Unit-3
Relation & Function

* Relation and function

In Relation, object related to one another

It is very important

By using output and other things we decide relation in typical program.

Represent given relation by graph and matrix.

Ex. father and son, mother and son, brother to sister and arithmetic relations. such as. "greater than", "less than" or equality between two no.

The relation between pair of objects or ordered pair called relation.

Any set of ordered pairs defines a binary relatio

We can write ordered pair $<x, y>$

$\forall$ R is relation
by writting xRy which may be read as
"x is an relation R to y".

In mathematatics relations are denoted by special symbol rather than real number.

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If two real numbers a and b these having $x > y$ then

$<x, y> \in >$ or $x > y$

$> = \{ <x, y> \mid x, y$ are real no & $x > y \}$

Relation of father to child denoted by F

$F = \{ <x, y> \mid x$ is father of $y \}$

$x =$ father $y =$ child

The set of intersecting or ordered pair
$<2, 4>, <1, 3>, <7, 6>$

$S = \{ <2, 4>, <1, 3>, <7, 6> \}$

* Binary Relation:

Let S be the binary relation. The set D(S) called Domain.

R(S) ⇒ Range 2nd element

$D(S) \Rightarrow \{ x \mid (\exists y)(<x, y> \in S) \}$

$D(S) = \{2, 1, \lambda\}$ —— ①

$R(S) = \{4, 3, 6\}$ —— ⑪

If X and Y are two sets with D(S) and R(S) and cartesion product C

$D(C) \subseteq X$
$R(C) \subseteq Y$

Here XXX called universal relation in X

By considering ①

$\{2, 1, \lambda\} \subseteq X$

$\{4, 3, 6\} \subseteq Y$

Now
$X \cup Y = \{\langle 2,4\rangle, \langle 1,3\rangle, \langle \lambda,6\rangle\}$

If there two relations R and S are there

$x(R \cap S)y \Leftrightarrow xRy \wedge xSy$

$x(R \cup S)y \Leftrightarrow xRy \vee xSy$

$x(R-S)y \Leftrightarrow xRy \wedge x \$ y$

$\forall\ x(\sim S)y \Leftrightarrow x \$ y$

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* Properties of Binary Relation in set :

① Reflective :

A binary relation R in set X is Reflective ~~in~~ ~~*~~ if, for every $x \in X$ $x R x$, i.e., $<x, x> \in R$

R is reflective in X $\Leftrightarrow (x)(x \in X \rightarrow xRx)$

Relation $\leq$ is reflective.

② Symmetric :

A relation R in set X is symmetric if, for every x and y in X whenever $x R y$ and $y R x$

$\cancel{x} \Leftrightarrow (x)(y)(x \in X \wedge y \in X \wedge xRy \rightarrow yRx)$

$\leq$, $<$ are not symmetric, $=$ is symmetric

③ Transitive :

A relation R in X is transitive if for every x, y, z in X whenever xRy, yRz then $zR\cancel{x}$ ie., R is called transitive

$X \Leftrightarrow (x)(y)(z)(x \in X \wedge y \in X \wedge z \in X \wedge$
$xRy \wedge yRz \longrightarrow xRz$

$\leq$, $<$, $=$ are transitive

* Matrix and Graph Representation

* Relation Matrix :

A relation from the finite set X to the finite set Y can also be reprene-ted by a matrix called relation matrix R

Let, $X = \{x_1, x_2, \ldots, x_m\}$

$Y = \{y_1, y_2, \ldots, y_m\}$

Then Relation from X to Y can be represented as

$R = \{\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \ldots, \langle x_m, y_n\rangle\}$

In a special case consider $m=3$, $n=2$ then

$R = \{\langle x_1, y_1\rangle, \langle x_2, y_1\rangle, \langle x_3, y_2\rangle, \langle x_2, y_2\rangle\}$

Table

	y_1	y_2
x_1	1	0
x_2	1	1
x_3	0	1

$$r_{ij} = \begin{cases} 1 & \text{if } x_1 R y_i \\ 0 & \text{if } x_i \not R y_i \end{cases}$$

Now we obtain matrix from it

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

① If relation is reflective, then all diagonal entries are 1

Ex : $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ it is reflective

$R = \{ <x_1, y_1>, <x_2, y_2>, <x_3, y_3>, <x_4, y_4> \}$

② If the relation is symmetric then relation matrix is symmetric

③ If $r_{ij} = 1$ and $r_{ij} = 0$ then relation is antisymmetric.

* Graph :

The relation are pictorically represented by graph.

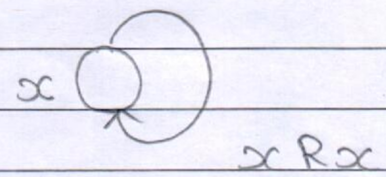
If R be the relation in set $X = \{x_1, ----x_m\}$

The elements of X are represented by points or circle called nodes

The nodes are also called as vertices if x_iRx_j is relation then $\langle x_i, x_j \rangle \in R$ then connect x_i and x_j with arrow

If x_iRx_i i.e., are starts with node x_i and returns with x_i the called as loop.

①



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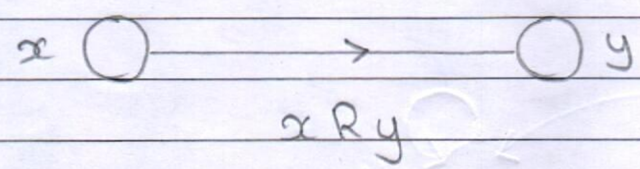
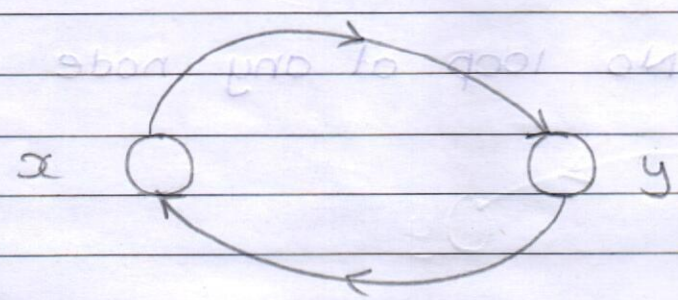
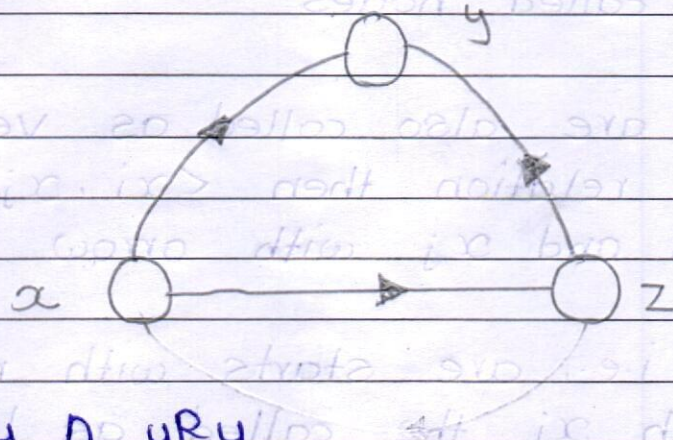


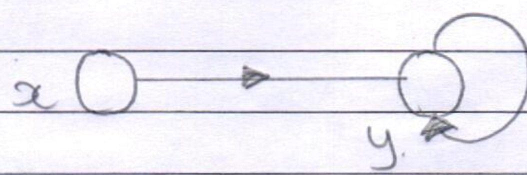
③ If Relation is xRy n yRx



④ If relation $xRy \cap yRz \cap zRx$

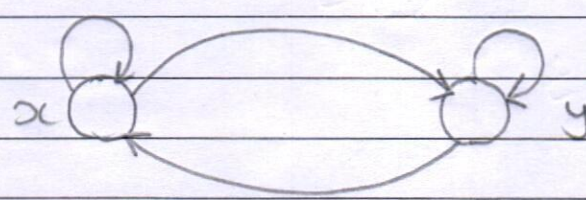


⑤ $xRy \cap yRy$



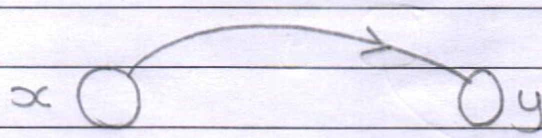
* Reflective Graph :

Must be loop at every node



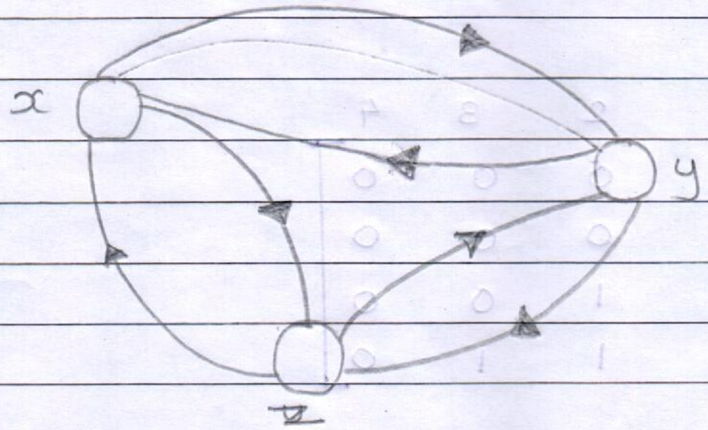
* Irreflexive :

No loop at any node



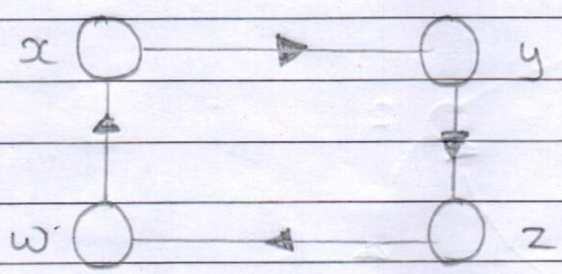
* Symmetric

The return path from connected path or nodes



* Antisymmetric :

There is no any direct path from one node to another in reverse direction



Ex : $X = \{1, 2, 3, 4\}$ and $R = \{ \langle x, y \rangle \mid x > y \}$

Draw the graph for R and also gives its matrix

→ No of Nodes in the graph are equal to no of elements in set

Here $R = \{ \langle 2,1 \rangle, \langle 3,2 \rangle, \langle 3,1 \rangle, \langle 4,1 \rangle, \langle 4,2 \rangle, \langle 4,3 \rangle \}$

We got total 6 ordered pairs hence no of 1's in the matrix will be 06 and Remaining elements will be 0

Matrix :

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

Plot the nodes | vertex
① Irreflexive
② Antisymmetric
③ Transitive

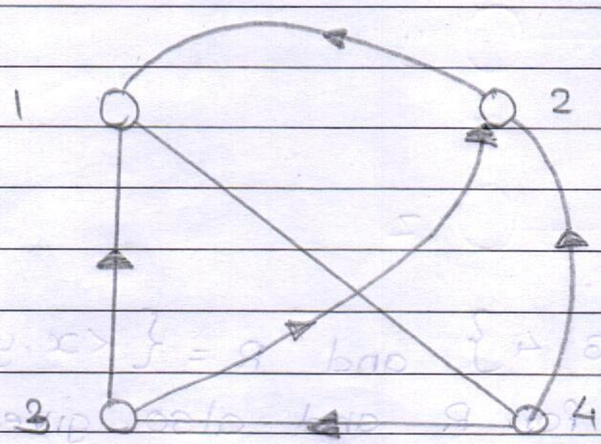


fig : Graphical representation of Matrix

The given relation is transitive antisymmetric and irreflexive

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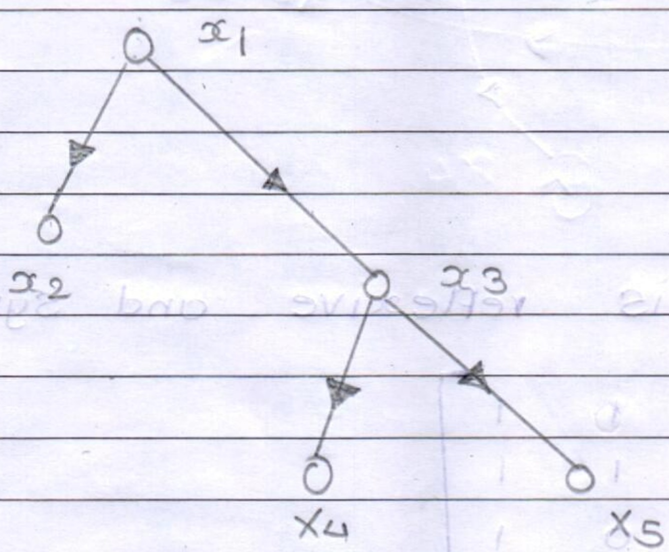
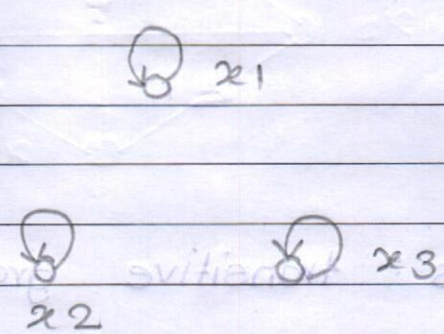
Ex: find out given relation type and matrix format to graph.

① $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$



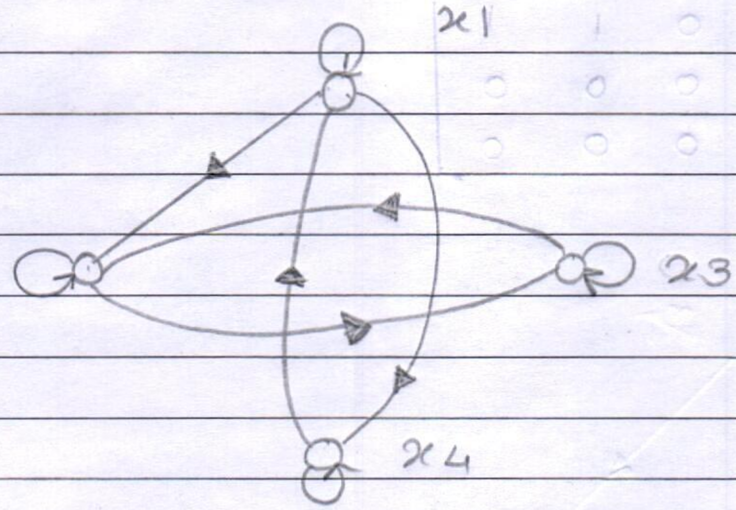
Ans: The graph is Antisymmetric

② $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



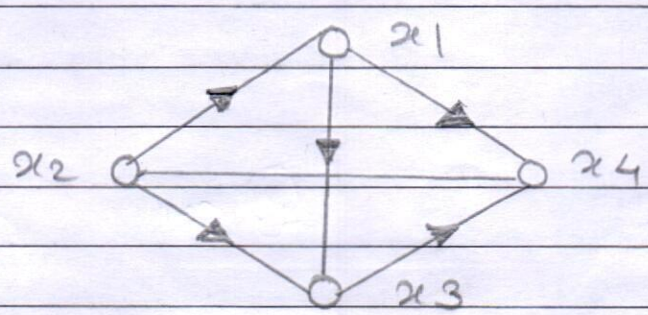
The graph is reflexive

(3) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$



The graph is reflexive and symmetric

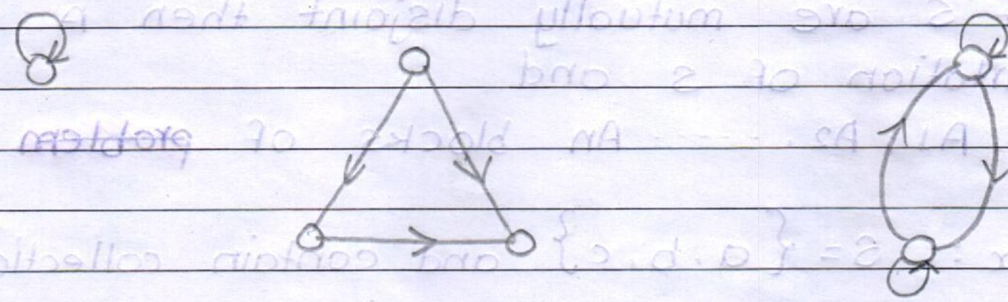
(4) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

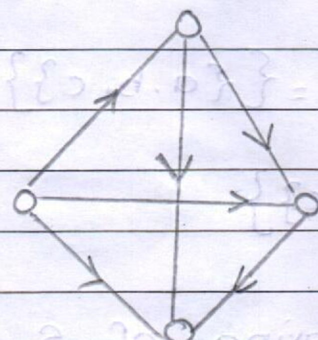


Ans : transitive graph

Transitive Relation :

If the given graph of the format





* Partition and covering of set :

Let S be the given set and A = {A1, A2, ---Am}
where each Ai ;
i = 1, 2, ---- m

$$\boxed{\bigcup_{i=1}^{m} A_i = S}$$

① A is called covering of S. A1, A2, --- Am are cover of S

② In addition if elements if A. which are subset of S are mutually disjoint then A is called partation of S and
A1, A2, ----- An blocks of partation

Ex : S = {a, b, c} and contain collection of S

$A = \{\{a, b\}, \{b, c\}\}$, $B = \{\{a\}, \{a, c\}\}$,

$C = \{\{a\}, \{b, c\}\}$, $D = \{\{a, b, c\}\}$, $E = \{\{a\}, \{b\}, \{c\}\}$

$F = \{\{a\}, \{a, b\}, \{a, c\}\}$

∀ A & F are covering of S
C, D, E are partitions of S as well as covering of S
B either partiation nor covering

Two partitions ofe called as equal if they are equal as sets.

Some partitions of Universal set E
if A is set then
$A \cup \sim A = E$

If A and B are any two subsets of E containing

$I_0 = \sim A \cap \sim B$

$I_1 = \sim A \cap B$

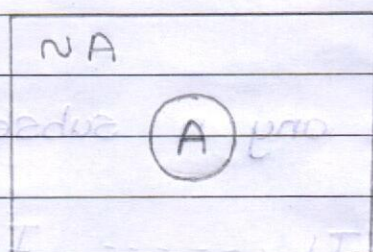
$I_2 = A \cap \sim B$

$I_3 = A \cap B$

I_0, I_1, I_2, I_3 are called complete intersections or minterms generated by A and B

$$E = I_0 \cup I_1 \cup I_2 \cup I_3 = \bigcup_{j=0}^{3} I_j$$

① $E = A \cup \sim A$



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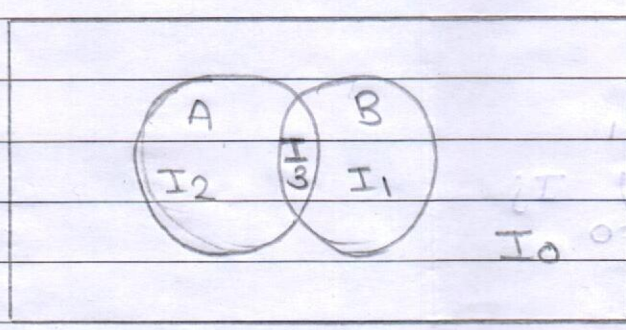


A & B are 2 subsets so minterms I_0, I_1, I_2, I_3

③ A, B, C are 3 subsets then minterms 2^3 denoted by $I_0, I_1, I_2, I_3 \text{ --------- } I_7$

$I_0 = \sim A \cap \sim B \cap \sim C$

$I_1 = \sim A \cap \sim B \cap C$

$I2 = \sim A \cap B \cap C$

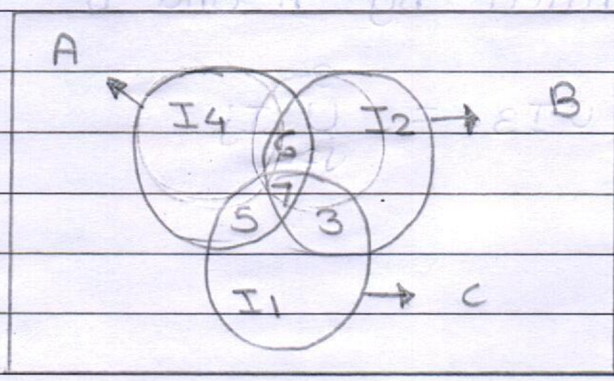
$I3 = \sim A \cap B \cap C$

$I4 = A \cap \sim B \cap \sim C$

$I5 = A \cap \sim B \cap C$

$I6 = A \cap B \cap \sim C$

$I7 = A \cap B \cap C$



If A1, A2, ------ An are any n subsets of universal set E then
complete minterms = $I_0, I_1, ------ I_{2^n-1}$
intersections /

$$\therefore \boxed{E = \bigcup_{j=0}^{2^n-1} I_j}$$

* Partation :

① IF S is the given set with A1, A2, A3 ---- An as subsets in such a way that

② All subsets are disjoint (mutually disjoint)

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* covering :

① It should contain all the elements singally in the given subset.

* Equivalence Relation

A Relation R is the set X is called as equivalence if it is

① Reflexive - diagonal elements are 1
② Symmetric - Path in reverse
③ transitive -

and other

If R is equivalance if in X Then domain of R is X it self.

i.e., $D(R) = X$

Ex of equivalance relations,

① Equality of number on set of real number
② Equality of subset on universal set
③ similarly of triangles on set of trangles
④ Relation of parallel line in plane
⑤ Relation of living in the same town on the set of persons living in canada.
⑥ Relation of statements being equivalent in set of statements.

* Ex : Let $X = \{1, 2, 3, 4\}$ and

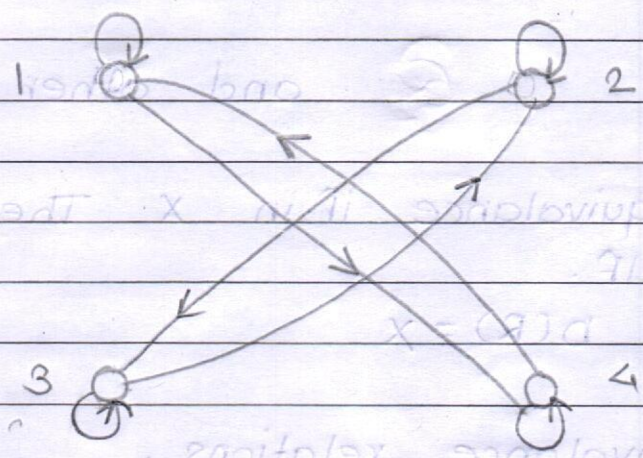
$R = \{\langle 1,1\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 4,4\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$

Write matrix and sketch graph

→ Matrix :

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Graph :



It is reflective, symmetric and transitive. so it is equivalence relation

* Ex : Let $X = \{1, 2, - - - - 7\}$ and

$R = \{\langle x, y\rangle \mid x-y$ is divisible by 3

Put $x = 1$ $y = 1$ $\langle 1,1\rangle$

$y = 4$ $\langle 1,4\rangle$

$y = 7$ $\langle 1,7\rangle$

$x = 2$ $\quad y = 2$ $\quad \langle 2, 2 \rangle$
$\quad\quad\quad\quad y = 5$ $\quad \langle 2, 5 \rangle$

$x = 3$ $\quad y = 3$ $\quad \langle 3, 3 \rangle$
$\quad\quad\quad\quad y = 6$ $\quad \langle 3, 6 \rangle$

$x = 4$ $\quad y = 1$ $\quad \langle 4, 1 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 4, 7 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 4, 4 \rangle$

$x = 5$ $\quad y = 2$ $\quad \langle 5, 2 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 5, 5 \rangle$

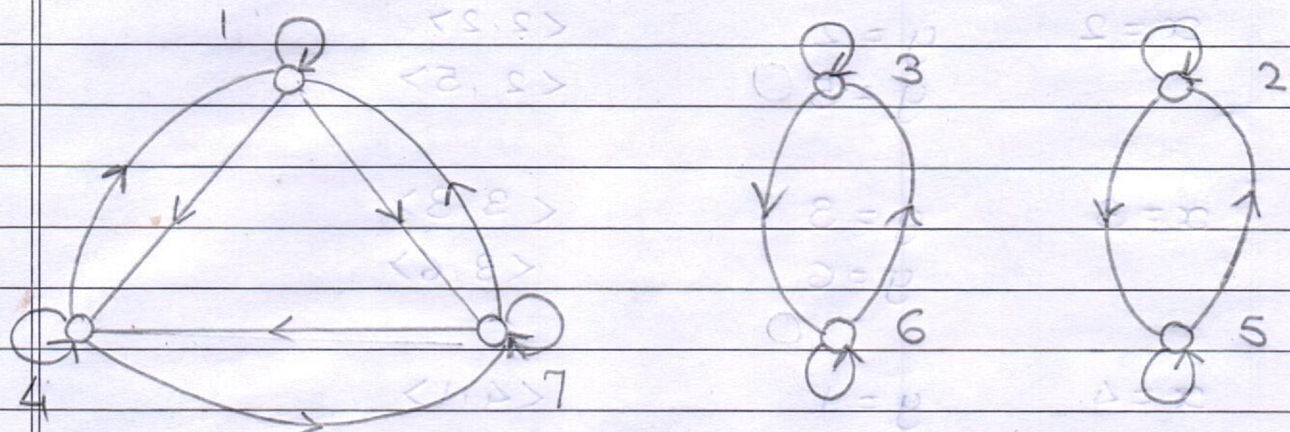
$x = 6$ $\quad\quad\quad \langle 6, 3 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 6, 6 \rangle$

$x = 7$ $\quad\quad\quad \langle 7, 1 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 7, 4 \rangle$
$\quad\quad\quad\quad\quad\quad\quad \langle 7, 7 \rangle$

$$R = \{ \langle 1, 1 \rangle, \langle 1, 4 \rangle, \langle 1, 7 \rangle, \text{- - - - - - - -} \langle 7, 1 \rangle, \langle 7, 4 \rangle, \langle 7, 7 \rangle \}$$

Matrix :

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$



① In Given graph every node contain loop

② Every node having reverse path

③ In matrix the diagonal elements are identical So, given relation is reflexive symmetric and transitive so it is equivalance relation

* Composition of Binary Relations

Union, intersections are single operations while Relation which are having more than one stage called composition of Binary relations

Let R be the relation from X to Y and S be the relation from Y to Z then relation R∘S called as composite relation of R and S.

$$R \circ S = \{ \langle x, z \rangle \mid x \in X \wedge z \in Z \wedge (\exists y)(y \in \wedge \langle x, y \rangle \in R \wedge \langle y, z \rangle \in S \}$$

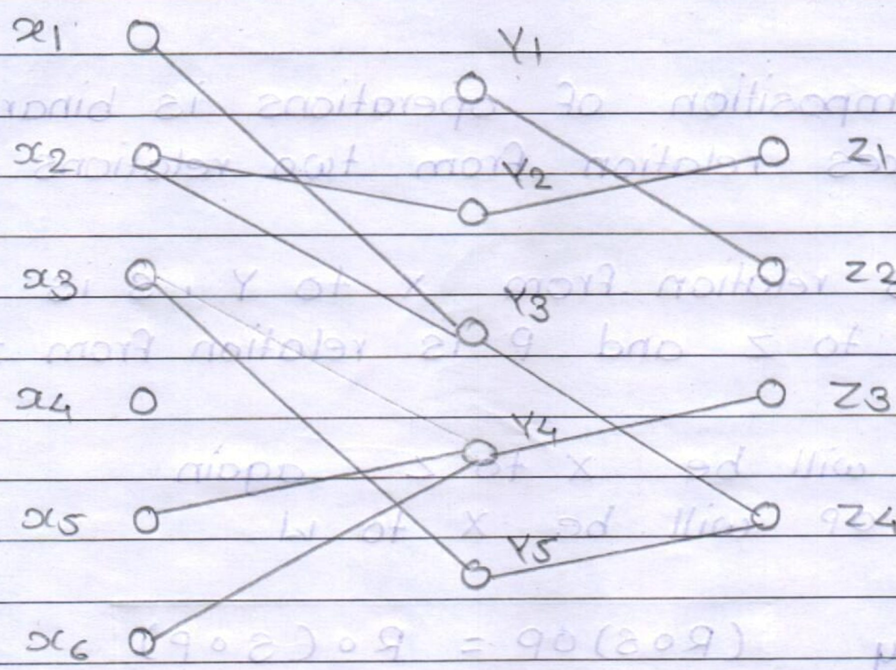
The operation of obtaining RoS from R and S is called composition of relation

① RoS ⇒ Intersection of Range of R and domain of S should be equal then take ordered pair as

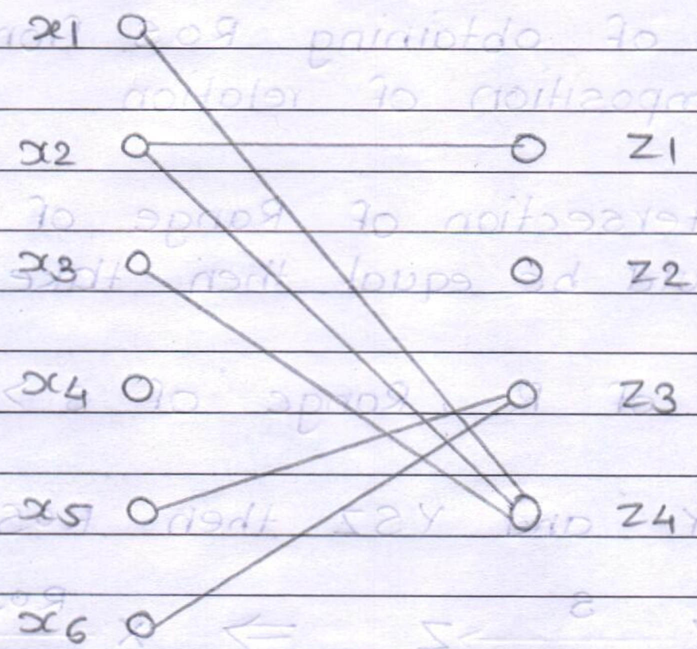
< Domain of R , Range of S >

Ex : IF XRY and YSZ then RoS ⇒ XZ

$$X \xrightarrow{R} Y \xrightarrow{S} Z \quad \Rightarrow \quad X \xrightarrow{RoS} Z$$



$$X \xrightarrow{R} Y \xrightarrow{S} Z$$



composition $X \xrightarrow{RoS} Z$

The composition of operations is binary relation it produces relation from two relations

If R is relation from X to Y, S is relation from Y to Z and P is relation from Z to ω then

RoS will be X to Z again

(RoS) oP will be X to W

property $\boxed{(RoS)oP = Ro(SoP)}$

The operation on composition is associative

i.e., $\boxed{(R \circ S) \circ P = R \circ (S \circ P) = R \circ S \circ P}$

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* Ex : $R = \{ \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 2,2 \rangle \}$

$S = \{ \langle 4,2 \rangle, \langle 2,5 \rangle, \langle 3,1 \rangle, \langle 1,3 \rangle \}$

Find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$, $R \circ R$, $S \circ S$ and $R \circ R \circ R$

① $R \circ S = \{ \langle 1,5 \rangle, \langle 3,2 \rangle, \langle 2,5 \rangle \}$

② $R \circ R \circ R = \{ \langle 1,2 \rangle, \langle 2,2 \rangle \}$

$R \circ S \neq S \circ R$

③ $S \circ R = \{ \langle 4,2 \rangle, \langle 3,2 \rangle, \langle 1,4 \rangle \}$

④ $R \circ (S \circ R) = \{ \langle 3,2 \rangle \}$

⑤ $(R \circ S) \circ R = \{ \langle 3,2 \rangle \}$

⑥ $R \circ R = \{ \langle 1,2 \rangle \}$

⑦ $S \circ S = \{ \langle 4,5 \rangle, \langle 3,3 \rangle, \langle 1,1 \rangle \}$

* Ex : Let R & S are two relations on a set of positive integers I

$R = \{ \langle x, 2x \rangle \mid x \in I \}$

$S = \{ \langle x, 7x \rangle \mid x \in I \}$

find , $R \circ S = \{\langle x, 14x\rangle \mid x \in I\}$

$R \circ R = \{\langle x, 4x\rangle \mid x \in I\}$

We know that the relation matrix of a Relation R from set $X = \{x_1, x_2, \dots x_m\}$ to set $Y = \{y_1, y_2, \dots y_n\}$ having matrix of m rows and n columns then relation matrix of R by M_R the entries of M_R are 0 & 1

The relation matrix M_S of a relation S from set Y to set $Z = \{z_1, z_2, z_3, \dots z_p\}$ then matrix will be M_S

The relation matrix $R \circ S$ could be found by M_R and $M_S \Leftrightarrow M_R \times M_S$

* Ex : If M_R and R_S are given find
  1. $M_{R \circ S}$
  2. $M_{S \circ R}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{R\circ S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad M_{S\circ R} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* Converse :

Given a Relation R from X to Y a relation $\tilde{R}$ from Y to X called converse of R.

The ordered pair of $\tilde{R}$ can be obtained by an interchanging members in each ordered pair of R this means $x \in X$ and $y \in Y$ i.e.,

$$x R y \Leftrightarrow y \tilde{R} x$$

$$\tilde{\tilde{R}} = R$$

The Relation matrix $M_{\tilde{R}}$ of $\tilde{R}$ can be obtained by simply interching the rows and columns of M_R such Matrix called transpose of M_R

$$M_{\tilde{R}} = \text{transpose of } M_R$$

If R be the relation from X to Y
S be the relation from Y to Z

then $\tilde{R}$ is relation from Y to X
$\tilde{S}$ is relation from Z to Y

$R \circ S$ will be x to z
$R \tilde{\circ} S$ will be z to x
$\tilde{S} \circ \tilde{R}$ will be z to x

$$R \tilde{\circ} S = \tilde{S} \circ \tilde{R}$$

The same condition in matrix format

$M_{\tilde{S}}$ = transpose of M_S

* Ex: Given the relation Matrices M_R and M_S find $M_{R \circ S}$, $M_{\tilde{R}}$, $M_{R \tilde{\circ} S}$ also show $M_{R \tilde{\circ} S} = M_{\tilde{R} \circ \tilde{S}}$

→ Given:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

① $M_{R \circ S} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

② $M_{\tilde{R}}$ = transpose of M_R

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

③ $M_{\tilde{S}}$ = transpose of S

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

④ $M_{R\tilde{o}S}$ = transpose of M_{RoS}

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

⑤ $M_{\tilde{S} o \tilde{R}} = M_{R \tilde{o} S}$

Ans : $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $M_{\tilde{S}} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

Relation's of R and S

① $\tilde{\tilde{R}} = R$

② $R = S \iff \tilde{R} = \tilde{S}$

③ $R \subseteq S \iff \tilde{R} \subseteq \tilde{S}$

④ $R \,\tilde{\cup}\, S = \tilde{R} \cup \tilde{S}$ ⑤ $R \,\tilde{\cap}\, S = \tilde{R} \cap \tilde{S}$

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* Partial ordering:

The binary relation R in a set P is called Partial order relation or partial ordering of P iff R is reflexive, antisymmetric & transitive.

symbol $\leq$ / This is not less than or equal to

If given ordering is reflexive the ordered pair $<P, \leq>$ called partially ordered set or POSET

Let $<P, \leq>$ be a partially ordered set, if for every $x, y \in P$, we have either $x \leq y \vee y \leq x$ then $\leq$ is called simple ordering or linear ordering on P and $<P, \leq>$ is called totally ordered or simply ordered set or a chain

If R is a partially ordering P then converse of R will be $\tilde{R}$

if R is denoted by $\leq$
$\tilde{R}$ will denoted by $\geq$

this means $<P, \leq>$ is partially ordered set then $<\tilde{P}, \geq>$ is also partially ordered and they called as duds of each other

If $x<y \Leftrightarrow x \leq y \cap x \neq y$ and
$x>y \Leftrightarrow x \geq y \cap x \neq y$ then it called as antisymmetric and transitive.

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To make given relation as reflexive some partial order relations are used.

① Less than or equal to and Greater than or equal to

Let R be the set of Real Numbers, the relation less than or equals to or $\leq$ is partial order on R

The converse of this relation is also greater than or equals to $\geq$ is also partial ordering on R

Associated Relations are $<$, $>$

② Inclusion:

Let $P(A) = 2^A = X$ will be power set of A. X is the set of the subset A

The relation $\subseteq$ is partial ordering then the proper inclusion $\subset$ with $\subseteq$ is irreflexive, antisymmetric, and transitive.

If $A = \{a, b, c\}$ then

$X = P^A = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, A\}$

③ Divide and Integral Multiple

If a and b are positive integers we say a divides b written $a|b$ iff there is an integer c such that,

$ac = b$ $\qquad\qquad$ $c = b/a$

b called integral multiple of a

* Ex : If $X = \{2, 3, 6, 8\}$ and let $\leq$ is divides on X then

$\leq \; = \{ \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 6,6 \rangle, \langle 8,8 \rangle, \langle 2,8 \rangle, \langle 2,6 \rangle, \langle 3,6 \rangle \}$

integral multiple $\geq$

$\geq \; = \{ \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 6,6 \rangle, \langle 8,8 \rangle, \langle 8,2 \rangle, \langle 6,2 \rangle, \langle 6,3 \rangle \}$

④ Lexicographics ordering :

If R be the set of real numbers and $p = R \times R$ the relation $\geq$ on R is assumed to be the usual relation of " greater than or equal to" for any two ordered pairs $\langle x_1, y_1 \rangle$ and $\langle x_2, y_2 \rangle$ in P

$$\langle x_1, y_1 \rangle \leq \langle x_2, y_2 \rangle \Leftrightarrow (x_1 > x_2) \vee ((x_1 = x_2)) \wedge (y_1 \geq y_2))$$

called lexicographic ordering

* Partially ordered set :- Representation and Associated Terminology

In partially ordered set $<P, \leq>$ $y \in P$ is called cover to $x \in P$ if $x<y$ and no exist any an element $z \in P$ such that $x \leq y$ and $z \leq y$; i.e.,

$$y \text{ covers } x \iff (x<y \wedge (x \leq z \leq y \Rightarrow x = z \vee z = y))$$

A partial ordering $\leq$ on a set P can be represented by means of diagram known as Hasse Diagram or partially ordered set diagram of $<P, \leq>$

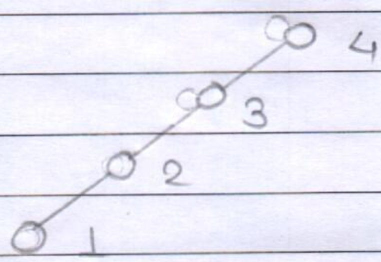
Each element denoted by small circle or dot.

The circle $x \in P$ is drawn below the $y \in P$ if $x<y$ and line drawn between x & y if y covers x

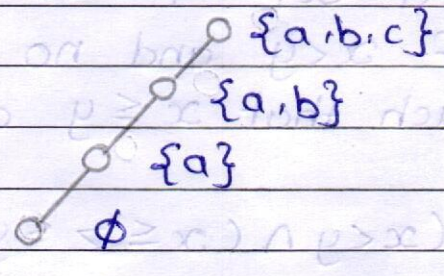
If $x<y$ but y doesn't cover x then x & y doest connected with single line.

Ex : If $P = \{1, 2, 3, 4\}$ and $\leq$ be the relation "less than or equal to" then draw hasse diagram

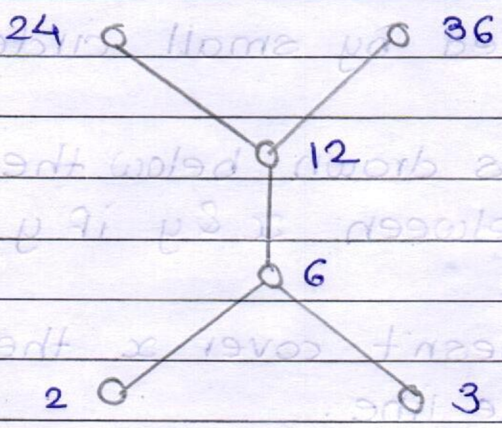
Totally ordered set is called chain



* Ex 2 : $P = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}\}$ and relation of inclusion $\subseteq$ on P then hasse diagram $\langle P, \subseteq \rangle$



* Ex : Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y draw Hasse diagram of $\langle X, \leq \rangle$



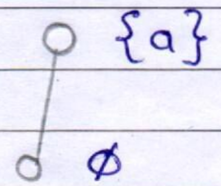
least no = 2
greatest no = 36

Hasse diagram of given relation

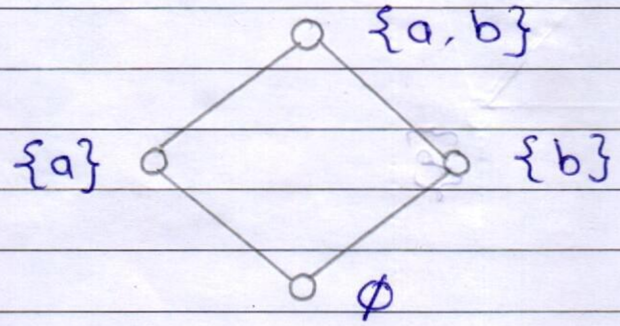
* Ex : Let A be any finite set and P(A) its power set $\subseteq$ is inclusion relation on elements of P(A). Draw Hasse diagram of $\langle P(A), \subseteq \rangle$ for

a) $A = \{a\}$
b) $A = \{a,b\}$
c) $A = \{a,b,c\}$
d) $A = \{a,b,c,d\}$

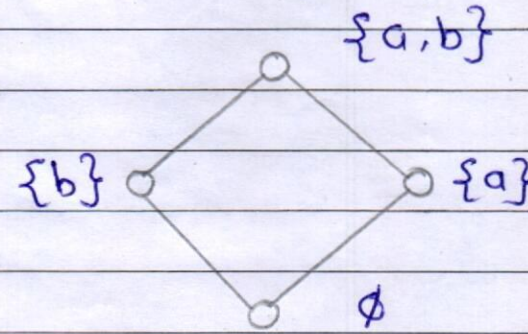
① $A = \{a\}$



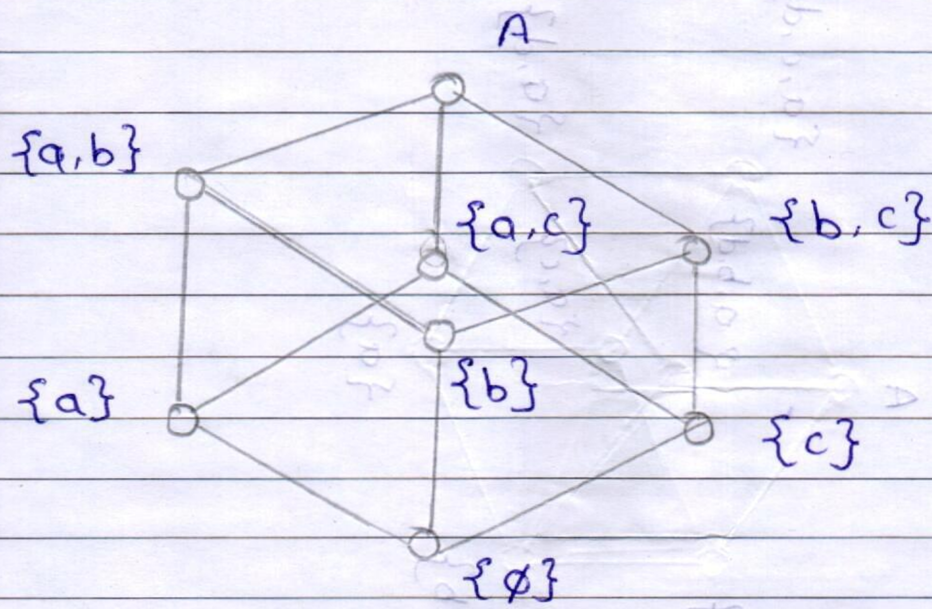
② $A = \{a, b\}$



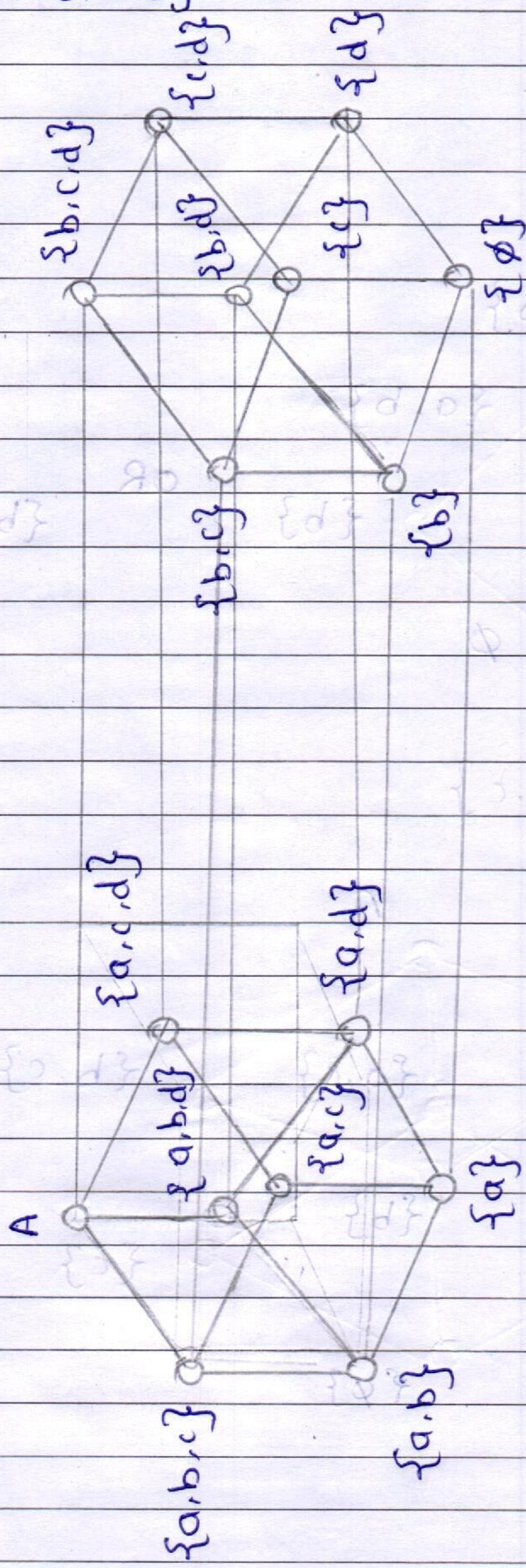
OR



③ $A = \{a, b, c\}$



④ $A = \{a, b, c, d\}$

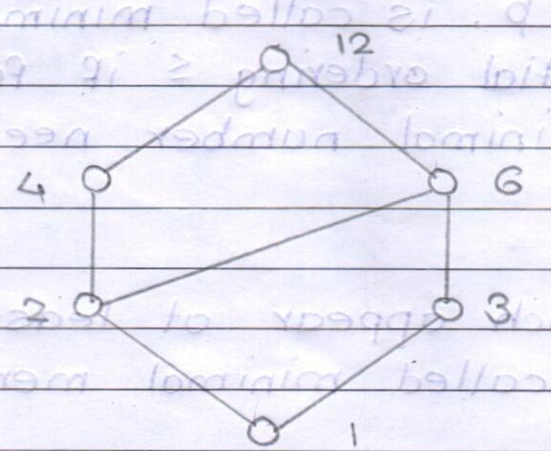


* Ex: Let A be the set of factors of a particular positive integer m and let $\leq$ be relation divides i.e.,

$$\leq = \{ \langle x, y \rangle \mid x \in A \wedge y \in A \wedge (x \text{ divides } y) \}$$

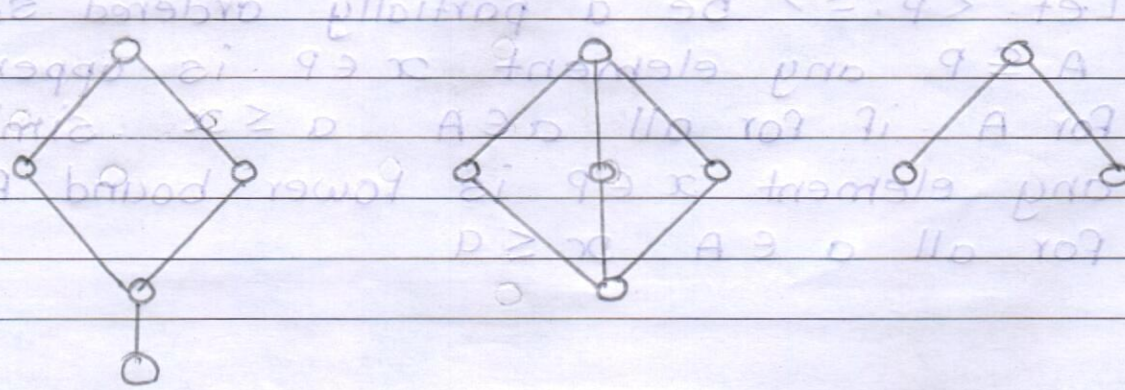
a) $m = 2$, $m = 6$, $m = 30$, $m = 270$, $m = 12$, $m = 45$

Now $m = 12$, factors of 12
1, 2, 3, 4, 6, 12



Hasse diagram also be drawn for relation antisymmetric and transitive but not necessary is reflexive

Ex:



If there exist an element $y \in P$ such that $y \leq x$ for all $x \in P$ then y is collect least member

it is single element

it is denoted by 0

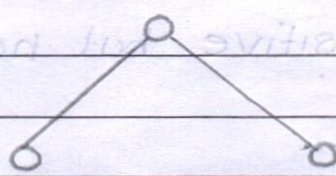
If $y \in P$ and $x \leq y$ for all $x \in p$ then y is called greatest member

greatest number denoted by $\perp$

An element $y \in p$, is called minimal member of P relative to partial ordering $\leq$ if for no $x \in P$ is $x < y$. A minimal number need not be unique

The member which appear at least level of hasse diagram called minimal member.

maximal member also vice verso to above



In given diagram 2 member are minimal 1 is maximal

Let $<P, \leq>$ be a partially ordered set and let $A \subseteq P$ any element $x \in P$ is upper bound for A. if for all $a \in A$ $a \leq x$. similiarly. any element $x \in P$ is lower bound for A if for all $a \in A$ $x \leq a$

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* Function :

Let x and Y be the two sets the relation f from X to Y called a function if for every $x \in X$ there is a unique $y \in Y$ and $\langle x, y \rangle \in f$

Two conditions :

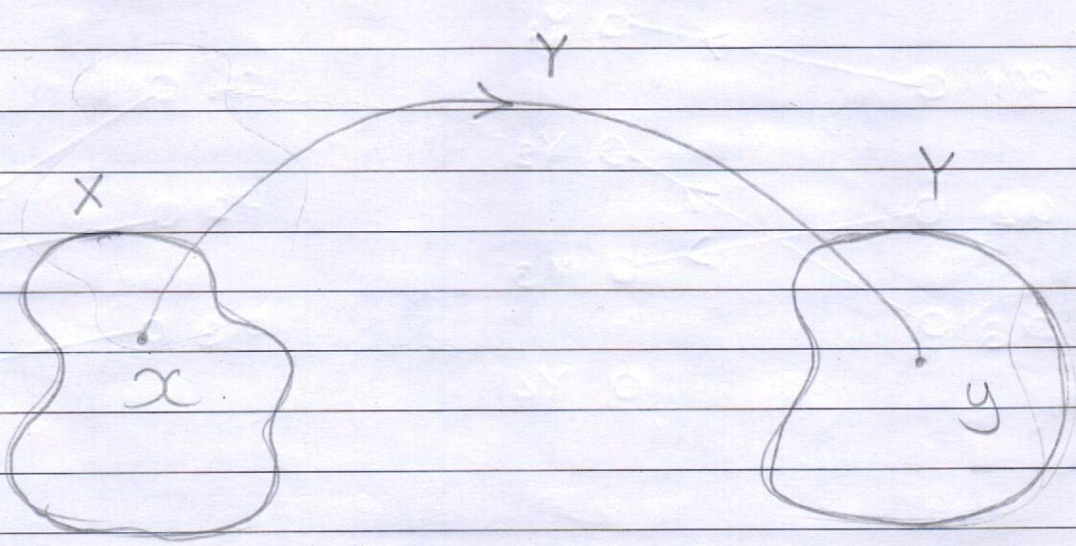
① $x \in X$ is related to some $y \in Y$ i.e., domain of f must be X and not meanly subset of X

② $\langle x, y \rangle \in f \wedge \langle x, z \rangle \in f \Rightarrow y = z$

Notation

$f : X \rightarrow Y$ OR $X \xrightarrow{f} Y$

Representation of Function



x is called argument
y is called image

$\langle x, y \rangle \in f$ will written as

$y = f(x)$ and read as
y the value of function f at x

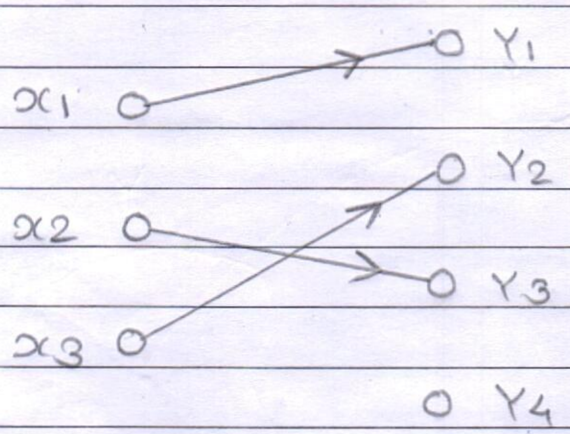
The domain could be represented by
$D_f = X$ and Range R_f

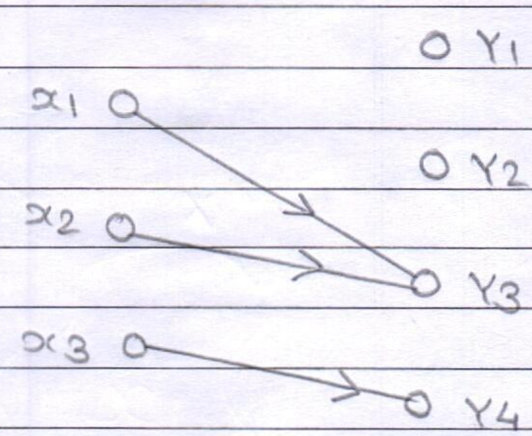
① If $X = \{1, 5, P, \text{Jacks}\}$ & $Y = \{2, 5, 7, a, \text{Jill}\}$
and $f = \{\langle 1,2 \rangle, \langle 5,7 \rangle, \langle P,9 \rangle, \langle \text{Jack}, 9 \rangle\}$
$D_f = X$
$R_f = \{2, 7, 9\}$

$f(1) = 2$, $f(5) = 7$, $f(P) = 9$, $f(\text{Jack}) = 9$

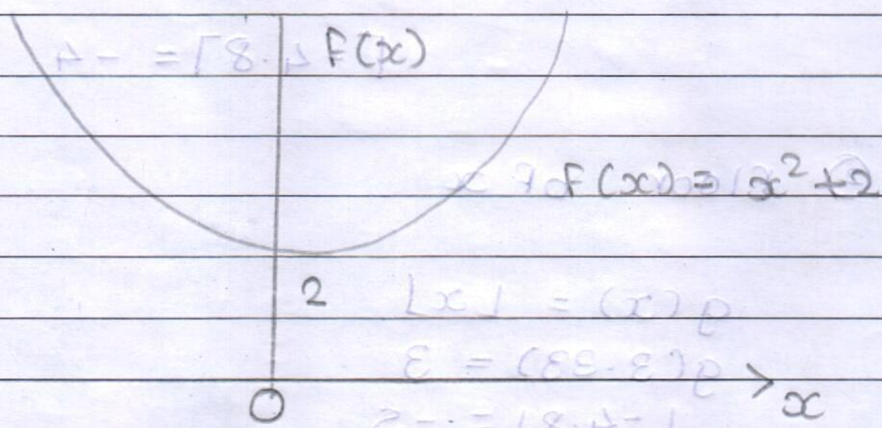
Ex:

$X \xrightarrow{f} Y$ $\quad\quad$ $X \xrightarrow{g} Y$





② IF $X = Y = R$ and $f(x) = x^2 + 2$ the parabola will be



③ IF E is a universal set then power set of E will P(E)

For any two sets A and B $A, B \in P(E)$ the intersection and union will mapping from P(E)

④ IF P is set of all positive integers and $\sigma : P \to P$ be such that $\sigma(n) = n+1$ $\forall\, n \in P$ then

$\sigma(1) = 2$

$\sigma(2) = 3$

The function called "Perano's sucessor" function

⑤ IF X be set of all statements and Y denote set $\{T, F\}$ then mapping from X to Y

⑥ IF f and g are two functions then

① ceiling of $f(x) = \lceil x \rceil$

$f(3.75) = \lceil 3.75 \rceil = 4$

$\lceil -4.8 \rceil = -4$

② Floor of x

$g(x) = \lfloor x \rfloor$

$g(3.33) = 3$

$\lfloor -4.8 \rfloor = -5$

⑦ The program written in high level language is transformed into m/c level by compiler, similarly the output from a computer is function of it inputs.

⑧ Let X and Y are two functions like $x \to Y$

$X = \{a, b, c\}$ $Y = \{a, 1\}$ then

$X \times Y = \{ \langle a,0 \rangle, \langle a,1 \rangle, \langle b,0 \rangle, \langle b,1 \rangle, \langle c,0 \rangle, \langle c,1 \rangle \}$

Here take Y^X as values of function

$2^3 = 8$ functions

$f_0 = \{\langle a,0\rangle, \langle b,0\rangle, \langle c,0\rangle\}$

$f_1 = \{\langle a,0\rangle, \langle b,0\rangle, \langle c,1\rangle\}$

$f_2 = \{\langle a,0\rangle, \langle b,1\rangle, \langle c,0\rangle\}$

$\vdots \qquad \vdots$

$f_7 = \{\langle a,1\rangle, \langle b,1\rangle, \langle c,1\rangle\}$

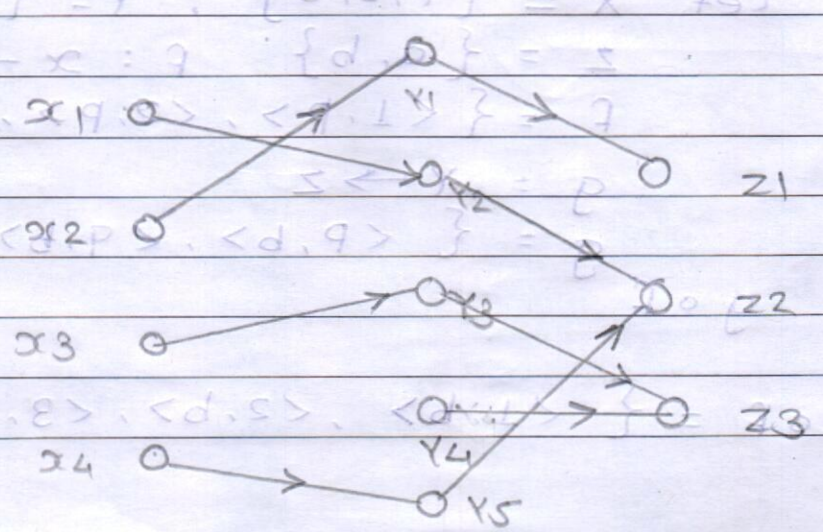
* Composition of function :

If $f : X \to Y$ and $g : Y \to Z$ the composite is called as $g \cdot f$

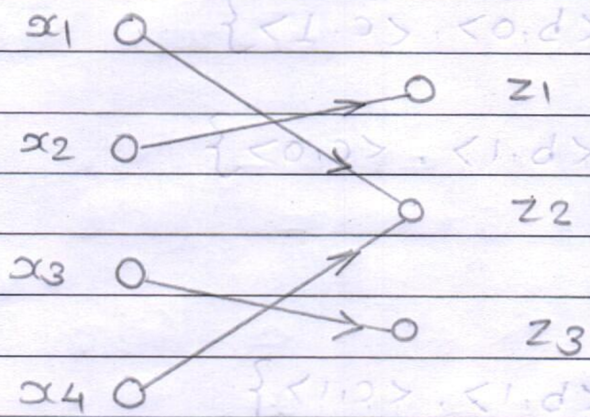
$g \circ f$ is reverse to $f \circ g$ in relation.

$g \circ f$ called as left composite of g with f

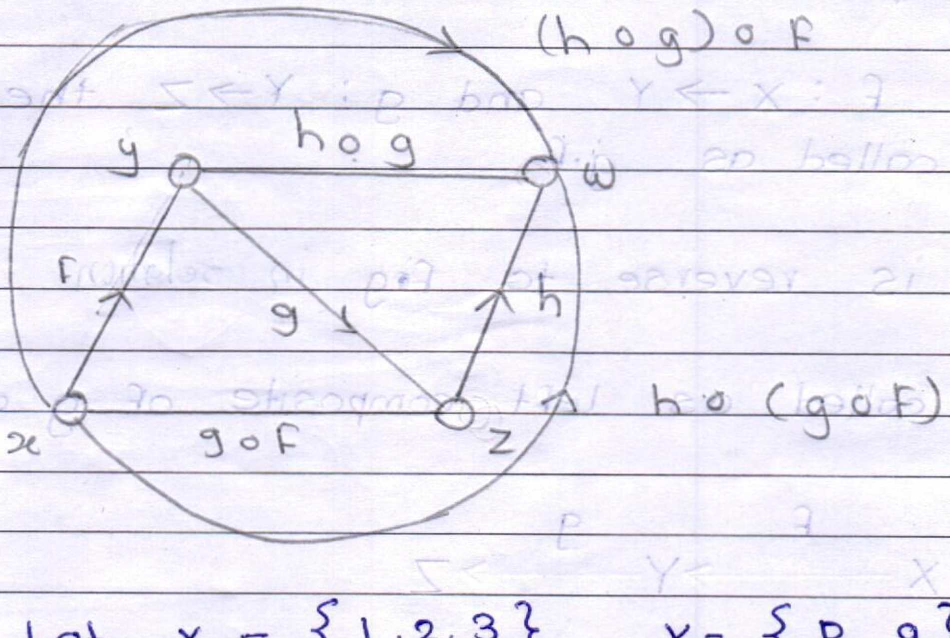
① $X \xrightarrow{f} Y \xrightarrow{g} Z$



$$X \xrightarrow{g \circ f} Z$$



Prove $(h \circ g) \circ f = h \circ (g \circ f)$ $(h \circ g) \circ f$



Ex : Let $X = \{1, 2, 3\}$, $Y = \{p, q\}$

$Z = \{a, b\}$ $f : X \rightarrow Y$

$f = \{\langle 1, p \rangle, \langle 2, p \rangle, \langle 3, p \rangle\}$ &

$g = Y \rightarrow Z$

$g = \{\langle p, b \rangle, \langle q, b \rangle\}$

Find $g \circ f$

$g \circ f = \{\langle 1, b \rangle, \langle 2, b \rangle, \langle 3, b \rangle\}$

* Inverse Function :

If relation R is from X to Y then converse of them is $\tilde{R}$ from Y to X

i.e., if $\langle x, y\rangle \in R \Leftrightarrow \langle y, x\rangle \in \tilde{R}$
i.e., interchanging of members

If we have given $X \xrightarrow{f} Y$ as function then $Y \xrightarrow{\tilde{f}} X$ will be converse of it not inverse.
i.e from Y to X

① If $X = \{1, 2, 3\}$, $Y = \{p, q, r\}$ and $f : X \longrightarrow Y$
then $f = \{\langle 1, p\rangle, \langle 2, q\rangle, \langle 3, q\rangle\}$ then
$\tilde{f} = \{\langle p, 1\rangle, \langle q, 2\rangle, \langle q, 3\rangle\}$
$\forall$ $\tilde{f}$ is only converse

② If R be the set of all Real numbers and $f : R \longrightarrow R$ be given by, $f = \{\langle x, x^2\rangle \mid x \in R\}$
Then,
$\tilde{f} = \{\langle x^2, x\rangle \mid x \in R\}$ is not function

③ If R is the set of real numbers
$f : R \longrightarrow R$ then
$f = \{\langle x, x+2\rangle \mid x \in R\}$ then
$\tilde{f} = \{\langle x+2, x\rangle \mid x \in R\}$ is function from R to R

④ Invertible :
If we have given to
$f : X \to Y$ then

$\tilde{f}$ become invertible iff
f is one to one and onto relation at such case $\tilde{f}$ is written as f^{-1}

$f^{-1} = Y \rightarrow X$

Identify of function :

$I_x : X \rightarrow X$ is identity map if

$I_x = \{ \langle x, x \rangle \mid x \in X \}$

If $f : X \rightarrow Y$ is invertible then

$f^{-1} \circ f = I_x$

$f \circ f^{-1} = I_y$

Let $f : X \rightarrow Y$ and $g : Y \rightarrow X$ the function g is equals to f^{-1} only if

$g \circ f = I_x$ and $f \circ g = I_y$

* prove :

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

→ Let, $\left.\begin{array}{l} f : X \rightarrow Y \\ g : Y \rightarrow Z \end{array}\right\}$ All are one to one and onto relation

The inverse f^{-1}, g^{-1} and $(g \circ f)^{-1}$ exists & have one to one and onto

Now,

$y = f(x)$

$z = g(y)$

$x = f^{-1}(y)$

$y = g^{-1}(z)$ so

$\langle z, x \rangle \in f^{-1} \circ g^{-1}$ it satisfy x, y, z so

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

① closure property :

① If $g, f \in f_x$ then $g \circ f$ & $f \circ g$ are also in f_x

② For any $f, g, h \in f_x$
$(f \circ g) \circ h = f \circ (g \circ h)$ associative

③ Identity, if $I_x \in f_x$ then for any :
$f \in f_x$
$I_x \circ f = f \circ I_x = f$.

④ Inverse : For every $f \in f_x$ exist an inverse function $f^{-1} \in f_x$ such that

$$f \circ f^{-1} = f^{-1} \circ f = I_x$$

Composite Function

A Composite function is created when one function is substituted into another function

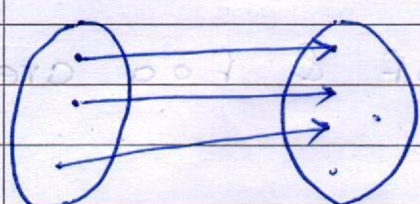
if $f(x) = 3x+5$

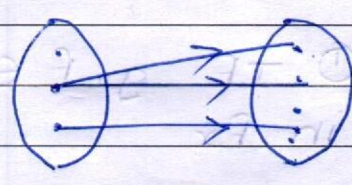
$g(x) = 2x+3$

then

$$f(x) \Rightarrow f(g(x))$$
$$= f(2x+3)$$
$$= 3(2x+3)+5$$
$$= 6x+9+5 = \underline{6x+14}$$

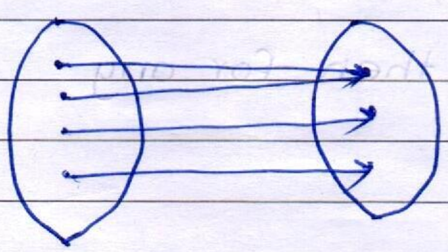
* Injective (one to one)





Not a valid func

* Surjective (onto)



* Bijective (one to one onto)