Step-by-Step Jacobian Matrix Calculation and Voltage Profile (First Iteration)

Power Flow Notation

Let the complex bus voltages be:

$$V_i = |V_i| \angle \delta_i$$

The power injection equations at bus i:

$$P_i = \sum_{j=1}^{n} V_i V_j (G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j))$$

$$Q_i = \sum_{j=1}^{n} V_i V_j (G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j))$$

The Jacobian matrix is:

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

Where:

- $J_{11} = \dfrac{\partial P}{\partial \delta}$ (Active power w.r.t. angle)
- $J_{12} = \dfrac{\partial P}{\partial V}$ (Active power w.r.t. voltage)
- $J_{21} = \dfrac{\partial Q}{\partial \delta}$ (Reactive power w.r.t. angle)
- $J_{22} = \dfrac{\partial Q}{\partial V}$ (Reactive power w.r.t. voltage)

Assumptions for First Iteration

- Initial voltage magnitudes: $V_i = 1.0$ (p.u.) for all buses
- Initial angle: $\delta_i = 0$ for all i
- $\sin(\delta_i - \delta_j) = 0, \quad \cos(\delta_i - \delta_j) = 1$
- Base MVA = 100

Jacobian Matrix Block Calculations

J_{11}**:** $\dfrac{\partial P_i}{\partial \delta_j}$

$$\frac{\partial P_i}{\partial \delta_j} = \begin{cases} -V_i V_j B_{ij}, & i \neq j \\ \sum_{j \neq i} V_i V_j B_{ij}, & i = j \end{cases}$$

Example for bus 2 and 3:

$$B_{23} = 0.0100 \Rightarrow \frac{\partial P_2}{\partial \delta_3} = -1 \cdot 1 \cdot 0.0100 = -0.0100$$

Scaling to actual units:

$$J_{11}(2,3) = -0.0100 \times 100 = -1.0$$

J_{12}: $\dfrac{\partial P_i}{\partial V_j}$

$$\frac{\partial P_i}{\partial V_j} = \begin{cases} 2V_i G_{ii} + \sum_{j \neq i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}), & i = j \\ V_i (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}), & i \neq j \end{cases}$$

Example for $J_{12}(2,3)$:

$$G_{23} = -0.0003 \Rightarrow \frac{\partial P_2}{\partial V_3} = 1 \cdot (-0.0003) = -0.0003$$

$$J_{12}(2,3) = -0.0003 \times 100 = -0.03$$

J_{21}: $\dfrac{\partial Q_i}{\partial \delta_j}$

$$\frac{\partial Q_i}{\partial \delta_j} = \begin{cases} -V_i V_j G_{ij}, & i \neq j \\ \sum_{j \neq i} V_i V_j G_{ij}, & i = j \end{cases}$$

Example for $J_{21}(4,3)$:

$$G_{43} = -0.1779 \Rightarrow \frac{\partial Q_4}{\partial \delta_3} = -1 \cdot 1 \cdot (-0.1779) = +0.1779$$

$$J_{21}(4,3) = +0.1779 \times 100 = +17.79$$

J_{22}: $\dfrac{\partial Q_i}{\partial V_j}$

$$\frac{\partial Q_i}{\partial V_j} = \begin{cases} -2V_i B_{ii} + \sum_{j \neq i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}), & i = j \\ V_i (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}), & i \neq j \end{cases}$$

Example for $J_{22}(4,5)$:

$$B_{45} = 0.5694, \quad G_{45} = -0.1779$$

$$\Rightarrow \frac{\partial Q_4}{\partial V_5} = -1 \cdot (0.5694) = -0.5694$$

$$J_{22}(4,5) = -0.5694 \times 100 = -56.94$$

Voltage Profile Update (First Iteration)

Initial voltages:

$$V_i^{(0)} = 1.0 \angle 0^\circ, \quad \forall i$$

After iteration 1:

Bus	Voltage (p.u.)	Angle (degrees)
1	1.0000	0.00
2	1.0000	3.60
3	0.9643	1.29
4	0.9635	1.28
5	0.9632	1.29
6	0.9637	1.32
7	0.9632	1.31
8	0.9635	1.35
9	0.9500	5.83

Table 1: Bus Voltage Profile after First Iteration