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* Kleenes theorem Part 2

A language accepted by finite automata is regular.

Proof :

If $M = (Q, \Sigma, q_0, A, \delta)$ is finite automata.

① If we consider p and q as two sets then by using mathematical induction we can take union of it. which is regular.

② If p is an initial state then transition from p to q should be going from p to q from p to q in k state, if k is large enough then L will be L (p, q).

③ Let consider n states in the from 1 to n and path going throughs. S

Now consider the string x, y, z and $x = yz$

$$\delta^*(p, y) = s$$

$$\delta^*(s, z) = q$$

Now,

$$p \xrightarrow{a} q \xrightarrow{b} r$$

If Language $L(P,q)$ for n states then

$$\boxed{L(P,q,n) = L(P,q)}$$

where n is not higher i.e., no of states higher than n.

if $L(P,q,n)$ is regular then $L(P,q,j)$ is also regular if

$$\boxed{0 \leq j \leq n}$$

if $j \leq n$ then

$$L(P,q,n) = L(P,q,j)$$

while proning basic steps i.e, $L(P,q,0)$ is regular

i.e., the state does not contain more than 0 state i.e., it contain only one symbol i.e., finite so, $L(P,q,0)$ is regular.

By induction hypothesis
if

$L(P,q,k)$ is regular.

then

$L(P,q,k+1)$ is also regular.

$L(p, q, k+1)$

The path from p to q doesn't contain path higher than (k+1) state.

Let consider ywz be the states in such a way.

y ⇒ from state p to 1st to q

w ⇒ from (k+1) to q

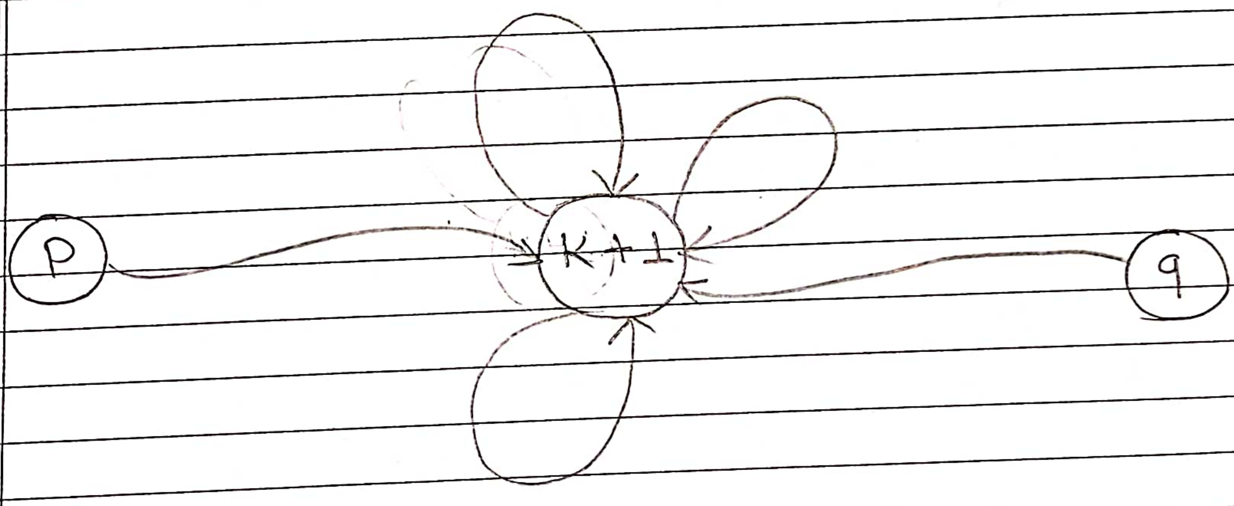
z ⇒ from (k+1) to back itself.

i.e.,

$y \in L(p, k+1, k)$

$w \in L(k+1, q, k)$

$z \in L(k+1, k+1, k)^*$



it follows either of following two cases

$x \in L(P,q,k) \cup L(P,k+1,k)\, L(k+1,q,k)\, L(k+1,k+1,k)^*$ — ①

on the other hand by using ①

$$\boxed{x \in L(P,q,k+1) = L(P,q,k) \cup L(P,k+1,k)\, L(k+1,q,k)\, L(k+1,k+1,k)^*}$$

from the above formula we have proved it is regular because it satisfy the union, concatenation and kleene operation.