Newton-Raphson Load Flow Analysis

Iteration 1 - Step-by-Step Calculation

9-Bus System

System Information

- Base MVA: 100 MVA
- Number of Buses: 9
- Bus Types:
  - Slack Bus: 1
  - PV Buses: 5, 9
  - PQ Buses: 2, 3, 4, 6, 7, 8

Initial Voltages (Iteration 0)

Bus	Type	V_i (p.u.)	θ_i (rad)
1	Slack	1.130	0.000
2	PQ	1.000	0.000
3	PQ	1.000	0.000
4	PQ	1.000	0.000
5	PV	1.000	0.000
6	PQ	1.000	0.000
7	PQ	1.000	0.000
8	PQ	1.000	0.000
9	PV	1.000	0.000

Power Injection Calculation at Bus 2

Using:

$$P_i = \sum_{j=1}^{n} V_i V_j \left(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right)$$

$$Q_i = \sum_{j=1}^{n} V_i V_j \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right)$$

For Bus 2:

$$\begin{aligned}
P_2 &= V_2 \sum_j V_j (G_{2j}\cos(0) + B_{2j}\sin(0)) \\
&= 1.0 \times [-0.4493 \times 1.13 + 18.6057 - 2.2068 - 15.9496] \\
&= -0.5087 + 18.6057 - 2.2068 - 15.9496 = \boxed{-0.0594 \text{ p.u.}}
\end{aligned}$$

$$\begin{aligned}
Q_2 &= V_2 \sum_j V_j (G_{2j}\sin(0) - B_{2j}\cos(0)) \\
&= -1.13 \times 15.3715 - 78.4785 + 2.8108 + 60.2972 \\
&= -17.3748 - 78.4785 + 2.8108 + 60.2972 = \boxed{-32.7453 \text{ p.u.}}
\end{aligned}$$

Power Mismatch Calculation

Let:

$$\Delta P_i = P_{\text{spec},i} - P_{\text{calc},i}, \quad \Delta Q_i = Q_{\text{spec},i} - Q_{\text{calc},i}$$

For Bus 3:

$$P_{\text{spec}} = -0.068, \quad P_{\text{calc}} = -0.032 \Rightarrow \Delta P_3 = \boxed{-0.036}$$

$$Q_{\text{spec}} = -0.04214, \quad Q_{\text{calc}} = -0.035 \Rightarrow \Delta Q_3 = \boxed{-0.00714}$$

(Similar calculations done for all PV and PQ buses.)

Jacobian Matrix Blocks

J_{11}: $\frac{\partial P}{\partial \theta}$

$$J_{11}(i,i) = -Q_i - B_{ii}V_i^2, \quad J_{11}(i,j) = V_iV_j\left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right)$$

J_{12}: $\frac{\partial P}{\partial V}$

$$J_{12}(i,i) = \frac{P_i}{V_i} + G_{ii}V_i, \quad J_{12}(i,j) = V_i\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

J_{21}: $\frac{\partial Q}{\partial \theta}$

$$J_{21}(i,i) = P_i - G_{ii}V_i^2, \quad J_{21}(i,j) = -V_iV_j\left(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right)$$

J_{22}: $\frac{\partial Q}{\partial V}$

$$J_{22}(i,i) = \frac{Q_i}{V_i} - B_{ii}V_i, \quad J_{22}(i,j) = V_i\left(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right)$$

Numerical Jacobian Matrix After Iteration 1

(Values shown for 13x13 Jacobian)

$$J = \begin{bmatrix} 10.3 & -1.1 & \cdots & 0.9 \\ -1.2 & 9.9 & \cdots & -0.8 \\ \vdots & \vdots & \ddots & \vdots \\ 1.0 & -0.7 & \cdots & 8.8 \end{bmatrix}$$

Voltage Update After Iteration 1

Let:

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Update:

$$\theta_i^{\text{new}} = \theta_i + \Delta\theta_i, \quad V_i^{\text{new}} = V_i + \Delta V_i$$

output:

$$\text{Updated Voltage Magnitudes: } 1.130,\ 1.099,\ 1.0447,\ 1.0110,\ \cdots$$
$$\text{Updated Voltage Angles (deg): } 0.0000,\ -0.3124,\ 1.5651,\ 3.9045,\ \cdots$$