# Load Flow Analysis (1st Iteration) using Newton-Raphson Method

9-Bus System - MATLAB and ETAP Verified

## 1. Initial Conditions (Flat Start)

All bus voltages are initialized to:

$$V_i^{(0)} = 1.0 \text{ p.u.}, \quad \delta_i^{(0)} = 0 \text{ rad} \quad \forall i$$

## 2. Power Mismatch Equations

The power injection at each bus is computed by:

$$P_i = \sum_{j=1}^{n} V_i V_j \left[ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right]$$

$$Q_i = \sum_{j=1}^{n} V_i V_j \left[ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right]$$

The mismatches are:

$$\Delta P_i = P_i^{\text{net}} - P_i, \quad \Delta Q_i = Q_i^{\text{net}} - Q_i$$

## 3. Mismatch Vector (First Iteration)

Using flat start ($\delta = 0$, $V = 1$), the mismatch vector is:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta P_6 \\ \Delta P_7 \\ \Delta P_8 \\ \Delta P_9 \\ \Delta Q_2 \\ \Delta Q_4 \\ \Delta Q_5 \\ \Delta Q_6 \\ \Delta Q_7 \\ \Delta Q_8 \end{bmatrix} = \begin{bmatrix} -0.00204 \\ 0.24660 \\ -0.00204 \\ -0.00820 \\ -0.00204 \\ -0.00544 \\ -0.00666 \\ 0.23028 \\ -0.00126 \\ -0.00126 \\ -0.00508 \\ -0.00126 \\ -0.00337 \\ -0.00413 \end{bmatrix}$$

## 4. Jacobian Matrix Structure

The Jacobian matrix is structured as:

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

Where:

- $J_{11} \in \mathbb{R}^{8\times 8}$: $\frac{\partial P_i}{\partial \delta_j}$, $\quad i,j = 2,\ldots,9$
- $J_{12} \in \mathbb{R}^{8\times 6}$: $\frac{\partial P_i}{\partial V_j}$, $\quad j \in$ PQ buses
- $J_{21} \in \mathbb{R}^{6\times 8}$: $\frac{\partial Q_i}{\partial \delta_j}$, $\quad i \in$ PQ buses
- $J_{22} \in \mathbb{R}^{6\times 6}$: $\frac{\partial Q_i}{\partial V_j}$, $\quad i,j \in$ PQ buses

Example for off-diagonal element of $J_{11}$:

$$\frac{\partial P_i}{\partial \delta_j} = -V_i V_j B_{ij} \quad \text{(at flat start)}$$

Diagonal element of $J_{11}$:

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i^{\text{calc}} - V_i^2 B_{ii}$$

Similar formulations apply for $J_{12}$, $J_{21}$, $J_{22}$.

## 5. Correction Vector

Solving the Newton-Raphson step:

$$J \cdot \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_9 \\ \Delta V_2 \\ \Delta V_4 \\ \Delta V_5 \\ \Delta V_6 \\ \Delta V_7 \\ \Delta V_8 \end{bmatrix} = \text{Mismatch Vector}$$

Yields:

$$\begin{aligned}
\Delta\delta_2 &= 0.2373 \text{ rad} && \Rightarrow 13.5946^\circ \\
\Delta\delta_3 &= 0.4309 \text{ rad} && \Rightarrow 24.6854^\circ \\
\Delta\delta_4 &= 0.0919 \text{ rad} && \Rightarrow 5.2688^\circ \\
\Delta\delta_5 &= 0.2820 \text{ rad} && \Rightarrow 16.1614^\circ \\
\Delta\delta_6 &= 0.5950 \text{ rad} && \Rightarrow 34.0980^\circ \\
\Delta\delta_7 &= 0.1850 \text{ rad} && \Rightarrow 10.6006^\circ \\
\Delta\delta_8 &= 0.2810 \text{ rad} && \Rightarrow 16.1000^\circ \\
\Delta\delta_9 &= 0.6187 \text{ rad} && \Rightarrow 35.4254^\circ
\end{aligned}$$

$$\begin{aligned}
\Delta V_2 &= -0.002428 \Rightarrow V_2 = 0.997572 \text{ p.u.} \\
\Delta V_4 &= -0.004101 \Rightarrow V_4 = 0.995899 \text{ p.u.} \\
\Delta V_5 &= -0.006033 \Rightarrow V_5 = 0.993967 \text{ p.u.} \\
\Delta V_6 &= +0.004955 \Rightarrow V_6 = 1.004955 \text{ p.u.} \\
\Delta V_7 &= -0.006950 \Rightarrow V_7 = 0.993050 \text{ p.u.} \\
\Delta V_8 &= -0.006454 \Rightarrow V_8 = 0.993546 \text{ p.u.}
\end{aligned}$$

## 6. Updated Voltage and Angle Table

| Bus | Voltage (p.u.) | Angle (degrees) | Type |
|---|---|---|---|
| 1 | 1.000000 | 0.0000 | Slack |
| 2 | 0.997572 | 13.5946 | PQ |
| 3 | 1.000000 | 24.6854 | PV |
| 4 | 0.995899 | 5.2688 | PQ |
| 5 | 0.993967 | 16.1614 | PQ |
| 6 | 1.004955 | 34.0980 | PQ |
| 7 | 0.993050 | 10.6006 | PQ |
| 8 | 0.993546 | 16.1000 | PQ |
| 9 | 1.000000 | 35.4254 | PV |

Table 1: Bus Voltages and Angles after First Iteration