

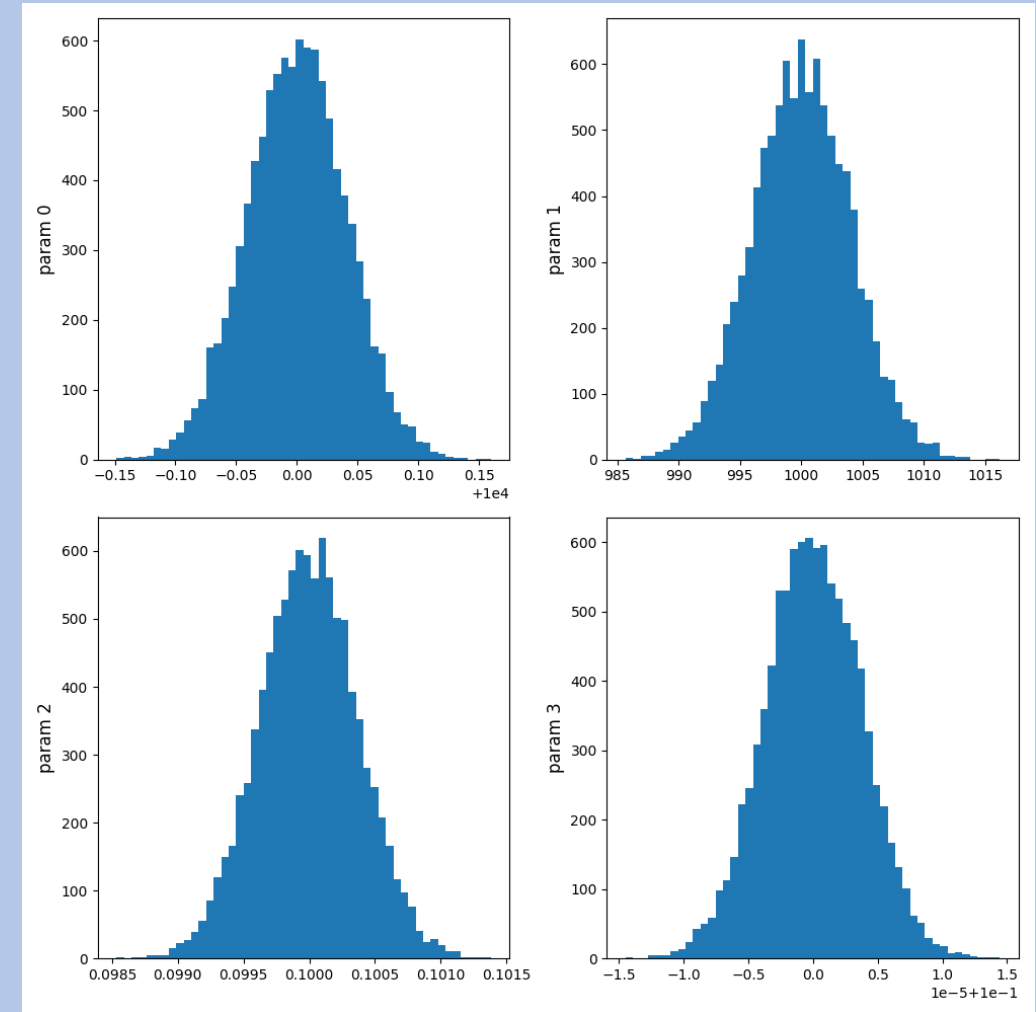
LM + MCMC on a known ARES curve

- Parameters used:
'pop_rad_yield_0_': 1E4,
'pop_rad_yield_2_': 1E3,
'fesc': 0.1,
'fX': 0.1}
- Inverse of Covariance Matrix for this Combination of Parameters:
1.64e-03, 1.96e-02, -4.39e-07, -5.37e-08,
1.96e-02, 1.60e+01, -1.44e-03, -5.94e-06,
-4.39e-07, -1.44e-03, 1.38e-07, 2.15e-10,
-5.37e-08, -5.94e-06, 2.15e-10, 1.38e-11
- Mean and standard deviation of the drawn samples
(As a sanity check to make sure the drawn samples are behaving as we expect):

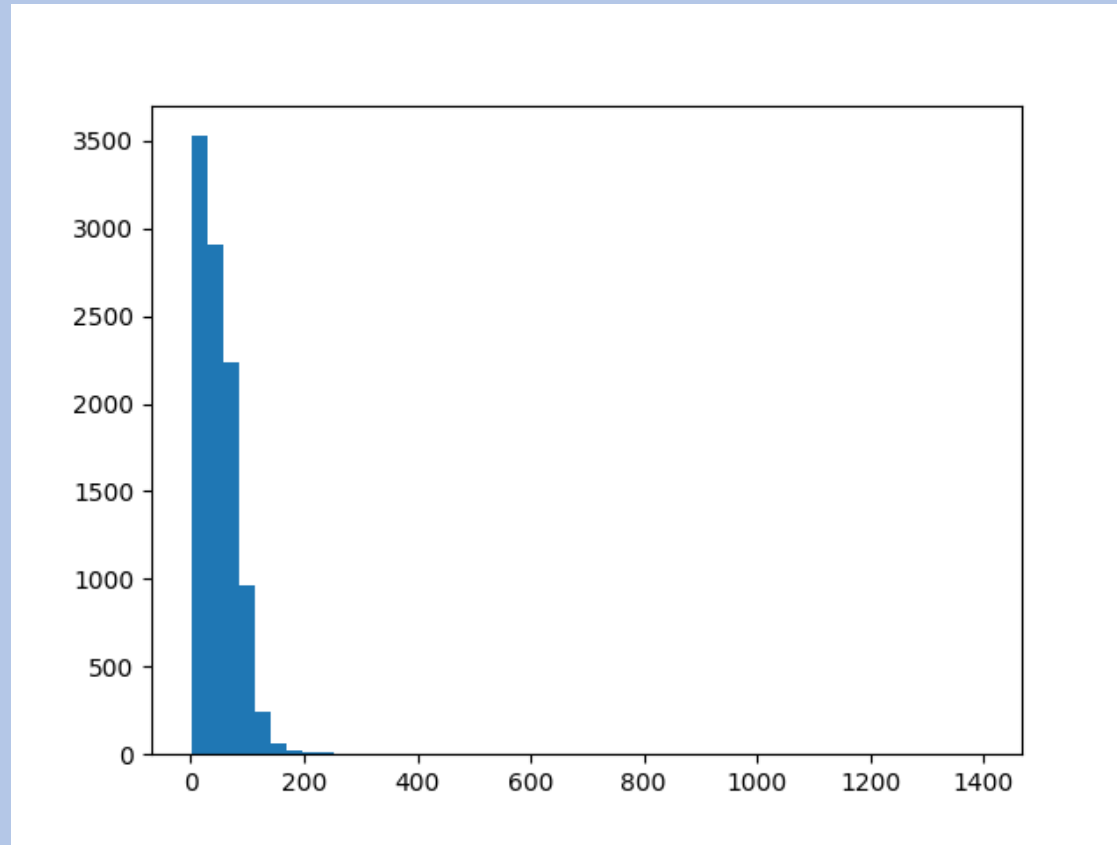
mean: [9.99999993e+03, 1.00002933e+03, 9.99962873e-02, 1.00000029e-01])

Std: [4.08174690e-02 4.03138658e+00 3.74407952e-04 3.72581029e-06]

Histogram of the drawn samples



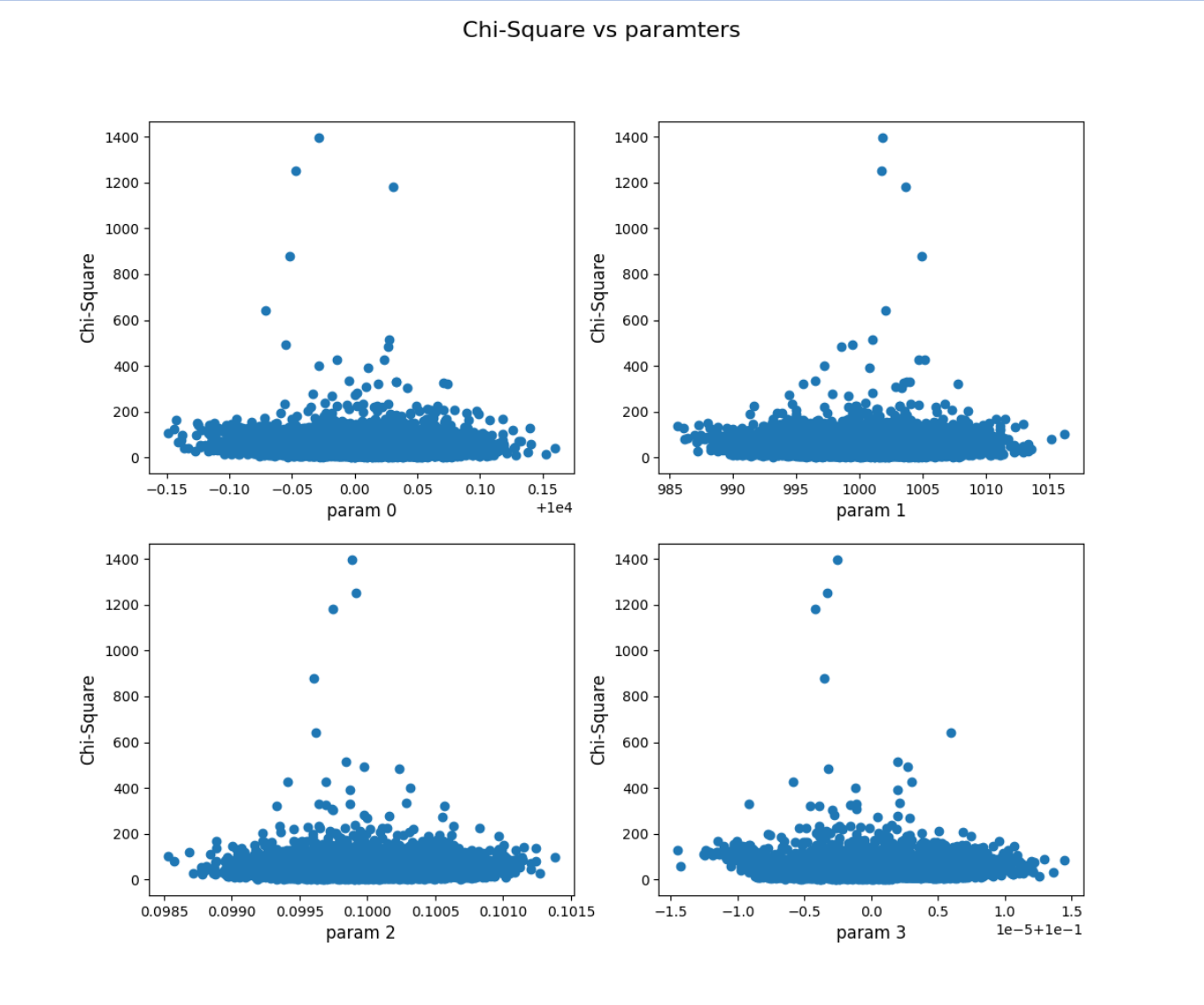
Histogram of Chi-Square of drawn samples



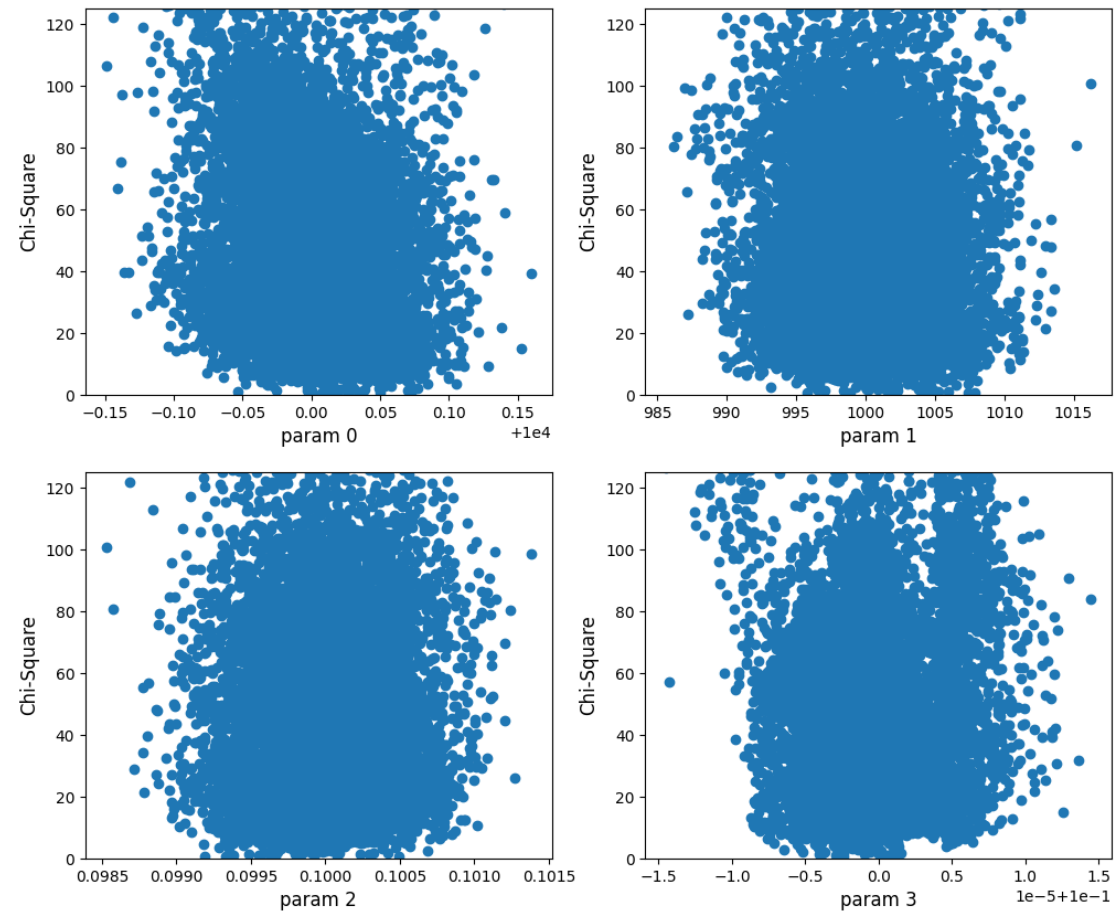
Mean: 49.20

Deviation: 41.98

Chi-Square of Drawn samples vs parameter values

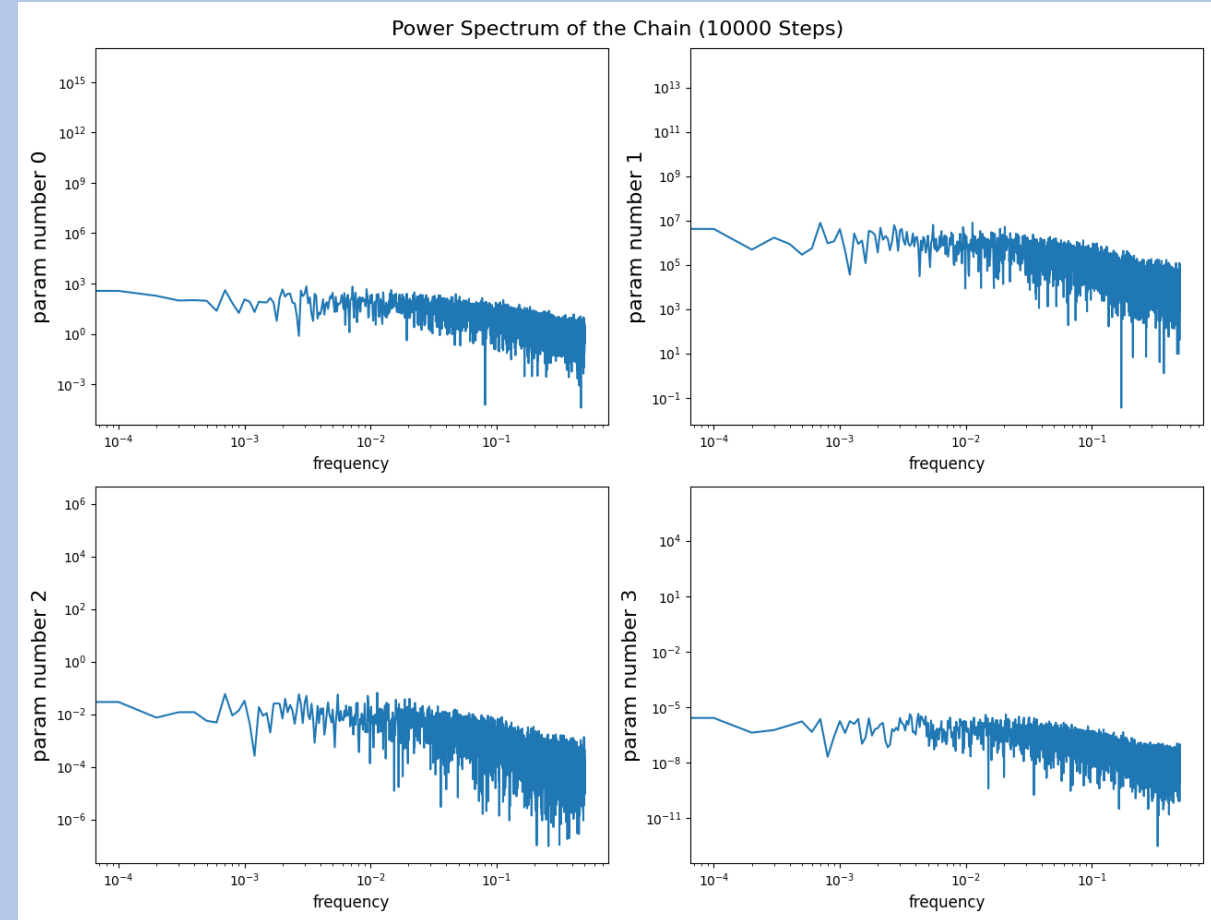
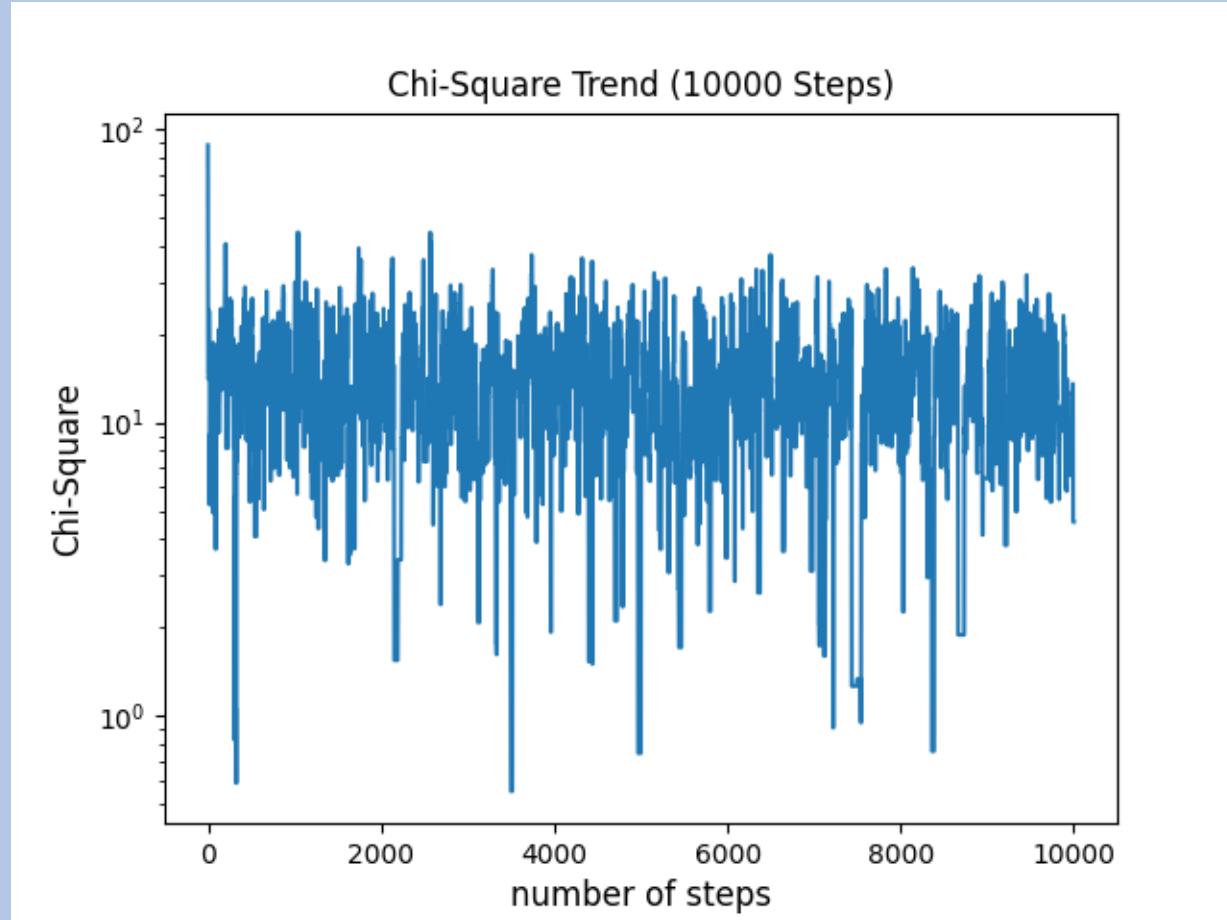


Chi-Square vs paramters



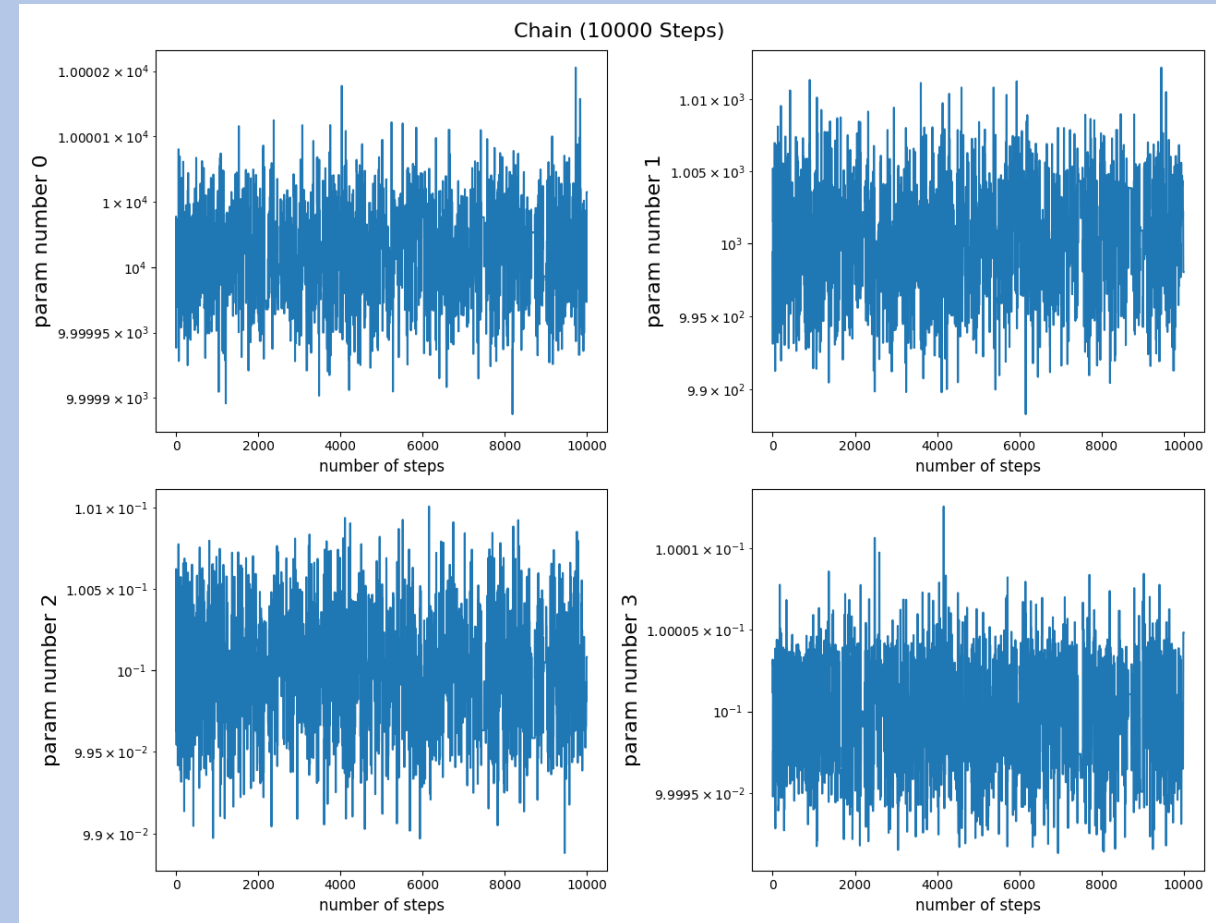
y-axis limit: (0, 125)

Feeding the samples to MCMC

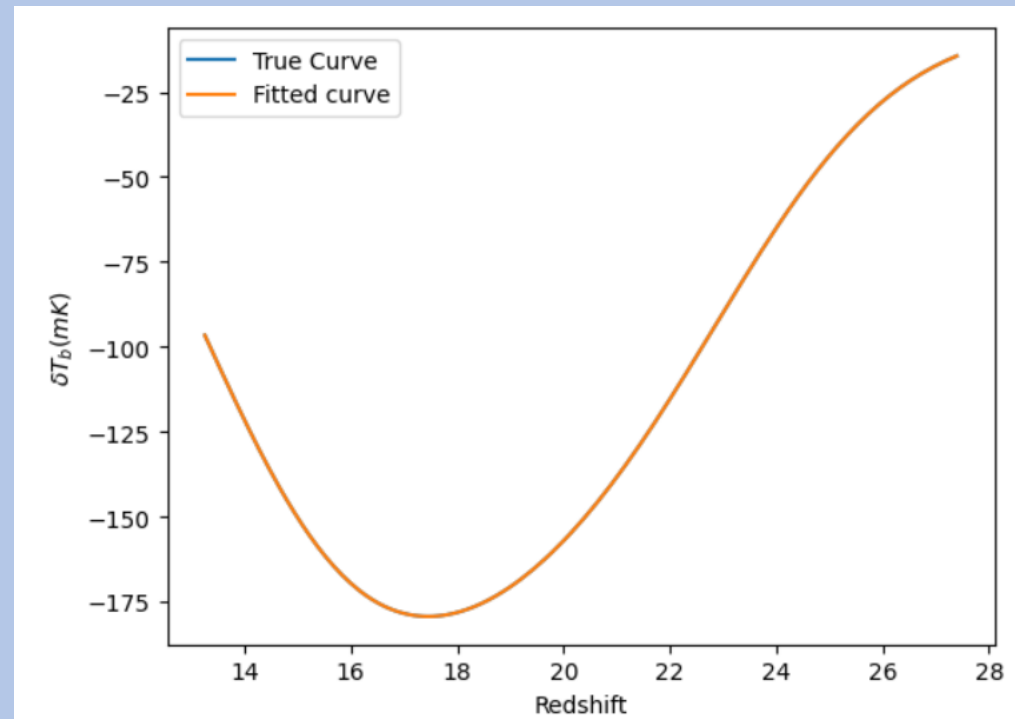
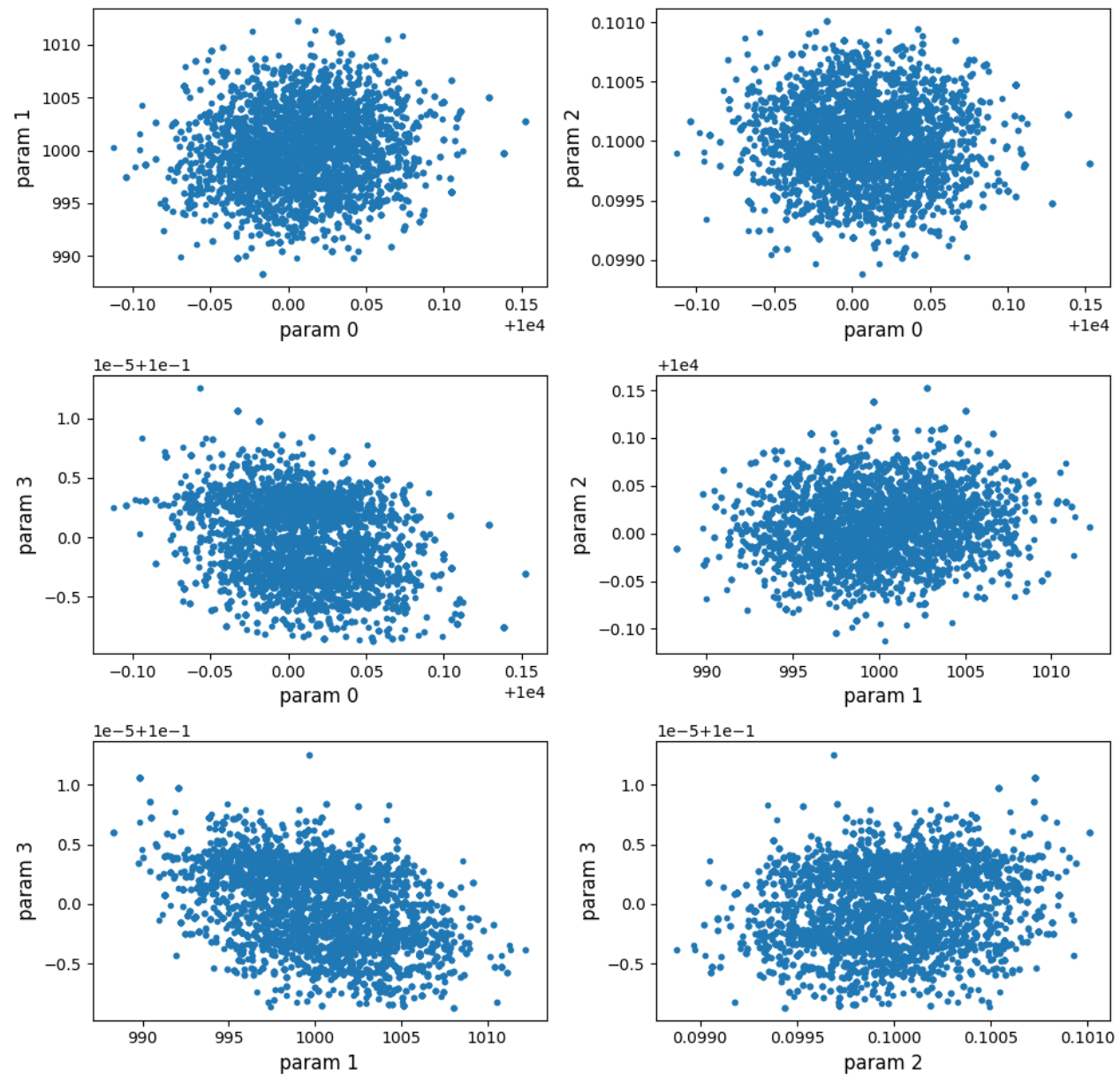


Acceptance ratio for 10000 Steps: 22.12%

- True Parameters:
1.e+04, 1.e+03,
1.e-01, 1.e-01
- Fitted parameters:
1.00000112e+04, 1.00028045e+03,
9.99804682e-02, 9.99997672e-02
- Error bar of fit:
3.50690161e-02, 3.77185397e+00,
3.48906276e-04, 3.35946795e-06



Corner plots of the chain



LM + MCMC on EDGES data

on iteration 0 chisq is 9747.329626187573 and lambda is 1
on iteration 1 chisq is 8159.367412755325 and lambda is 0.6666666666666666
on iteration 2 chisq is 8159.367412755325 and lambda is 1.5
on iteration 3 chisq is 8074.891683514397 and lambda is 1.0
on iteration 4 chisq is 8074.891683514397 and lambda is 2.25
on iteration 5 chisq is 8056.520525545141 and lambda is 1.5
on iteration 6 chisq is 8028.450223507109 and lambda is 1.0
on iteration 7 chisq is 8011.247617830618 and lambda is 0.6666666666666666
on iteration 8 chisq is 8011.247617830618 and lambda is 1.5
on iteration 9 chisq is 8011.247617830618 and lambda is 3.375
on iteration 10 chisq is 8011.247617830618 and lambda is 7.59375
on iteration 11 chisq is 8010.907299395982 and lambda is 5.0625
on iteration 12 chisq is 8008.634292903948 and lambda is 3.375
on iteration 13 chisq is 8007.347490614724 and lambda is 2.25
on iteration 14 chisq is 8006.679031433592 and lambda is 1.5
on iteration 15 chisq is 8006.679031433592 and lambda is 3.375
on iteration 16 chisq is 8006.679031433592 and lambda is 7.59375
on iteration 17 chisq is 8006.679031433592 and lambda is 17.0859375
on iteration 18 chisq is 8006.679031433592 and lambda is 38.443359375
on iteration 19 chisq is 8006.655661443841 and lambda is 25.62890625

- **Best-fit Point:**

4.54933009e+03, 2.47592394e+03, 3.70100011e-01, 1.36397790e-01

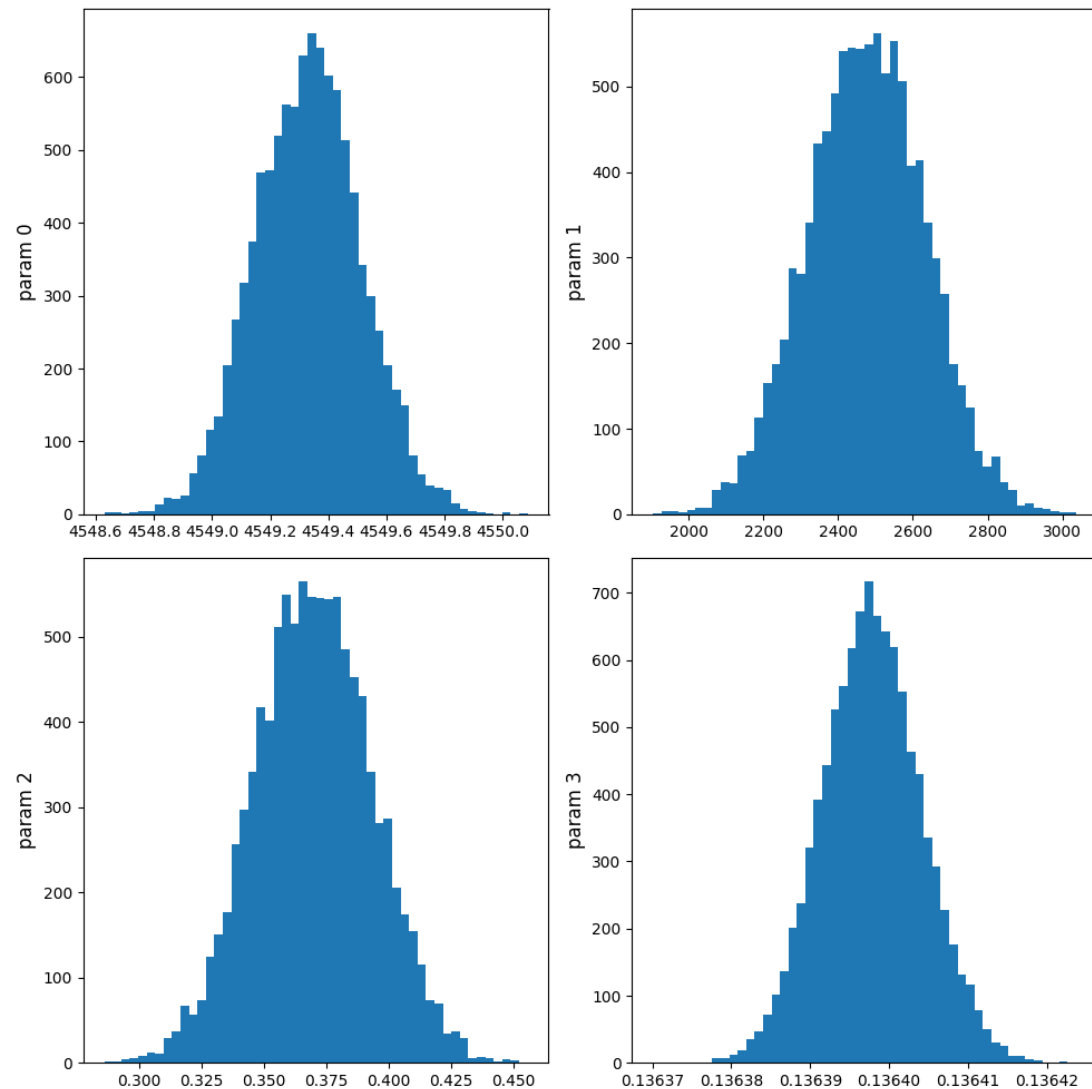
- **Chi-Square at the best-fit point:**

8006.655661443841

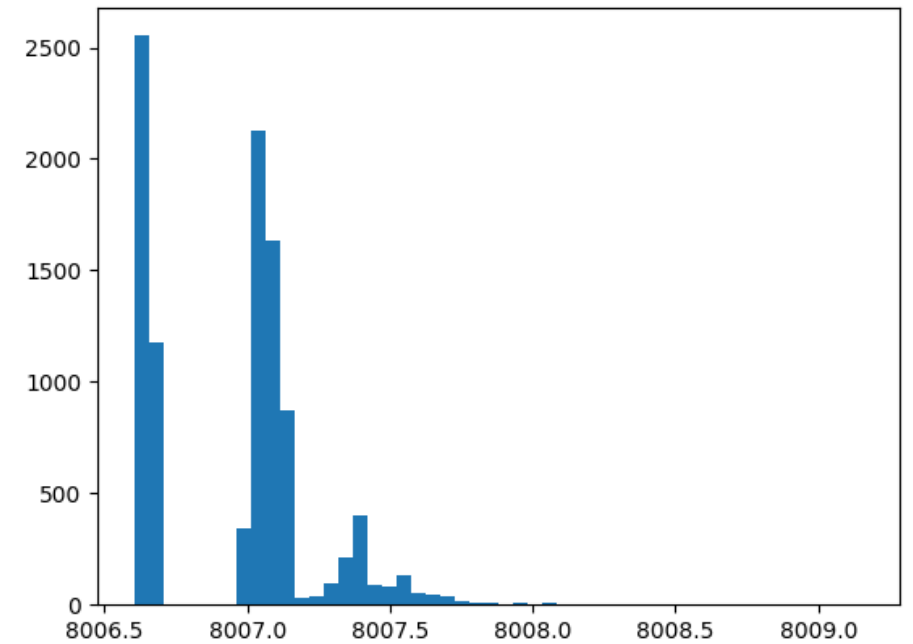
- **Inverse of Covariance Matrix:**

3.24667471e+04, 5.68112552e+06, -8.51686688e+02, 5.65856615e-01,
5.68112552e+06, 2.43358043e+10, -3.63874836e+06, 5.64163649e+00,
-8.51686688e+02, -3.63874836e+06, 5.44075269e+02, -9.80887059e-04,
5.65856615e-01, 5.64163649e+00, -9.80887059e-04, 3.92215290e-05

Histogram of the drawn samples

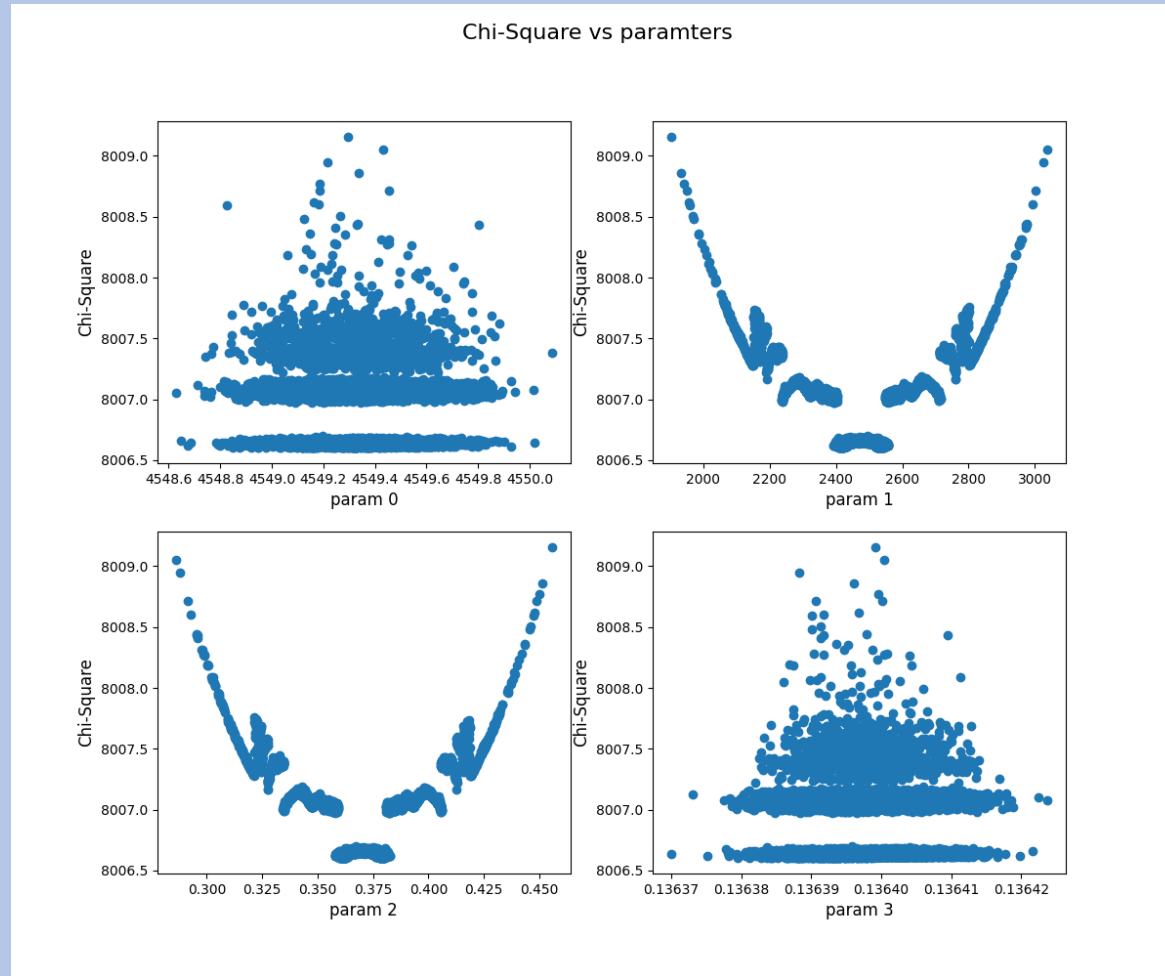


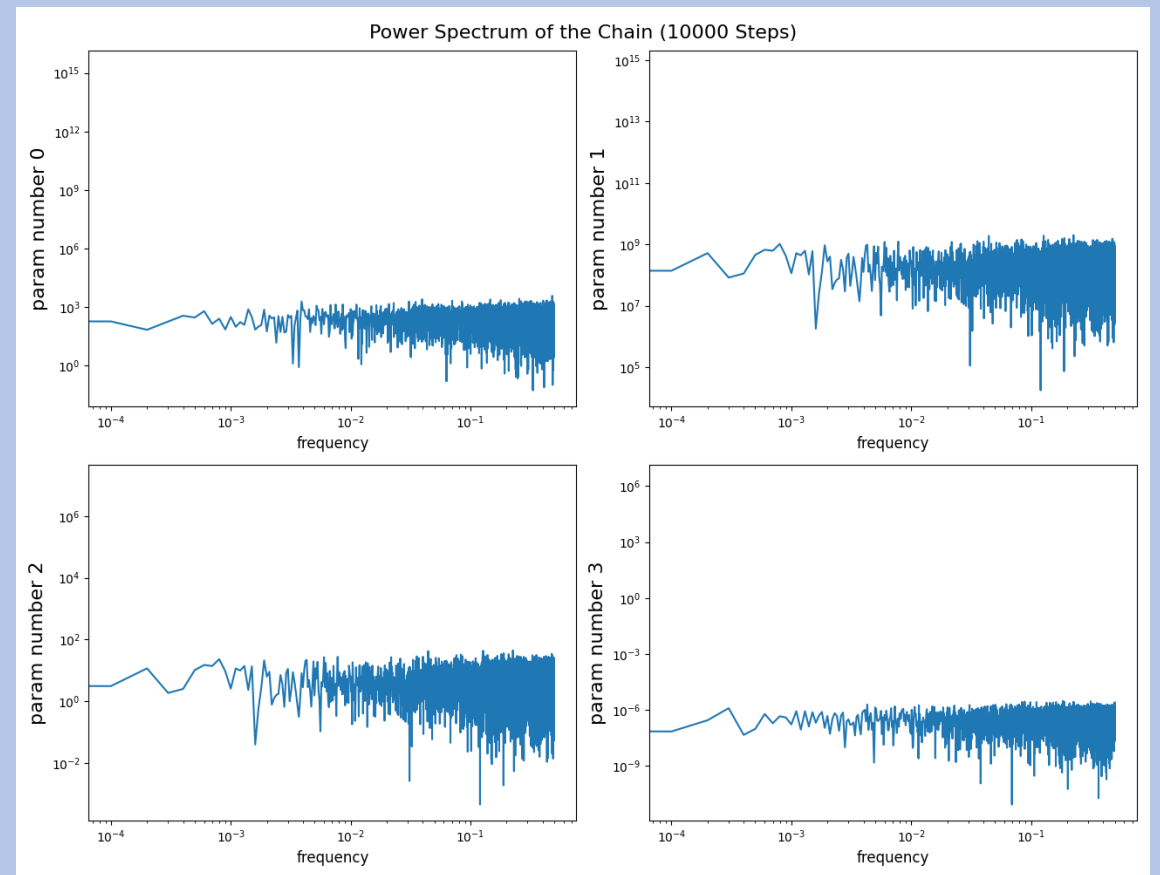
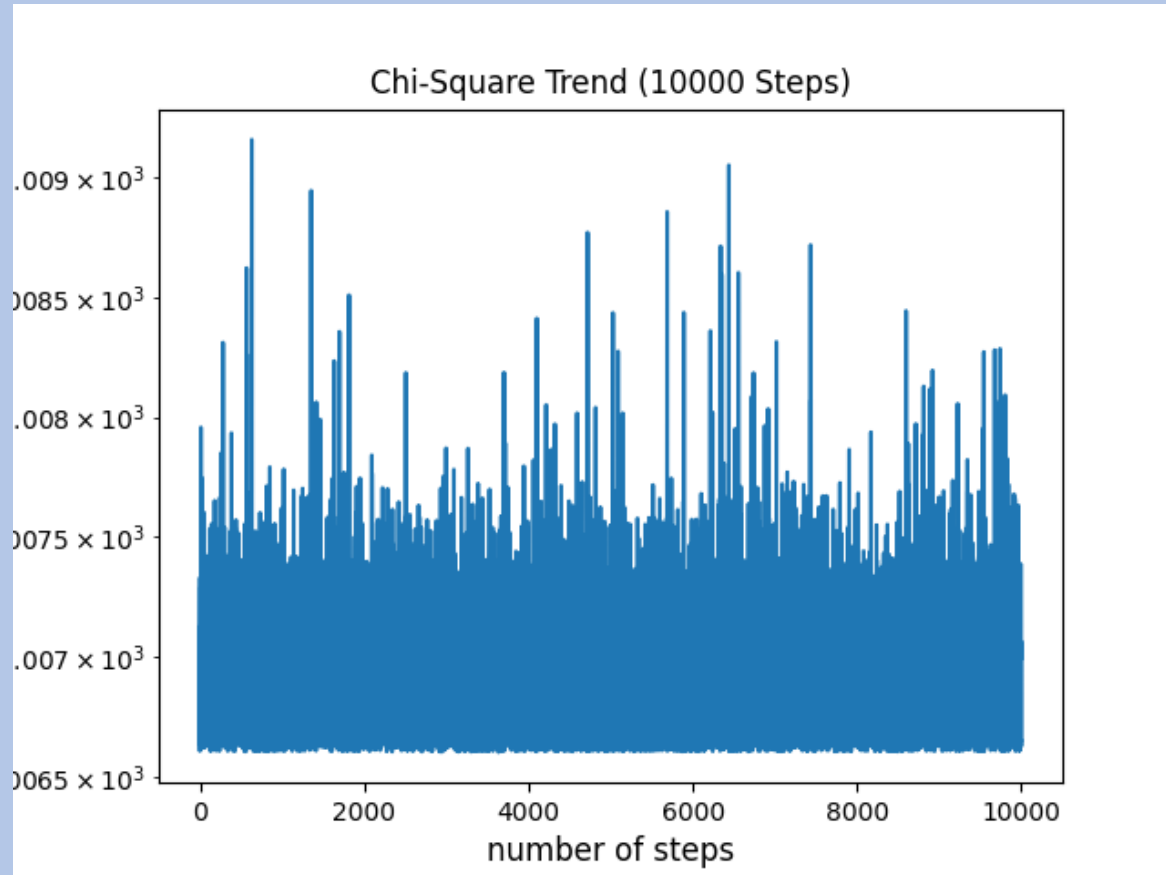
Histogram of difference between the chi-square of drawn samples



Mean: 0.305
Deviation: 0.289

Chi-Square of Drawn samples vs parameter values





Chi-Square of the starting point of MCMC (result of LM): 8006.655661443841

Chi-Square of MCMC best-fit point:

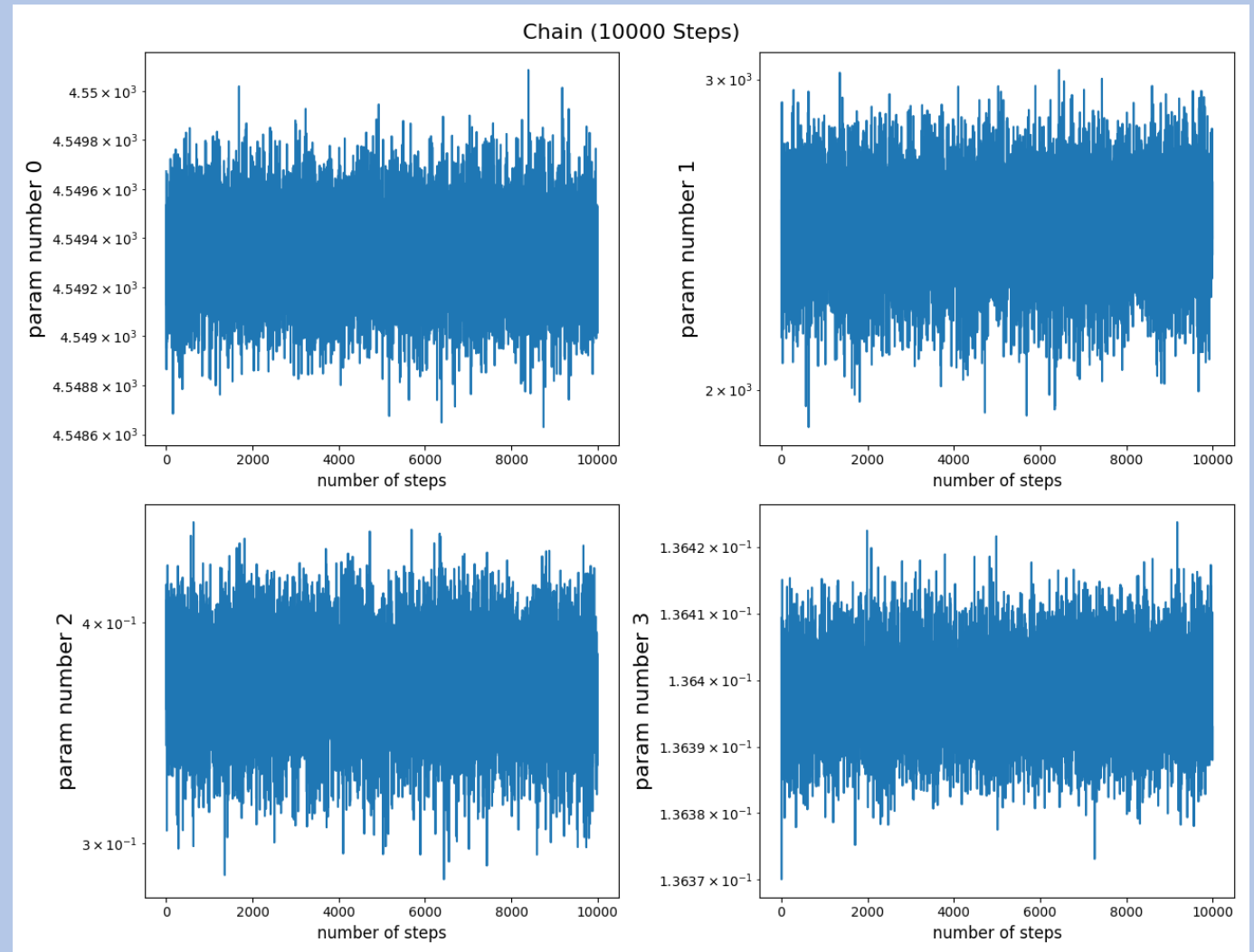
Acceptance ratio for 10000 Steps: 22.12%

Fitted Parameters:

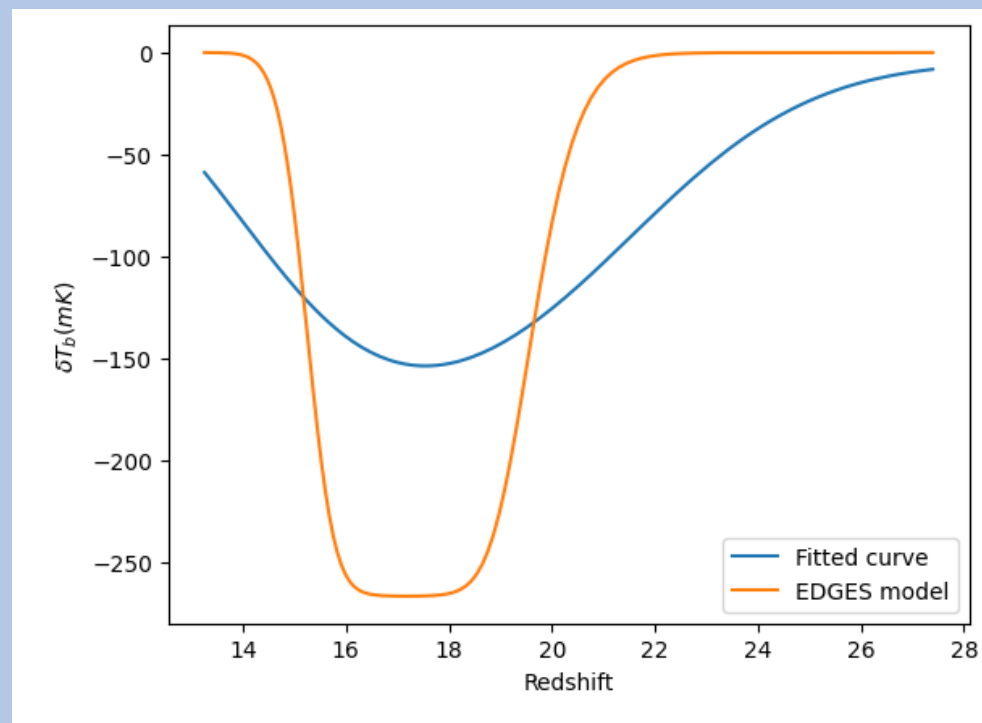
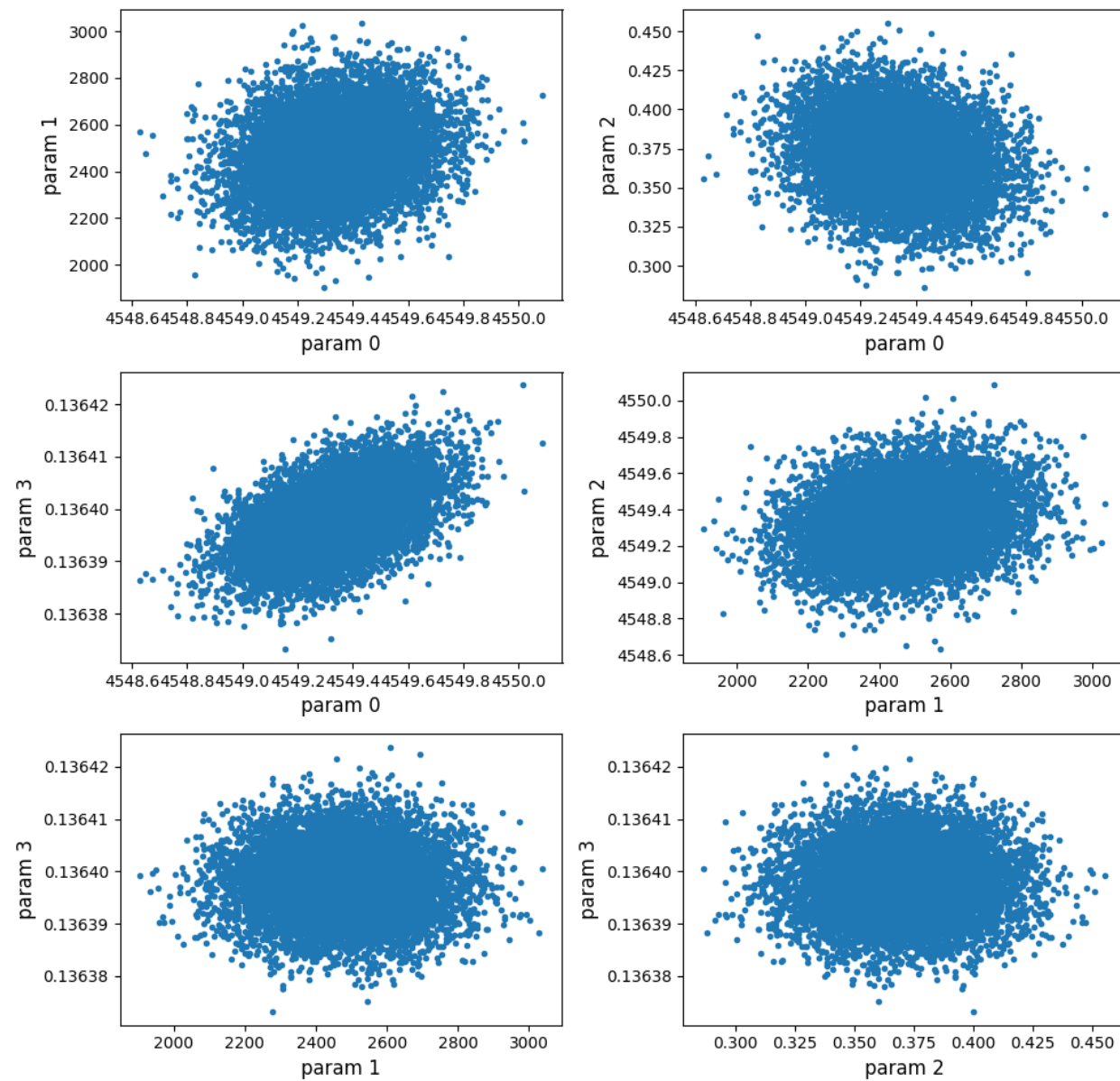
[4.54933238e+03 2.47654876e+03
3.70006283e-01 1.36397845e-01]

Error Bars:

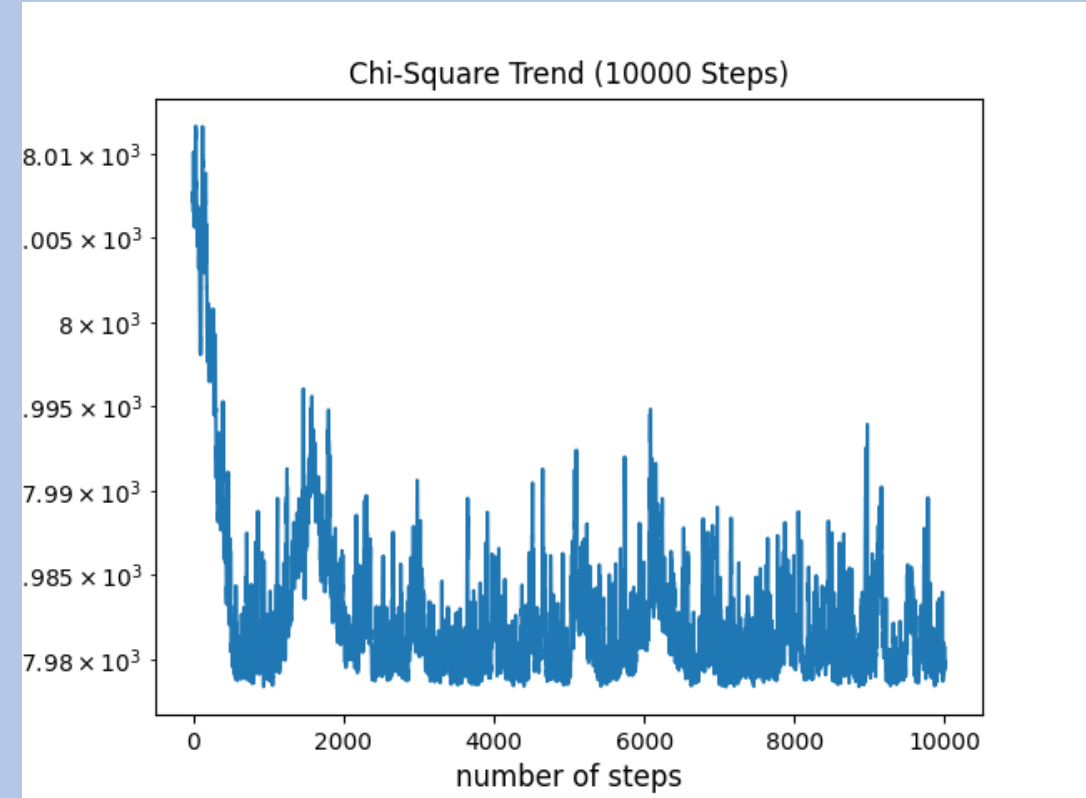
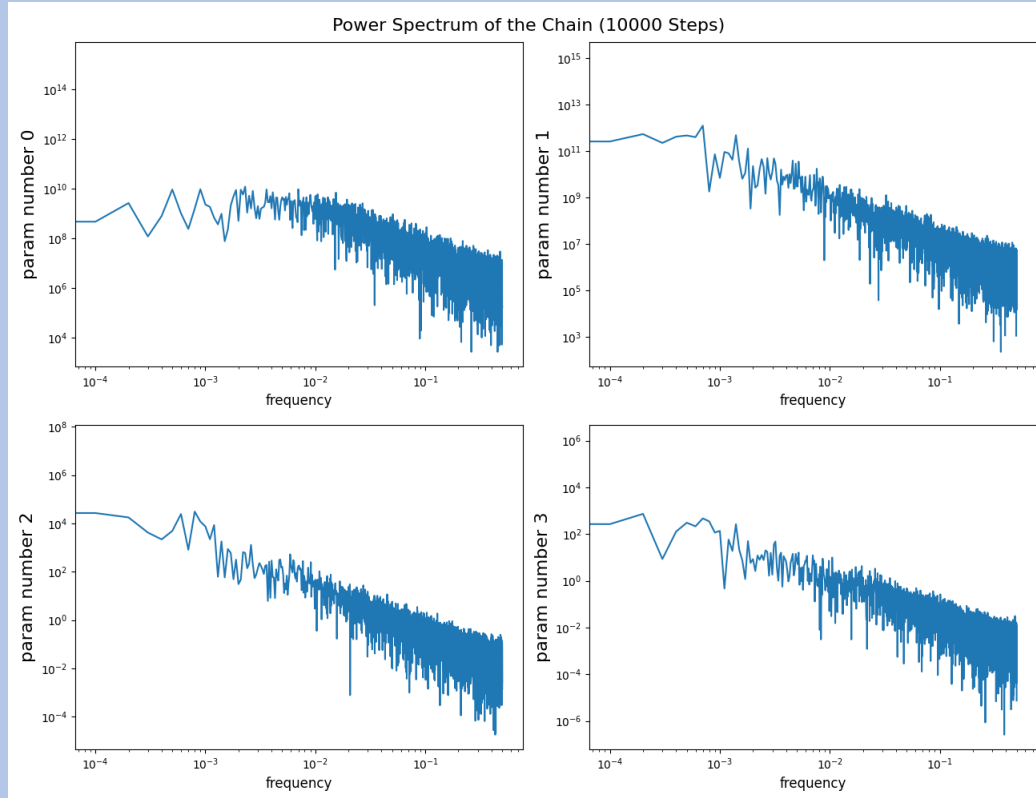
[1.81616137e-01 1.56788160e+02
2.34431702e-02 6.32705006e-06]



Corner plots of the chain



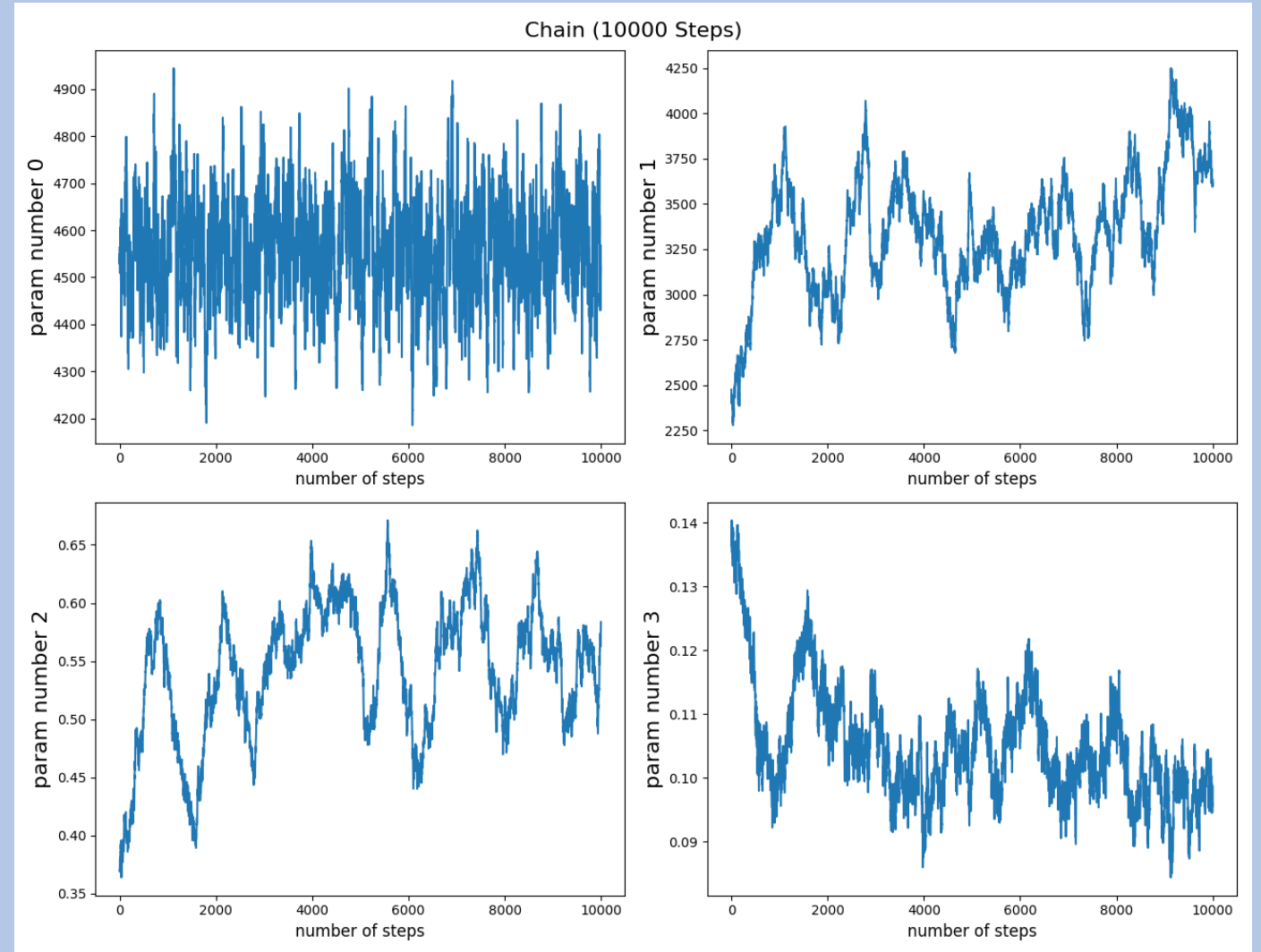
Simple MCMC



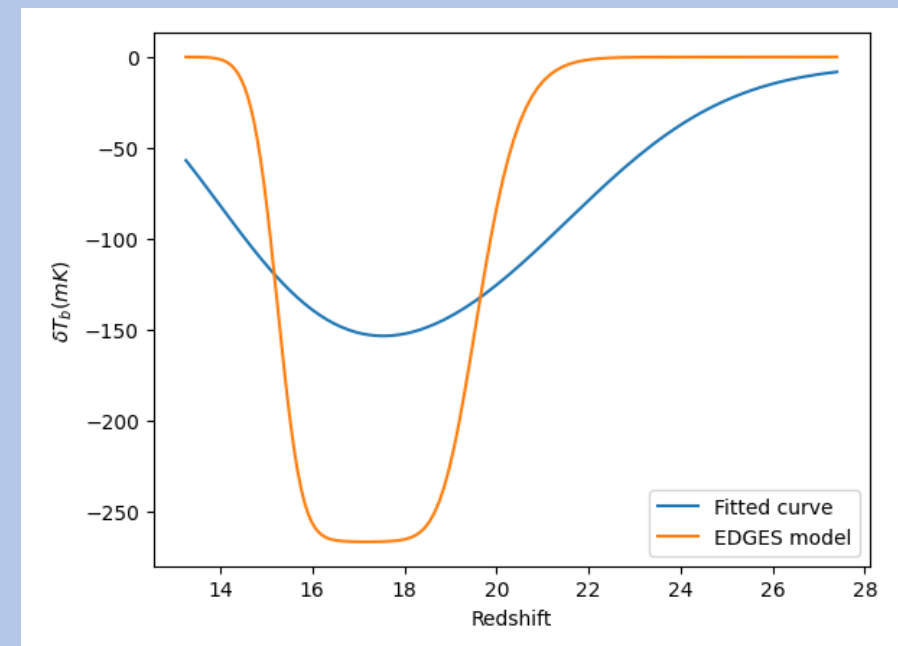
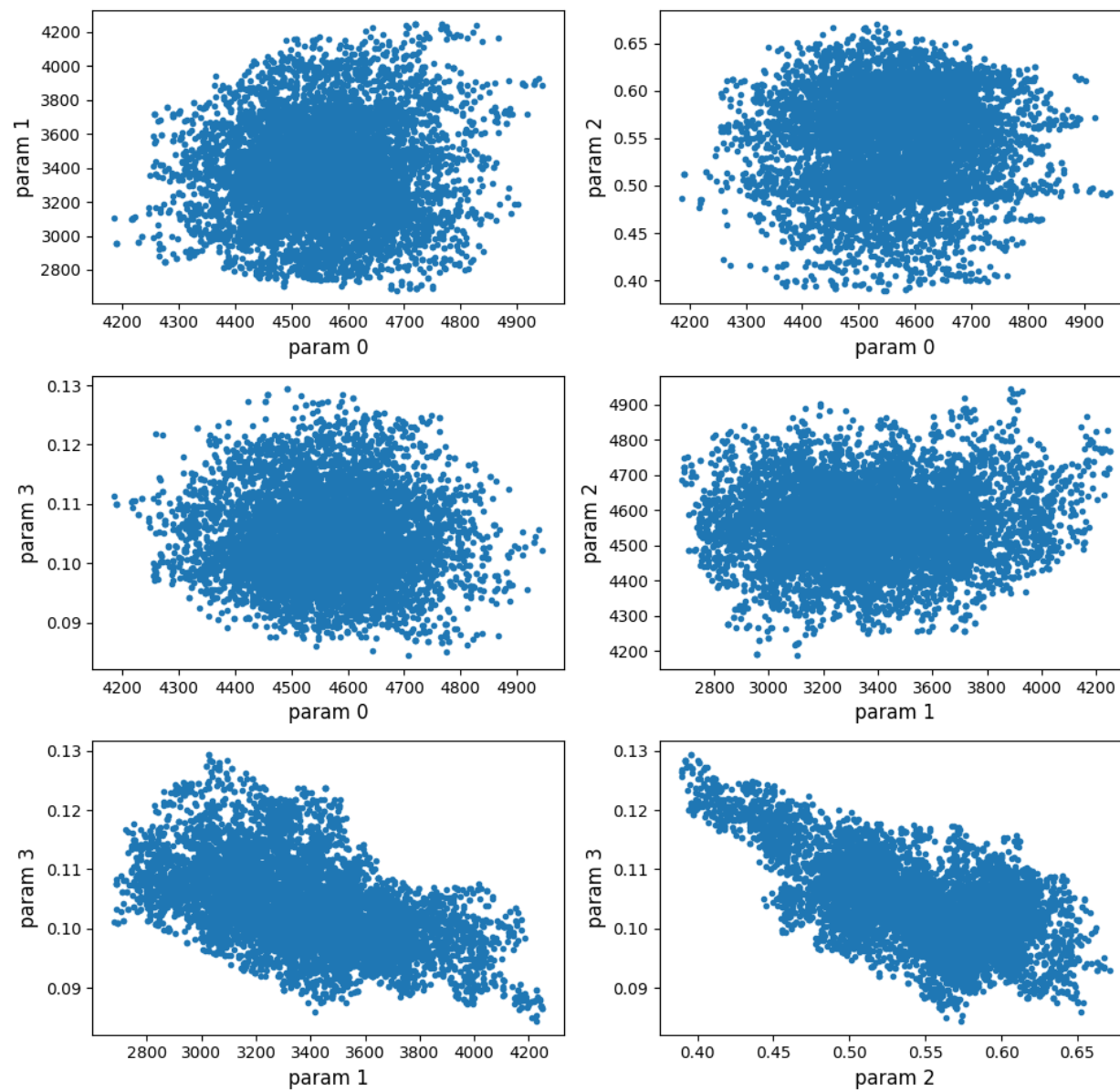
- Samples are not drawn from the covariance matrix
- Only the starting point comes from LM
- Chi-Square of the starting point of MCMC (result of LM): 8006.655
- Chi-Square of MCMC best-fit point: 7978.887
- Acceptance ratio for 10000 Steps: 78.3%

Fitted Parameters:

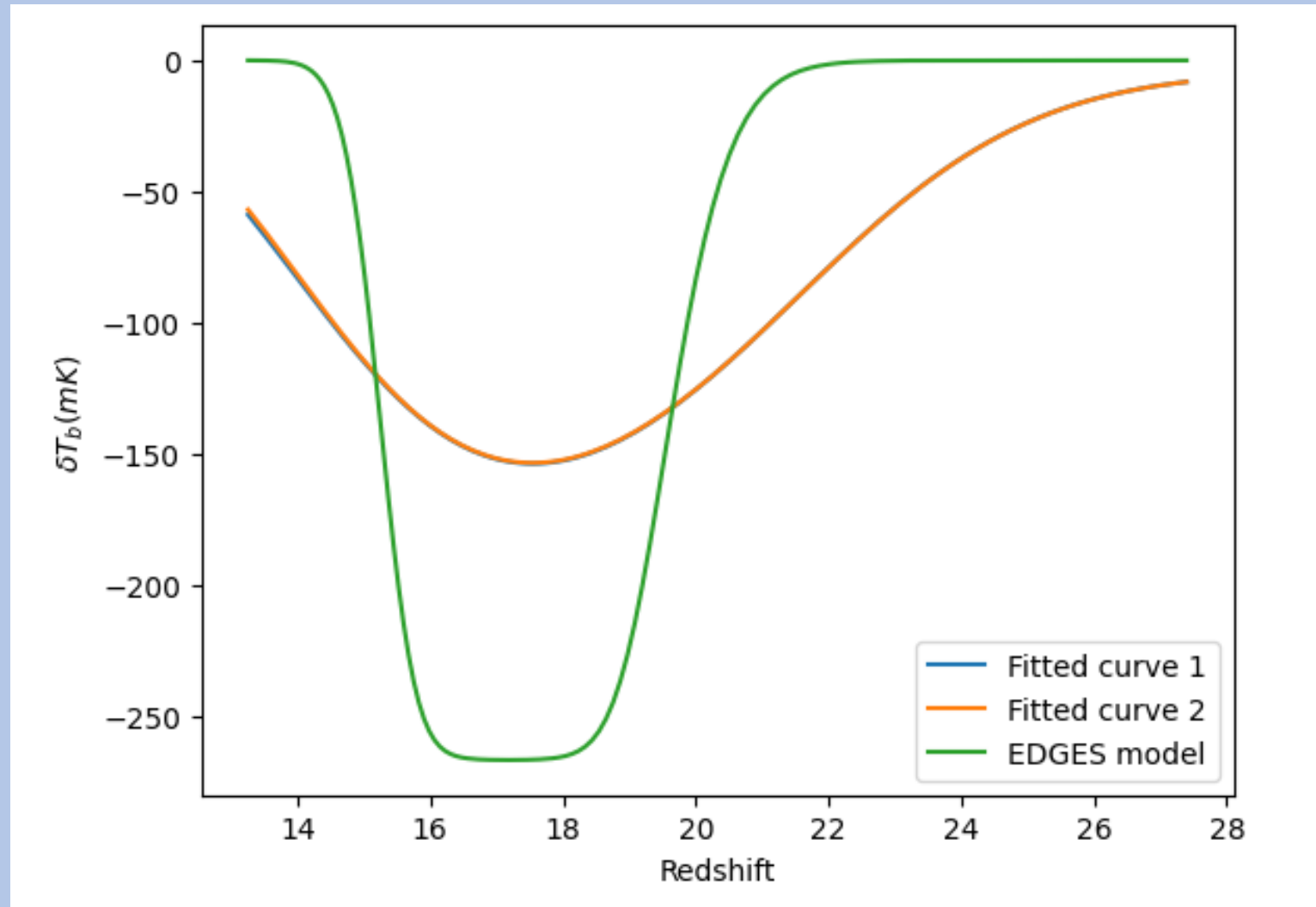
[4.55952291e+03, 3.33655523e+03,
5.41704221e-01, 1.04942202e-01]



Corner plots of the chain



Comparison between the two fits



Physical Explanation of Parameters:

- **f_{esc}**
fraction of ionizing photons that escape their host galaxy into the IGM
- **N_{ion}**
mean number of ionizing photons produced per baryon of star formation

$$\bar{x}_i = \zeta f_{\text{coll}} / (1 + \bar{n}_{\text{rec}}),$$

Globally-averaged
ionizing efficiency

$$\zeta = A_{\text{He}} f_{\star} f_{\text{esc}} N_{\text{ion}}.$$

Ionizing efficiency

Furlanetto 2006, eq 82 & 83

- **f_X:**

high- redshift renormalization factor in the relation between X-ray luminosity and star formation rate (Furlanetto 2006, eq 68)

$$L_X = 3.4 \times 10^{40} f_X \left(\frac{\text{SFR}}{1 \text{ M}_\odot \text{ yr}^{-1}} \right) \text{ erg s}^{-1},$$

- **N_{lw}:**

number of photons emitted in the Lyman-Werner band (photon energy in the range of 11.2 to 13.6 eV) per baryon of star formation