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Section: D
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Tutorial-4

Date.........

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol^n.

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a \geq 1,\ b > 1$

On comparing

$a = 3,\ b = 2,\ f(n) = n^2$

Now,

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

Sol^n:

$a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

3. $T(n) = T(n/2) + 2^n$

Sol^n:

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

Sol^n: $a = 2^n$

$b = 2,\ f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

Sol^n: $a = 16,\ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

Sol^n: $a = 2,\ b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $n\log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n\log n)$

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7. $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$\underline{Sol^n}$: $a=2$, $b=2$, $f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$\underline{Sol^n}$: $a=2$, $b=4$, $f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + 1/n$

$\underline{Sol^n}$: $a = 0.5$, $b = 2$

Since acc. to Master theorem $a \geq 1$, but here a is 0.5 so we cannot apply master theorem.

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$\underline{Sol^n}$: $a=16$, $b=4$, $f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

11. $4T(n/2) + \log n$

$\underline{Sol^n}$:

$a=4$, $b=2$, $f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

12. $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$\underline{Sol^n}$: $a = \sqrt{n}$, $b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

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13. $T(n) = 3T(n/2) + n$

Sol: $a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$\therefore n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \theta(n^{1.5849})$

14. $T(n) = 3T(n/3) + sqrt(n)$

Solⁿ: $a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

As $sqrt(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

15. $T(n) = 4T(n/2) + cn$

Solⁿ: $a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn < n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

Solⁿ: $a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

Solⁿ: $a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

18. $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2\log n$

$\therefore T(n) = \theta(n^2\log n)$

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19. $T(n) = 4T(n/2) + n/\log n$

Solⁿ: $a = 4$, $b = 2$, $f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

Solⁿ: $a = 64$, $b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$\therefore n^c = n^2$

$\therefore n^2 \log n > n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

21. $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

Solⁿ: $a = 1$, $b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$