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Unit 3 : Context free Grammers

* Context free Grammer :

Context free Grammar is a formal grammer which is used to generate all possible strings in a given formal language.

Grammar is set of rules. It check correctness of sentence or syntax.

FA and Grammar are classified by chomsky.

Context free Grammar G is 4 tuple.

$G = (V, \Sigma, S, P)$

Where,

$V \Rightarrow$ variable, Non Terminal symbol

$\Sigma \Rightarrow$ Terminal symbol

$S \Rightarrow$ start symbol

$P \Rightarrow$ Production in grammar.

Draw the regular language of the following

$L = \{0,1\}^* \{10\}$

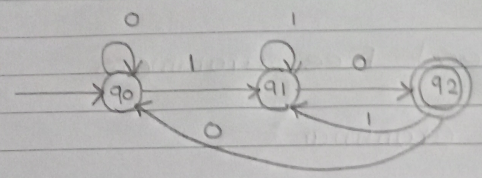

fig: DFA for $\{1.0\}^* \{10\}$

consider the language which is accepted by given DFA i.e.

W = 1100010

Initial string	state
$\wedge$	q_0
1	q_1
11	q_1
110	q_2
1100	q_0
11000	q_0
110001	q_1
1100010	q_2

Now convert given table into CFG form

$q_0 \rightarrow 1q_1$ / $q_0 \xrightarrow{1} q_1$

$q_1 \rightarrow 1q_1$ / $q_1 \xrightarrow{1} q_1$

$q_1 \rightarrow 0q_2$ / $q_1 \xrightarrow{0} q_2$

so it having shorthand notation

$q_0 \longrightarrow a.q_1$

where $a \in \Sigma$ and q_0 and q_1 are non-terminal symbol

$q_0, q_1 \in V$

When last production is their then we can take $q_0 \rightarrow b$ where $q_0 \in V$ and $b \in \Sigma$.

Where V = non terminal symbol

Σ = Terminal symbol

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* Union, Concatenation and kleene on CFG

① Union :

If L_1 and L_2 are two context free language which having,

$G_1 = (V_1, \Sigma, S_1, P_1)$

$G_2 = (V_2, \Sigma, S_2, P_2)$

union of G_1 and G_2 will be noted as,

$\boxed{G_u = G_1 \cup G_2}$

$\boxed{L(G_u) = L_1 \cup L_2}$

$G_u = (V_u, \Sigma, S_u, P_u)$

where, $\boxed{V_u = V_1 \cup V_2 \cup \{S_u\}}$

$S_u = S_0$ starting symbol in new union CFL

$\boxed{P_0 = P_1 \cup P_2 \cup \{S_0 \rightarrow S_1 \mid S_2\}}$

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② Concatenation :

We have G_1 and G_2 as two CFG

$G_c = (V_c, \Sigma, S_c, P_c)$

$$\boxed{V_c = V_1 \cup V_2 \cup \{S_c\}}$$

S_c = start symbol in G_c

$$\boxed{P_c = P_1 \cup P_2 \cup \{S_c \rightarrow S_1 S_2\}}$$

③ kleene :

$G_1 = (V_1, \Sigma, S_1, P_1)$

then

$G_K = (V_K, \Sigma, S_K, P_K)$

where,

$$\boxed{V_K = V_1 \cup \{S_K\}}$$

S_K = new starting symbol

$$\boxed{P_K = P_1 \cup \{S_K \rightarrow S_1 S_K \mid \Lambda\}}$$

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* Union, concatenation and kleen's of CFL's

If L_1 and L_2 are CFL then $L_1 \cup L_2$, $L_1 \cdot L_2$, L_1^* also CFL

By contractive method,

consider $G_1 = \{V_1, \Sigma, S_1, P_1\}$ and

$G_2 = \{V_2, \Sigma, S_2, P_2\}$

G_1 and G_2 are generating L_1 and L_2 respectively.

① Union :

$G_u = \{V_u, \Sigma, S_u, P_u\}$ generating $L_1 \cup L_2$

If necessary we rename the elements of V_2 then

$V_1 \cap V_2 = \phi$

$V_u = V_1 \cup V_2 \cup \{S_u\}$

where, S_u = new start symbol not in V_1 & V_2

$P_u = P_1 \cup P_2 \cup \{S_u \rightarrow S_1 | S_2\}$

if x is in L_1 or L_2 then $S_u \overset{*}{\Longrightarrow} x$ is in grammar G_u

because we start the derivation in

$S_u \to S_1$ or $S_u \to S_2$ and continue the derivation of x in G_1 or G_2

$\therefore L_1 \cup L_2 \subseteq L(G_u)$

If x is derivable from S_u in G_u the first step in any derivation must be

$S_u \Rightarrow S_1$ or $S_u \Rightarrow S_2$

In the first case all subsequent production are must be production in because no variable in V_2 are involved and thus $x \in L_1$ and second case $x \in L_2$

$$\boxed{L(G_u) \subseteq L_1 \cup L_2}$$

② Concatenation :

A Grammar $G_c = (V_c, \Sigma, S_c, P_c)$ generating $L_1 \cdot L_2$

where $V_1 \cap V_2 = \phi$

$V_c = V_1 \cup V_2 \cup \{S_c\}$

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$P_c = P_1 \cup P_2 \cup \{ S_c \rightarrow S_1 . S_2 \}$

if $x \in L_1 . L_2$ then $x = x_1 x_2$ where $x_i \in L_i$ for i
we may then derive x in G_c as follows.

$S_c \Rightarrow S_1 S_2 \overset{*}{\Rightarrow} x_1 S_2 \overset{*}{\Rightarrow} x_1 x_2 = x$

x_1 in G_1 and
x_2 in G_2

if x is derived from S_c then $S_c \Rightarrow S_1 . S_2$ x
must be derived from $x = x_1 x_2$

where for each i, x_i can be derived from S_i in G_c

$\because V_1 \cap L_2 = \phi$, being derivable from S_i in G_c
means being derivable from S_i in G_i and so

$$\boxed{x \in L_1 L_2}$$

③ kleene :

A Grammar $G^* = \{ V, \Sigma, S, P \}$ generating L_1^*

$V = \{ V_1 \cup \{ S_{*1} \} \}$

where, $S_1 \notin V_1$

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L_1^* contains string of the form

$x = x_1, x_2, \ldots x_k$

where $x_1 \in L_1$

Each x_i is derived from S_1, then to derive x from S. it is enough to be able to derive a string of k S_1's

$S \rightarrow S_1 S \mid \Lambda$ in P

$P = P_1 \cup \{S \rightarrow S_1 S \mid \Lambda\}$

if $x \in L(G^*)$ then on other hand $x = \Lambda$ or x can be derived from some string of the form S_1^k in G^*, second case only production in G^* beginning with S_1 are those G_1 we conclude

$$\boxed{x \in L(G_1)^k \subseteq L(G_1)^*}$$

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* Derivation tree and ambiguity :

Like any language english or other there is some fixed strcture like that in CFG it contain also fixed pattern called Derivation tree or Parse tree.

Rules :

① Root node in Parse/Derivation tree is always tabled with start symbol

② All the terminal symbols are labeled with some terminal symbols of the grammar. So nodes are called terminal or leaf node

③ If string contain n symbols then it arranged from left to right

④ Non terminal symbols labbled with nonterminal string.

⑤ If the string contain $\{\wedge\}$, ϵ, λ empty symbol then it will be ignored.

Ex : ① If the Grammar $G = \{V, \Sigma, S, P\}$ in such a way that

① $V = \{S, A, B\}$

② $T / \Sigma = \{a, b\}$

③ $P \Rightarrow \{ S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow b \}$

④ $S \Rightarrow$ starting symbol

Derivation tree

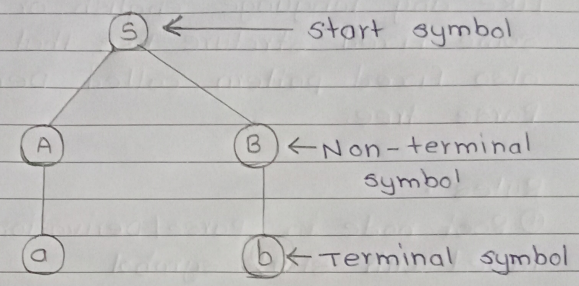

fig: Derivation tree for string ab

Ex ②: Draw the derivation tree for string aabbaa using following grammar:

① $G = (V_n, \Sigma, P, S)$

② $V_n = \{S, A\}$, $\Sigma = \{a, b\}$

③ $P = \{$

$S \rightarrow aAS$

$A \rightarrow SbA$

$S \rightarrow a$

$A \rightarrow ba$

$\}$

④ $S \Rightarrow$ starting symbol

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Now, S is start symbol so any string genera-
-ted will stored at S

$S \rightarrow aAS$

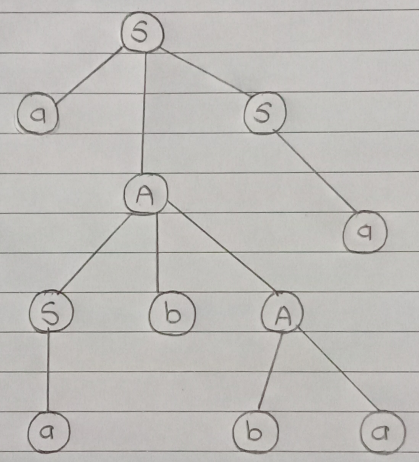

fig: Derivation tree

Reading from left the string is aabbaa hence
Ans.

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* Ambiguity / Ambigous Grammar :

In case of CFG they called as Ambigous if,

① It contain terminal string with more than one parse tree.

② More than one leftmost derivation and rightmost derivation.

Ex : ① If $G = (V, \Sigma, P, S)$ be a grammar with

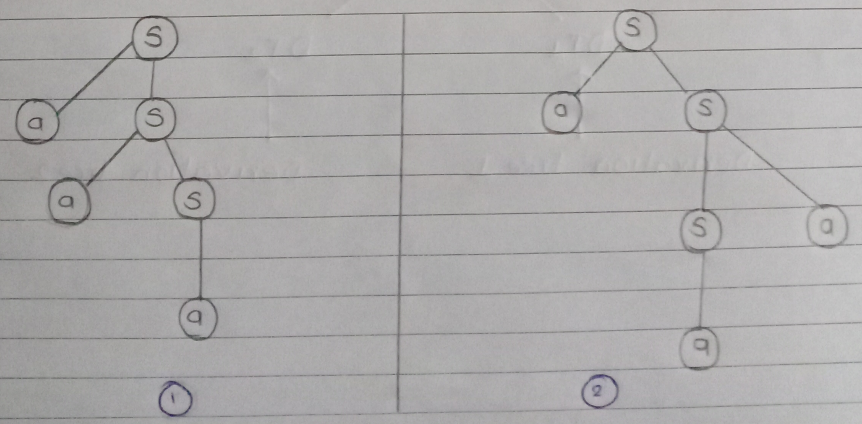
$G = (\{S\}, \{a\}, P, S)$

$V = \{S\}$

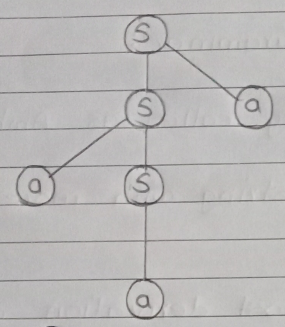
$\Sigma / T = \{a\}$

$P = \{S \rightarrow aS \mid SaS\}$ find out

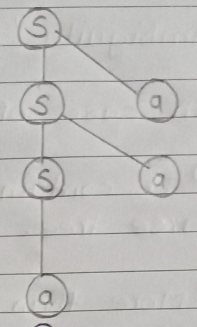
Parse tree for aaa



① ②



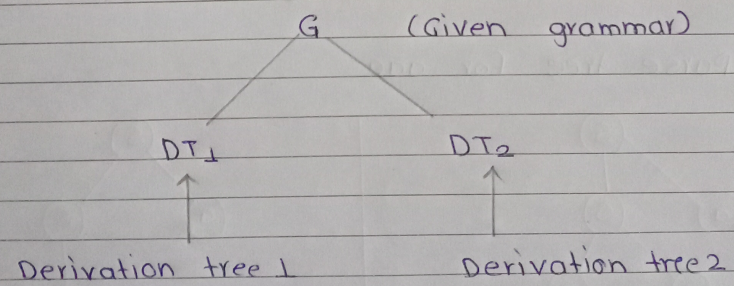
③



④

by above 4 way we can represent the aaa so it is ambigous grammar.

If given grammar having more than one derivation tree then it is called as an Ambigous grammar.



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Ex ②: The following grammar:

$S \rightarrow A$
$S \rightarrow bSa$
$A \rightarrow \epsilon$
$A \rightarrow AcA$

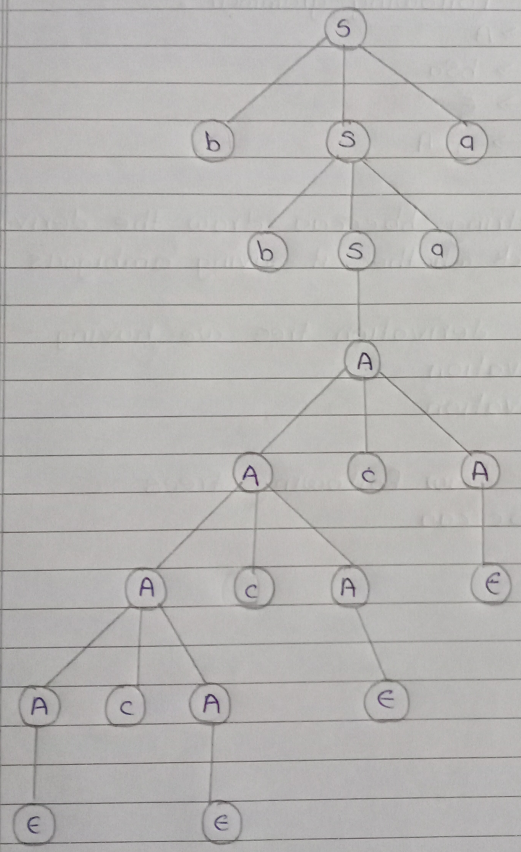
It accepts string bbcccaa draw the derivation tree and check whether it having ambigous or not.

→ Now to find derivation tree we having-
① left derivation
② Right derivation

Now we can draw following trees
String ⇒ bbcccaa

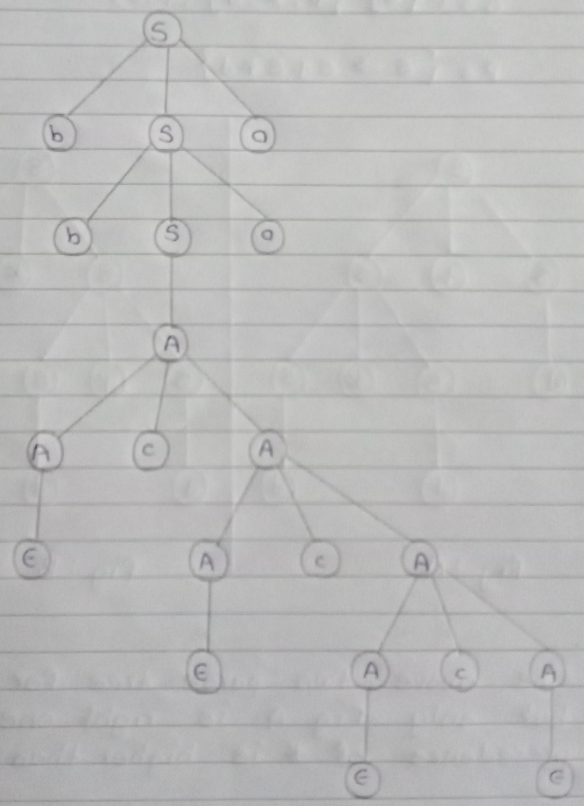
$S \rightarrow A$
$S \rightarrow bSa$
$A \rightarrow e$
$A \rightarrow AcA$

Tree no 1:



bbcccaa string accepting.

Tree No 2 :



bbccc aa string accepting

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Ex ③: If $G = (V, \Sigma, S, P\}$ be with grammar

$G = (\{S\}\ \{+, *, d\}, P, S)$

$P = \{S \rightarrow S+S*d\}$

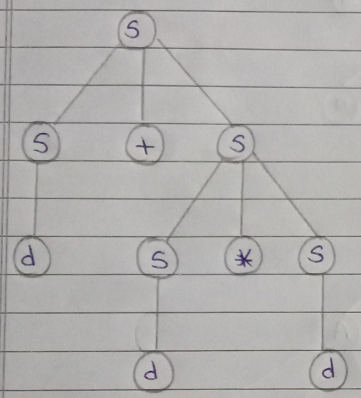

fig : A

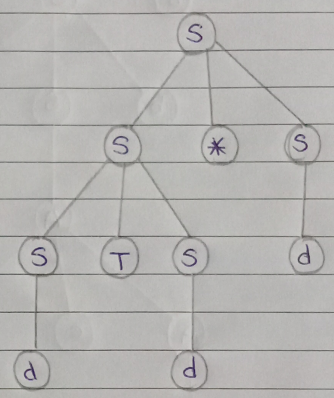

fig : B

The above two parse tree for d+d*d but only fig A is right one because presedence of * is higher than +

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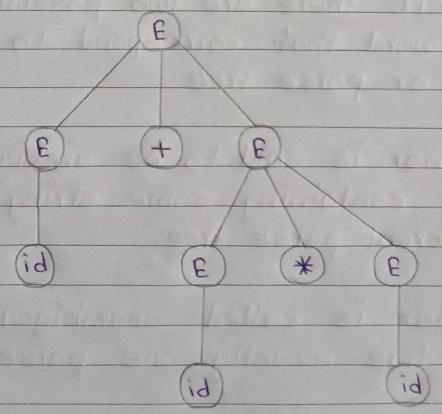
$V = \{E\}$
$\Sigma = \{id\}$
$S = E$
$P \Rightarrow E \rightarrow E+E$
$E \rightarrow E*E$
$E \rightarrow id$

By using above 3 productions we can construct

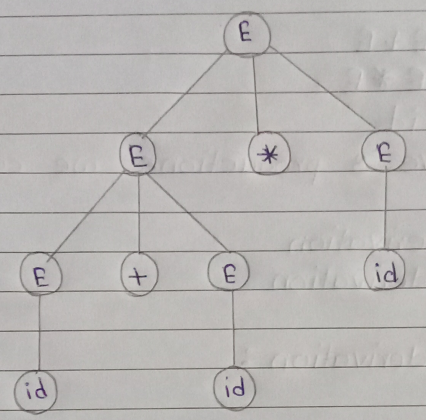
① Leftmost derivation
② Rightmost derivation

① Leftmost derivation :

$E \rightarrow E+E$



② Rightmost derivation :



A CFG is ambigous if for atleast one word in its CFL there are two possible derivation of the word that correspond to two different syntax or parse tree

A grammar is said to be ambigous if its language containing some string that has two different parse tree.

A grammar is called unambigous if its language has exactly one parse tree

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* Simplification of CFG :

Simplification of CFG :

① Eliminating useless symbol

② Eliminating unit productions

③ Eliminate $\wedge$ production / ϵ

* Useful and useless symbol :

Useful	Useless
① Derives a string of terminals	Does not derive a string of terminals
② It is used in derivation of atleast one ω in L (CFG)	It does not occur in derivation sequence of any ω in L (CFG)

Ex : find useful and useless symbol in

$G = (V, \Sigma, S, P)$

$V = \{S, A, B\}$

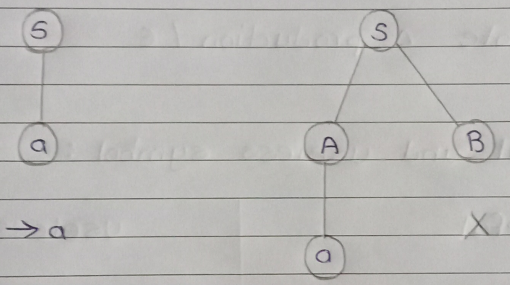
$T / \Sigma = \{a\}$

$P \Rightarrow \{ S \rightarrow a \mid AB$ and

$A \rightarrow a \}$

The starting symbol S can be terminate at ① $S \to a$

② If we take ② $S \to AB$
Hence A & B are not useful are useless symbol



① $S \to a$

useful symbol : S
useless symbol : A, B ② $S \to AB$
final production will $S \to a$

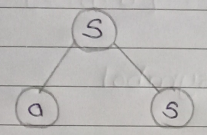
Ex ② $G = (V, T, P, S)$
$V = \{S, A, B, C\}$
$T/\Sigma = \{a, b\}$
S = starting symbol

$P : \Rightarrow$ $S \to aS \mid A \mid C$
$A \to a$
$B \to aa$
$C \to acb$

As S is starting symbol. The starting symbol is always useful.

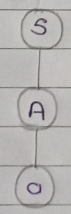
Now check reachability from start state

① $S \rightarrow aS$



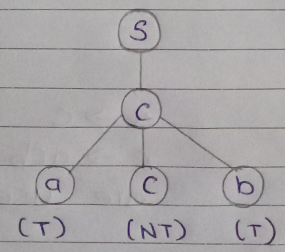
$S \rightarrow aS$ is useful

② $A \rightarrow a$



$A \rightarrow a$ is useful

③ $S \rightarrow C$
$C \rightarrow acb$
$B \rightarrow aa$



Not useful
$S \rightarrow C$
$C \rightarrow acb$
$B \rightarrow aa$

Ex ③ : P ⇒ $S \to aS$

$S \to A$

$S \to C$

$A \to a$

$B \to aa$

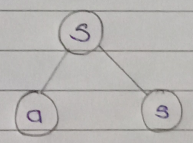
$C \to aCb$

S = starting symbol

$V = \{A, B, C, S\}$ $T / \Sigma = \{a, b, s\}$

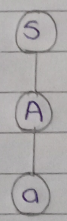
→ useful and useless symbol

① starting symbol always useful



$S \to aS$ is useful

② $S \to A$

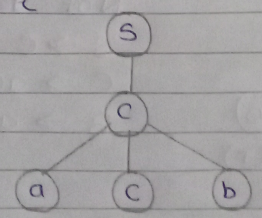


A is useful

$S \to A$

$A \to a$

③ $S \to C$



$S \to C$

$C \to aCb$

are useless

$B \to aa$ is useless

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Ex: $G = (V, \Sigma, S, P)$

$V = \{S, A, B, C\}$

$T = \{a, b\}$

$P \Rightarrow S \rightarrow AB \mid CA$

$B \rightarrow BC \mid AB$

$A \rightarrow a$

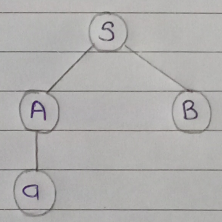
$C \rightarrow aB \mid b$

S = starting symbol

S is starting symbol. The starting symbol is useful variable.

① $S \rightarrow AB$

$B \rightarrow AB$



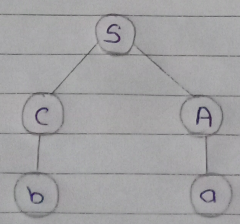
$S \rightarrow AB$

$B \rightarrow AB$ are useless

② $S \rightarrow CA$

$A \rightarrow a$

$C \rightarrow b$



$S \rightarrow CA$

$A \rightarrow a$

$C \rightarrow b$ are useful

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* Elimination of Λ productions :

The partation of the form $A \to \Lambda$ is called null production. It may also represent like,

$$A \to \Lambda$$
$$A \to \epsilon$$

Ex :

$G = (V, \Sigma, S, P)$
$V = \{S, A, B, C, D\}$
$\Sigma = \{a, b, \Lambda\}$
S = starting symbol
Find nullable variable :

① $S \to ABCBCDA$
$A \to CD$
$B \to Cb$
$C \to a | \Lambda$
$D \to bb | \Lambda$

$\to$ ① $C \to \Lambda$ and $D \to \Lambda$ so C and D are nullable variable

② $S \to ABCBCDA$
$\to AB\Lambda B\Lambda\Lambda A$
$\to ABBA$
$\to CDcbcbCD$
$\to \Lambda\Lambda\Lambda b\Lambda b\Lambda\Lambda$
$S \to bb$
S is not nullable

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③ $A \to CD$
$A \to \Lambda\Lambda$
$A \to \Lambda$
A is nullable

④ $B \to cb$
$B \to \Lambda b$
$B \to b$
B is not nullable

Finally
Nullable variable = A, C, D
Not nullable variable = B, S

Ex :
$S \to ACB \mid cbB \mid Ba$
$A \to da \mid BC$
$B \to gC \mid \Lambda$
$C \to ha \mid \epsilon$

Find : ① Remove null production
② Find nullable variable

→ Null productions are $B \to \Lambda$
$C \to \Lambda$

$S \to ACB \mid cbB \mid Ba$

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$S \rightarrow ACB \mid AC \mid AB \mid A$
$S \rightarrow cbB \mid cb$
$S \rightarrow Ba \mid a$

② $A \rightarrow da \mid BC$
$A \rightarrow da$
$A \rightarrow Bc \mid B \mid C$

③ $B \rightarrow gc \mid \Lambda$
$B \rightarrow gc \mid g$

④ $C \rightarrow ha$

Nullable variables.

$V = \{S, A, B, C\}$
$B \rightarrow \Lambda$ and $C \rightarrow \Lambda$
B and C are nullable

$S \rightarrow ACB \mid cbB \mid Ba$
$S \rightarrow A\Lambda\Lambda \mid cb\Lambda \mid \Lambda a$
$S \rightarrow A \mid cb \mid a$
S is not nullable

$C \rightarrow ha \mid \epsilon$
C is nullable

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$A \rightarrow da \mid BC$
$A \rightarrow da \mid nn$
$A \rightarrow da$

A is not nullable

Nullable are : B and C

Not nullable are : A and S

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* Remove Null / Epsilon Product

Ex : $S \rightarrow AB$
$A \rightarrow aA \mid a$
$B \rightarrow bB \mid \epsilon$

Step I : Now we have epsilon symbol

$B \rightarrow \epsilon$
Initially
$S \rightarrow AB$
$A \rightarrow aA \mid a$
$B \rightarrow bB$

Step II : Now replace $B \rightarrow \epsilon$ whenever applicable at right side

$S \rightarrow A\epsilon$
$A \rightarrow aA \mid a$
$B \rightarrow b\epsilon$

i.e $S \rightarrow A$
$A \rightarrow aA \mid a$
$B \rightarrow b$

Hence final result will be

$S \rightarrow AB \mid A$
$A \rightarrow aA \mid a$
$B \rightarrow bB \mid b$

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* Removing Unit Production:

If $G = (V, \Sigma, S, P)$ is CFG and we have a production such a way that

$$V_1 \rightarrow V_2$$

Where V_1 and V_2 are non terminal symbols then $V_1 \rightarrow V_2$ called unit production.

$$V_1, V_2 \in V$$

Ex: ①

①	$S \rightarrow A$	Unit production
②	$S \rightarrow a$	Not unit production
③	$S \rightarrow \wedge \mid \epsilon$	Null production
④	$S \rightarrow Aa$	Not unit production

Ex ②:

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow c \mid b$
$C \rightarrow D$
$D \rightarrow E$
$E \rightarrow a$

Identify unit and non unit production

unit production :

1 N.T → 1 N.T symbol symbol

unit production :

B → C
C → D
D → E

Non unit production :

S → AB
A → a
B → b
E → a

Ex ③ : S → ABC / Λ
B → a/b
C → D
E → aB
F → AB

unit production	Null production	non unit production
C → D	S → Λ	S → ABC B → a/b E → aB F → AB

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* Removing Unit production :

Production of the form $A \to B$, $B \to S$ are called unit production

Ex :

$S \to A \mid bb$ — ①
$A \to B \mid b$ — ②
$B \to S \mid a$ — ③

from ① and ②

$B \to bb \mid a$ — ④

from ② and ④

$A \to bb \mid a \mid b$ — ⑤

from ① and ② and ⑤

$S \to b \mid bb$

The final result will be

$S \to bb$
$A \to b$
$B \to a$

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* Eliminating useless variable from a Context free grammar:

A context free grammar is $\{V, \Sigma, S, P\}$ combines some Nonterminal (V) and terminal (Σ / T) symbol

If the given Nonterminal symbol is not in use called as useless symbol or variable

Now consider following grammar

$G = \{V, \Sigma, S, P\}$
$V = \{A, B, S\}$
$\Sigma = \{a, b\}$

$S \rightarrow AA \mid 1A0 \mid B \mid 0A1 \mid 01$ —— ①

A and B are nonterminals and 0 and 1 are terminal symbol.

$S \rightarrow AA \mid 1A0 \mid 0A1 \mid 01$ —— ②

B is useless symbol

both ① and ② are identical

Remove those string whose string contain least one non used string.

Ex : $G = \{V, T, S, P\}$

$V = \{S, C, D\}$

$T = \{d\}$

$P \Rightarrow S \rightarrow CD \mid d$

$D \rightarrow d$

→ From above production we can say C is useless

So, we can remove C

$S \rightarrow d$

$D \rightarrow d$

$G = \{\{S\}, \{d\}, \{S\}, \{P\}\}$

Now we observe $S \rightarrow d$ doesn't contain any nonterminal symbol D in it so it become,

$S \rightarrow d$ only by removing $D \rightarrow d$

Ex ② : $S \rightarrow a \mid aA \mid B \mid C$

$A \rightarrow aB \mid \epsilon$

$B \rightarrow Aa$

$C \rightarrow cCD$

$D \rightarrow fff$

find : ① unit production ④ ∧ Production

② useful variable ⑤ Nullable variable

③ useless variable

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① unit Production :

$G = (V, \Sigma, S, P)$ and we have production such a way that

$$\boxed{\perp NT \rightarrow \perp NT}$$

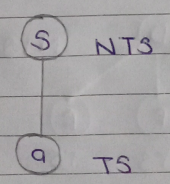
$$\boxed{NT \in V}$$

∴ unit production :- $S \rightarrow B \mid C$

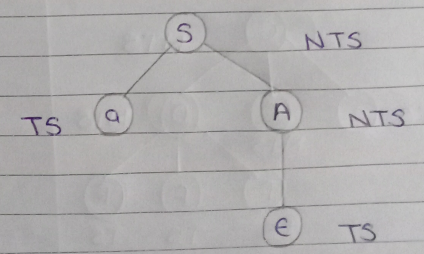
Non Unit production :- $S \rightarrow a \mid aA$
$A \rightarrow aB$
$B \rightarrow Aa$
$C \rightarrow cCD$
$D \rightarrow fff$

② useful and useless variable :

① $S \rightarrow a$



② $S \rightarrow aA$



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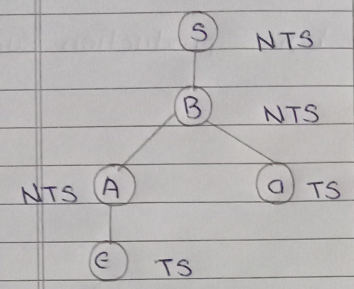
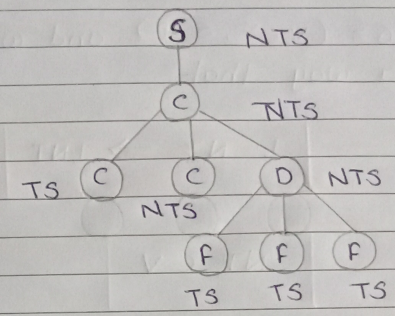
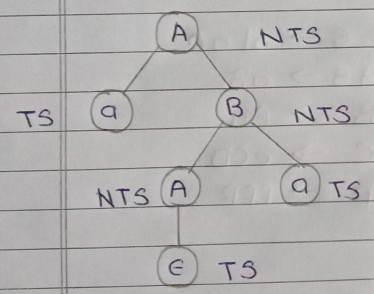
③ $S \rightarrow B$



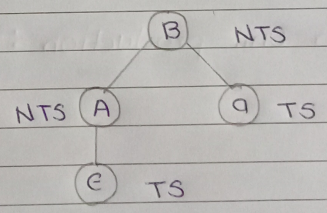
④ $S \rightarrow C$



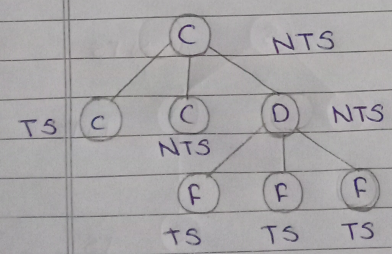
⑤ $A \rightarrow aB$



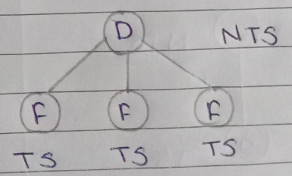
⑥ $B \rightarrow Aa$



⑦ $C \rightarrow cCD$



⑧ $D \rightarrow fff$



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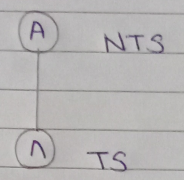
⑨ $A \rightarrow \epsilon \mid \wedge$



useful variable : S, A, B

useless variable : C, D

③ Null production :

$A \longrightarrow \epsilon$

N.T symbol — Null / Epsilion Symbol

In given production $A \rightarrow \epsilon$ is null production

④ Nullable variable :

$A \rightarrow \epsilon$ ∴ A is nullable

∴ Nullable variable : A

Not Nullable variable : S, B, C, D

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* Chomskey Normal form :

In a CFG the production is of the form

$A \rightarrow BC$ $\quad$ $A \rightarrow 2NT$

$A \rightarrow a$ $\quad$ $S \rightarrow TS$

i.e.,

Nonterminal $\rightarrow$ string of two non terminals

Nonterminal $\rightarrow$ Terminal symbol

then they called chomskey normal form

Ex: If two productions $A \rightarrow aAb$ and

$B \rightarrow ab$ and

replace $x_a \rightarrow a$, $x_b \rightarrow b$

$A \rightarrow x_a A x_b$

$B \rightarrow x_a x_b$

Also

$A \rightarrow BCDBGE$ would be replace by

$A \rightarrow BY_1$

$Y_1 \rightarrow CY_2$

$Y_2 \rightarrow DY_3$

$Y_3 \rightarrow BY_4$

$Y_4 \rightarrow CE$

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Hence, Now variable Y_1, Y_2, Y_3, Y_4 are production. so it is CNF

If CFG $G = (V, \Sigma, S, P)$ be in CNF and $G' = (V', \Sigma, S, P')$ then

$$L(G') = L(G) - \{\Lambda\}$$

Ex ②:

$S \rightarrow bA \mid aB$
$A \rightarrow bAA \mid aS \mid a$
$B \rightarrow aBB \mid bS \mid a$

① $S \rightarrow bA$

$S \rightarrow bA$ —— ①

Take $b = S_1$ in eq^n ①

b = terminal symbol
S_1 = Non terminal symbol

$\therefore S \rightarrow S_1 A$ CNF

② $S \rightarrow aB$

put $a = S_2$

$S \rightarrow S_2 B$ CNF

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③ $A \rightarrow bAA$
$A \rightarrow S_1AA$ —②

put $S_3 = S_1A$ in ②

$\therefore A \rightarrow S_3A$ CNF

④ $A \rightarrow aS$
$A \rightarrow S_2S$ CNF

⑤ $A \rightarrow a$ already CNF

⑥ $B \rightarrow aBB$
$B \rightarrow S_2BB$

$S \rightarrow SB$ CNF

⑦ $B \rightarrow bS$
$B \rightarrow S_1S$ CNF

⑧ $B \rightarrow a$ already CNF

Required CNF

$V = \{S, A, B, S_1, S_2, S_3\}$

$T/\Sigma = \{a, b\}$

$P = \{ S \rightarrow S_1A \mid S_2B$ S = starting symbol
$A \rightarrow S_3A \mid S_2S \mid a$
$B \rightarrow SB \mid S_1S \mid a \}$

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* Converting CFG to CNF

Let G be the grammar of the production

Ex: $S \rightarrow AACD$
$A \rightarrow aAb \mid \Lambda$
$C \rightarrow aC \mid a$
$D \rightarrow aDa \mid bDb \mid \Lambda$
convert into CNF

① Eliminating Λ production:

In above example nullable variable are A & D

$S \rightarrow AACD \mid ACD \mid AAC \mid CD \mid AC \mid C$

$A \rightarrow aAb \mid ab$

$C \rightarrow aC \mid a$

$D \rightarrow aDa \mid bDb \mid aa \mid bb$

② Eliminating unit production:

Now from production ① and ③ it is

$S \rightarrow aC \mid a$
where we delete $S \rightarrow C$

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③ Restricting the right side of production to single terminals or string of two or more variable

$S \rightarrow AACD \mid ACD \mid AAC \mid CD \mid AC \mid X_aC \mid a$

$A \rightarrow X_aAX_b \mid X_aX_b$

$C \rightarrow X_aC \mid a$

$D \rightarrow X_aDX_a \mid X_bDX_b \mid X_aX_a \mid X_bX_b$

$X_a \rightarrow a$

$X_b \rightarrow b$

④ Final step to CNF

$S \rightarrow AT_1 \qquad T_1 \rightarrow AT_2 \qquad T_2 \rightarrow CD$

$S \rightarrow AU_1 \qquad U_1 \rightarrow CD$

$S \rightarrow AV_1 \qquad V_1 \rightarrow AC$

$S \rightarrow CD \mid AC \mid X_aC \mid a$

$A \rightarrow X_a\omega_1 \qquad \omega_1 \rightarrow AX_b$

$A \rightarrow X_aX_b$

$C \rightarrow X_a C | a$

$D \rightarrow X_a Y_a \qquad Y_1 \rightarrow D X_a$

$D \rightarrow X_b Z_1 \qquad Z_1 \rightarrow D X_b$

$D \rightarrow X_a X_b | X_b X_b$

$X_a \rightarrow a$

$X_b \rightarrow b$

Ex ②: $S \rightarrow ASA$
$S \rightarrow BSB$
$S \rightarrow AA$
$S \rightarrow BB$
$S \rightarrow a$
$S \rightarrow b$
$A \rightarrow a$
$B \rightarrow b$

① $S \rightarrow AX_1$
where $X_1 \rightarrow SA$

② $S \rightarrow BX_2$
where $X_2 \rightarrow SB$

Hence required CNF is

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$S \to AX_1$
$S \to BX_2$
$S \to AA$
$S \to BB$
$S \to a$
$S \to b$
$A \to a$
$B \to b$

③ Ex :

$S \to ABC$
$A \to a|b$
$B \to Bb|bb$
$C \to aC|cc|ba$

$G = (V, \Sigma, S, P)$

$V = \{S, A, B, C\}$

$\Sigma = \{a, b\}$

S = starting symbol

→ CNF : 1NT → 2NT
1NT → 1TS

① Take $S \to ABC$ —①
put $S_1 \to AB$ in ①
$S \to S_1C$ CNF

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② $A \rightarrow a|b$ CNF

③ $B \rightarrow Bb|bb$

take $A \rightarrow a|b$

$B \rightarrow BA|AA$ CNF

③ $C \rightarrow ac|cc|ba$ —— ③

where $A \rightarrow a|b$ in ①

$C \rightarrow Ac|cc|AA$ CNF

Non terminal symbols.

$V = \{S, S_1, A, B, C\}$