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POPL Test

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Q.2.(i) The valuation function maps a language's abstract syntax structures to meanings drawn from semantic domains. It determines the meaning of a derivation tree by determining the meaning of its subtree and combining them into a meaning for the entire tree

Q.1. $T = 0 \mid 1 \mid 2 \mid T0 \mid T1 \mid T2 \mid T \oplus T$

(i) Denotional Semantics

$M[[0]] = 0$
$M[[1]] = 1$
$M[[2]] = 2$
$M[[x0]] = 3 * M[[x]]$
$M[[x1]] = 3 * M[[x]] + 1$
$M[[x2]] = 3 * M[[x]] + 2$
$M[[x \oplus y]] = M[[x]] + M[[y]]$

(ii) Axiomatic Semantics

$0 \oplus 0 = 0$
$0 \oplus 1 = 1$
$0 \oplus 2 = 2$
$1 \oplus 1 = 2$
$1 \oplus 2 = 10$
$2 \oplus 2 = 11$
$0x = x$
$x \oplus y = y \oplus x$
$x \oplus (y \oplus 2) = (x \oplus y) \oplus 2$
$x0 \oplus y0 = (x \oplus y)0$
$x1 \oplus y0 = (x \oplus y)1$

$X2 \oplus Y0 = (X \oplus Y)\,2$

$X1 \oplus Y1 = (X \oplus Y)\,2$

$X2 \oplus Y1 = (X \oplus Y \oplus 1)\,0$

$X2 \oplus Y2 = (X \oplus Y \oplus 1)\,1$

(3) fib : Nat → $Nat_\perp$

and

fib(n) = λn. n equals zero → zero [] n equals one → one [] (fib(n minus one) + fib(n minus two))

(i) ~~Also, till comp~~ ⇒ Finite unfolding :-

fibo (zero unfolding) : no argument $n \in Nat$

~~graph~~

graph (fib_0) = { }

graph (fib_1) = {(zero, zero)}

graph (fib_2) = {(zero, zero), (one, one)}

graph (fib_3) = {(zero, zero), (one, one), (two, one)}

graph (fib_4) = {(zero, zero), (one, one), (two, one), (three, two)}

(ii) ~~So, we can formulate~~ the Functional F

F : (Nat → $Nat_\perp$) → (Nat → $Nat_\perp$)

F = λf : λn n equals zero → zero [] n equals one → one [] (f(n minus one)) (f(n minus two))

thus, graph(fib) = $\bigcup_{i=0}^{\infty}$ graph($F^i \phi$)

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2.(ii) $P[[X:=5;\ Y:=X+1;\ \text{if } (A=5) \text{ then diverge};\ Z:=Y+X]]$

$= \text{let } s = (\text{update } [[A]]\ \text{one } \cancel{\text{not}}\ \text{newstore}) \text{ in}$
$\quad \text{let } s' = C[[X:=5;\ Y:=X+1;\ \text{if } (A=5) \text{ then diverge};\ Z:=Y+X]]\ s$

$\quad \text{in } (\text{access } [[Z]]\ s')$

Now $s' = C[[X:=5;\ Y:=X+1;\ \text{if } (A=5) \text{ then diverge};\ Z:=Y+X]]\ ([[[A]] \to \text{one}]\ \text{newstore})$

$\quad \text{in access } [[Z]]\ s$

$\text{let, } s_1 = [\ [[A]] \to \text{two}]\ \text{newstore}$

$\therefore C[[X:=5;\ Y:=X+1;\ \text{if } (A=5) \text{ then diverge};\ Z:=Y+X]]\ s_1$

$= (\lambda s.\ C[[\text{if } (A=5) \text{ then diverge};\ Z:=Y+X]]\ (C[[X:=5;\ Y:=X+1]]\ s_1)$

Now,
$C[[X:=5;\ Y:=X+1]]\ s_1$
$= C[[Y:=6;\ X:=5]]\ s_1$ (shown in class)
$= \lambda s.\ C[[X:=5]]\ (C[[Y:=6]]\ s)\ s_1$

$= [[[X]] \to \text{five}]\ [[[Y]] \to \text{six}]\ [[[A]] \to \text{one}]\ \text{newstore}$
$= s_2$

Thus,
$C[[$if $(A=5)$ then diverge; $Z:=Y+X]]S_2$

$= C[[Z:=Y+X]]$ $(B[[A=5]]S_2 \longrightarrow C[[\text{diverge}]]S_2 [\,]S_2)$

Now,
$B[[A=5]]S_2$
$=$ (access $[[A]]S_2$) equals five

And, access $[[A]]$ S_2
$= S_2[[A]]$
$=$ one

$\therefore B[[A=5]]S_2 =$ false

$\hookrightarrow C[[$if $(A=5)$ then diverge$]]$ $S_2 = S_2$

$\therefore C[[Z:=Y+X]]S_2$

Using access $[[Y]]$ and access $[[X]]$ we have,
access $[[Z]]$ $([[Z]] \rightarrow$ eleven$)$ S_2

$=$ eleven