \neq (P \downarrow Q) \downarrow R$.

3] Exclusive OR (Ex-OR)

P	Q	$P \rightleftarrows Q \Leftrightarrow P \oplus Q$.
T	T	F
T	F	T
F	T	T
F	F	F

- Symbol



- It follows commutative law,
  i.e $P \oplus Q = Q \oplus P$.

- It also follows associative law,
  i.e $(P \oplus Q) \oplus R = P \oplus (Q \oplus R)$.

4] Exclusive NOR (Ex-NOR).

P	Q	$P \oplus Q$	$P \odot Q \Leftrightarrow \overline{P \rightleftarrows Q}$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T.

- Symbol

- It follows commutative law,
  i.e $P \odot Q = Q \odot P$

- It also follows associative law,
  i.e $P \odot (Q \odot R) = (P \odot Q) \odot R$.

Q2] List out all equivalence formulas.

$\Rightarrow$ 1. $\neg\neg P \Leftrightarrow P$

2. $\left.\begin{array}{l}(P \wedge Q) \Leftrightarrow (Q \wedge P) \\ (P \vee Q) \Leftrightarrow (P \vee Q)\end{array}\right\}$ Commutative law

3. $\left.\begin{array}{l}P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R \\ P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R\end{array}\right\}$ Associative law

4. $\left.\begin{array}{l}\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q \\ \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q\end{array}\right\}$ De morgan's law

5. $\left.\begin{array}{l}P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R) \\ P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)\end{array}\right\}$ Distributive law.

6. $P \wedge T \Leftrightarrow P$
7. $P \vee T \Leftrightarrow T$
8. $P \wedge F \Leftrightarrow F$
9. $P \vee F \Leftrightarrow P$
10. $P \rightleftarrows Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$
11. $P \to Q \Leftrightarrow \neg P \vee Q$
12. $P \to Q \Leftrightarrow P \wedge \neg Q$
13. $P \vee P \Leftrightarrow P$
14. $P \wedge P \Leftrightarrow P$
15. $R \vee (P \wedge \neg P) \Leftrightarrow R$
16. $R \wedge (P \vee \neg P) \Leftrightarrow R$

17. $R \vee (P \vee \neg P) \Leftrightarrow T$

18. $R \wedge (P \wedge \neg P) \Leftrightarrow F$

19. $\neg(P \to Q) \Leftrightarrow \neg(\neg P \vee Q)$

20. $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R.$

21. $P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Q3] write all implication formulas.

⇒ 1. $P \wedge Q \Rightarrow P$

2. $P \wedge Q \Rightarrow Q$

3. $P \Rightarrow P \vee Q$

4. $Q \Rightarrow P \vee Q$

5. $\neg P \Rightarrow P \to Q$

6. $Q \Rightarrow P \to Q$

7. $\neg(P \to Q) \Rightarrow P$

8. $\neg(P \to Q) \Rightarrow \neg Q$

9. $P, Q \Rightarrow P \wedge Q.$

10. $\neg P, P \vee Q \Rightarrow Q$

11. $P, P \to Q \Rightarrow Q$

12. $\neg Q, P \to Q \Rightarrow P$

13. $P \to Q, Q \to R \Rightarrow P \to R$

14. $P \vee Q, P \to R, Q \to R \Rightarrow R.$

Q4] write a note on tautology, contradiction & contigency with suitable example.

⇒ Tautology

All the truth values in the last column are True, Hence it is tautology.

Eg :- $P \vee \neg P$

	P	$\neg P$	$P \vee \neg P$
t	T	F	T
	F	T	T

2] Contradiction

All the truth values in the last column are False. It is contradiction.

Eg :- $\neg(P \vee \neg P)$

P	$\neg P$	$P \vee \neg P$	$\neg(P \vee \neg P)$
T	F	T	F
F	T	T	F

3] Contigency

All truth value in the last column are not identical, they are neither all true values nor all false values it is combination of true or false values in last column

Eg :- $P \wedge \neg Q$

P	Q	$P \wedge \neg Q$
T	T	F
T	F	T
F	T	F
F	F	F

Q5] Explain in detail well formed formulae.

⇒ In mathematical logic well formed formula (WFF) is an expression consisting of variable (capital letter), parenthese & connective symbols. An expression is basically a combination of operands & operators are the connective symbols.

Below are the possible connective Symbol

1. Negation ($\neg$)
2. Conjuction ($\wedge$)

3. Disjunctive (~~∧~~) (V)
4. Conditional ($\rightarrow$)
5. Biconditional ($\rightleftarrows$)

Also,
a] A single statement is well formed formula
b] Negative of given statement is well form--ed formula
c] $(P \wedge Q)$, $(P \vee Q)$, $(P \rightarrow Q)$, $(P \rightleftarrows Q)$ are also well formed formulae.
d. Combination of single statement, Negation, conjuction, Disjunction, conditional, Biconditional are also WFF.

Q6.] Explain in detail duality law.

$\Rightarrow$ Duality Principle states that for any true statement, the dual statement obtained by interchanging conjuctions into Disjunctions & vice versa is also true.

If we interchange the symbol & get this statement itself, it is known as self-dual statement.

Here,
1.] Conjuction is replaced by Disjunction.
2.] Disjunction is replaced by conjuction.
3.] Truth value 'True' is replaced by Truth ~~Table~~ value 'False'.
4.] Truth value 'False' is replaced by Truth value 'True'.

Q7] What are connectives. Explain in detail.

⟹ Connectives are used to connect 2 sentences

They are of 2 types:-

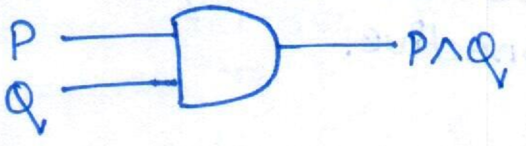
Connectives

Unary	Binary
Negation	Conjuction
	Disjunction
	Conditional
	Biconditional.

1.] Conjuction (Anding)

when 2 statement are combined with an 'and'.

- Also known as CAP & ANDING.
- Denoted by $P \wedge Q$ i.e '$\wedge$'.



P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

- Properties followed by conjuction are:-

a.] Associative ⟹ $P \wedge (Q \wedge R) = (P \wedge Q) \wedge R$.

b.] Commutative ⟹ $(P \wedge Q) = (Q \wedge P)$

2. Disjunction.

when two statements are combined with an 'OR'.

- Also known as CUP & ORING.
- Denoted by '$\vee$' & $P \vee Q$

2.] $P \wedge (Q \rightleftarrows R)$

$P \wedge ((Q \rightarrow R) \wedge (R \rightarrow Q))$

$P \wedge ((\neg Q \vee R) \wedge (\neg R \vee Q))$

$P \wedge (\neg(Q \wedge \neg R) \wedge \neg(R \wedge \neg Q))$

3] $((P \vee Q) \wedge R) \rightarrow (P \vee R)$

$\neg((P \vee Q) \wedge R) \vee (P \vee R)$

$(\neg(P \vee Q) \vee \neg R) \vee (P \vee R)$

Q.9] Explain normal forms in detail.

⟹ The problem of finding whether a given statement is tautology or contradiction or satisfiable in a finite no. of steps is called the Decision Problem. For Decision Problem, construction of truth table may not be participate / practical always. We consider an alternate procedure known as reduction to normal forms.

There are 4 such forms:-

1. Disjunctive Normal Form
2. Principal Disjunctive Normal Form.
3. Conjuctive Normal Form.
4. Principal Conjuctive Normal Form.

1. Disjunctive Normal Form.

If p, q are two statements, then "p or q" is a compound statement, denoted by pvq & referred as the disjunction of p & q. The disjunction of p & q is true whenever at least one of the two statement is true, & it is false only when both p & q is false.

- It contains 'Maxterms'.

Eg:-

If p is "4 is a positive integer" & q is "$\sqrt{5}$ is a rational numbers", then $p \vee q$ is true as statement p is true, although statement q is false.

2. Principal Disjunctive Normal Form (PDNF)
   - PDNF is also known as conical Disjunctive Normal Form.
   - It refers to the sum of products i.e SOP.
   - Eg:-
     - If P, Q, R are the variable then.

$$(P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R).$$

Here '$\vee$' i.e sum is main operator

- The key diff. betn PDNF & DNF is that in case of DNF, it is not necessary that the length of all variables in the expression is same.

Eg:-

1.] $(P \wedge \neg Q \wedge R) \vee (P \wedge Q)$ is a example of an expression in DNF but not in PDNF.

2.] $(P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)$ is an eg. of an expression which is both in PDNF & DNF.

- Every example of PDNF is an example of DNF &
  Every example of DNF can NOT be an example of PDNF.

3] Conjuctive Normal Form.

If p, q are 2 statements, then "p & q" is a compound statements, denoted by 'p∧q'. The conjuction of p & q is true only when both p & q are true, otherwise, it is false.

It contains 'Min terms.'

Eg:-

If statement p is "6<7" & q is "-3>-4" then the conjuction of p & q is true as both p & q are true statement.

P	Q	P∧F.
T	T	T
T	F	F
F	T	F
F	F	F.

4] Principal Conjuctive Normal Form.

- Also known as Conical Conjuctive Normal Form.
- It refers to 'Product of Sums' i.e POS.

Eg:- If P, Q, R are variables, then.

$(P \vee \neg Q, \vee R) \wedge (\neg P \vee Q, \vee R) \wedge (P \vee Q, \vee \neg R)$.

Here '∧' i.e product is main operator.

Also,

The key difference betn PCNF & CNF is that in case of CNF, it is not necessary that the length of all variables in the expression is same.

Eg:- 1] $(P \vee \neg Q, \vee R) \wedge (P \vee \neg R)$ is example of CNF but not PCNF

2] $(P \vee \neg Q \vee R) \wedge (P \vee Q, \vee \neg R) \wedge (\neg P \vee Q \vee R)$. is an example of both CNF & PCNF.

• Every example of PCNF can be example of CNF but
Every example of CNF can not be an eg. of PCNF.

Q10. Difference betⁿ

1] DNF & CNF

DNF	CNF
1. It refers to Disjunctive Normal Form.	It refers to Conjuctive Normal Form.
2. DNF is sum of Products	CNF is ~~sum~~ Product of sum.
3. Take a disjunction (that is v) of all satisfying truth assignments	Take a conjuction (∧) of negations of satisfying truth assignments.
4. Eg:- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R)$	Eg:- $(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)$

2] PDNF & PCNF.

PDNF	PCNF
1. It refers to Principal Disjunctive Normal Form	1. It refers to Principal Conjuctive Normal Form.
2. It refers to Sum of Products i.e SOP.	It refers to Product of Sum i.e POS.
3. The combination of disjunction is known as 'Minterms'.	The combination of conjuctions is known as 'Maxterms'.

4. Also known as Conical Conjuctive Normal Form.	4. Also known as Conical Disjunctive Normal Form
5. Eg:- $(P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R)$.	5. Eg:- $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R)$.

Q12] Give ~~P~~CNF

a] $(\neg R \rightarrow (\neg Q \rightarrow \neg P))$

$\Rightarrow (\neg R \rightarrow (\neg\neg Q \vee \neg P))$

$(\neg R \rightarrow (Q \vee \neg P))$

$(\neg\neg R \vee (Q \vee \neg P))$

$(R \vee (Q \vee \neg P))$

$\neg(\neg R \wedge \neg(Q \vee \neg P))$.

b] $(\neg(\neg P \vee \neg Q) \rightarrow R)$

$\Rightarrow (\neg\neg(\neg P \vee \neg Q) \vee R)$.

$((\neg P \vee \neg Q) \vee R)$

$\neg(\neg(\neg P \vee \neg Q) \wedge \neg R)$.

Q13] Construct PCNF & PDNF.

Extra

a] $(\neg R \rightarrow (\neg Q \rightarrow \neg P))$

$\Rightarrow$ PCNF :- $(\neg R \rightarrow (\neg\neg Q \vee \neg P))$

$(\neg\neg R \vee (\neg\neg Q \vee \neg P))$

$(R \vee (Q \vee \neg P))$

$\neg(\neg R \wedge \neg(Q \vee \neg P))$.

a.] $(\neg R \to P) \land ((Q \to P) \land (P \to Q))$.

$\Rightarrow (\neg\neg R \lor P) \land ((\neg Q \lor P) \land (\neg P \lor Q))$

$(R \lor P) \land ((\neg Q \lor P) \land (\neg P \lor Q))$

$(R \lor P) \land \neg(\neg(\neg Q \lor P) \lor \neg(\neg P \lor Q))$. $\Rightarrow$ CNF.

DNF :-

$(\neg R \to P) \land ((Q \to P) \land (P \to Q))$

$(\neg\neg R \lor P) \land ((\neg Q \lor P) \land (\neg P \lor Q))$

$(R \lor P) \land ((\neg Q \lor P) \land (\neg P \lor Q))$

$\neg(\neg(R \lor P) \lor \neg((\neg Q \lor P) \land (\neg P \lor Q)))$

$= \neg((\neg R \land \neg P) \lor \neg((\neg Q \lor P) \land (\neg P \lor Q)))$.

Q14] Show that following implication without constructing truth table.

$$(Q \to (P \land \neg P)) \to (R \to (P \land \neg P)) \Rightarrow (R \to Q)$$

$\Rightarrow (\neg Q \lor (P \land \neg P)) \to (R \to (P \land \neg P))$

$(\neg Q \lor (P \land \neg P)) \to (\neg R \lor (P \land \neg P))$

$\neg(\neg Q \lor (P \land \neg P)) \lor (\neg R \lor (P \land \neg P))$.

$\neg(\neg Q \lor F) \lor (\neg R \lor F)$

$\neg(\neg Q) \lor (\neg R)$

$Q \lor \neg R$.

$\therefore \neg R \lor Q$

$\therefore R \to Q$

$=$ R.H.S.

Q15] Show the Following implication without constructing truth table.

i.] $(P \to Q) \to Q \Rightarrow P \vee Q$.

$\Rightarrow$ $(P \to Q) \to Q$

$(\neg P \vee Q) \to Q$

$\neg(\neg P \vee Q) \vee Q$

$(\neg\neg P \wedge \neg Q) \vee Q$

$(P \wedge \neg Q) \vee Q$

$\therefore (P \vee Q) \wedge (\neg Q \vee Q)$

$(P \vee Q) \wedge T$

$= P \vee Q$

$=$ R.H.S.

ii.] $(Q \to (P \wedge \neg P)) \to (\neg(P \wedge \neg P) \to \neg R) \Rightarrow (\neg R \vee Q) \Rightarrow$

$\Rightarrow (\neg Q \vee (P \wedge \neg P)) \to (\neg\neg(P \wedge \neg P) \vee \neg R)$ $(\neg R \vee Q)$

$\Rightarrow (\neg Q \vee F) \to (\neg\neg(P \wedge \neg P) \vee \neg R)$

$\neg(\neg Q \vee F) \vee ((P \wedge \neg P) \vee \neg R)$

$\neg(\neg Q) \vee (F \vee \neg R)$

$Q \vee \neg R$

$\therefore \underline{\neg R \vee Q}$

$\Rightarrow R \to Q$

$\Rightarrow$ R.H.S

Q16. Define well formed formula state wheather the following are WFF's.

The combination of $\{\neg, \vee, \wedge\}$ is a well formed formula.
The combination of $\{\neg, \vee\}$ & $\{\neg, \wedge\}$ is a WFF.

i. $(A \to B) \vee (B \to C)$.

$\Rightarrow$ $(\neg A \vee B) \vee (\neg B \vee C)$

$(\neg A \vee B) \vee (\neg B \vee C)$

It is a WFF as it is combination of $\{\neg, \vee\}$.

2] $((\neg B) \rightarrow (P \rightarrow Q))$

$\Rightarrow ((\neg\neg B) \veebar (P \rightarrow Q))$

$((\neg\neg B) \vee (\neg P \vee Q))$

$= (B \vee (\neg P \vee Q))$

$\therefore$ It is a WFF as it is a combination of $\{\neg, \vee\}$.

Q17] Show that.

1. $\neg(P \uparrow Q) \Leftrightarrow \neg P \downarrow \neg Q$.

P	Q	$\neg P$	$\neg Q$	$P \uparrow Q$	$\neg(P \uparrow Q)$	$\neg P \downarrow \neg Q$.
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F.

II] $\neg(P \downarrow Q) \Leftrightarrow \neg P \uparrow \neg Q$.

P	Q	$\neg P$	$\neg Q$	$P \downarrow Q$	$\neg(P \downarrow Q)$	$\neg P \uparrow \neg Q$.
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F.

Q18] Construct the truth table for

1. $(\neg P \land (\neg Q \land R)) \lor (Q \land R) \lor (P \land R)$

P	Q	R	$\neg Q$	$\neg P$	$(\neg Q \land R)$	$(\neg P \land (\neg Q \land R))$	$(Q \land R)$
T	T	T	F	F	F	F	T
T	T	F	F	F	F	F	F
T	F	T	T	F	T	F	F
T	F	F	T	F	F	F	F
F	T	T	F	T	F	F	T
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	F
F	F	F	T	T	F	F	F.

$(P \land R)$	$(Q \land R) \lor (P \land R)$	$(\neg P \land (\neg Q \land R)) \lor (Q \land R) \lor (P \land R)$.
T	T	T
F	F	F
T	T	T
F	F	F
F	T	T
F	F	F
F	F	T
F	F	F.

b. $((P \land Q) \lor (\neg P \land Q)) \lor ((P \land \neg Q) \lor (\neg P \land \neg Q))$

P	Q	$\neg P$	$\neg Q$	$P \land Q$	$\neg P \land Q$	$((P \land Q) \lor (\neg P \land Q))$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	T	T
F	F	T	T	F	F	F.

$P \land \neg Q$	$\neg P \land \neg Q$	$((P \land \neg Q) \lor (\neg P \land \neg Q))$
F	F	F
T	F	T
F	F	F
F	T	T

$((P \land Q) \lor (\neg P \land Q)) \lor ((P \land \neg Q) \lor (\neg P \land \neg Q))$.
T
T
T
T

$\therefore$ It is example of tautology.

Q19] Obtain PCNF

$Q \land (P \lor \neg Q)$

$\Rightarrow \quad Q \land (P \lor \neg Q)$

$\cancel{(Q \land P) \lor (Q \land \neg Q)}$

$\cancel{Q \land P} \quad Q \lor (P \land \neg P) \quad \land (P \lor \neg Q)$

$(Q \lor P) \land (Q \lor \neg P) \land (P \lor \neg Q)$

$\therefore (P \lor Q) \land (\neg P \lor Q) \land (P \lor \neg Q)$

Q21] Explain the following binary relations in detail with example.

i. Reflexive.

If R is the relation with ordered pair $\langle x, y \rangle$ such a way that the value of domain & range are same there it is represented as $\boxed{xRx}$ which said to be reflexive relation for $=$,

- It may be said as it follows commutative law as both domain & range have same value. $R = \{(a,a) \in R \ \forall a \in A\}$

Eg:- $\{\langle 1,1 \rangle, \langle 2,2 \rangle \langle 3,3 \rangle.\}$.

Matrix :-

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

graph :-

- The relation $\subseteq$, $\leq$ & $=$ are reflexive relation

ii] Symmetric.

If R is the relation on the set x such a way that for every element x & y in x is said to be symmetric, if it contains xRy then yRx.

Eg:- $\{\langle 1,2 \rangle, \langle 2,1 \rangle\}$.

Matrix :-

$$\begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

Graph :-

3. Transitive.

If Relation R in the set 'x' is said to be transitive if $x, y, z \in X$ & whenever xRy, yRz thn it gives xRz.

In the set of real nos. $<, \leq, =, \subseteq, \subset$ are the Transitive relation.

Eg :- $X = \{1, 2\}$

$R = \{x, y \in R / x - y$ is divisible by $1\}$

$$R = \{\langle 1,2\rangle\ \langle 2,1\rangle\ \langle 1,1\rangle\ \langle 2,2\rangle\}$$

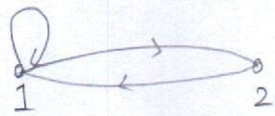
$\therefore \langle 1,2\rangle \in R$

$\langle 2,1\rangle \in R$

also $\langle 1,1\rangle \in R$.

$$\text{Matrix} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

Graph =



Q22] Explain the following

1] Anti - Symmetric.

If R is the relation on set X. For Every $x, y \in R$. whenever xRy & yRx, then $x = y$ R is called as Anti-symmetric in x.

or It is antisymmetric if either $(x, y) \notin R$ or $(y, x) \notin R$ whenever $x \neq y$.

Eg :- $\{\langle 1,1\rangle, \langle 2,2\rangle\}$.

Graph :- 

The relation, <, >, = are antisymmet-ric.

Matrix :- $\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$

ii] Irreflexive.

If R is the Relation on set 'x' is said to be irreflexive for every $x \in X$ with ordered pair $\langle x, x \rangle \notin R$.

Any relation which is not reflexive is not necessarily irreflexive & vice versa.

Eg :- $A = \{1, 2, 3\}$

$R = \{ \langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 2, 3 \rangle \}$

It is a irreflexive relation as

$1 \in A$

but $\langle 1, 1 \rangle \notin R$.

Matrix :- $\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix} \end{array}$

graph :-

Q23] Let I be the set of integer show that $R = \{\langle x, y\rangle / x-y$ is divisible by $m\}$. $(x, y \in R)$, m is positive integer) R is an equivalence relation

$\Rightarrow$ $R = \{\langle x, y\rangle / x-y$ is divisible by $m\}$

Let m be 1.

& $I = \{1, 2, 3\}$

$\therefore R = \{\langle x, y\rangle / x-y$ is divisble by $1\}$.

$$R = \begin{Bmatrix} \langle 1, 2\rangle, & \langle 1, 3\rangle, & \langle 1, 1\rangle, \\ \langle 2, 1\rangle, & \langle 2, 2\rangle, & \langle 2, 3\rangle, \\ \langle 3, 1\rangle, & \langle 3, 2\rangle, & \langle 3, 3\rangle, \end{Bmatrix}$$

① Reflexive

$1 \in I$

also, $\langle 1, 1\rangle \in R$.

$\therefore$ R is a reflexive relation. — ①

② Symmetric

$\langle 1, 2\rangle \in R$

also, $\langle 2, 1\rangle \in R$

$\therefore$ It is a symmetric Relation. — ②

③ Transitive.

$\langle 1, 3\rangle \in R$,

$\langle 3, 1\rangle \in R$

also,

$\langle 1, 1\rangle \in R$.

$\therefore$ It is a transitive relation — ③

From 1, 2 & 3,

R is an equivalence relation.

Q25. $X = \{1, 2, 3, 4, 5\}$

$R = \{\langle x, y\rangle \mid x > y\}$

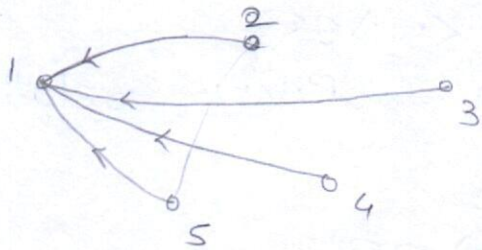
Draw graph & matrix for above example.

$\Rightarrow$ $R = \{\langle 2,1\rangle, \langle 3,1\rangle, \langle 4,1\rangle, \langle 5,1\rangle,$
$\langle 3,2\rangle, \langle 4,2\rangle, \langle 5,2\rangle, \langle 3,4\rangle,$
$\langle 4,3\rangle, \langle \cancel{4}\, 5,3\rangle, \langle 5,4\rangle\}$

Matrix :-

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Graph :-



Incomplete

Q.26] Show that.

1.] $D(P \cup Q) = D(P) \cup D(Q)$

2] $R(P \cap Q) \subseteq R(P) \cap R(Q)$ for following set.

$P = \{\langle 1,2\rangle, \langle 2,4\rangle, \langle 3,3\rangle\}$

$Q = \{\langle 1,3\rangle, \langle 2,4\rangle, \langle 4,2\rangle\}$.

&

Find $P \cup Q$, $P \cap Q$, $D(P)$, $D(Q)$, $D(P \cup Q)$, $R(P)$, $R(Q)$, $R(P \cap Q)$.

$\Rightarrow$ i. $P \cup Q = \left\{ \begin{array}{l} \langle 1,2\rangle, \langle 2,4\rangle, \langle 3,3\rangle \\ \langle 1,3\rangle, \cancel{\langle 2,4\rangle}, \langle 4,2\rangle \end{array} \right\}$

ii. $P \cap Q = \{\langle 2,4\rangle\}$

iii. $D(P) = \{1, 2, 3\}$

iv. $D(Q) = \{1, 2, 4\}$

v. $D(P \cup Q) = \{1, 2, 3, 4\}$

vi. $R(P) = \{2, 4, 3\}$

vii $R(Q) = \{3, 4, 2\}$

viii. $R(P \cap Q) = \{4\}$.

$D(P \cup Q) = \{1, 2, 3, 4\}$ — (1)

$D(P) \cup D(Q) = \{1, 2, 3\} \cup \{1, 2, 4\}$

$= \{1, 2, 3, 4\}$ — (2)

From 1 & 2

$D(P \cup Q) = D(P) \cup D(Q)$.

Range.

$R(P \cap Q) = \{4\}$ —— ①

$R(P) = \{2, 3, 4\}$
$R(Q) = \{2, 3, 4\}$

$R(P) \cap R(Q) = \{2, 3, 4\}$ —— ②

$\therefore$ from 1 & 2.
$R(P \cap Q) \subseteq R(P) \cap R(Q)$.

Q27] $\angle$ denotes the relation ($\leq$) D denotes the relation divides. Both relation $\angle$ & D defined on set X.

$X = \{1, 2, 3, 6\}$

Find out $\angle \cap D$, $\angle \cup D$

$\Rightarrow$ $\angle = \{x, y \in R / x \leq y\}$

$\angle = \{ \langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 1,6\rangle$
~~⟨3,2⟩, ⟨3,1⟩, ⟨3,3⟩, ⟨6,1⟩~~
$\langle 2,2\rangle, \langle 2,3\rangle, \langle 2,6\rangle,$
$\langle 3,3\rangle \quad \langle 3,6\rangle$
$\langle 6,6\rangle \}$

$R = \{x, y \in R / x/y\}$

$D = \{ \langle 2,1\rangle, \langle 2,2\rangle, \langle 1,1\rangle$
$\langle 3,1\rangle, \langle 3,3\rangle,$
$\langle 6,1\rangle \quad \langle 6,6\rangle, \langle 6,3\rangle \}$

$\angle \cup D = \{ \langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 1,6\rangle$
$\langle 2,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,6\rangle$
$\langle 3,1\rangle, \langle 3,3\rangle, \langle 3,6\rangle$
$\langle 6,1\rangle, \langle 6,6\rangle, \langle 6,3\rangle \}$

$\therefore C \cap D = \begin{Bmatrix} \langle 1,1 \rangle, & \langle 2,2 \rangle \\ \langle 3,3 \rangle, & \langle 6,6 \rangle \end{Bmatrix}$

Q.28] Let $X = \{1, 2, 3, 4\}$

Find $(R \cup S)$, $(S \cap R)$

$R = \{ \langle x,y \rangle \mid (x \in X) \wedge (y \in X), \wedge (x-y)$ is an integral nonzero multiple of 2 $\}$.

$S = \{ \langle x,y \rangle \mid (x \in X) \wedge (y \in X) \wedge (x-y)$ is divisible by 3 $\}$.

$\Rightarrow$ $X = \{1, 2, 3, 4\}$

$R = \begin{Bmatrix} \langle 4,2 \rangle, & \langle 2,4 \rangle \\ \langle 3,1 \rangle, & \langle 1,3 \rangle \end{Bmatrix}$

$S = \begin{Bmatrix} \langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 4,4 \rangle \\ \langle 4,1 \rangle, \langle 1,4 \rangle \end{Bmatrix}$

$R \cup S = \begin{Bmatrix} \langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle \\ \langle 4,4 \rangle\ \langle 4,1 \rangle, \langle 4,2 \rangle\ \langle 1,4 \rangle \\ \langle 2,4 \rangle, \langle 3,1 \rangle, \langle 1,3 \rangle \end{Bmatrix}$

$R \cap S = \phi \Rightarrow \{ \}$.

Q30. Explain set ~~&~~ types of set with example.

$\Rightarrow$ Set is a collection of well defined objects.

Types of set.

1. Finite Set

The set which we can count / countable set.

Eg:- Set of ~~even~~ no.s from 1 to 10

$A = \{ x / x$ is ~~even~~ no., $1 \leq x \leq 10 \}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2] Infinite Set:-

· The set which we cannot count/ uncountable set.

Eg:- Set of Natural No.

$R = \{x / x \in N\}$

$R = \{1, 2, 3, 4 ---\}$

Q31· Explain Inclusion & proper Inclusion.

1. Two sets A & B are given such a way that all members of A present in B then we can say that A is included in B.

$A \subseteq B$.

Eg:- $A = \{1, 2, 3, 4\}$
$B = \{1, 2\}$

$\therefore B \subseteq A$

2. Proper Inclusion

If A & B are two sets such a way that all members of A present in B with the conditional at least 1 more additional memb- -er rather than A should present in B then A is said to be properly included in B.

Eg:-

$A = \{1, 2\}$

$B = \{1, 2, 3\}$

$\therefore A \subset B$

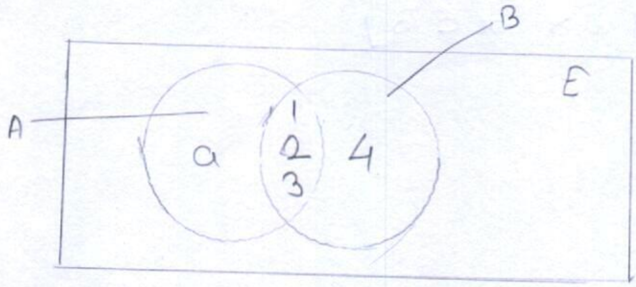
Q32. Explain

1. Universal Set.

A set E is said to be universal set if & only if it covers all other set under the dicussion.

- $A = \{1, 2, 3\}$
  $B = \{1, 2, 3, 4\}$
  $E = \{1, 2, 3, 4\}$

Here E is universal set which covers both sets A & B.



2. Empty sets

A set which does not contain any el-element.

Eg:-
$A = \{ \}$.

3. Power Set:

If A is given set such a way that all the possible subsets of A together in a set is called as power set of A.

$A = \{x / x \in A\}$
$A = \{a, b, c\}$

$P(A) = 2^A = 2^3 = 8$.

$P(A) = \left\{ \begin{matrix} \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\} \\ \phi, \{a, b, c\} / A \end{matrix} \right\}$.

Q33. Explain operations on sets.

⟹ In set theory, there are 3 major types of operations performed on sets, such as:-

1. Union of sets (∪)
2. Intersection of sets (∩)
3. Difference of sets (−)

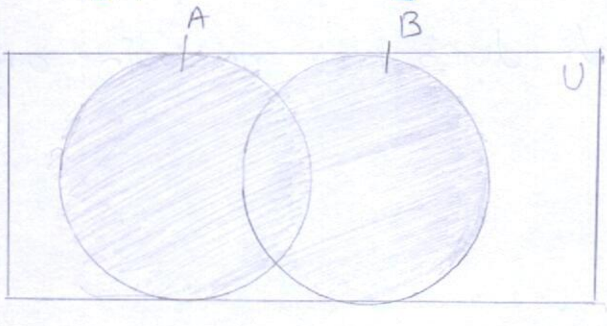
1. Union of set (∪)

If two set A & B are given, then the union of A & B is equal to set that contains all the elements present in set A & B.

$B \cup A = \{x : x \in A \text{ or } x \in B\}$.

Eg:- $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6\}$



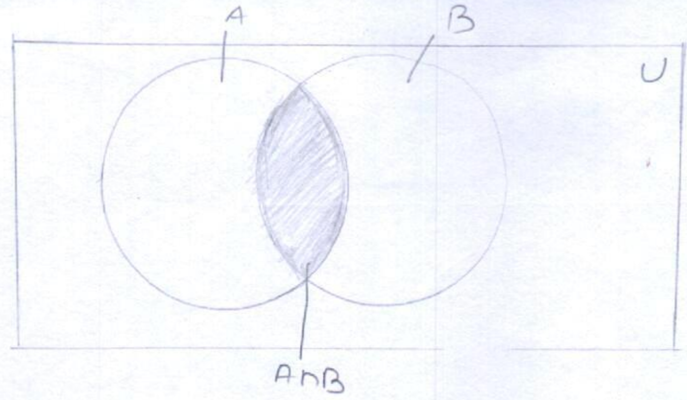
2. Intersection of sets.

If two sets A & B are given, then the intersection of A & B is the subset of U i.e universal set which consist of elements common to both A & B

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

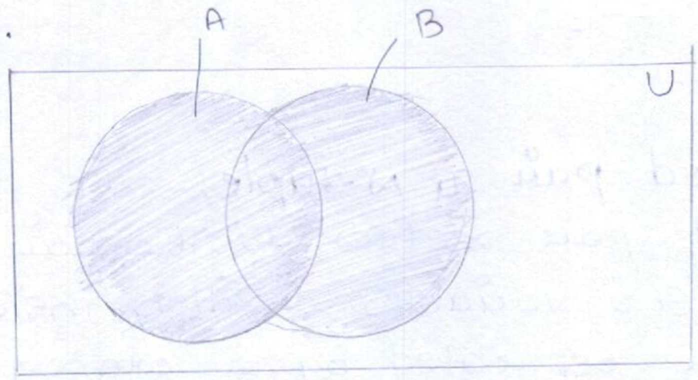
Eg:-
$A = \{1, 2, 3\}$ $B = \{2, 9\}$

$A \cap B = 2$

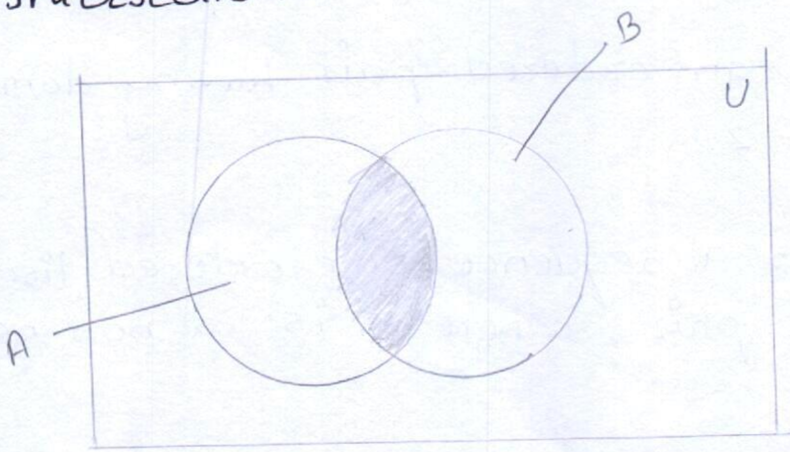


Q35 Draw venn diagram of union & intersection

1. Union.

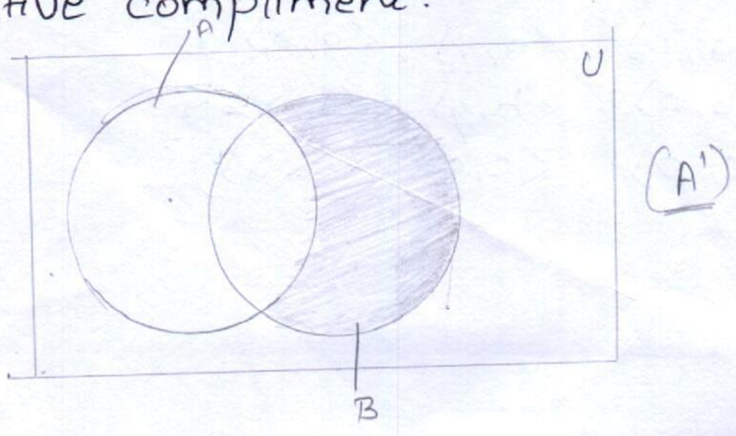


2. Intersection

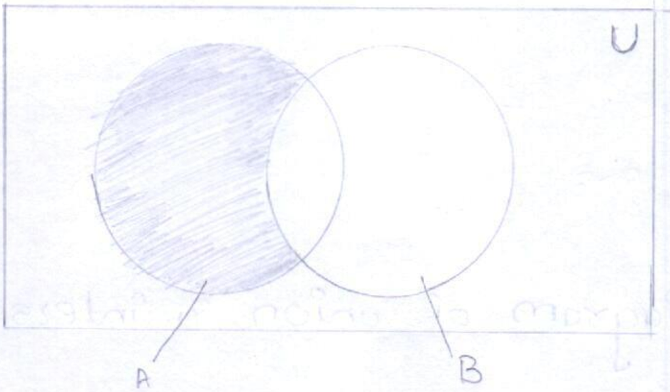


Q36. Venn diagram of.

1. Relative compliment.



2] Boolean Sum.



A - B

Q37. Explain ordered pair & N-tuple.

⟹ An ordered pair refers to the pair of two numbers (or variables) written inside brackets & are separated by a comma.

In set theory, it represents an element of a relation / cartesian product.

Eg:-

$\langle 1, 2 \rangle$ is an ordered pair having domain = 1 & range = 2.

An-n-tuple is a sequence (or ordered list) of n ordered pair, where n is a non-negative integer.

Q38. $X = \{a, b, c\}$, $Y = \{p, q, r\}$, $Z = \{m, n, o\}$

Find i. $X \times Y$, $Y \times X$, $X \times Z$ check $X \times Y = Y \times X$.

⟹ $X \times Y = \begin{Bmatrix} \langle a, p \rangle, \langle a, q \rangle, \langle a, r \rangle \\ \langle b, p \rangle, \langle b, q \rangle, \langle b, r \rangle \\ \langle c, p \rangle, \langle c, q \rangle, \langle c, r \rangle \end{Bmatrix}$

$$Y \times X = \left\{ \begin{matrix} \langle p,a\rangle, & \langle p,b\rangle, & \langle p,c\rangle \\ \langle q,b\rangle, & \langle q,a\rangle, & \langle q,c\rangle \\ \langle r,a\rangle, & \langle r,c\rangle, & \langle r,b\rangle \end{matrix} \right\} \quad \text{—— } 2$$

From 1 & 2

$$X \times Y \neq Y \times X$$

$$X \times Z = \left\{ \begin{matrix} \langle a,m\rangle, & \langle a,n\rangle, & \langle a,o\rangle \\ \langle b,m\rangle, & \langle b,n\rangle, & \langle b,o\rangle \\ \langle c,m\rangle, & \langle c,n\rangle, & \langle c,o\rangle \end{matrix} \right\}$$

Q39] $A = \{1,2,3\}$; $B = \{4\}$; $C = \{m,n\}$

Find. i. $A \cup B$ ii. $A \cap B$ iii $\bar{A}$ iv. $A-B$.

I. $A \cup B = \{1,2,3\} \cup \{4\}$
$= \{1,2,3,4\}$

II. $A \cap B = \{1,2,3\} \cap \{4\}$
$= \{\}$

III. $\bar{A} = \{4, m, n\}$

IV. $A - B = \{1,2,3\} - \{4\}$
$A - B = \{1,2,3\}$

Q40. $A = \{x \mid x \in 2n, (n<7), n \in N\}$ Find $A+B$
$B = \{x \mid x \in 2n+1, (n<7), n \in N\}$ $A-B$

=> $A = \{2,4,6,8,10,12\}$

$B = \{3,5,7,9,11,13\}$

$A+B = (A-B) \cup (B-A)$
$= \{2,4,6,8,10,12\} \cup \{3,5,7,9,11,13\}$
$A+B = \{2,4,6,8,10,12,3,5,7,9,11,13\}$

iii] $A - B = \cancel{\{\}}\ \{2, 4, 6, 8, 10, 12\}$

Q41. $A = \{1\}$ $\quad B = \{m, n\}$, $C = \{4, 5\}$

Find $A \times B$, $A \times B \times C$, A^2, B^2.

$\Rightarrow$ $A \times B = \{1\} \times \{m, n\}$

$= \{\langle 1, m\rangle, \langle 1, n\rangle\}$

$(A \times B) \times C = \{\langle 1, m\rangle, \langle 1, n\rangle\} \times \{4, 5\}$

$$= \begin{Bmatrix} \langle\langle 1, m\rangle, 4\rangle, & \langle\langle 1, m\rangle, 5\rangle \\ \langle\langle 1, n\rangle, 4\rangle, & \langle\langle 1, n\rangle, 5\rangle \end{Bmatrix}$$

$A^2 = A \times A$

$= \{1\} \times \{1\}$

$= \{\langle 1, 1\rangle\}$

$B^2 = B \times B$

$= \{m, n\} \times \{m, n\}$

$$= \begin{Bmatrix} \langle m, m\rangle, & \langle m, n\rangle \\ \langle n, m\rangle, & \langle n, n\rangle \end{Bmatrix}$$

Q42. $A = \{2, 3\}$ $\quad B = \{m, n\}$ $\quad C = \{p, q\}$

check whether $(A \times B) \times C = A \times (B \times C)$

$\Rightarrow$ $A \times B = \begin{Bmatrix} \langle 2, m\rangle, & \langle 2, n\rangle \\ \langle 3, m\rangle, & \langle 3, n\rangle \end{Bmatrix}$

$$(A \times B) \times C = \begin{Bmatrix} \langle\langle 2, m\rangle, p\rangle, & \langle\langle 2, m\rangle, q\rangle \\ \langle\langle 2, n\rangle, p\rangle, & \langle\langle 2, n\rangle, q\rangle \\ \langle\langle 3, m\rangle, p\rangle, & \langle\langle 3, m\rangle, q\rangle \\ \langle\langle 3, n\rangle, p\rangle, & \langle\langle 3, n\rangle, q\rangle \end{Bmatrix} \quad \text{--- (I)}$$

$B \times C = \begin{Bmatrix} \langle m, p\rangle, & \langle m, q\rangle \\ \langle n, p\rangle, & \langle n, q\rangle \end{Bmatrix}$

~~(B×C~~

$$A\times(B\times C) = \left\{\begin{array}{l} \langle 2,\langle m,p\rangle\rangle,\ \langle 2,\langle m,q\rangle\rangle, \\ \langle 3,\langle m,p\rangle\rangle,\ \langle 3,\langle m,p\rangle\rangle, \\ \langle 2,\langle n,p\rangle\rangle,\ \langle 2,\langle n,q\rangle\rangle, \\ \langle 3,\langle n,q\rangle\rangle,\ \langle 3,\langle n,p\rangle\rangle \end{array}\right\} \quad \text{—— (2)}$$

$\therefore$ from 1 & 2

$(A\times B)\times C \neq A\times(B\times C)$.

Q43. $A = \{3, 4, 5\}$ $B = \{2, 4\}$

Find $P(A)$ $P(B)$ $P(A\cup B)$ $P(A\cap B)$

=> $P(A) = \left\{\begin{array}{l} \phi, \{3\}, \{4\}, \{5\}, \{3,4\} \\ \{3,5\}, \{4,5\}\ \{3,4,5\}/A \end{array}\right\}$

$P(B) = \left\{\begin{array}{l} \phi, \{2\}, \{4\} \\ \{2,4\}/B \end{array}\right\}$

$A\cup B = \{3,4,5,2\}$

$P(A\cup B) = \left\{\begin{array}{l} \phi, \{3\}, \{4\}, \{5\}, \{2\} \\ \{3,4\}, \{3,5\}, \{3,2\}, \{4,5\}, \{4,2\} \\ \{5,2\}, \{3,4,5\}, \{3,4,2\}, \{3,5,2\} \\ \{4,5,2\}, \{2,3,4,5\}/A\cup B \end{array}\right\}$

$A\cap B = \{4\}$

$P(A\cap B) = \{\phi, \{4\}\}$.

Q 44. $A = \{\{a\}, p, m\}$

$B = \{\{\phi\}, \phi\}$

$C = \{1, 2, 9, \{\phi\}\}$

Find (i) $P(A)$ (ii) $P(B)$ $P(A \cup B)$ $P(A \cup C)$.

$\Rightarrow$ $P(A) = \{\phi, \{\{a\}\}, \{p\}, \{m\}, \{\{a\}, p\}, \{\{a\}, m\}, \{p, m\}, \{\{a\}, p, m\}\}$

$P(B) = \{\{\{\phi\}\}, \{\phi\}, \phi, \{\{\phi\}, \phi\}\}$

$P(A \cup B) = \{\{a\}, m, p, \{\phi\}, \phi\}$

$$P(A \cup B) = \{\phi, \{\{a\}\}, \{m\}, \{p\}, \{\{\phi\}\}, \{\phi\}, \{\{a\}, m\}, \{\{a\}, p\}, \{\{a\}, \{\phi\}\}, \{\{a\}, \phi\}, \{m, p\}, \{m, \{\phi\}\}, \{m, \phi\}, \{p, \{\phi\}\}, \{p, \phi\}, \{\{\phi\}, \phi\}, \{\{a\}, m, p\}, \{\{a\}, m, \{\phi\}\}, \{\{a\}, m, \phi\}, \{\{a\}, p, \{\phi\}\}, \{\{a\}, p, \phi\}, \{m, p, \{\phi\}\}, \{m, p, \phi\}, \{p, \{\phi\}, \phi\}, \{m, \{\phi\}, \phi\}, \{\{a\}, m, p, \{\phi\}\}, \{\{a\}, m, p, \phi\}, \{\{a\}, p, \{\phi\}, \phi\}, \{m, p, \{\phi\}, \phi\}, \{\{a\}, m, p, \{\phi\}, \phi\}\}$$

$A \cup C = \{\{a\}, p, m, 1, 2, 9, \{\phi\}\}$

$P(A \cup C) = \{ \phi, \{\{a\}\}, \{p\}, \{m\}, \{1\}, \{2\}, \{9\}, \{\{\phi\}\},$
$\{\{a\}, p\}, \{\{a\}, m\}, \{\{a\}, 1\}, \{\{a\}, 2\}, \{\{a\}, 9\}$
$\{\{a\}, \{\phi\}\}, \{p, m\}, \{p, 1\}, \{p, 2\}, \{p, 9\},$
$\{p, \{\phi\}\}, \{m, 1\}, \{m, 2\}, \{m, 9\}, \{m, \{\phi\}\},$
$\{1, 2\}, \{1, 9\}, \{1, \{\phi\}\}, \{2, 9\}, \{2, \{\phi\}\},$
$\{9, \{\phi\}\}, \{\{a\}, p, m\} \{\{a\}, p, 1\}, \{\{a\}, p, 2\}$
$\{\{a\}, p, 9\} \{\{a\}, p, \{\phi\}\}, \{\{a\}, m, 1\}, \{\{a\}, m, 2\}$
$\{\{a\}, m, 9\}, \{\{a\} m, \{\phi\}\}, \{\{a\}, 1, 2\} \{\{a\}, 1, 9\}$
$\{\{a\}, 1, \{\phi\}\}, \{\{a\}, 2, 9\}, \{\{a\}, 2, \{\phi\}\},$
$\{\{a\}, 9, \{\phi\}\}, \{p, m, 1\}, \{p, m, 2\}, \{p, m, 9\}$
$\{p, m, \{\phi\}\}, \{p, 1, 2\}, \{p, 1, 9\}, \{p, 1, \{\phi\}\},$
$\{p, 2, 9\} \{p, 2, \{\phi\}\}, \{p, 9, \{\phi\}\}, \{m, 1, 2\}$
$\{m, 1, 9\}, \{m, 1, \{\phi\}\}, \{m, 2, 9\}, \{m, 2, \{\phi\}\},$
$\{1, 2, 9\}, \{1, 2, \{\phi\}\}, \{2, 9, \{\phi\}\}$

(Incomplete)

Q45] $A = \{x / x^2 - 8x + 12, x \in N\}$

$B = \{x / x^3 < 100, x \in \omega\}$

$C = \{x / x \in \frac{n-1}{n+1}, n \in \text{Integer}, 0 < n < 5\}$

$\Rightarrow$ $A = \{5, 0, -3, -4, -3 - -\}$ / $\cancel{\{1, 2, 3, 4 -\}}$ $\{2, 6\}$

$B = \{0, 1, 8, 27, 64\}$ $\cancel{125, 216}$ / $\{0, 1, 2, 3, 4\}$

$C = \{0, 0.33, 0.5, 0.6\}$ $\cancel{/ \{1, 2, 3, 4\}}$

• check whether True or False.

① $\{\phi\} \subseteq C$ ~~True~~ / False

② $C \in N$ ~~False~~ / ~~True~~ False.

③ $B \in N$ ~~True~~ / False

④ $B \cap C$ is single set True / ~~False~~

⑤ $\{3, 4\} \subseteq B$. ~~False~~ / True

Q46]

$\Rightarrow$ $A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5\}$

$X = \{1, 2, 3\}$

$Y = \{\}$

$\therefore A \neq X$ & $B \neq Y$

Q 47. Prove that $A \cup \phi = A$ & $A \cap \phi = \phi$ using formula method.

$\Rightarrow$ 1. Let $A = \{1, 2\}$

$\therefore n(A) = 2$

$\phi = \{\}$

$n(\phi) = 0$

$A \cup \phi = \{1, 2\}$ $\quad$ $A \cap \phi = \{\}$

$n(A \cup \phi) = 2$. $\quad$ $n(A \cap \phi) = 0$

$\therefore n(A \cup \phi) = n(A) + n(\phi) - n(A \cap \phi)$

$2 = 2 + 0 - 0$.

$\underline{\underline{2 = 2}}$

OR

$A \cup \phi = A + \phi - (A \cap \phi)$

$= A + \phi - \phi$

$\therefore \underline{\underline{A \cup \phi = A}}$

II. $n(A \cap \phi) = n(A) + n(\phi) - n(A \cup \phi)$

$0 = \cancel{2} + 0 - \cancel{2}$

$\underline{\underline{0 = 0}}$

OR

$A \cap \phi = A + \phi - A \cup \phi$

$A \cap \phi = \cancel{A} + \phi - \cancel{A}$

$\therefore \underline{\underline{A \cap \phi = \phi}}$

Q48. $A = \{1,2,3,4,5\}$ $\quad B = \{1,2\}$ $\quad C = \{a,b\}$

Find (i) $R = \{\langle x,y\rangle \mid (x=y) \wedge (x,y \in A)\}$

(ii) $A^2 \times B$

(ii) $B^2 \times A$

(iii) $A \times B \times C$.

$\Rightarrow$ $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 5,5\rangle\}$

ii $A^2 = A \times A$

$= \{1,2,3,4,5\} \times \{1,2,3,4,5\}$

$$= \left\{ \begin{array}{l} \langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \langle 1,5\rangle \\ \langle 2,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 2,5\rangle \\ \langle 3,1\rangle, \langle 3,2\rangle, \langle 3,3\rangle, \langle 3,4\rangle, \langle 3,5\rangle \\ \langle 4,1\rangle, \langle 4,2\rangle, \langle 4,3\rangle, \langle 4,4\rangle, \langle 4,5\rangle \\ \langle 5,1\rangle, \langle 5,2\rangle, \langle 5,3\rangle, \langle 5,4\rangle, \langle 5,5\rangle \end{array} \right\}$$

$$A^2 \times B = \left\{ \begin{array}{l} \langle\langle 1,1\rangle,1\rangle, \langle\langle 1,1\rangle,2\rangle, \langle\langle 1,2\rangle,1\rangle, \langle\langle 1,2\rangle,2\rangle, \\ \langle\langle 1,3\rangle,1\rangle, \langle\langle 1,3\rangle,2\rangle, \langle\langle 1,4\rangle,1\rangle, \langle\langle 1,4\rangle,2\rangle, \\ \langle\langle 1,5\rangle,1\rangle, \langle\langle 1,5\rangle,2\rangle, \langle\langle 2,1\rangle,1\rangle, \langle\langle 2,2\rangle,2\rangle, \\ \langle\langle 2,2\rangle,1\rangle, \langle\langle 2,2\rangle,2\rangle, \langle\langle 2,3\rangle,1\rangle, \langle\langle 2,4\rangle,1\rangle, \\ \langle\langle 2,5\rangle,1\rangle, \langle\langle 2,3\rangle,2\rangle, \langle\langle 2,4\rangle,2\rangle, \langle\langle 2,5\rangle,2\rangle, \\ \langle\langle 3,1\rangle,1\rangle, \langle\langle 3,1\rangle,2\rangle, \langle\langle 3,2\rangle,1\rangle, \langle\langle 3,2\rangle,2\rangle, \\ \langle\langle 3,3\rangle,1\rangle, \langle\langle 3,3\rangle,2\rangle, \langle\langle 3,4\rangle,1\rangle, \langle\langle 3,4\rangle,2\rangle, \\ \langle\langle 3,5\rangle,1\rangle, \langle\langle 3,5\rangle,2\rangle, \langle\langle 4,1\rangle,1\rangle, \langle\langle 4,1\rangle,2\rangle, \\ \langle\langle 4,2\rangle,1\rangle, \langle\langle 4,2\rangle,2\rangle, \langle\langle 4,3\rangle,1\rangle, \langle\langle 4,3\rangle,2\rangle \\ \langle\langle 4,4\rangle,1\rangle, \langle\langle 4,4\rangle,2\rangle, \langle\langle 4,5\rangle,1\rangle, \langle\langle 4,5\rangle,2\rangle, \\ \langle\langle 5,1\rangle,1\rangle, \langle\langle 5,2\rangle,2\rangle, \langle\langle 5,2\rangle,1\rangle, \langle\langle 5,2\rangle,2\rangle, \\ \langle\langle 5,3\rangle,1\rangle, \langle\langle 5,3\rangle,2\rangle, \langle\langle 5,4\rangle,1\rangle, \langle\langle 5,4\rangle,2\rangle, \\ \langle\langle 5,5\rangle,1\rangle, \langle\langle 5,5\rangle,2\rangle. \end{array} \right\}$$

ii. $B^2 = B \times B$

$= \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}$

iii. $B^2 \times A = \{\langle\langle 1,1\rangle,1\rangle, \langle\langle 1,1\rangle,2\rangle, \langle\langle 1,1\rangle,3\rangle, \langle\langle 1,1\rangle,4\rangle, \langle\langle 1,1\rangle,5\rangle, \langle\langle 1,2\rangle,1\rangle, \langle\langle 1,2\rangle,2\rangle, \langle\langle 1,2\rangle,3\rangle, \langle\langle 1,2\rangle,4\rangle, \langle\langle 1,2\rangle,5\rangle, \langle\langle 2,1\rangle,1\rangle, \langle\langle 2,1\rangle,2\rangle, \langle\langle 2,1\rangle,3\rangle, \langle\langle 2,1\rangle,4\rangle, \langle\langle 2,1\rangle,5\rangle, \langle\langle 2,2\rangle,1\rangle, \langle\langle 2,2\rangle,2\rangle, \langle\langle 2,2\rangle,3\rangle, \langle\langle 2,2\rangle,4\rangle, \langle\langle 2,2\rangle,5\rangle\}$

iv. $A \times B = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle, \langle 3,1\rangle, \langle 3,2\rangle, \langle 4,1\rangle, \langle 4,2\rangle, \langle 5,1\rangle, \langle 5,2\rangle\}$

$A \times B \times C = \{\langle\langle 1,1\rangle,a\rangle, \langle\langle 1,1\rangle,b\rangle, \langle\langle 1,2\rangle,a\rangle, \langle\langle 1,2\rangle,b\rangle, \langle\langle 2,1\rangle,a\rangle, \langle\langle 2,1\rangle,b\rangle, \langle\langle 2,2\rangle,a\rangle, \langle\langle 2,2\rangle,b\rangle, \langle\langle 3,1\rangle,a\rangle, \langle\langle 3,1\rangle,b\rangle, \langle\langle 3,2\rangle,a\rangle, \langle\langle 3,2\rangle,b\rangle, \langle\langle 4,1\rangle,a\rangle, \langle\langle 4,1\rangle,b\rangle, \langle\langle 4,2\rangle,a\rangle, \langle\langle 4,2\rangle,b\rangle, \langle\langle 5,1\rangle,a\rangle, \langle\langle 5,2\rangle,b\rangle, \langle\langle 5,2\rangle,a\rangle, \langle\langle 5,2\rangle,b\rangle\}$

Q49] $A = \{0, 2, 4, 6, 8, 10\}$

$B = \{0, 1, 2, 3, 4, 5, 6\}$

$C = \{4, 5, 6, 7, 8\}$

$D = \{a, b, c\}$

Find. i] $\bar{A}$ 2] $\bar{\bar{A}}$ 3] $\bar{A} \cap \bar{\bar{A}}$ iv. $\bar{A} \cup \bar{\bar{A}}$

$\Rightarrow$ $\bar{A} = \{1, 3, 5, 7, a, b, c\}$

$\bar{\bar{A}} = \{0, 2, 4, 6, 8, 10\}$

$\therefore \bar{A} \cap \bar{\bar{A}} = \{\}$

$\therefore \bar{A} \cup \bar{\bar{A}} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 10, a, b, c\}$.

Q50. Find DNF of following.

1. $(P \to Q) \to R$.

$\Rightarrow (\neg P \vee Q) \to R$

$\neg(\neg P \vee Q) \vee R$

$(P \wedge \neg Q) \vee R$

2. $(P \wedge Q) \to R$.

$= \neg(P \wedge Q) \vee R$

~~$\neg P \vee \neg Q$~~

$= \neg(P \wedge Q) \vee R$

wrong

3. $(\neg P \to R) \wedge (Q \rightleftarrows P)$

$\Rightarrow (\neg\neg P \vee R) \wedge (Q \to P) \wedge (P \to Q)$

$(\neg\neg P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$(P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$\neg(\neg(P \vee R) \vee \neg(\neg Q \vee P) \vee \neg(\neg P \vee Q))$

$\neg((\neg P \wedge \neg R) \vee (Q \wedge \neg P) \vee (P \wedge \neg Q))$

3] $(\neg P \to R) \wedge (Q \rightleftarrows P)$

$(\neg\neg P \vee R) \wedge ((Q \to P) \wedge (P \to Q))$

$(P \vee R) \wedge ((\neg Q \vee P) \wedge (\neg P \vee Q))$

$\neg(\neg(P \vee R) \vee (\neg((\neg Q \vee P) \wedge (\neg P \vee Q))))$

$= \neg((\neg P \wedge \neg R) \vee (\neg((\neg Q \vee P) \wedge (\neg P \vee Q))))$

4] $Q \wedge (P \vee \neg Q)$

$Q \wedge (P \vee \neg Q)$

$\neg(\neg Q \vee \neg(P \vee \neg Q))$

$= \neg(\neg Q \vee (\neg P \wedge Q))$

Q12] Give PCNF

1. $(\neg R \to (\neg Q \to \neg P))$

$(\neg\neg R \lor (\neg Q \to \neg P))$

$(R \lor (\neg\neg Q \lor \neg P))$

$(R \lor (Q \lor \neg P))$

$\neg(\neg R \land (P \land \neg Q))$

$\neg(\neg R \land \neg(\neg P \lor Q))$.

1.] $\neg R \lor (P \land \neg P)$

$(P \lor \neg R) \land (\neg P \lor \neg R)$

a. $(P \lor \neg R) \lor (Q \land \neg Q)$

$(P \lor Q \lor \neg R) \land (P \lor \neg Q \lor \neg R)$.

b. $(\neg P \lor \neg R) \lor (Q \land \neg Q)$

$(\neg P \lor Q \lor \neg R) \land (\neg P \lor \neg Q \lor \neg R)$

$(P \lor Q \lor \neg R) \land (P \lor \neg Q \lor \neg R) \land (\underline{\neg P \lor Q \lor \neg R}) \land (\neg P \lor \neg Q \lor \neg R)$.

2.] $(Q \lor \neg P)$

$(Q \lor \neg P) \lor (R \land \neg R)$

$(\neg P \lor Q \lor R) \land (\underline{\neg P \lor Q \lor \neg R})$.

$\neg(((P \lor Q \lor \neg R) \land (P \lor \neg Q \lor \neg R) \land (\neg P \lor Q \lor \neg R) \land (\neg P \lor \neg Q \lor \neg R))$

$\land \neg((\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R)))$

$\neg(\neg(\neg(P \lor Q \lor \neg R) \lor \neg(P \lor \neg Q \lor \neg R) \lor \neg(\neg P \lor Q \lor \neg R) \underline{\lor}$

$\neg(\neg P \lor \neg Q \lor \neg R)) \underline{\land} (\neg(\neg P \lor Q \lor R) \underline{\lor} \neg(\neg P \lor Q \lor \neg R))$

b. $(\neg(\neg P \vee \neg Q) \rightarrow R)$

$\Rightarrow (\neg\neg(\neg P \vee \neg Q) \vee R)$

$((\neg P \vee \neg Q) \vee R)$

$\neg(\neg R \wedge \neg(\neg P \vee \neg Q))$.

1.] $\neg R \vee (P \wedge \neg P)$

$(P \vee \neg R) \wedge (\neg P \vee \neg R)$

(a) $(P \vee \neg R) \vee (Q \wedge \neg Q)$

$(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$

(b) $(\neg P \vee \neg R) \vee (Q \wedge \neg Q)$

$(\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)$.

$(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\underline{\neg P} \vee \neg Q \vee \neg R)$.

2] $(\neg P \vee \neg Q) \vee (R \vee \neg R)$

$(\neg P \vee \neg Q \vee R) \wedge (\neg \underline{P \vee} \neg Q \vee \neg R)$

$\neg(((P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R))$

$\wedge \neg((\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)))$

$\neg(((\neg(\neg(P \vee Q \vee \neg R) \vee \neg(P \vee \neg Q \vee \neg R) \vee \neg(\neg P \vee Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee \neg R))$

$\wedge (\neg(\neg P \vee \neg Q \vee R) \;\cancel{\wedge}\, \vee\; \neg(\neg P \vee \neg Q \vee \neg R)))$.

Q13] Construct PCNF & PDNF.

a. $(\neg R \rightarrow P) \wedge ((Q \rightarrow P) \wedge (P \rightarrow Q)))$

$\Rightarrow$ PCNF :-

$(\neg\neg R \vee P) \wedge ((\neg Q \vee P) \wedge (\neg P \vee Q))$

$(R \vee P) \wedge \neg(\neg(\neg Q \vee P) \vee \neg(\neg P \vee Q))$

1. $(R \vee P) \vee (Q \wedge \neg Q)$
   $(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)$

2. $(\neg Q \vee P) \vee (R \wedge \neg R)$
   $(P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$

3. $(\neg P \vee Q) \vee (R \wedge \neg R)$
   $(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$.

$((P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)) \wedge \neg(\neg((P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)) \vee \neg((\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)))$.

$((\neg(\neg(P \vee Q \vee R) \vee \neg(P \vee \neg Q \vee R)) \wedge \neg((\neg(P \vee \neg Q \vee R) \vee \neg(P \vee \neg Q \vee \neg R)) \vee (\neg(\neg P \vee Q \vee R) \vee \neg(\neg P \vee Q \vee \neg R))))$

$((\neg(\neg(P \vee Q \vee R) \vee \neg(P \vee \neg Q \vee R)) \wedge (\neg(\neg(P \vee \neg Q \vee R) \vee \neg(P \vee \neg Q \vee \neg R)) \wedge \neg(\neg(\neg P \vee Q \vee R) \vee \neg(\neg P \vee Q \vee \neg R)))$.

Q20] show that

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$

$\Rightarrow$

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$((\neg P \wedge \neg Q) \wedge R) \vee (R \wedge (Q \vee P))$

$R \wedge ((\neg P \wedge \neg Q) \vee (P \vee Q))$

$R \wedge ((\neg P \vee (P \vee Q)) \wedge (\neg Q \vee (P \vee Q)))$.

$R \wedge ((\neg P \vee P) \vee Q) \wedge ((\neg Q \vee Q) \vee P)$.

$R \wedge ((T \vee Q) \wedge (T \vee P))$

$R \wedge (T \wedge T)$

$= R \wedge T$

$= \underline{R}$

$= \underline{\underline{R.H.S}}$

[34]

⇒ The symmetric difference of two sets, also known as disjunctive union, is the set of elements which are in either of the sets, but not in their intersection.

Eg:-

$A = \{1, 2, 3\}$

$B = \{a, 2, b\}$

$A - B = \{1, 3\}$