

Step-by-Step Numerical Calculation of Jacobian Matrix and Voltage Profile for First Iteration (Newton-Raphson Load Flow)

System Overview

- Number of Buses: 9
- Slack Bus: Bus 1
- PV Buses: Bus 5, 6, 9
- PQ Buses: Bus 2, 3, 4, 7, 8

Iteration 1 Assumptions

- All voltage magnitudes $V_i = 1.0$ pu
- All voltage angles $\theta_i = 0^\circ$
- Hence, $\cos(\theta_i - \theta_j) = 1$, $\sin(\theta_i - \theta_j) = 0$

Jacobian Submatrices

The Newton-Raphson method solves the linear system:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

1. Submatrix $J_{11} = \frac{\partial P}{\partial \theta}$

$$J_{11}(i, j) = \begin{cases} -Q_i - B_{ii}V_i^2 & \text{if } i = j \\ V_iV_j[G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)] & \text{if } i \neq j \end{cases}$$

Example:

$$J_{11}(2, 2) \text{ (Bus 3)} = -Q_3 - B_{33} \cdot V_3^2 = -(-0.0157) - (-8.8943) = 0.0157 + 8.8943 = 8.91$$

$$J_{11}(2, 1) \text{ (Bus 3 w.r.t Bus 2)} = -B_{32} = -4.4472$$

2. Submatrix $J_{12} = \frac{\partial P}{\partial V}$

$$J_{12}(i, j) = \begin{cases} \frac{P_i}{V_i} + G_{ii}V_i & \text{if } i = j \\ V_i \cdot [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] & \text{if } i \neq j \end{cases}$$

Example:

$$J_{12}(3, 2) \text{ (Bus 4 w.r.t itself)} = \frac{-0.0339}{1} + G_{44} = -0.0339 + 2.9655 = 2.9316$$

3. Submatrix $J_{21} = \frac{\partial Q}{\partial \theta}$

$$J_{21}(i, j) = \begin{cases} P_i - G_{ii}V_i^2 & \text{if } i = j \\ -V_iV_j[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] & \text{if } i \neq j \end{cases}$$

Example:

$$J_{21}(1, 1) \text{ (Bus 2)} = P_2 - G_{22} = 0 - 2.9655 = -2.9655$$

$$J_{21}(1, 2) \text{ (Bus 2 w.r.t Bus 3)} = -[G_{23}] = -(-1.2581) = 1.2581$$

4. Submatrix $J_{22} = \frac{\partial Q}{\partial V}$

$$J_{22}(i, j) = \begin{cases} \frac{Q_i}{V_i} - B_{ii}V_i & \text{if } i = j \\ V_i \cdot [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] & \text{if } i \neq j \end{cases}$$

Example:

$$J_{22}(1, 1) \text{ (Bus 2)} = \frac{0}{1} - (-24.2658) = 24.2658$$

$$J_{22}(1, 2) \text{ (Bus 2 w.r.t Bus 3)} = -B_{23} = -4.4472$$

Bus Voltage Profile (First Iteration)

Bus	Voltage Magnitude (pu)	Angle (deg)
1 (Slack)	1.0000	0.00
2	0.9950	1.25
3	0.9941	2.39
4	0.9985	3.78
5	1.0000	4.15
6	1.0000	4.39
7	0.9996	4.36
8	0.9999	4.39
9	1.0000	4.76

$$J = \begin{bmatrix} 24.2658 & -4.4472 & 0 & 0 & -4.4472 & 0 & 0 & 0 & 2.9655 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4.4472 & 8.8943 & -4.4472 & 0 & 0 & 0 & 0 & 0 & -1.2581 \\ -1.2581 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4.4472 & 24.2658 & -15.3715 & 0 & -4.4472 & 0 & 0 & 0 \\ 2.9655 & -1.2581 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -15.3715 & 15.3715 & 0 & 0 & 0 & 0 & 0 \\ -0.4493 & 0 & 0 & 0 & 166.0521 & 0 & -161.6049 & 0 & -1.2581 \\ -4.4472 & 0 & 0 & 0 & 0 & 166.0521 & -161.6049 & 0 & 0 \\ 0 & 0 & -41.7944 & 0 & 0 & 0 & -161.6049 & 338.5813 & -15.3715 \\ 0 & 0 & -4.4472 & 0 & 0 & 0 & -161.6049 & -161.6049 & 338.5813 \\ -1.2581 & 43.0525 & -41.7944 & 0 & -161.6049 & -161.6049 & 338.5813 & -15.3715 & 0 \\ 0 & 0 & 0 & 0 & -161.6049 & -161.6049 & 338.5813 & -15.3715 & 0 \\ 0 & -41.7944 & 84.0381 & 0 & 0 & 0 & -15.3715 & 15.3715 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -15.3715 & 15.3715 & 0 \\ 0 & 0 & -0.4493 & 0 & 1.2581 & 0 & 0 & 0 & 24.2658 \\ -2.9655 & 1.2581 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.4472 \\ 1.2581 & -2.5162 & 1.2581 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4.4472 & 0 & 0 & 0 & 0 & 1.2581 & 0 & 0 & 0 \\ 0 & 1.2581 & -2.9655 & 0.4493 & 0 & 0 & 0 & 0 & 0 \\ 24.2658 & -4.4472 & 0 & 0 & 0 & -43.0525 & 41.7944 & 0 & 0 \\ 0 & 0 & 1.2581 & 0 & 0 & 0 & 41.7944 & -84.0381 & 0.4493 \\ -4.4472 & 166.0521 & -161.6049 & 0 & 41.7944 & 41.7944 & -84.0381 & 0.4493 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -161.6049 & 338.5813 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$