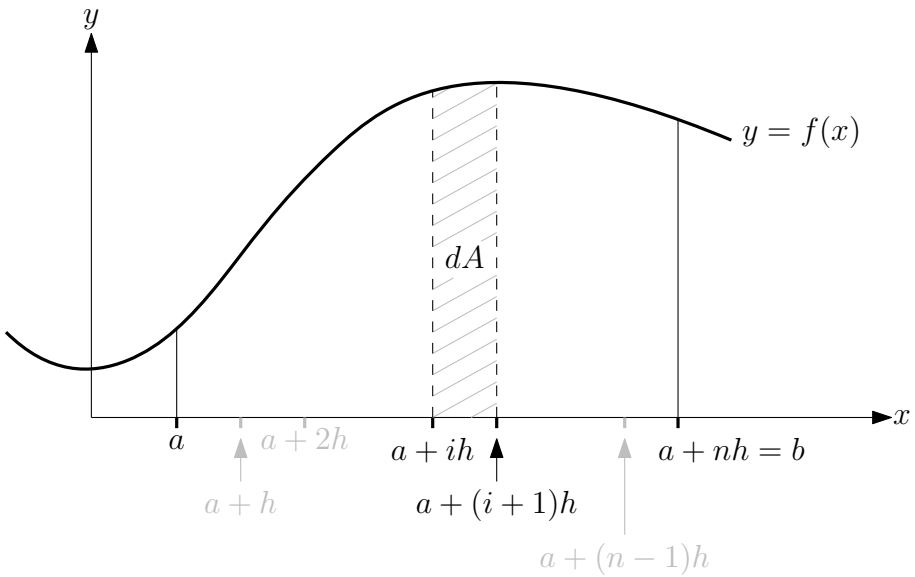
Assignment p3d

Definite integral by trapezoidal rule



We divide the interval $[a, b]$ into n subintervals of length $h = \dfrac{b-a}{n}$. Thus, $b = a + nh$.

The differential area is $dA = \dfrac{h}{2}\Big(f\big(a+ih\big) + f\big(a+(i+1)h\big)\Big)$. Hence, the total area is

$$\begin{aligned} A &= \int_a^b f(x)dx \\ &= \sum_{i=0}^{n-1} dA \\ &= \frac{h}{2}\Big(\big(f(a)+f(a+h)\big) + \big(f(a+h)+f(a+2h)\big) + \big(f(a+2h)+f(a+3h)\big) + \ldots \\ &\qquad\qquad + \big(f(a+(n-1)h)+f(a+nh)\big)\Big) \\ &= \frac{h}{2}\left(f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b)\right). \end{aligned}$$