Shivaji University, Kolhapur

Question Bank for Mar.2022 (summer) Examination

Subject Code: 73276 Subject Name: Applied Mathematics

Unit No. 1 Correlation, Regression and Curve Fitting

1 If the equations of two lines of regressions are $3x+2y=26,$ and $6x+y=31$. Find the means of X and Y and the correlation coefficien**t.**

2 Find equations of two lines of regressions. Also find coefficient of correlation.

x: 5 6 7 8 9 10 11

y: 11 14 14 15 12 17 16

3 Obtain the two equation of line of regression from the following data.

x: 2 4 6 8 12 14

y: 4 2 5 10 11 12

4 Fit a straight line $y = a + bx$ to following data.

x:	1	2	3	4	6	8
y:	2.4	3	3.4	4	5	6

5 Find the lines of regression for the following data and estimate value of y when x =14.5

X:	10	14	19	26	30	34	39
Y:	12	16	18	26	29	35	38

6 Fit a straight line to the following data.

x: 6 2 10 4 8

y: 9 11 5 8 7

7 Fit a second degree parabola to following data:

X	1	2	3	4	5
Y	25	28	33	39	46

8 Fit a second degree parabola to the following data taking x as the independent variable.

x:	0	1	2	3	4
y:	1	1.5	1.5	2.5	3.5

Fit a curve of the type $y = ae^{bx}$ to the following data by method of least squares.

9

x:	1	5	7	9	12
y:	10	15	12	15	21

10 Fit a second degree parabola to the following data.

x:	1	2	3	4	5	6	7	8	9
y:	2	6	7	8	10	11	11	10	9

11. Fit a straight line to the following data.

x:	1	2	3	4	5
y:	1	5	11	8	14

12. If the equations of two lines of regressions are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Find the means of X and Y and the correlation coefficient.

13 Find equations of line of regression for the following data.

x:	10	14	19	26	30	34	39
y:	12	16	18	26	29	35	38

14 Find coefficient of correlation from following data.

x:	1	2	3	4	5	6	7	8	9
y:	9	8	10	12	11	13	14	16	15

15 Find coefficient of correlation from following data.

x:	10	12	14	15	16	17	18	10	14	15
y:	17	16	15	12	10	9	8	15	13	12

Unit No. 2 Probability Distribution

1 A random variable X has the following probability distribution.

X	1	2	3	4	5	6	7
p(X)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

2 Find k if f(x) is probability density function of continuous random variable x

$$f(x)=\begin{cases} kx^2(1-x^3) & 0\le x\le 1 \\ 0 & \text{otherwise} \end{cases}$$

3 A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6
p(X)	k	3k	5k	7k	9k	11k	13k

Find i) k, ii) P(1< X < 5)

4 From a box containing 100 transistors 20 of which are defective , 10 are selected at random. Find the probability that

i. All will be defective. ii. All will be non defective

iii. At least one is defective.

5 The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that i) exactly two will be defective,

ii) At least two will be defective.

6 10% of the tools produced in a certain manufacturing process turn out to be defective. Find probability that out of 20 tools selected at random there are (i) exactly 2 defectives, (ii) At least 2 defectives.

7 The probability that a missile will strike the target is 1/5. if 6 such missiles are fixed. Find the probability that i) exactly two, ii) at least 2 will strike the target.

8 If 3% of the bulbs manufactured by a company are defective , assuming Poisson distribution find probability that in a pack of 100 bulbs zero, two are defective.

9 If the probability that an individual suffer a bad reaction from certain injection is 0.001. Determine the probability that out of 2000 individual i) exactly 3, ii) more than 2 will suffer a bad reaction

10 It is 1 in 1000 that an article is defective. There are in box 100 articles of this type. Assuming Poisson distribution find probability that the box contains i) no defective, ii) two or more defectives.

11 In certain factory turning out blades, there is small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing, no defective , one defective and two defective blades in a consignment of 10000 packets.

12 In a test on 2000 electrical blubs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate number of bulbs likely to burn i) more than 2150 hours, ii) less than 1950 hours iii) more than 1920 hours but less than 2160 hours.

[Given: for S.N. V. Z, area under the normal curves from Z = 0 to Z = 1.5 is 0.4332, From z = 0 to z=2 is 0.4772 and z = 0 to z =1.83 is 0.4664]

13 The customer accounts of a certain departmental store have an average balance of Rs. 120 and a standard deviation of 40 Rs. Assuming that the distribution of account balances is normal, find the probability of account balance is i) Over Rs. 150, ii) Between Rs. 100 & Rs. 150, iii) between Rs. 60 & Rs. 90

[Given: for S.N. V. Z, area under the normal curves between Z = 0 to Z = 0.75 is 0.2734, from Z = 0 to Z=0.5 is 0.1915 and that of between Z = 0 and Z = 1.5 is 0.4332]

14 An aptitude test for selecting engineers in an industry is conducted on 100 candidates. Average score is 42 and standard deviation is 24. Assuming normal distribution for the score , find probability of score of candidate selected randomly i) lies between 30 to 60, ii) more than 60.

[Given: for S.N. V. z area from z = 0 to 0.5 is 0.1915, from z = 0 to z = 0.75 is 0.2734]

15 In a sample of 1000 student the mean and standard deviation of marks obtained by the student in a certain test are 14 and 2.5. Assuming the distribution to be normal find the number of students getting marks (i) between 12 and 15, (ii) above 18, (iii) below 8.

(Given: For a S.N.V. z area between z = 0 and z = 0.4 is 0.1554, that between z = 0 and z = 0.8 is 0.2881, that between z = 0 and z = 1.6 is 0.4452 that between z = 0 and z = 2.4 is 0.4918)

16 The income distribution of group of 10000 persons was found to be normal with mean Rs. 7500 & standard deviation Rs. 500. What is the number of persons of this group who have income

i) exceeding Rs. 6680 , ii) exceeding Rs. 8320.

[Given: for S.N. V. Z, area under the normal curves between the ordinates ± 1.64 is 0.8990]

Unit No.03: Numerical Integration

1 Evaluate $\int_0^2 e^{x^2}\,dx$ using Trapezoidal Rule, Taking n = 10 subintervals.

2 Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{sinx + cosx}\,dx$ using Trapezoidal Rule.

3 Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ using Trapezoidal Rule.

4 Evaluate $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Trapezoidal Rule.

5 Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1+3sin^2x}\,dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule.

6 Evaluate $\int_0^2 e^{-x^2}\,dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule, taking h=0.25.

7 Evaluate $\int_0^1 \sqrt{1+x+x^2}\,dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule.

8 Evaluate $\int_{0.2}^{1.4} (sinx - logx + e^x)\,dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule.

9 Evaluate $\int_0^9 \frac{1}{1+x^3}\,dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule, Taking n = 9 subintervals.

10 Evaluate $\int_0^{\pi} \sqrt{1+3cos^2x}\,dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule.

11 Evaluate $\int_{0.2}^{5.2} logx\,dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule.

12 Evaluate $\int_4^1 (sinx - logx + e^x)\,dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule.

13 Evaluate $\int_0^6 \frac{e^x}{1+x}\,dx$ using Weddle's Rule.

14 Evaluate $\int_0^{\pi} \sqrt{1+3cos^2x}\,dx$ using Weddle's Rule.

15 Evaluate $\int_{0.2}^{1.4} (sinx - logx + e^x)\,dx$ using Weddle's Rule.

Unit No.04: Introduction to Fuzzy set

Q.1) Find α-cut and strong α-cut for α =0.2, 0.3, 0.4, 0.5 for the fuzzy set defined by

C(x) = $\frac{x}{x+1}$, x∈ {1,2,3,4,5}

Q.2) IF the fuzzy set A and B are defined by the following membership functions:

$A(x) = \frac{0.1}{1} + \frac{0.6}{2} + \frac{0.8}{3} + \frac{0.9}{4} + \frac{0.7}{5}$, $B(x) = \frac{0.9}{1} + \frac{0.7}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.1}{5}$.

Calculate $\bar{A}$, $\bar{B}$, $\overline{A \cup B}$, $\overline{A \cap B}$,

Q.3) Find the fuzzy cardinality of the fuzzy set defined by

A(x) = $\frac{35-x}{15}$, x∈ {20,22,24,26,28,30,32,34}

Q.4) Consider the fuzzy sets:

A(x) = $\frac{x}{x+2}$, B(x) = $\frac{x}{x+5}$ x∈ {1,2,3,4,5} .Find$|A|$,$|B|$,$S(A,B), S(B,A)$

Q.5) If $A(x) = \frac{0.1}{x} + \frac{0.6}{y} + \frac{0.8}{z} + \frac{0.9}{w}$, $B(x) = \frac{0.9}{x} + \frac{0.7}{y} + \frac{0.5}{z} + \frac{0.2}{w}$.Find $S(A,B), S(B,A)$.

Q.6) C(x) = $\frac{x}{x+1}$, x∈ {1,2,3,4,5}

D(x) = $1 - \frac{x}{10}$, x∈ {1,2,3,4,5}. Find$S(C,D), S(D,C)$.

Q.7) Find α-cut and strong α-cut for α =0.2, 0.4, 0.6, 0.8 for the fuzzy set defined by

B(x) =$\frac{0.2}{1} + \frac{0}{2} + \frac{0.65}{3} + \frac{0.7}{4} + \frac{0.35}{5}$

Q.8) C(x) = $\frac{x}{x+1}$, x∈ {0,1,2,3, …,10}

Find α-cuts and strong α-cuts for α =0.2, 0.4, 0.5, 0.6, 0.8

Q.9) X=$\{x_1, x_2, x_3, x_4, x_5\}$ $A = X \to [0,1]$ defined by$A(x_1) = 0.2$, $A(x_2) = 0.4$,$A(x_3) = 0.6$,$A(x_4) = 0.8$,$A(x_5) = 1$. Find level set of A.

Q.10) IF the fuzzy set A and B are defined by the following membership functions:

$A(x) = \frac{0.1}{1} + \frac{0.6}{2} + \frac{0.8}{3} + \frac{0.9}{4} + \frac{0.7}{5} + \frac{0.1}{6}$, $B(x) = \frac{0.9}{1} + \frac{0.7}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.1}{5} + \frac{0}{6}$.

Calculate $\bar{A}$, $\bar{B}$, $\bar{A} \cup B$, $A \cup \bar{B}$

Q.11) If $A(x) = \frac{0.2}{x} + \frac{0.4}{y} + \frac{0.6}{z} + \frac{0.8}{w} + \frac{1}{p}$, $B(x) = \frac{0.9}{x} + \frac{0.7}{y} + \frac{0.5}{z} + \frac{0.2}{w} + \frac{1}{p}$. Determine $\bar{A} \cup \bar{B}$.

Q.12) Define

i) Scaler Cardinality

ii) Membership function

iii) Fuzzy set.

Q.13) Define

i) Fuzzy intersection

ii) Height of fuzzy set

iii) Degree of subset hood $S(C, D)$ for the fuzzy sets $C\ and\ D$.

Q.14) Let the fuzzy sets $C\ and\ D$ be defined on the same universal set X by

C(x) = $\frac{x}{x+2}$, D(x) = $\frac{x}{x+5}$, x∈ {0,1,2,3,4,5}.Find $|C|, |D|, |C \cap D|$

Q.15) Let the fuzzy sets $A\ and\ B$ be defined on the same universal set X by

A(x) = $\frac{2x}{2x+5}$, B(x) = $\frac{x}{x+1}$, x∈ {6,7,8,9,10}.

Find 1) $A \cup \overline{A \cap B}$.

2) $\propto -cut\ of\ A \cup \overline{A \cap B}\ for\ \propto = 0.9$

Unit No.05: Fuzzy Arithmetic:

Q.1) Find fuzzy cardinality of $A(x) = \frac{1}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.7}{4} + \frac{0.2}{5}$

Q.2) Let $A(x) = \frac{0.3}{1} + \frac{0.9}{2} + \frac{0.7}{3}$ and $B(x) = \frac{0.2}{-1} + \frac{0.4}{0} + \frac{0.5}{1}$ Also if $f: X \times X \to X$ is defined for all $x_1, x_2 \epsilon\ X$ such that $f(x_1, x_2) = x_1 - x_2$ then using extension principle find $f(A, B)$.

Q.3) $A(x) = \frac{0.5}{-1} + \frac{1}{0} + \frac{0.5}{1} + \frac{0.3}{2}$ and $B(x) = \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.3}{5}$ Let a function $f: X \times X \to X$ is defined for all $x_1, x_2 \epsilon\ X$ such that $f(x_1, x_2) = x_1 \times x_2$. Calculate $f(A, B)$.

Q.4) $A(x) = \frac{0.5}{-1} + \frac{1}{0} + \frac{0.5}{1} + \frac{0.3}{2}$ and $B(x) = \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.3}{5}$ Let a function $f: X \times X \to X$ is defined for all $x_1, x_2 \epsilon\ X$ such that $f(x_1, x_2) = x_1 + x_2$. Calculate $f(A, B)$.

Q.5) Using extension principle, find $f(A, B)$ where $f(x_1, x_2) = x_1 \times x_2$. for the fuzzy numbers

$A(x) = \frac{0.5}{1} + \frac{0.6}{2} + \frac{0.9}{3} + \frac{1}{4} + \frac{0.2}{5}$, $B(x) = \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.3}{5}$

Q.6) Using extension principle, find $f(A, B)$ where $f(x_1, x_2) = 2x_1 + x_2$. for the fuzzy numbers $A(x) = \frac{0.5}{1} + \frac{0.6}{2} + \frac{0.9}{3} + \frac{1}{4} + \frac{0.2}{5}$, $B(x) = \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4} + \frac{0.3}{5}$.

Q.7) If A be a fuzzy set defined by :

$A(x) = \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5}$ then list all $\alpha - cuts$ and strong $\alpha - cuts$ of A and show That $A = \cup_\alpha\ A$ where $\alpha \in [0,1]$ and α^A denotes special fuzzy set of A.

Q.8) Define fuzzy number and determine whether the following fuzzy set is a fuzzy number

$$A(x) = \begin{cases} sinx & 0 \le x \le \pi \\ 0 & otherwise \end{cases}$$

Q.9) Find which fuzzy set defined by following functions are fuzzy numbers

$$a)\ A(x) = \begin{cases} x & 0 \le x \le 1 \\ 0 & otherwise \end{cases} \quad b)\ A(x) = \begin{cases} 1 & 0 \le x \le 10 \\ 0 & otherwise \end{cases}$$

Q.10) Find which fuzzy set defined by following functions are fuzzy numbers

$$a)\ A(x) = \begin{cases} Min(1, x) & x \ge 0 \\ 0 & x < 0 \end{cases} \quad b)\ A(x) = \begin{cases} 1 & x = 5 \\ 5 & otherwise \end{cases}$$

Q.11) Calculate the fuzzy number $A+B$, $A.B$ where,

$$A(x)=\begin{cases}\frac{3+x}{3} & -3\le x\le 0\\ \frac{3-x}{3} & 0\le x\le 3\\ 0 & othewise\end{cases} \qquad B(X)=\begin{cases}\frac{x-3}{3} & -3\le x\le 6\\ \frac{9-x}{3} & 6\le x\le 9\\ 0 & othewise\end{cases}$$

Q.12) Calculate the fuzzy number $A+B$, $A-B$ where,

$$A(x)=\begin{cases}\frac{x+2}{2} & -2\le x\le 0\\ \frac{2-x}{2} & 0\le x\le 2\\ 0 & othewise\end{cases} \qquad B(X)=\begin{cases}\frac{x-2}{2} & 2\le x\le 4\\ \frac{6-x}{2} & 4\le x\le 6\\ 0 & othewise\end{cases}$$

Q.13) Calculate the fuzzy number $A+B, A-B$ where,

$$A(x)=\begin{cases}\frac{x+1}{2} & -1\le x\le 1\\ \frac{3-x}{2} & 1\le x\le 3\\ 0 & othewise\end{cases} \qquad B(X)=\begin{cases}\frac{x-1}{2} & 1\le x\le 3\\ \frac{5-x}{2} & 3\le x\le 5\\ 0 & othewise\end{cases}$$

Q.14) Solve the equation $A+X=B$ where:

$$A=\frac{0.2}{[0,1)}+\frac{0.6}{[1,2)}+\frac{0.8}{[2,3)}+\frac{0.9}{[3,4)}+\frac{1}{4}+\frac{0.5}{(4,5]}+\frac{0.1}{(5,6]}$$

$$B=\frac{0.1}{[0,1)}+\frac{0.2}{[1,2)}+\frac{0.6}{[2,3)}+\frac{0.7}{[3,4)}+\frac{0.8}{[4,5)}+\frac{0.9}{[5,6)}+\frac{1}{6}+\frac{0.5}{(6,7]}+\frac{0.4}{(7,8]}+\frac{0.4}{(7,8]}+\frac{0.2}{(8,9]}+\frac{0.1}{(9,10]}$$

Q.15) If $A,B\in f(x)$ then prove that $|A|+|B|=|A\cap B|+|A\cup B|$

Unit No.06: Assignment Problem:

Q.1) Solve the following assignment problem

	To				
From		I	II	III	IV
	A	8	26	17	11
	B	13	28	4	26
	C	18	29	18	15
	D	19	23	24	10

Q.2) Solve the following assignment problem

	Jobs				
Tasks		I	II	III	IV
	A	10	12	19	11
	B	5	10	7	8
	C	12	14	13	7
	D	8	15	11	9

Q.3) Solve the following assignment problem

	operators						
Machines		I	II	III	IV	V	VI
	A	12	10	15	22	18	8
	B	10	18	15	15	16	12
	C	11	10	1	8	5	9
	D	6	14	10	13	13	12
	E	8	12	11	7	13	10

Q.4) Four salesmen are to be assigned to four districts. Estimates of the sales revenue in thousand of rupees for each salesman are as under:

	District				
Salesman		A	B	C	D
	1	32	35	40	28
	2	40	25	30	22
	3	42	27	34	30
	4	25	39	41	35

Q.5) A company has 4 machines which do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table. What are the job assignments which will minimize the cost?

	Machines				
Jobs		M1	M2	M3	M4
	J1	18	24	28	32
	J2	8	13	17	18
	J3	10	15	19	22

Q.6) Following table represent expected time required (in min.) to five different groups for completing four different tasks. Determine assignment schedule in order to minimize the time of completion of tasks.

	Group					
Tasks		A	B	C	D	E
	I	35	24	28	26	31
	II	34	32	35	24	32
	III	29	25	38	35	33
	IV	28	26	27	33	32

Q.7) Following table represent profit earned by workers from different jobs.Find assignment Schedule to maximize profit.

	Jobs				
Tasks		A	B	C	D
	I	5	4	8	6
	II	4	2	5	4
	III	9	5	8	5
	IV	8	1	7	3

Q.8) Solve the assignment problem for maximization

	I	II	III	IV
A	2	3	4	5
B	11	12	9	7
C	21	35	15	7
D	9	15	13	14

Q.9) Solve the following assignment problem

	A	B	C	D
I	3	6	2	6
II	7	1	4	4
III	3	8	5	8
IV	4	6	3	7
V	5	7	8	4

Q.10) Solve the following assignment problem and find the cost of assignment in lakhs.

	R1	R2	R3	R4
C1	9	14	19	15
C2	7	17	20	19
C3	9	18	21	18
C4	10	12	18	19
C5	10	15	21	26

Q.11) Solve the assignment problem for minimization

	Machines					
Operators		I	II	III	IV	V
	A	8	20	19	31	25
	B	26	37	39	41	15
	C	9	11	24	5	7
	D	29	31	41	45	50
	E	71	19	21	31	45

Q.12) Solve the traveling salesman problem in the matrix

	A1	A2	A3	A4	A5
B1	-	4	7	3	4
B2	4	-	6	3	4
B3	7	6	-	7	5
B4	3	3	7	-	7
b5	4	4	5	7	-

Q.13) Solve the traveling salesman problem in the matrix

	1	2	3	4	5
1	∞	2	5	7	1
2	6	∞	3	8	2
3	8	7	∞	4	7
4	12	4	6	∞	5
5	1	3	2	8	∞

Q.14) Solve the traveling salesman problem in the matrix

	1	2	3	4	5
1	∞	1	8	3	4
2	1	∞	8	2	3
3	1	3	∞	5	1
4	2	5	6	∞	5
5	5	3	7	6	∞

Q.15) Solve the traveling salesman problem in the matrix

	1	2	3	4	5
1	∞	6	12	6	4
2	6	∞	10	5	4
3	8	7	∞	11	3
4	5	4	11	∞	5
5	5	2	7	6	∞