

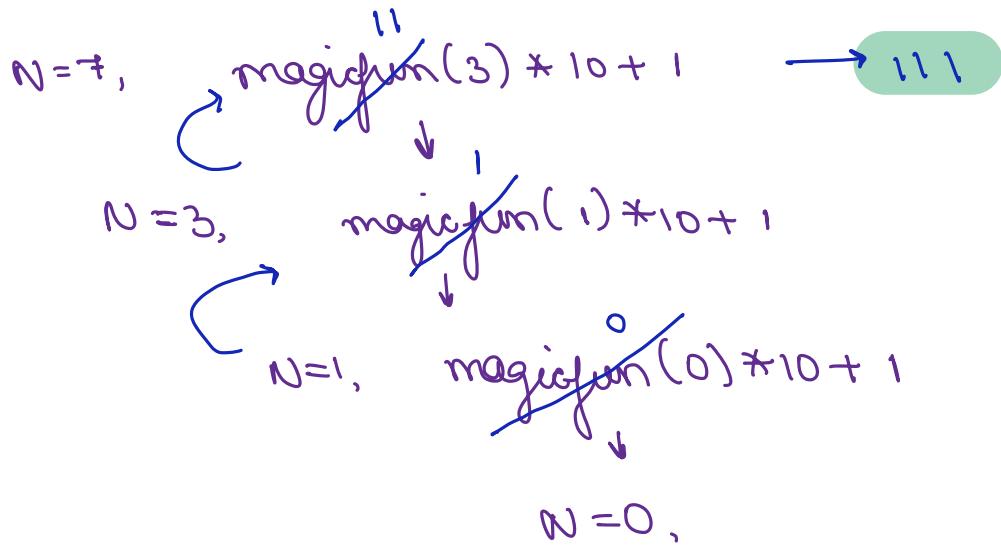
## Agenda :

1. What is backtracking
2. Intro to subset.
3. Return all subset array in an array.
4. Return all permutation in an array.

Quiz 1:

```
int magicfun(int N){  
    1. if (N==0)  
        return 0;  
    2. return magicfun(N/2)*10 + (N%2);  
}
```

$\Theta(\log N)$



Quiz 2:

T.C.

$$N \rightarrow N/2 \rightarrow N/4 \rightarrow N/8 \dots 0$$

$$\Theta(\log_2 N)$$

[First value pointed]

Quiz 3: void fun(char s[], int x) { SCROLL, 0

```

    print(s);
    char temp;
    if (x < s.length() {
        temp = s[x];
        s[x] = s[s.length - x - 1];
        s[s.length - x - 1] = temp;
        fun(s, x+1);
    }
}

```

<sup>0 1 2 3 4 5</sup>  
SCROLL, 0.

$0 < 6/2 \rightarrow \text{True}$

swap(0, 5)



<sup>0 1 2 3 4 5</sup>  
(LCROLS, 1)

$1 < 6/2 \rightarrow \text{True}$

swap(1, 4)



<sup>0 1 2 3 4 5</sup>  
(LRLDCS, 2)

$2 < 6/2$

swap(2, 3)



(LLORCS, 3)

$3 < 6/2 \rightarrow \text{False}$

Output  
SCROLL  
LCROLS  
LRLDCS  
LLORCS

Quiz 4: T.C.  $(N/2) \rightarrow o(N)$

()

Ques. Print all valid parenthesis of length  $2N$  for a given value of  $N$ .

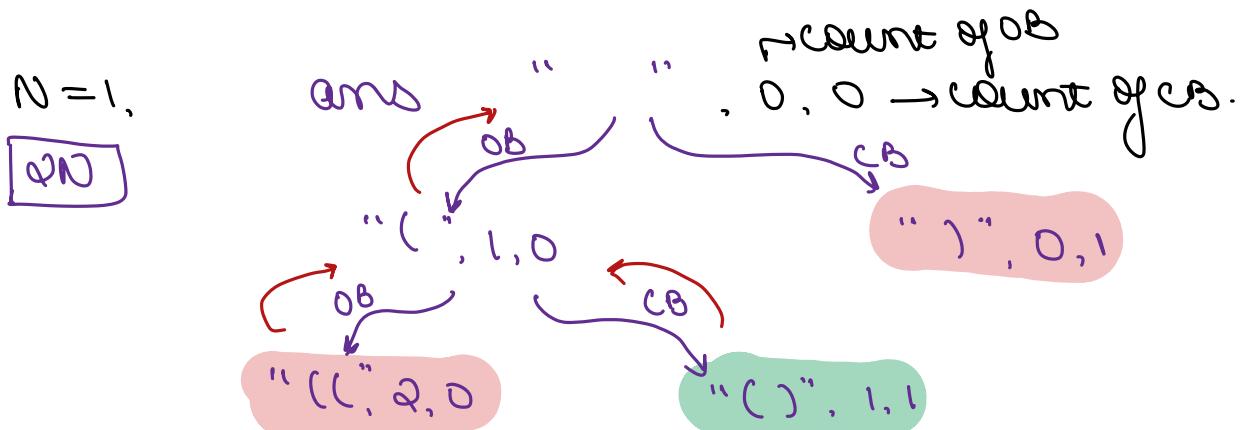
$\hookrightarrow$  equal no. of opening & closing brackets

$N=1$       ( ) , ) (

$N=2$       ( )( ) , (( )) , ))(( , ))( ) , ))))

$N=3$       ( )( )( ) , (( ( )) ) , (( ( ) ) ) , ( ( )( ) ) , ( ( ) )( )

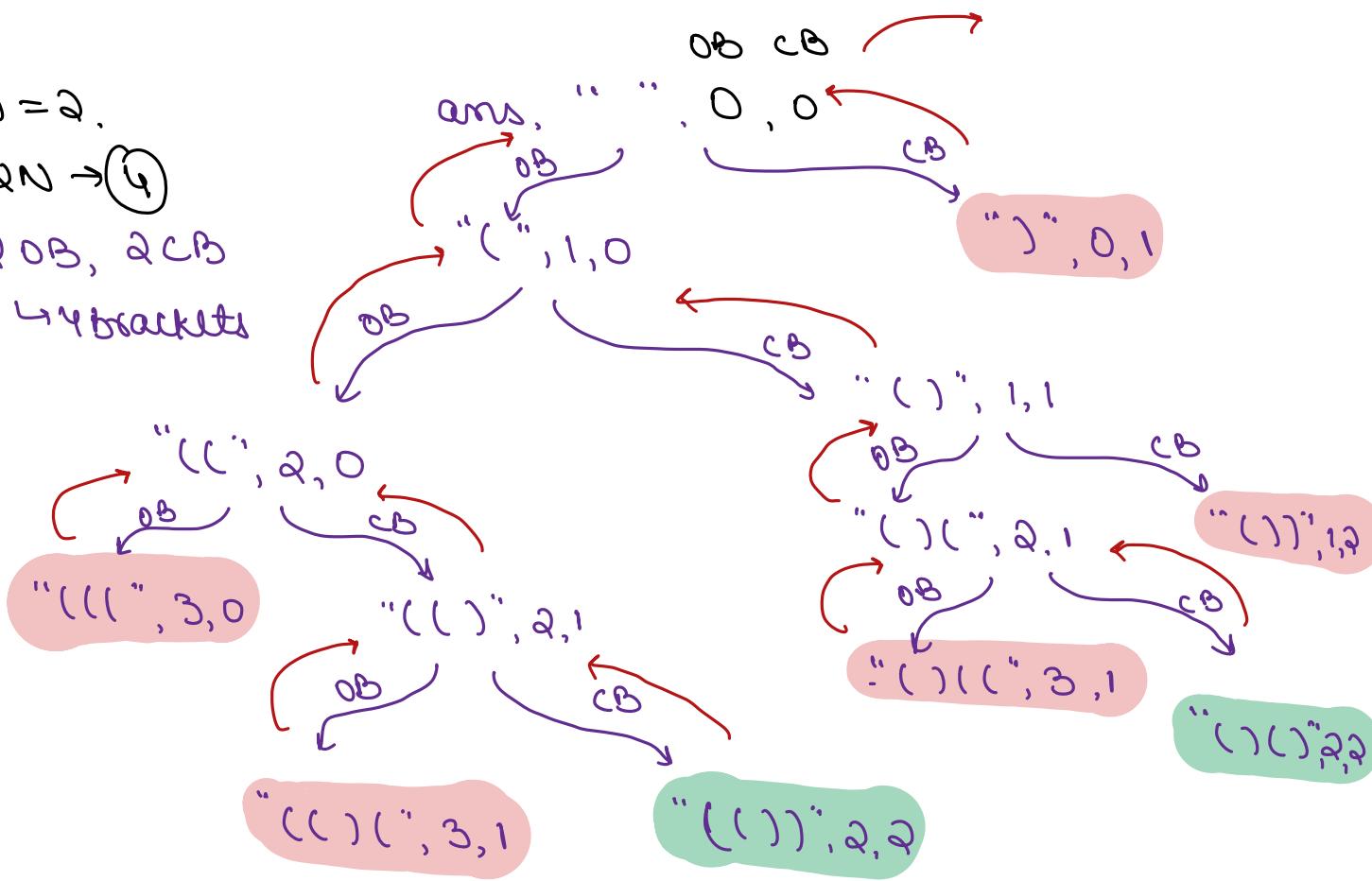
Idea: Start generating string starting from "" till  $len = 2N$



### Invalid

- 1. Opening  $> N$
- 2. Closing  $>$  Opening

$N=2$ ,  
 $2N \rightarrow 4$   
 $2OB, 2CB$   
 $\hookrightarrow 4$  brackets



Code:

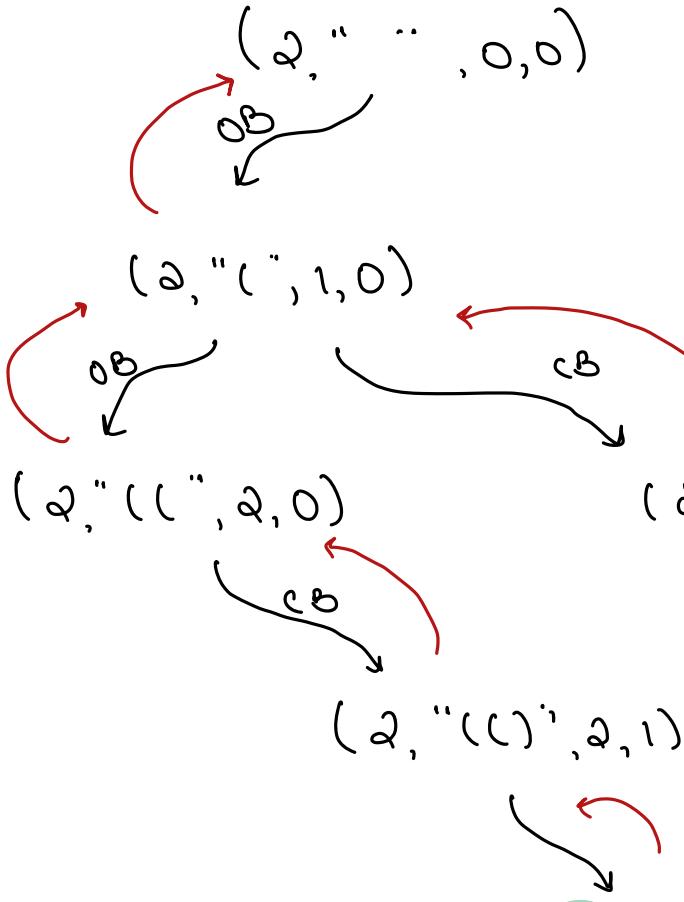
```

void solve(int n, string ans, int open, int close){
    if(ans.length() == 2 * n){
        cout << ans;
        return;
    }

    if(open < n){
        solve(n, ans + "(", open + 1, close);
    }

    if(close < open){
        solve(n, ans + ")", open, close + 1);
    }
}
  
```

$$N=2,$$



```

if(ano.length() == 2 * n) {
    sopln(ano);
    return;
}

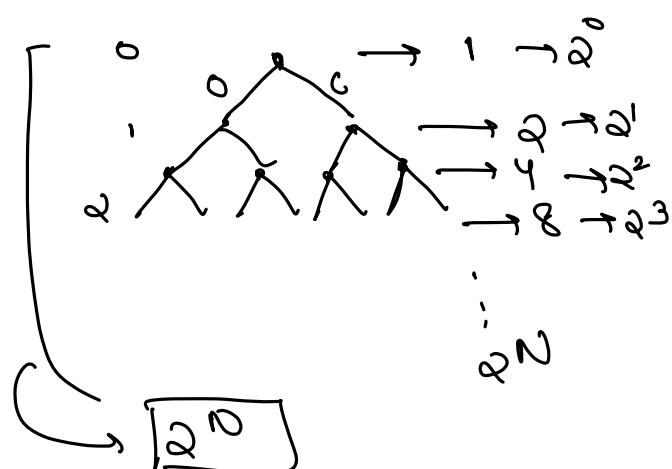
if(open < n) {
    solve(n, ano + "(", open + 1, close);
}

if(close < open) {
    solve(n, ano + ')', open, close + 1);
}

```

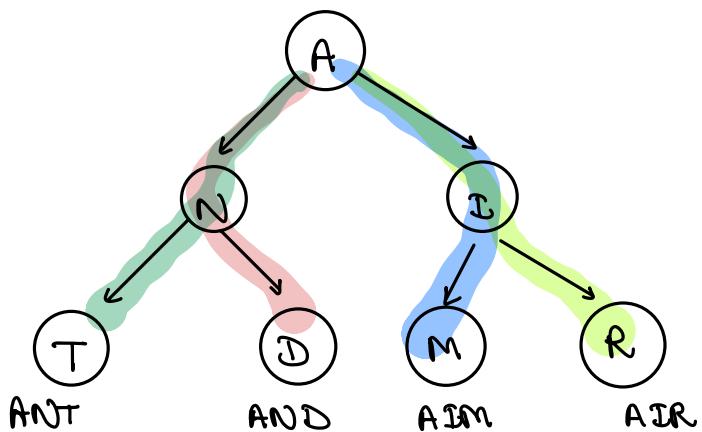
TODO :  $n=3$

$$\begin{aligned}TC &: O(2^N) \\SC &: O(N)\end{aligned}$$



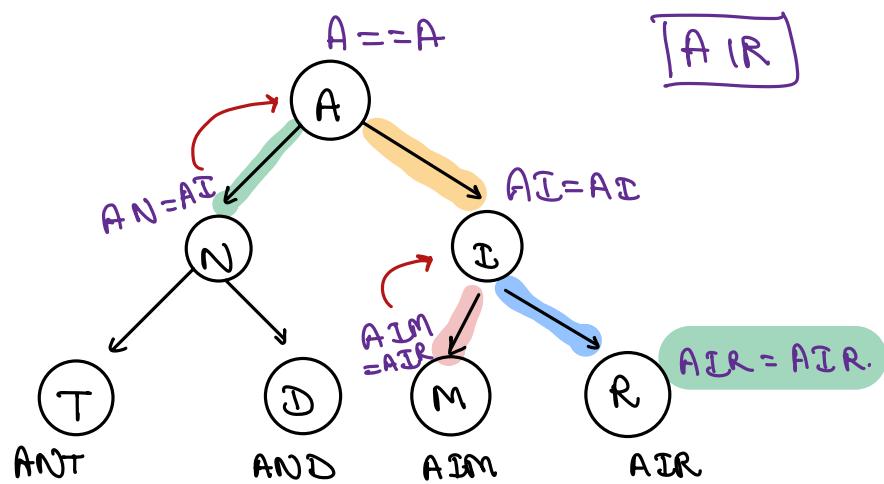
## # Backtracking :

backtracking is like trying different paths. When you hit a dead end, you backtrack to the last choice and try a different route.

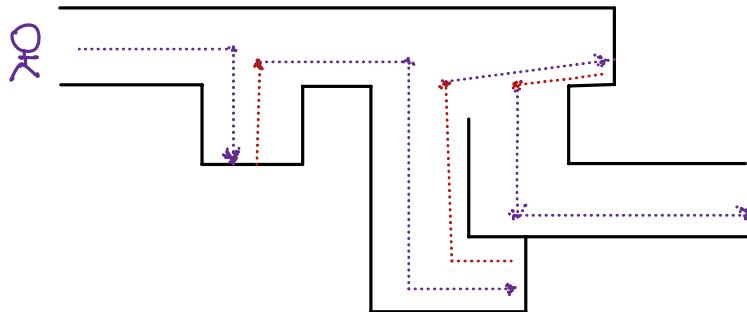


final word "AIR"

Efficient way: Backtracking  
↳ Backtracking optimizes the search, avoiding unnecessary exploration



Maze:



Idea: while generating all solutions, choose path but if we cannot go any further, backtrack and take another path.

## Subsets

Subarray: Continuous part of array.

Subset: Any possible combination of the original array is a subset.

- ↳ Set can be obtained by deleting 0 or more elements from the array.
- ↳ Empty set is a valid set
- ↳ Order does not matter here.

ar: {3, 2, 1, 9, 6, 8}

## Subsets

$$\{1, 6, 8\} \xrightarrow{\text{Subset}} \{*, *, 1, *, 6, 8\} \quad \checkmark$$

$$\{9, 6, 3\} \xrightarrow{\text{Subset}} \{3, *, *, 9, 6, *\} \quad \checkmark$$

$$\{2, 9, 8\} \xrightarrow{\text{Subset}} \{*, 2, *, 9, *, 8\} \quad \checkmark$$

Every subarray  $\longrightarrow$  subset  
vice versa not valid

## Subsequence

obtained by deleting elements from an array without changing order.

e.g.: arr: [1, 2, 3, 4, 5]

	Subarray	subset	subsequence
{1, 2, 3, 4}	✓	✓	✓
{2, 3, 5}	✗	✓	✓
{4, 1, 2}	✗	✓	✗
{2, 3}	✓	✓	✓
{1, 3, 5}	✗	✓	✓

- Note:
1. A subset may or may not be a subsequence
  2. A subsequence is always a subset
  3. A subarray is also a subsequence

Q → Given an array with distinct integers.

Print all subsets using recursion.

↪ order does not matter (all possible combinations)

e.g.  $\text{arr} = \{1, 2, 3\}$

Subsets:  $\{\}, [1], [1, 2], [1, 2, 3], [2], [2, 3], [3], [1, 3]$

$\text{arr} = \{6, 10\}$

Subsets:  $\{\}, [6], [6, 10], [10]$

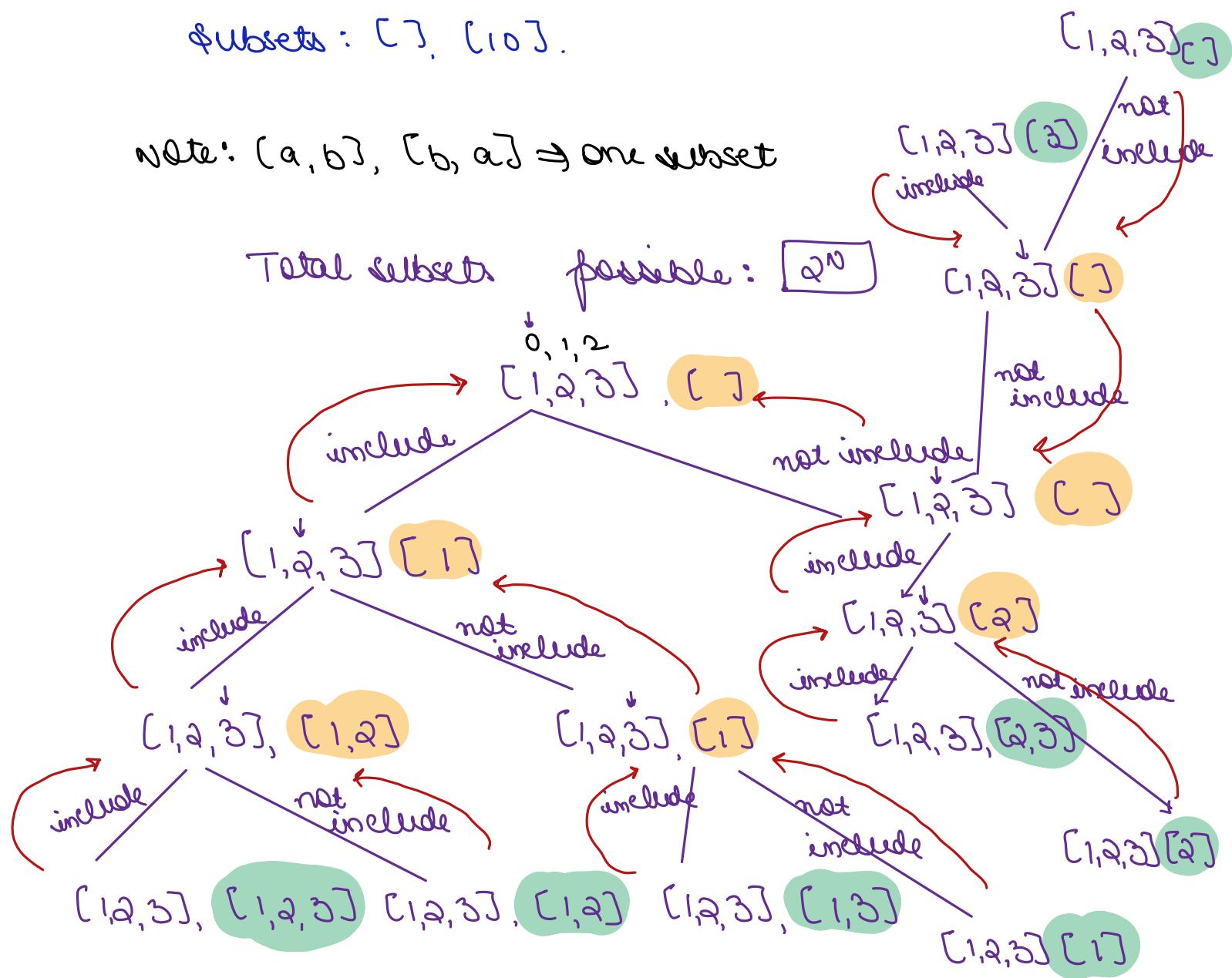
$\text{arr} = \{10\}$

Subsets:  $\{\}, [10]$

Note:  $[a, b], [b, a] \Rightarrow$  one subset

Total subsets possible:

$$2^N$$



Idea:

list<sup>→ 2D list</sup>  
list<ans>

TC: O(2^N)  
SC: O(N)

void subsets(list A, list <> curset, int idx) {

```
if (idx == A.length) {
    ans.add(curset);
    return;
}
```

|| for every ele , we have 2 choices

|| choice 1 → keep it in curset.

```
curset.add(A[idx]);
subsets(A, curset, idx + 1);
```

|| choice 2 → Don't keep in curset

```
curset.remove(A[idx]);
```

```
subsets(A, curset, idx + 1);
```

}

[[ ]]

[2, 6, 9], [], 0

Day Run.

$ids = 0$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

$ids = 1$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

$ids = 2$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

$ids = 3$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

$ids = 4$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

$ids = 5$

```
if (ids == A.length) {
    ans.add(wset);
    return;
}

wset.add(A[ids]);
subset(A, wset, ids + 1);
wset.remove(A[ids]);
subset(A, wset, ids + 1);
```

wset [2, 9]



idn = 3

```
if(idn == A.length){  
    ans.add(weset);  
    return;  
}  
  
weset.add(A[idn]);  
subset(A, weset, idn+1);  
weset.remove(A[idn]);  
subset(A, weset, idn+1);
```

idn = 3

```
if(idn == A.length){  
    ans.add(weset);  
    return;  
}  
  
weset.add(A[idn]);  
subset(A, weset, idn+1);  
weset.remove(A[idn]);  
subset(A, weset, idn+1)
```

ans: [[2, 6, 9], [2, 6]]

~~TODO~~ complete  
~~dry run~~

{ Backtracking → changes you have done while going down in recursion. Undo that changes

a, b, c → length 3

abc

acb

bac

bca

cab

cba

→ [6]

$|3| \Rightarrow 6$

$N!$

ab

$2! \Rightarrow 2$

ab

ba

## Permutations:

Given a character array with distinct elements.

Print all permutations of it without modifying it.

іlp: [a, b, c]

## Permutations ,

abc

ach

bar

bea

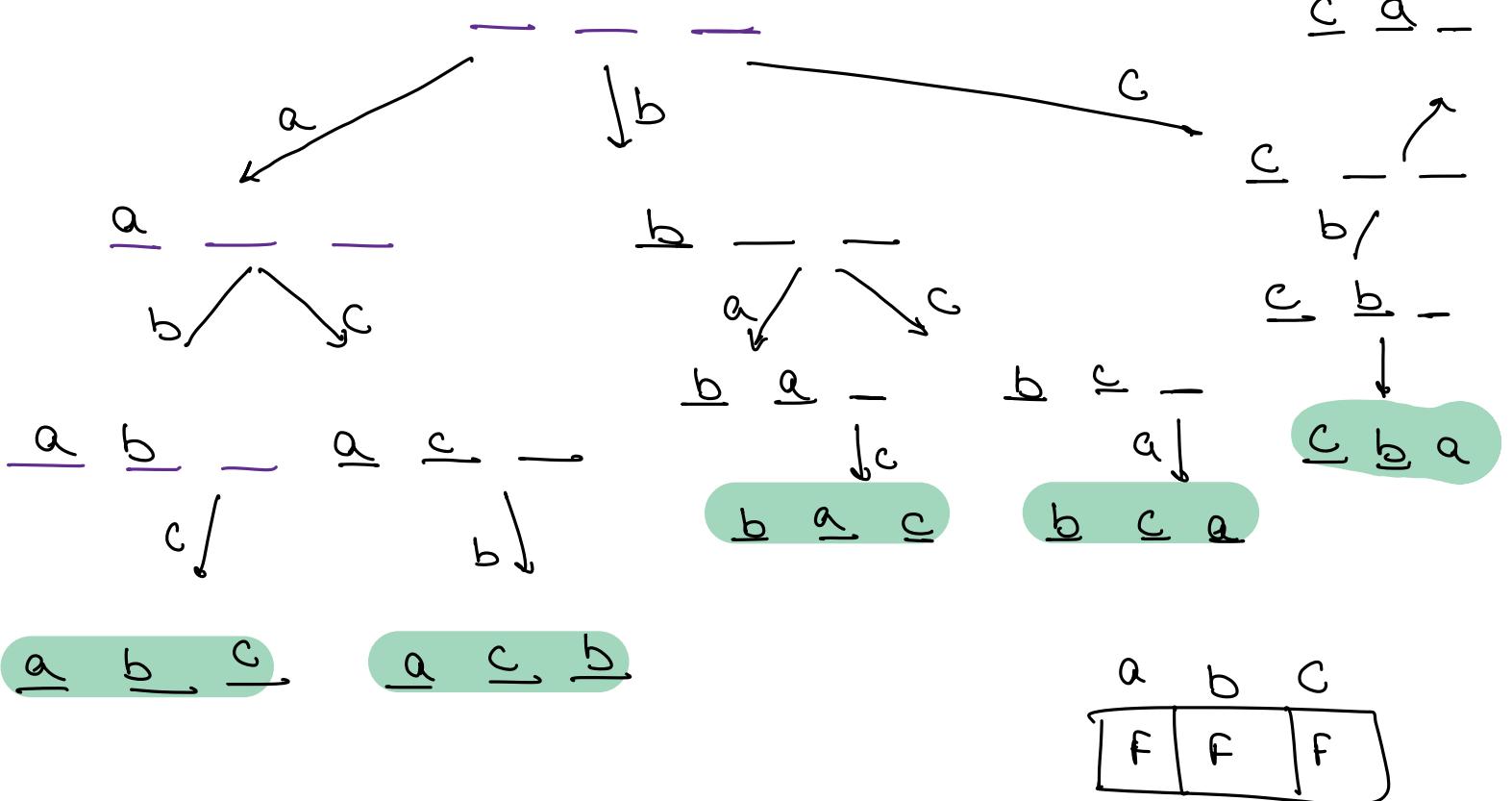
cab

cha

$N=3 \rightarrow$  6 permutations

3

Approach:



## Code

```
void permutations(char[] arr, ans[N], idn, visited[]) {  
    if (idn == arr.length) {  
        print (ans);  
        return;  
    }  
    for (int i=0; i<N; i++) { // all possibilities  
        if (visited[i] == false) {  
            visited[i] = true;  
            ans[idn] = arr[i];  
            permutations (arr, ans, idn+1, visited[]);  
            visited[i] = false; // undo changes.  
            ↳ backtracking  
        }  
    }  
}
```

0	1	2
F	T	F

$$TC: O(N!)$$

$$SC: O(N)$$

```

if (idr == ar.length) {
    print (ans);
    return;
}

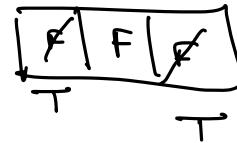
```

$i = 0$

```

for (int i=0; i< N; i++) { // all possibilities
    if (visited[i] == false) {
        visited[i] = true;
        ans[i] = ar[i];
        permutations(ar, ans, idr+1, visited[]);
        visited[i] = false; // undo changes.
    }
}

```



```

if (idr == ar.length) {
    print (ans);
    return;
}

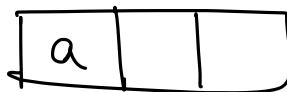
```

$i = 1$

```

for (int i=0; i< N; i++) { // all possibilities
    if (visited[i] == false) {
        visited[i] = true;
        ans[i] = ar[i];
        permutations(ar, ans, idr+1, visited[]);
        visited[i] = false; // undo changes.
    }
}

```



$i = 2$



```

if (idr == ar.length) {
    print (ans);
    return;
}

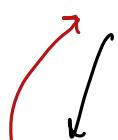
```

$i = 3$

```

for (int i=0; i< N; i++) { // all possibilities
    if (visited[i] == false) {
        visited[i] = true;
        ans[i] = ar[i];
        permutations(ar, ans, idr+1, visited[]);
        visited[i] = false; // undo changes.
    }
}

```



$i=3$

```

if (idk == arr.length) {
    print (ans);
    return;
}

for (int i=0; i<N; i++) { // all possibilities
    if (visited[i] == false) {
        visited[i] = true;
        ans[i] = arr[i];
        permutations (arr, ans, idk+1, visited[]);
        visited[i] = false; // undo changes.
}

```

$a, b, c$

