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Q1) Prove that

$$P_t = PM^t - \gamma\left[\frac{M^t - 1}{M-1}\right]$$ is true for $t \geq 0$ by induction method.

1) Basis step:-

Now

$$P_t = PM^t - \gamma\left[\frac{M^t - 1}{M-1}\right]$$

put $t=0$

$$P_0 = PM^0 - \gamma\left[\frac{M^0 - 1}{M-1}\right]$$

$$\boxed{P_0 = P}$$

Now $P = P_0$ hence we have proved the basis step.

2) Induction step:-

Now for every $k \geq 0$ assume the formula is true

Now Assume formula is true for $\underline{t=k}$

Now check for $\underline{t = k+1}$

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$$P_k = PM^k - Y\left[\frac{M^k - 1}{M-1}\right]$$

Now we have to prove

$$P_{k+1} = PM^{k+1} - Y\left[\frac{M^{k+1} - 1}{M-1}\right]$$

Now we know that

$$\boxed{P_{k+1} = P_k M - Y}$$

$$P_{k+1} = \left[P \cdot M^{k} - Y\left[\frac{M^k - 1}{M-1}\right]\right] \cdot M - Y.$$

$$= PM^{k+1} - Y\left[\frac{M^{k+1} - M}{M-1}\right] - Y\left[\frac{M-1}{M-1}\right]$$

$$P_{k+1} = PM^{k+1} - Y\left[\frac{M^{k+1} - \cancel{M} + \cancel{M} - 1}{M-1}\right]$$

$$\boxed{P_{k+1} = PM^{k+1} - Y\left[\frac{M^{k+1} - 1}{M-1}\right]}$$

Hence o/p by induction method.

Q) Show that $\sqrt{2}$ is irrational.

$\Rightarrow$ Now initially assume that $\sqrt{2}$ is rational

so

$$\sqrt{2} = m/n$$

$$2 = \frac{m^2}{n^2}$$

$$2n^2 = m^2 \quad \text{--- (1)}$$

Now we know that if m & n are 2 intergers then

m^2 is twice of n^2 so

m^2 is even

m is even

Now put $m = 2k$

eq'n (1) becomes

$$2n^2 = 4k^2$$

$$n^2 = 2k^2$$

Hence k is also even

so n is also even

As Both m & n are even but it is contradiction hence $\frac{m}{n} = \sqrt{2}$ is a irrational

9) Mechanism to design FA. with sutiable example

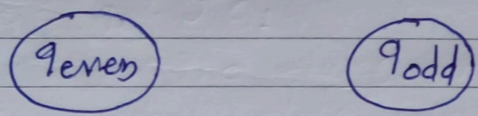
→ A language is said to be regula language if some finite automata recognis it.

Consider yourself as machine.
Try to design
Consider whole design process
Take a string & read from one by one symbol
Finite Automata - finite memory.

Now Take alphebet {0,1} such a way to find even & odd no of 1's.

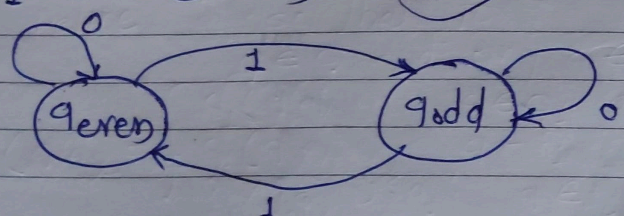
qeven	qodd
even possiblity	odd possiblity

Two states



Two states qeven & qodd

0 = Transition contain same state
1 = Contain odd state



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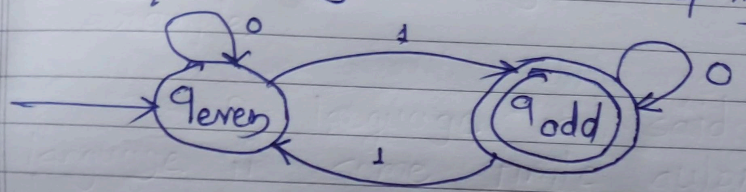
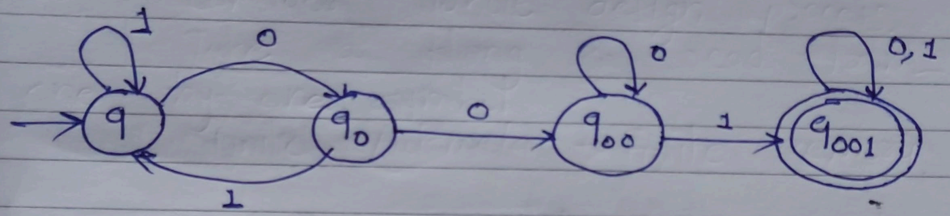
By Taking odd as accepting state



eg string / Accepting string 001



Regular Operations

1) Union $A \cup B = \{x \mid x \in A \wedge x \in B\}$

2) concatenation $A \circ B$

3) Kleene / closure A^*

Binary operations

Unary

9) Prove by induction that every $n \geq 0$

i) $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2) $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

1) $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Proof No - 01

for $n \geq 0$

$$0+1+2+\cdots+n = \frac{n(n+1)}{2}$$

Now

when $n=0$ $\quad 0 = \frac{0(0+1)}{2}$ ——— (I)

$n=1$ $\quad 0+1 = \frac{0(1)}{2} + 0+1$

$$= \frac{0(1)+2(1)}{2}$$

$$= \frac{1(1+1)}{2} \text{ ——— (II)}$$

when $n=2$

$$0+1+2 = \frac{1(2)}{2} + 2$$

$$= \frac{1(2)+2(2)}{2} = \frac{2(2+1)}{2}$$

so on we will got.

$$n = n = \frac{n(n+1)}{2}$$

$$\boxed{0+1+2+\cdots+n = \frac{n(n+1)}{2}}$$

Now

<u>Proof</u> <u>No - 02</u>

we know that

$$0+1+2+\cdots+n = \frac{n(n+1)}{2}$$

when $n=k$.

$$0+1+2+\cdots+k = \frac{k(k+1)}{2}$$

For $\underline{\underline{n=k+1}}$

$$\underline{0+1+2+\cdots+k+k+1}$$

we know that

$$0+1+2+\cdots+k = \frac{k(k+1)}{2}$$

$$= \frac{k(k+1)}{2} + k+1$$

$$0+1+\cdots+k+1 = \frac{(k+1)(k+2)}{2}$$

hence it is proof.

Principale of mathematical induction

If $p(n)$ statement involving an integer n, Then to prove that $p(n)$ is true for every $n \geq n_0$ Here two things are necessary

1) $p(n_0)$ is true.
2) For any $k \geq n_0$ if $p(k)$ is true then $p(k+1)$ is true.

3) **Proof No-03** By Induction

Let $p(n)$ be stmt then

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$p(n)$ is true for every $n \geq 0$.

1) we must show that $P(0)$ is true.

$p(0)$ is stmt $0 = \frac{0(0+1)}{2} = 0$ & it is true

2) Induction hypothesis

if $k \geq 0$ and $1+2+3+\cdots+k = \frac{k(k+1)}{2}$

3) stmt to be shown in induction step.

$$1+2+\cdots+k+1 = \frac{(k+1)((k+1)+1)}{2}$$

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Proof By induction step.

$1+2+3+\cdots+k+(k+1)$

$= \underbrace{1+2+\cdots+k}_{A} + \underbrace{k+1}_{B}$

by putting value of A.

$= \frac{k.(k+1)}{2} + k+1$

$= \frac{k(k+1)+2(k+1)}{2}$

$$\boxed{1+2+\cdots+k+1 = \frac{(k+1)(k+2)}{2}}$$

2) By induction step proove.

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

① Basis step. by taking $n=0$

$P(0) = 0$ $\quad 0^2 = 0$

$P(1) = \frac{1(1+1)(2+1)}{6} = \frac{6}{6}$ $\quad 1^2 = 1$

$p(2) = \frac{2(3)(4+4)}{6} = 5$

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Now
Induction hypothesis

$k \geq 0 \quad 1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

stmt to be shown by induction step.

$n = k+1$

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= k+1\left[\frac{k(2k+1)}{6} + k+1\right]$$

$$= k+1\left[\frac{2k^2 + k + 6k + 6}{6}\right]$$

Q) Prove using determinism that the class of regular languages is closed under the union operation.

DFA :-

Deterministic finite automata is a 5 tuple

$M = (Q, \Sigma, q_0, A, \delta)$

Q = Finite No of states
Σ = set of alphebets
q_0 = initial state
A / F = Accepting or final state
δ = Transition funin

Now Assume

$M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$ &
$M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$ be the two DFA recognizes a regular language A_1 & A_2

Now we have to prove

$A_1 \cup A_2$

Now this is proof by construction
As we know

$A = A_1 \cup A_2$

ie $M = (Q, \Sigma, q_0, A, \delta)$ is a union of $M_1 \cup M_2$ in such a way that it recognize A.

Now Assume

M_1 contain K_1 No of states

M_2 contain K_2 No of states

so M contain $K_1 \times K_2$ No of states in it.

Let.

M_1 recognize A_1 where $M_1 = (Q_1, \Sigma, q_1, \delta_1, A_1)$

$M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$

$M = A_1 \cup A_2$ (recognize) &

$M = (Q, \Sigma, \delta, q_0, A)$

1) Now $Q = Q_1 \times Q_2$ ie cartesian Product

if $r_1 \in Q_1$ & $r_2 \in Q_2$ then cartesian product

$Q = \{ \langle r_1, r_2 \rangle \mid r_1 \in Q_1 \wedge r_2 \in Q_2 \}$

ie

r_1 from Q_1

r_2 from Q_2

2) Alphebet Σ.
is same in M_1 & M_2

It contain same input symbol in both M_1 & M_2
if the automata having different alphebets
ie M_1 having Σ_1
M_2 having Σ_2 then it remain same proof by

$$\boxed{\Sigma = \Sigma_1 \cup \Sigma_2}$$

3) q_0 ie initial state
initial state of M_1 &
initial state of M_2 ie q_0 is such a pair.

$$q_0 = (q_1, q_2)$$

4) A/F ie accepting state or final state.
If M_1 & M_2 are two automatas
then accepting state is in either M_1 or M_2

$$A/F = \{ \langle r_1, r_2 \rangle \mid r_1 \in A_1 \vee r_2 \in A_2 \}$$

$$A/F = \{ (A_1 \times Q_1) \cup (Q_2 \times A_2) \}$$

it is not same as

$$\boxed{F = F_1 \times F_2}$$

3) If δ is transition funn in such a way that $(r_1, r_2) \in Q$ & $a \in \Sigma$

$$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$$

Concatenation Operation

If M_1 & M_2 are two automatas then

$$M = M_1 \circ M_2$$

M recognizes $A_1 \circ A_2$

M accept if either M_1 or M_2 then
M_1 accept first part
M_2 accept second part.
it doesn't know where to split
it uses Nondeterminism technique