TOPICS IN COSMIC STRING PHENOMENOLOGY AND STRING COSMOLOGY

Written By

Samuel Laliberte

Department of Physics

McGill University, Montreal

February, 2020

A thesis submitted to McGill University in partial fulfillment of the requirements of the degree of

Master of Science, Physics

Aknowledgements

First and foremost, I wish to thank my supervisor Robert Brandenberger, who has always been available for meetings and various inquiries during the three years I worked with him. I thank him for believing in me and allowing me to follow my intuition on a variety of research topics.

All my achievements wouldn't have been possible without the valuable input of the professors and students in the department of physics at McGill University. Among these people, I wish to thank my current and past office mates: Omar Alarayi, Bryce Cyr, Heliudson Bernardo, Suddhasattwa Brahma, Gabrielle Mitchell, Ziwei Wang, Vincent Comeau, Disrael Cunha, Jerome Quintin, and Elisa Ferreira. You guys have always been a motivation for me to come to the office every day, and were always open to discuss research or just chat about our daily lives.

I would also like to thank my family for believing in me throughout my studies and inspiring me to be better every day. A special thanks to my girlfriend Laurence, who lets me work late every evening and tries to keep me healthy with her delicious meals.

As a special mention, I would like to thank the organizers of SIFTS 2019 for their hospitality and lecturers of the event for their insightful lectures on string theory and quantum gravity. This was definitely one of the best academic experiences of my life, and I learned a lot from lectures and discussions with other students attending the event.

Finally, I would like to thank all the contributors who made my research projects possible. I thank NSERC for a graduate scholarship, MSI for a graduate award, and the Canada Research Chair program for its generous contributions.

Abstract

In this thesis, we explore some implications of cosmic strings and string theory for early time cosmology. Our work comes in two parts. First, we study how cosmic strings produce charged particles which can, by emitting electromagnetic radiation, partially ionize neutral hydrogen during the dark ages. We find that corrections to the ionization fraction of neutral hydrogen induced by cosmic strings lead to new constraints on the string tension around $G\mu \sim 10^{-16} - 10^{-20}$ for values of the primordial magnetic field greater than $B_0 \sim 10^{-11}$ Gauss. Second, we study some aspects of moduli stabilization using string gases in the context of the Swampland. The goal of work was to find if string gas cosmology is consistent with the "Swampland conjectures", a series of conjectures which effective field theories must obey to be consistent with string theory. In the framework which we derive, the matter Lagrangian for string gases yields a potential for the size moduli which satisfies the de Sitter conjecture with the condition $\frac{|\nabla V|}{V} \geq \frac{1}{\sqrt{p}}\frac{1}{M_p}$, where p is the number of compactified dimensions. Moreover, the moduli find themselves stabilized at the self-dual radius, and gravity naturally emerges as the weakest force.

Abrégé

Dans cette thèse, nous explorons certains impacts des cordes cosmiques et de la théorie des cordes sur la cosmologie primordiale. Notre travail est effectué deux parties. Premièrement, nous étudions comment les cordes cosmiques produisent des particules chargées qui peuvent, en émettant un rayonnement électromagnétique, partiellement ioniser l'hydrogène neutre lors de le période de l'univers précédant l'âge de la réionisation. Nous concluons que les corrections à la fraction d'ionisation de l'hydrogène neutre induite par les cordes cosmiques conduisent à de nouvelles contraintes sur la tension des cordes autour de $G\mu \sim 10^{-16}-10^{-20}$ pour les valeurs du magnétique primordial champ supérieur à $B_0 \sim 10^{-11}$ Gauss. Deuxièmement, nous étudions certains aspects de la stabilisation de dimensions supplémentaires à l'aide de gaz à cordes dans le contexte du "Swampland". Le but du travail était de déterminer si la cosmologie des gaz à cordes est cohérente avec les "Swampland conjectures", une série de conjectures auxquelles les théories des champs à basse énergie doivent obéir afin d'être cohérentes avec la théorie des cordes. Dans le cadre théorique que nous dérivons, l'équation de Lagrange pour les gaz à cordes procure un potentiel relié au dimensions supplémentaires qui satisfait la "de Sitter conjecture" avec la condition $\frac{|\nabla V|}{V} \geq \frac{1}{\sqrt{p}}\frac{1}{M_p}$, où p est le nombre de dimensions supplémentaires. De plus, les modules se trouvent stabilisés au rayon auto-dual, et la gravité émerge naturellement comme la force la plus faible.

Statement of contribution

This manuscript based thesis contains two published articles that are original. These are presented in their original form, including the bibliography. We state below the contribution of the author to each of the included works.

Contributions of the author

S. Laliberte and R. Brandenberger, "Ionization from cosmic strings at cosmic dawn," Phys. Rev. D **101**, no. 2, 023528 (2020) [arXiv:1907.08022 [astro-ph.CO]]. Ref. [1] in the bibliography.

This article is presented in Chapter 4. As the first author of this paper, I carried out all the computations and revised them with the help of my advisor Robert Brandenberger. I also produced all the figures and wrote the major portion of the paper except the introduction, which was written by Robert. Robert also helped to review the paper and contributed to fixing minor issues.

S. Laliberte and R. Brandenberger, "String Gases and the Swampland," arXiv:1911.00199 [hep-th]. Ref. [2] in the bibliography.

This article is presented in Chapter 5. Again, as a first author, I carried out all the computations and revised them with the help of my advisor Robert Brandenberger. This time I also wrote the entire paper. Then, Robert contributed by clarifying statements in the introduction and adding useful footnotes. Robert also helped to review the paper and contributed to fixing minor issues.

Additional Content

The two articles we present in this thesis are written for an audience with knowledge of cosmology and string theory, and more particularly the topics of cosmic strings and the Swampland. Hence, we deemed appropriate to include a review section covering the knowledge necessary to understand our contributions. Here is a list, sorted by topic, of documents that helped write this review.

Big bang cosmology and inflation

- [3] R. H. Brandenberger, "Inflationary cosmology: Progress and problems," hep-ph/9910410.
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String gas cosmology

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String theory and the Swampland

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- [14] D. Tong, "String Theory," arXiv:0908.0333 [hep-th].
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Contents

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Chapter 1

Introduction

Cosmology is the study of the evolution of the universe as a whole. More precisely, it is the scientific study of the origin, evolution, and eventual fate of the universe. With the current era of precision cosmology, our understanding of the history of the universe is becoming increasingly precise. Given the current data, we know that the universe began about 13.8 years ago as a hot plasma. About 300 000 later, this plasma led place to a gas of hydrogen which then evolved into the stars and galaxies we observe today. Despite technological advances, our answers to the most fundamental questions remain speculative. For example, we still ignore what physical process led to the creation of the universe, and we are unsure how the universe will meet his end. Theories beyond standard big bang cosmology may be able to answer these questions. However, significant theoretical progress remains to be done to reach a satisfying answer.

In this thesis, we make small progress into understanding theories beyond the standard model which might give us some insight on the beginning of the universe. First, we discuss some new experimental constraints related to cosmic strings [18]. These string are defects that could have formed during a phase transition in the early universe. Studying these strings might help us constrain the parameter space of some field theories beyond the standard model. Second, we show how string gas cosmology [9] is consistent with the recent Swampland conjectures (See [15] for a review). As a result, string gas cosmology might be a correct description of the late universe consistent with string theory.

Before digging into the topics of our thesis. one might wonder why cosmology is a good ground to study high energy physics. Hence, we will begin by a review of some important issues to modern physics, and motivate why we should seek answers in cosmology.

1.1 The current status of particle physics

The standard model of particle physics has been tested to remarkable accuracy. Perhaps one of the most important recent successes of the theory is the discovery of the Higgs boson at 125 GeV [19] [20], which explains how the W and Z bosons acquire their mass at low temperatures. However, there are many reasons to think that our understanding of fundamental physics is incomplete. For example, there are no experimentally verified mechanism which can explain the neutrino masses, the stability of the Higgs mass is unsolved and the standard model of particle physics still suffers from the strong CP problem. Furthermore, astrophysical observations suggest the existence of exotic species of matter such as dark matter and dark energy, which cannot be explained by known physics.

Many attempts to answer the shortcomings of the standard model involve exotic particles which are part of grand unified theories operating at energy scales of 10^{16} GeV or higher. These particles are either too heavy to be produced in our most powerful accelerators, which operate at an energy scale lower than 10^3 GeV, or too weakly coupled to the standard model to be produced in a sufficiently large number. This problem becomes worse if we try to reconcile the standard model with a quantum theory of gravity. Such theories are believed to operate at an energy scale of 10^{19} GeV, where quantum fluctuations of space-time itself should be taken into account. This energy scale is far out of reach of current accelerators, and potentially unreachable on earth.

On the scale of the universe, however, phenomena involving beyond the standard model of particle physics are abundant. In the hot environment of the early universe, heavier and weakly coupled particles can be produced in large quantities and leave their imprint on the evolution of the universe. The gravitational influence of these particles can be detected during phase transitions of the universe (e.g. during the electroweak phase transition), and their interaction with standard model particles can leave distinct signatures on cosmological observables. Given these facts, it is likely that the next big discoveries in theoretical physics will come from cosmological observations, and not from experiments on earth.

1.2 Cosmic strings and probing for new physics in cosmology

Probing for effects caused by elementary particles is not the only way to constrain high energy physics. Many field theory models admit the existence of extended objects named topological defects. These topological defects come into four forms: domain walls, cosmic strings, monopoles, and textures. Topological defects gravitate and can leave an impact on cosmology. In return, cosmological observables provide means to constrain the field theory giving rise to the defect.

Let us consider cosmic strings for example. The energy density of cosmic strings is related to the symmetry breaking scale η via the relationship

$$\mu \sim \eta^2 . \tag{1.1}$$

Conversely, most properties of a string network depend on the dimensionless number $G\mu$ where G is Newton's constant. Hence, by constraining $G\mu$, we can rule out or confirm field theories with a symmetry scale η.

Effects by elementary particles and topological defects can be constrained using data from the cosmic microwave background (CMB). This background describes a time of the universe where light decoupled from the primordial plasma and started propagating freely in the universe.

Big bang cosmology suggests that the universe began in a highly dense and hot state. In this primordial plasma made of fundamental particles, the interaction rate of photons with other standard model particles was high and light was confined by repeated interaction with other particles. As the universe expanded and cooled down, the interaction rate of particles became lower than the rate of expansion of the universe, and light became allowed to move freely. This decoupling happened roughly 300,000 years ago and remains observable today in the microwave band of the electromagnetic spectrum, hence the name "cosmic microwave background".

Relic radiation from the CMB reveals a great deal of information about the early universe. For example, its homogeneity and isotropic features are a great indicator that the universe is homogeneous and isotropic when viewed on large enough scales. State of the art measurement by the Planck satellite [16] [21] [22] show an average temperature of $\bar{T} = 2.7255$ K today [23]. Meanwhile, temperature fluctuations are on the order of

$$\frac{\delta T}{\bar{T}} \sim 10^{-4}\,, \tag{1.2}$$

which shows a remarkable agreement with a homogeneous and flat universe. These small

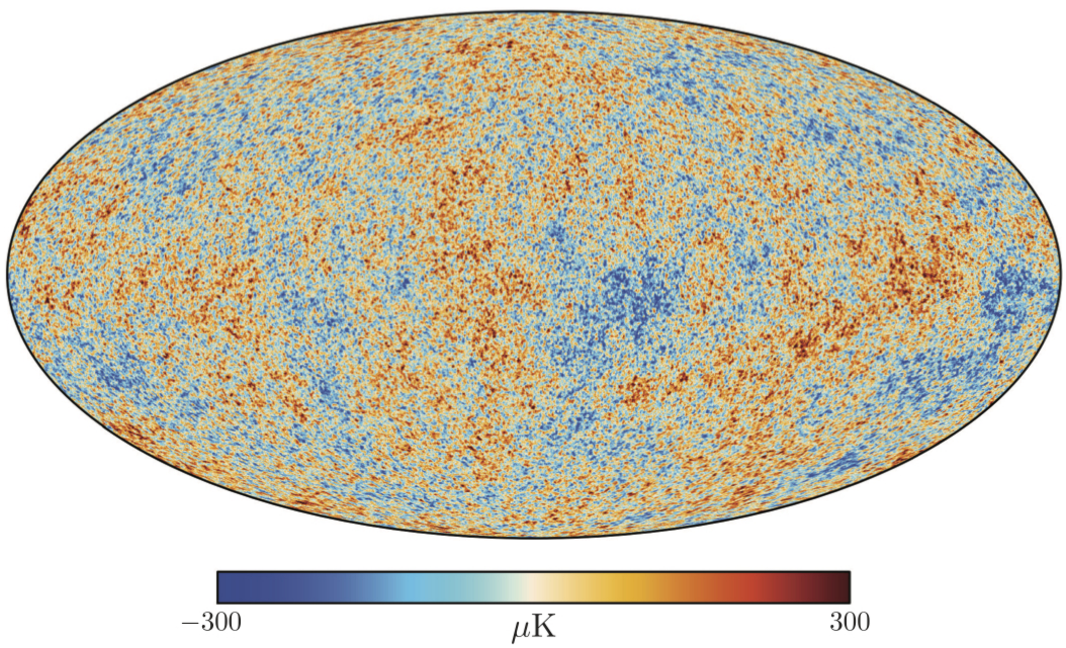

Figure 1.1: Planck 2015 CMB sky map as presented in [16]. Varying colors show fluctuations in temperature in μ K.

fluctuations, which can be seen in Figure 1.1, can be studied using two-point correlation functions. We first decompose the temperature fluctuations a sum of spherical harmonics $Y_{lm}(\theta, \phi)$

$$\frac{\delta T}{\bar{T}}(\hat{n}) = \sum_{l=l}^{\infty} \sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\theta, \phi)\,, \tag{1.3}$$

weighted by a coefficient a_{lm}, which encodes the size of fluctuations with respect to spherical angles θ and ϕ on the sky. Here, $\hat{n}$ is a unit radial vector. Then, we evaluate the correlation functions and obtain

$$\langle \delta T(\hat{n}) \delta T(\hat{n}') \rangle = \sum_{l=1}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos\theta)\,, \tag{1.4}$$

where $\hat{n} \cdot \hat{n}' = \cos\theta$ and P_l are Legendre polynomials. The expansion coefficents defined by

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \langle a_{lm}^* a_{lm} \rangle \tag{1.5}$$

describe the temperature-temperature angular power spectrum of the universe. This power spectrum is plotted in Figure 1.2. The CMB is known to be a black body to remarkable

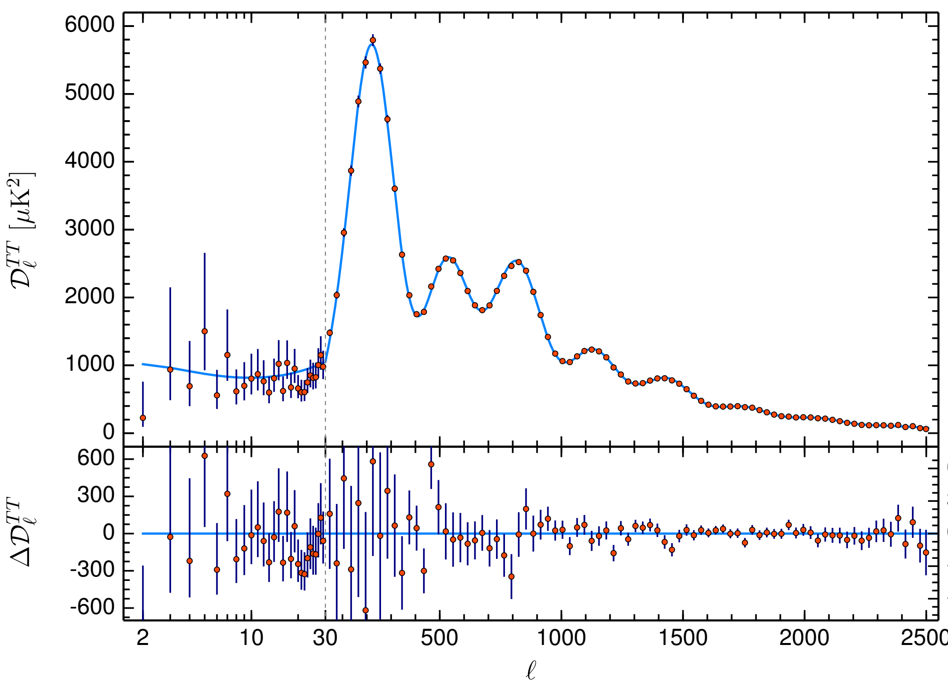

Figure 1.2: Fit of the angular power spectrum as measured by Planck (2018 release) [17]. Here, the data points describe the quantity $D_l^{TT} = l(l+1)C_l/(2\pi)$ with error bars of 1σ.

accuracy, any effects from models beyond the standard model are expected to be heavily constrained by the data.

Again, let us discuss cosmic strings as an example. Such strings leave linear non-gaussianities in the CMB which scale as

$$\frac{\delta T}{T} \sim 8\pi G\mu v\gamma(v)\,, \tag{1.6}$$

where v is the transverse string velocity with the gamma factor $\gamma(v) = (1-v^2)^{-1/2}$. Hence, we expect strings to leave features on the power spectrum which scales as $G\mu$. The absence of these features leads to the constraint

$$G\mu < 1.7 \times 10^{-7} \tag{1.7}$$

as shown in [24].

Direct modification of temperature fluctuations is merely an example of how new physics can be constrained by the CMB. Fitting the power spectrum requires precise knowledge of the foreground and physical processes at the time of recombination. Hence, any new physics which modifies fundamental parameters used for the fit of the CMB is expected to be constrained by the CMB data. The parameters are numerous: abundance of matter species Ω_m, amplitude of scalar perturbations A_s, scalar tilt n_s, optical depth τ, Hubble expansion rate H_0, etc. Modification of these parameters by new physics will change the fit of the power spectrum, and hence provide constrain the new physics.

1.3 The case for string theory and string gas cosmology

String theory is a theory with a very ambitious goal: unify all the force of nature and all the particles into a single framework. At the center of this theory are strings, one-dimensional objects whose oscillations give rise to a wide range of particles.

There are very good reasons to think string theory is the correct description of quantum gravity and physics at higher energies:

1. Every string theory contains a massless spin-2 state, which corresponds to a graviton. At low energy, graviton dynamics reduce to general relativity.

2. String theory is a consistent theory of quantum gravity or at least in perturbation theory. Graviton interactions can be computed, whereas in quantum field theory gravitational interactions are non-renormalizable.

3. String theory leads to gauge groups large enough to include the standard model of particle physics. Since we expect forces to unify at high energies, this means string theory has the potential to unify all forces at high energies.

4. String theory has no free parameters. Fields play the role of couplings in the theory and their evolution is understood perturbatively.

5. Although there are multiple descriptions of the theory (Type II A/B , Type I, Heterotic, etc), the descriptions unify into a single framework called M-theory. Hence, one can argue string theory is a unique, universal description of the universe.

Given the properties above, there are many reasons to think string theory could provide an accurate description of early universe cosmology. First, we expect the general relativity to break down at the beginning of the universe as a result of a cosmological singularity. We expect this singularity should be resolved by quantum gravity, and hence string theory. Second, we expect forces to unify as the temperature of the universe increases in the early universe. There again, string theory provides a unified description of physics which may provide a correct description of the early universe. Third, because the CMB is opaque, we cannot directly look at physics taking place at the very beginning of the universe. Hence, we must rely on the most self-consistent theory at hand to make predictions related to physics beyond the standard model, which string theory provides.

One of the challenges of modern cosmology has been to find a consistent description of early universe cosmology consistent with string theory. In the past, the paradigm of string cosmology has been to realize inflation within the framework of string theory. While some approaches have been relatively successful [25], there exist a variety of no-go theorem which forbid the existence of a de Sitter vacua in a variety of scenarios (e.g. [26]), recent advances of the Swampland program have contributed to constrain inflation in string theory even more.

The Swampland [27] is a recent program in string theory which aims to distinguish low energy theories that are consistent with string theory (the landscape) and those that aren't (the swampland). The program is based on a set of conjectures, the "Swampland conjectures", which low energy theories must obey to be consistent with string theory. The conjectures have recently proven to be a powerful tool to study cosmology. For example, using the de Sitter conjecture [28] [29], it was recently shown that inflation might not realizable in string theory. This is because most inflationary models rely on a mechanism named "slow roll" [3] in which a scalar field slowly rolls down a potential well. For inflation to be effective, the slow-rolling field must reach Trans-Planckian values. However, these Trans-Planckian values are forbidden by the swampland conjectures and more recently the Trans-Planckian cosmic censorship conjecture [30]. As a result, inflation is severely restrained and suffers from a fine-tuning problem [31]. This might indicate that we must look for alternatives to inflation which are consistent with string theory.

String gas cosmology is an early universe scenario based on symmetries from string theory. In this scenario, the dynamics of the early universe are described by a gas of strings and their thermodynamics. Because it relies on T-duality, the theory has the potential to solve the singularity problem of cosmology. Hence, it has the potential to provide a consistent theory of the early universe, and consequently an alternative scenario to inflation. On this topic, string gas cosmology has a key advantage on the inflationary scenario. Because string gas cosmology borrows from string symmetries, we expect the theory to be consistent with the Swampland conjecture. Proving this statement would put the theory on a stronger footing compared to inflation.

1.4 Outline

Our thesis will be organized as follows. Part I is a review of cosmology, cosmic strings, string theory, and the Swampland. This knowledge is essential to understand Part II, which presents original research papers publishes by the author of this thesis.

In chapter 2, we give a short review of standard big bang cosmology and highlight some of its issues. Then, we shortly explain the inflationary scenario and the string gas scenario, which present a solution to these issues. We also give a short introduction to cosmic strings, and more specifically the topic of string formation and network evolution.

In chapter 3, we review the basics of bosonic string theory, present its low energy action and briefly explain string compactification. Then, we motivate three Swampland conjectures, namely the distance conjecture, the weak gravity conjecture, and the de Sitter conjecture.

In chapter 4, we discuss new experimental constraints related to cosmic strings. More precisely, we show that cosmic strings emit charged particles that can ionize neutral hydrogen in the dark ages. The ionization fraction of hydrogen is constrained by the optical depth.

Hence, we obtain constraints related to the cosmic string tension and the primordial magnetic fields.

In chapter 5, we show how the potential of string gases satisfies the Swampland de Sitter conjecture. This result suggests the late time description of string gas cosmology is consistent with string theory.

Finally, in chapter 6, we summarise the key results of our thesis, point out ways to improve our results and discuss new avenues of research. We also reflect on the current state of theoretical cosmology and give some take-away messages.

1.5 Conventions

In our work, we use the units given to us by the gods, otherwise known as the "natural units". In these units the speed of light and Planck's constant are set to unity: $c = \hbar = 1$. Greek indices are used for spacetime coordinates, $\mu, \nu \in \{0, 1, 2, ...\}$ and Latin indices are uses for spatial coordinates, $i, j \in \{0, 1, 2, ...\}$. In the context of string compactification, Greek indices $\mu, \nu \in \{0, 1, 2, ..., d\}$ are associated to non-compact dimentions, Latin indices $a, b \in \{d + 1, d + 2, ..., D\}$ are associated to compact dimensions and capital Latin indices $M, N \in \{0, 1, ..., D\}$ span compact and non-compact dimensions. Here, d is the number of non-compact dimensions and D is the total number of dimension. In cosmology, we use the metric signature $\eta_{\mu\nu}$ = diag(1,-1-,1,-1) while in string theory we use $\eta_{\mu\nu}$ = diag(-1,1,1,1). Finally, we define the d-dimensional Planck mass M_p^d as the coefficient of the d-dimensional Ricci scalar R^d in the Einstein-Hilbert action

$$S = \int d^d x \sqrt{-g} \left[\frac{(M_p^d)^{d-2}}{2} R^d + ... \right] , \tag{1.8}$$

where in four dimensions we obtain

$$M_p = \sqrt{\frac{1}{8\pi G_N}} \sim 10^{18} GeV \, . \tag{1.9}$$

Here, G_N is Newton's constant. Often, we will set M_p to one for convenience.

Part I

Review of cosmology and string theory

Chapter 2

A brief review of cosmology and cosmic strings

As we mentioned before, cosmology is the science of the evolution of the universe as a whole. In the following sections, we will cover the basics of standard big bang cosmology, discuss some of its problems and introduce scenarios, such as inflation and string gas cosmology, which try to answer these problems. Then, we provide a short introduction to cosmic strings and cosmic string network evolution.

2.1 Standard big bang cosmology

As in every gravitational system, the study of cosmology begins with the Einstein field equations for general relativity,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \, . \tag{2.1}$$

Here, $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, G is Newton's constant and $T_{\mu\nu}$ is the energy-momentum tensor. The Ricci tensor and Ricci scalar encode information about the curvature of space-time while the energy-momentum tensor encodes information about the distribution of matter in space-time. Hence, the Einstein equations describe how matter curves space-time.

The Einstein equations can be derived from the Einstein-Hilbert action with matter,

$$S = \frac{1}{8\pi G}\int d^4xR + S_{matter}\,, \tag{2.2}$$

where the energy momentum tensor is defined by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{matter}}{\delta g^{\mu\nu}}\,. \tag{2.3}$$

If matter is conserved, $T_{\mu\nu}$ satisfies the condition

$$\nabla_\mu T^{\mu\nu} = 0\,. \tag{2.4}$$

We will assume this to be the case in a cosmological setup.

In cosmology, the universe is a gravitational system that obeys the cosmological principle. This principle states that the spatial distribution of matter is homogeneous and isotropic when viewed on a large enough scale. The most general metric which expresses these properties, called the Friedmann-Robertson-Walker (FRW) metric, is described by

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right), \tag{2.5}$$

where $a(t)$ is a scaling factor and $k = -1, 0, 1$ is a constant related to the curvature of spacetime. Given equation 2.5, the most general stress-energy tensor $T^\mu{}_\nu$ we can write as the diagonal form $\mathrm{diag}(\rho(t), -p(t), -p(t), -p(t))$. Given then constraints, the equations of motion of the universe (FRW equations) follow from stress-energy conservation and the Einstein field equations.

$$\frac{8\pi G}{3}\rho = \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} \tag{2.6}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{2.7}$$

$$0 = \dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) \tag{2.8}$$

An important quantity which arises naturally from the equations above is the Hubble parameter

$$H = \frac{\ddot{a}}{a}\,, \tag{2.9}$$

This quantity describes the expansion rate of the universe, where $H > 0$ corresponds to an expanding universe and $H < 0$ corresponds to a collapsing universe. The Hubble parameter is very useful in cosmology since it sets a fundamental scale for observables on the scale of the universe. Some useful scales are for example the Hubble time $t_H \sim H^{-1}$ and the Hubble length $l_H \sim H$.

The FRW equations can be solved using the equation of state $p = \omega\rho$, where ω is a constant which depends on the type of matter being considered. The constant takes the value $\omega = 1/3$ for radiation, $\omega = 0$ for matter and $\omega = -1$ for dark energy. Solving the FRW equations, we obtain the following energy density and scale factor:

$$\rho \propto a^{-3(t+\omega)} \qquad a(t) \propto \begin{cases} t^{2/[3(1+\omega)]} & \omega \neq -1 \\ e^{Ht} & \omega = 1 \end{cases} \tag{2.10}$$

Given the solutions above, we conclude that the universe is expanding at a rate that depends on the matter content of the universe. As a consequence, the proper wavelength λ of photons find itself redshifted at the rate

$$\lambda \propto a(t)\,. \tag{2.11}$$

Using the relationship above, we will define the redshift z as

$$1 + z = \frac{\lambda_0}{\lambda_{t_{em}}} = \frac{a(t_0)}{a(t_{em})}\,, \tag{2.12}$$

where λ_0 and $a(t_0)$ are the wavelengths and scale factors today and λ_{em} and $a(t_{em})$ are the wavelengths and scale factors at the time of photon emission. This redshift is often used as a measure of time in cosmology, where $z = 0$ is the redshift today and $z \to \infty$ is the redshift at the creation of the universe.

If more than one matter species contribute significantly to the energy density and pressure we must replace the pressure and energy density by the sum of the pressure and energy density of each species

$$\rho = \sum_i \rho_i \qquad p = \sum_i p_i\,. \tag{2.13}$$

In this case, the solutions to the equations of motion must be found numerically. Nonetheless, we can express ρ as a function of z to determine which type of matter dominates at each stage of the universe. Comparing the different terms in the sum 2.13, we find the universe was first dominated by radiation, then matter and finally dark energy (See Figure 2.1). The

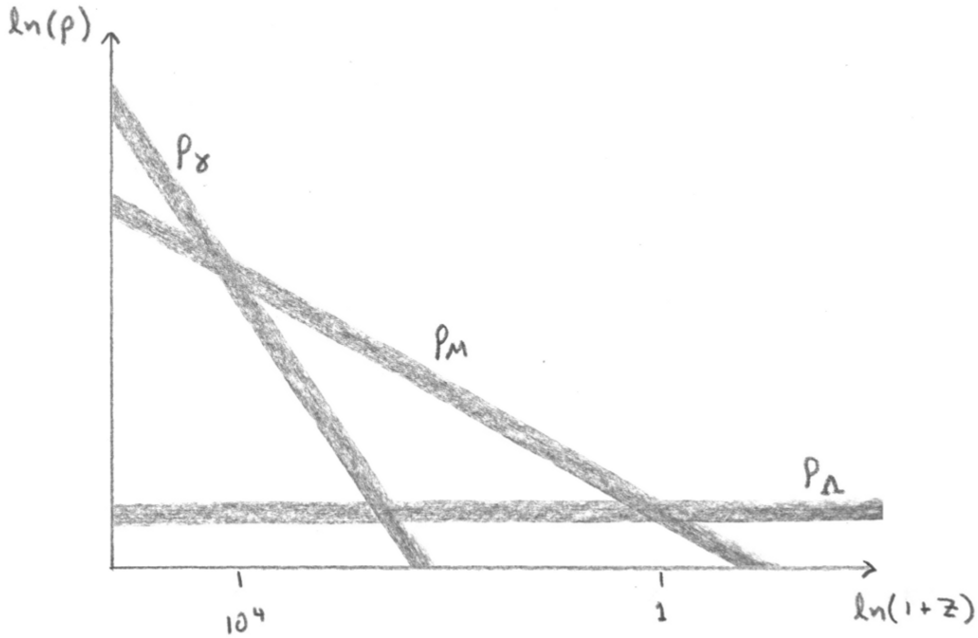

Figure 2.1: Schematic description of how each type of matter scale as a function of the redshift z (Taken from [4]). At early times, radiation dominates followed by matter and vacuum energy.

fraction of each matter component of the universe is often expressed in terms of the critical density

$$\rho_c = \frac{3}{8\pi G} H^2 \,, \tag{2.14}$$

which describes the total energy of a flat universe (when $k = 0$). Current observations give

$$\Omega_i \equiv \frac{\rho_i}{\rho_c} = \begin{cases} 0.7\,, & \text{Vacuum} \\ 0.3\,, & \text{Matter} \\ 10^{-4}\,, & \text{Radiation} \end{cases} \tag{2.15}$$

for the distribution of matter today, where the energy fraction Ω_i satisfies $\Omega \equiv \sum_i \Omega_i = 1$ for a flat universe. More precise values can be found in [17]. For curved universes, the sum of the energy fractions reads

$$\Omega - 1 = \frac{k}{(aH)^2}, \tag{2.16}$$

where the right-hand side of the equation corresponds to energy stored in the form of curvature.

Although standard big bang cosmology has been successful in describing the late-time evolution of the universe, it suffers from some problems. We will end this section by discussing some of these problems, namely the horizon, the flatness, and the singularity problem, before exploring solutions such as inflation and string gas cosmology in the next sections.

2.1.1 The horizon problem

The horizon problem comes from a mismatch between the future and past comoving regions with respect to the time of recombination. Standard big bang cosmology predicts the universe was created a finite amount of time ago, when the universe was dominated by radiation. However, this conflicts with the observed homogeneity of the cosmic microwave background.

This can be seen by computing our past light cone $l_p(t_{rec})$ and the future light cone at the moment of the big bang $l_f(t_{rec})$ at the time of recombination. Starting from the time of creation $t = 0$, the maximal comoving distance over which microphysical forces could have caused the homogeneity of the cosmic microwave background (CMB) is given by the future comoving distance

$$l_f(t_{rec}) = \int_0^{t_{rec}} a(t)^{-1} dt \approx 3{t_0}^{\frac{2}{3}} {t_{rec}}^{\frac{1}{3}} , \tag{2.17}$$

where t_{rec} is the time of reionization and t_0 is the age of the universe. In comparison, the comoving distance $l_f(t_{rec})$ over which the CMB is observed to be homogeneous is given by the past comoving distance

$$l_p(t_{rec}) = \int_{t_{rec}}^{t_0} a(t)^{-1} dt \approx 3t_0 \left(1 - \left(\frac{t_{rec}}{t_0}^{\frac{1}{3}} \right) \right) \approx 3t_0 - l_f(t_{rec}). \tag{2.18}$$

As we can see in Figure 2.2, l_p is much bigger than l_f. Therefore, Standard big bang cosmology cannot explain the apparent homogeneity of the CMB.

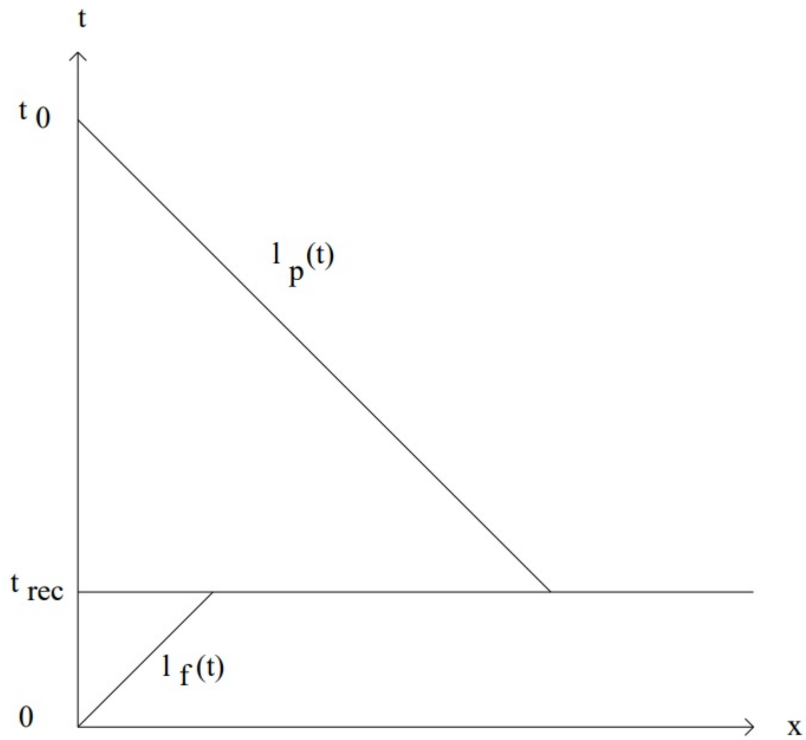

Figure 2.2: Schematic description of our past light cone l_p and the future light cone at the time of creation l_f (Taken from [3]). Our past ligh cone is much larger than l_f at t_{rec}, the time of recombination.

2.1.2 The flatness problem

In standard big bang cosmology, the universe is observed to be approximately flat ($\Omega \simeq 1$). However, flat universes are an unstable fixed point in the theory, and achieving spatial flatness today requires an incredible amount of fine-tuning in the early universe.

This can be seen by considering a curved universe, and rewriting the Friedmann equation as

$$H^2 + \epsilon T^2 = \frac{8\pi G}{3}\rho \tag{2.19}$$

with

$$\epsilon = \frac{k}{(aT)^2}. \tag{2.20}$$

In standard cosmology, ϵ is a constant so we can estimate the variation of energy density as

$$\frac{\rho - \rho_c}{\rho} = \frac{3}{8\pi G}\frac{\epsilon T^2}{\rho_c} \sim T^{-2}\,. \tag{2.21}$$

The equation above means that as the temperature of the universe decrease with time, the energy density of the universe deviates more and more from the critical value. Considering the age of the universe and the change in temperature since the Big Bang, the universe had to be extremely close to a perfectly flat universe. For example, at $T = 10^{15}$ GeV, the deviation from flatness gives

$$\frac{\rho - \rho_c}{\rho} \sim 10^{-13} \,. \tag{2.22}$$

These extremely fine-tuned conditions cannot be explained by standard cosmology, hence the "flatness" problem.

2.1.3 The singularity problem

In standard big bang cosmology, the scale factor $a(t)$ of the universe starts from an initially small value then increases following a power law or an exponential law depending on what type of matter dominates the universe. This poses a problem in the limit $t \to 0$, where $a(t)$ vanishes and the cosmological description of the universe is expected to break down. This can be seen by computing the Ricci scalar of the theory

$$R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \,. \tag{2.23}$$

In the limit where $a(t) \to 0$, the Ricci scalar blows up to infinity, which signals the breakdown of general relativity. We conclude there exists a cosmological singularity at $t = 0$, hence the name "singularity" problem.

2.2 Inflation

The inflationary scenario was suggested as a solution to the horizon and flatness problems [32]. The idea is to assume there existed a lapse of time, near the beginning of the cosmological history, where the universe was exponentially expanding. This inflating phase can be described by the scale factor

$$a(t) \sim e^{Ht} \,, \tag{2.24}$$

where the Hubble parameter H is constant. Assuming inflation lasts for a sufficiently long time (we will come back to this later), the causal horizon during the inflationary phase becomes large enough to explain the homogeneity of the CMB and the universe emerges sufficiently flat to explain the current flatness of the universe (See Figure 2.3). One can

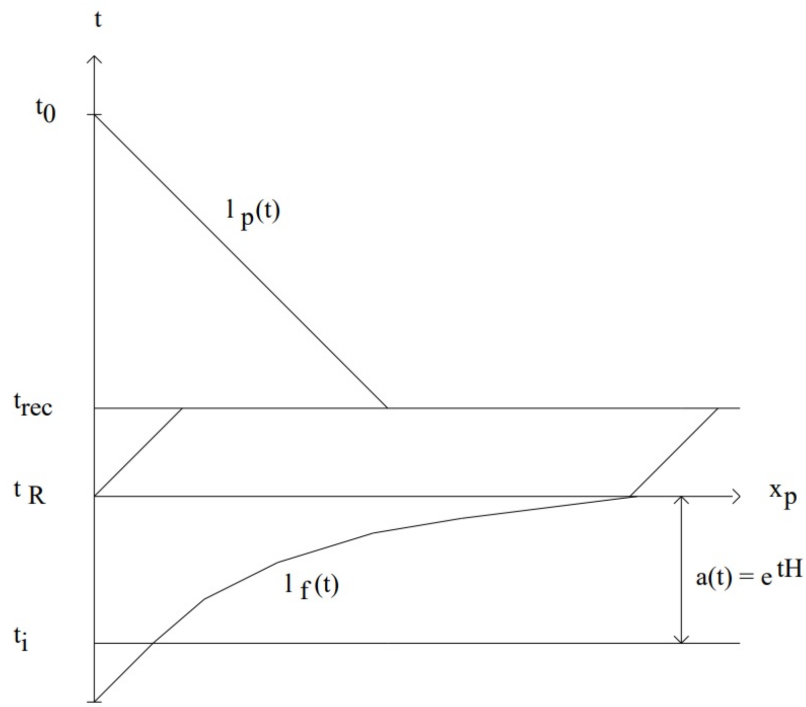

Figure 2.3: By adding a phase of inflation in the early universe, the future light cone can become greater than our past light cone at the time of recombination. Hence, inflation can explain the homogeneity of the CMB. The figure is taken from [3].

obtain inflation by considering a slowly rolling scalar field with the matter action

$$S_{matter} = \int dx^4 \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] . \tag{2.25}$$

and the equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 . \tag{2.26}$$

By "slowly-rolling", we mean the condition $|\ddot{\phi}| \ll |3H\dot{\phi}|$ such that ϕ slowly decreases linearly along the slope of the potential.

In this system, the cosmological principle imposes that ϕ be only dependant on time, such that we obtain the following energy density and pressure

$$\rho = \frac{1}{2}\dot{\phi} + V(\phi)\,, \tag{2.27}$$

$$p = \frac{1}{2}\dot{\phi} - V(\phi)\,. \tag{2.28}$$

Given the equation of state

$$\omega = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi} - V(\phi)}{\frac{1}{2}\dot{\phi} + V(\phi)}\,, \tag{2.29}$$

we conclude that accelerated expansion is possible ($p \approx -\rho$) if the potential energy V dominates over the kinetic energy $\frac{1}{2}\dot{\phi}$. This statement can be made more rigorous by defining a set of parameters that have to be small for inflation to occur. A first parameter can be found by rewriting the second FRW equation to obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = H^2(1-\epsilon)\,, \tag{2.30}$$

where

$$\epsilon = \frac{3}{2}\left(\frac{p}{\rho} - 1\right) = 4\pi G\left(\frac{\dot{\phi}}{H}\right)^2 \tag{2.31}$$

is the so-called slow-roll parameter. Exponential acceleration occurs if $\epsilon < 1$, which corresponds to the condition

$$\left(\dot{\phi}\right)^2 \ll V(\phi) \tag{2.32}$$

found earlier. The slow-roll condition $|\ddot{\phi}| \ll |3H\dot{\phi}|$ also imposes the smallness of a second slow-roll parameter

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1\,, \tag{2.33}$$

such that the accelerated expansion can be sustained for a sufficiently long time. Altogether, the conditions $\epsilon, \eta \ll 1$ can be expressed as a condition on the shape of the inflationary potential

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2\,, \tag{2.34}$$

$$\eta = M_p^2\frac{V''}{V}\,. \tag{2.35}$$

Hence, inflation ends when ϕ reaches a region of the potential where ϵ or η becomes large.

By comparing the particle horizon during inflation to the past light cone related to the surface of last scattering, we can compute the number of e-folds necessary to solve the horizon problem. We find

$$N = \int_{t_i}^{t_f} H dt \approx 60\,, \tag{2.36}$$

where t_i and t_f are respectively the beginning and end of the inflationary phase. Given the number of e-foldings above, the flatness problem finds itself trivially solved. Recall the energy consistency relation

$$\Omega - 1 = \frac{k}{(aH)^2}\,, \tag{2.37}$$

found in the context of standard big bang cosmology. Through 60 e-folds of inflation, the universe rapidly expands as much as 10^{26} times its initial size ($a_f/a_i \sim 10^{26}$) which leaves the universe in a very flat state at the end of inflation.

2.3 String gas cosmology

Although inflationary cosmology can solve the flatness and horizon problems, it still suffers from the singularity problem. String gas cosmology is a scenario that tries to answer this problem while taking inspiration from string theory. This cosmological scenario is based on new symmetries such as T-duality, which will be discussed later in our review of string theory. For now, we will simply review some basics relevant to the cosmological scenario.

The starting point of string gas cosmology is a gas of closed superstrings moving in 10 dimensions. At the beginning of the universe, all the spatial dimensions of the universe are taken to be compact with radius R, and the dynamics of the theory are described by string thermodynamics. Each string in the string gas has three energy modes which contribute to the thermodynamics: the momentum modes which scale as

$$E_n = \frac{n}{R}\,, \tag{2.38}$$

the winding modes which scale as

$$E_w = wR\,, \tag{2.39}$$

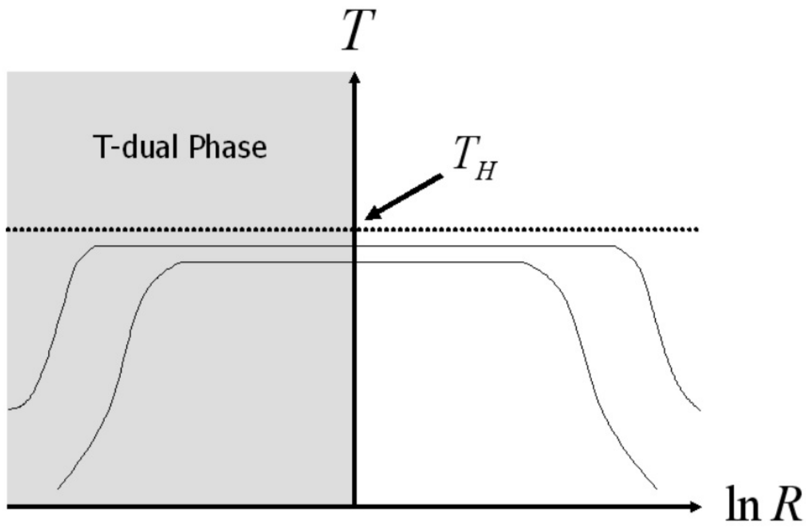

Figure 2.4: Temperature of a string gas as a function of radius. The string gas temperature reaches the Hagedorn temperature at the self-dual radius $R = 1$. The figure is taken from [8].

and the oscillatory modes which are independent of R. These modes transform under T-duality

$$R \leftrightarrow \frac{1}{R} \quad , \quad n \leftrightarrow m \,, \tag{2.40}$$

which relates the momentum modes to the winding modes and vice-versa [1]. Depending on the radius R, thermodynamics will favor the less massive states in the theory. For example, at large R, the theory favors momentum modes, while in the opposite limit the theory favors winding modes. It follows there exists a maximum temperature, the Hagedorn temperature, at the self-dual radius $R = 1$ where the momentum modes and winding modes are equally massive (See Figure 2.4). In this global picture, oscillatory modes don't play an important role since they don't depend on the size of extra dimensions. They will however become important near the Hagedorn temperature, where all massive states must be considered.

The fact that string gases have a maximum temperature is a prime indication that string gas cosmology might be able to solve the singularity problem. In standard big bang cosmology, the temperature in a radiation dominated universe is inversely proportional to the scale

[1] In non-supersymmetric theories, the T-duality transformations also pick up α' corrections. We expect these corrections to vanish in the low temperature limit, as considered in Chapter 5.

factor

$$T \sim \frac{1}{a(t)}\,. \tag{2.41}$$

Hence, at the beginning of the universe, the temperature reaches infinity as a consequence of the singularity problem. In string gas cosmology, the scale factor cannot decrease further than the inverse of the Hagedorn temperature $T_H \sim 1/a(T_H)$. Hence, we expect the cosmology to be non-singular.

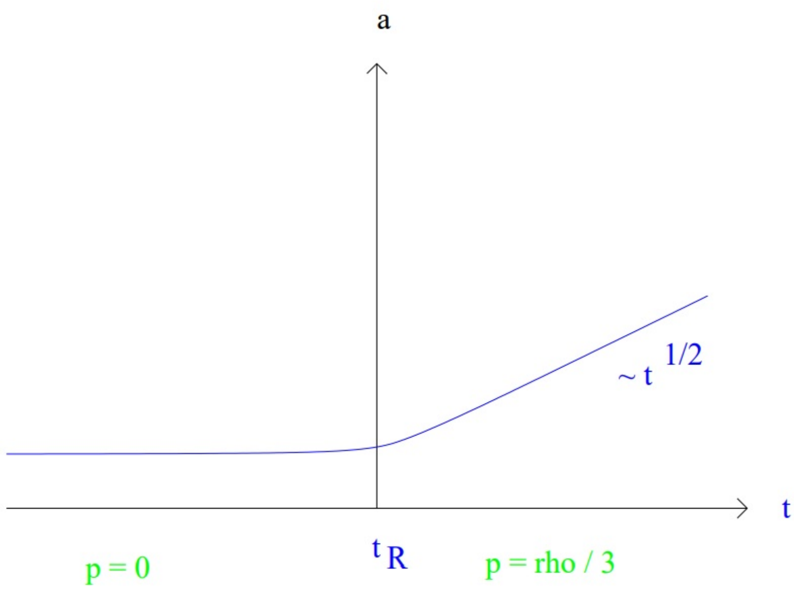

Figure 2.5: Time evolution of the scale factor in string gas cosmology. Near the Hagedorn phase, the scale factor stays approximately constant. After some time t_R, the universe leaves the Hagedorn phase into the radiation dominated phase of Standard Big Bang Cosmology. The figure is taken from [8].

Given the dynamics we described above, one can imagine a cosmological scenario where the universe starts in a metastable phase near the Hagedorn temperature, then cools down to a radiation dominated phase like in standard big bang cosmology (See Figure 2.5). The transition is a consequence of the annihilation of string winding modes into string loops. For this interaction to take place, three spatial dimensions must decompactify while the six other dimensions remain small (See [9] for more details). Hence, string gas cosmology has the potential to explain why three spatial dimensions remain large. The stabilization of the other six compact dimensions is still up to debate, but may find an answer in string gas cosmology [33] [34].

At late times, the dynamics of the theory is described by the low energy dilaton gravity action

$$S_D = \frac{1}{2\kappa_0^2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left[R + 4\partial_\mu\Phi\partial^\mu\Phi\right] + S_{matter}\,, \tag{2.42}$$

where k_0 is some normalization constant, Φ is the dilaton field and S_{matter} is a matter action that is invariant under T-duality. Usually, we take S_m to the hydrodynamical action of a string gas. It is known that the string gas action can stabilize the size of extra dimensions as a consequence of T-duality. We will offer a review of this process in our paper "String gases and the swampland", which will be presented later in the thesis.

2.4 Cosmic strings

Cosmic strings are stable configurations of matter formed at phase transitions in the very early universe. These defects arise in unified particle physics and models of strong, weak and electromagnetic interactions and can act as seeds for galaxies and large scale structures. This is possible because these structures carry energy, which in return leads to an extra attractive gravitational force. In the next sections, we go over the basics of cosmic string formation and cosmic string network evolution. These notions will be useful to understand chapter 4, where we study cosmic strings in the context of reionization.

2.4.1 Cosmic string formation

Cosmic string defects arise naturally in some field theory models as a consequence of spontaneous symmetry breaking. This symmetry breaking occurs when scalar field (e.g. Higgs fields) changes its expectation value as temperatures decreases, hence breaking the symmetry of the theory at higher temperature.

To understand cosmic string formation, consider for example a theory in which matter consists of a complex scalar field ϕ with the Lagrangian

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi^* - V_T(\phi)\,. \tag{2.43}$$

and the finite temperature effective potential

$$V_T(\phi) = V(\phi) + \frac{1}{2}\tilde{\lambda}T^2|\phi|^2 \,. \tag{2.44}$$

Here, we will assume the vacuum potential $V(\phi)$ takes "Mexican hat" shape

$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - \eta^2\right)^2 . \tag{2.45}$$

such that $V_T(\phi)$ can be described as

$$V_T(\phi) = \frac{1}{4}\lambda|\phi|^4 - \frac{1}{2}\left(\lambda\eta^2 - \tilde{\lambda}T^2\right)|\phi|^2 + \frac{1}{4}\lambda\eta^4 \,. \tag{2.46}$$

Here, η is called the symmetry braking scale which sets the critical temperature

$$T_c = \tilde{\lambda}^{-1/2}\lambda^{1/2}\eta \tag{2.47}$$

at which the mass term of the theory vanishes. When $T > T_c$, the effective potential has as a minimum at $|\phi| = 0$. As the temperature decreases below T_c, the effective potential takes the "Mexican hat" shape and the field amplitude stabilizes at the vacuum expectation value $|\phi| \approx \eta$.

Let us now take a closer look at the vacuum manifold. When $T \ll T_c$, the scalar field stabilizes at values of constant amplitude η and varying phase θ

$$\phi(r, \theta) = \eta e^{i\theta} \,. \tag{2.48}$$

As a result, there exist solutions where the phase winds around a single point on a plane as described in Figure 2.6. In this configuration, the phase at the center point is undermined. Hence, the field must leave the vacuum manifold and take the value $\phi = 0$. Because of translational symmetry, the center point extends a line of points where $\phi = 0$, which forms a one-dimensional object called a cosmic string. The width of the cosmic string can be estimated using the Compton wavelength of ϕ particles. We obtain

$$\omega \sim \lambda^{-1/2}\eta^{-1}, \tag{2.49}$$

which implies a mass per unit length μ of

$$\mu \sim \eta^2 \tag{2.50}$$

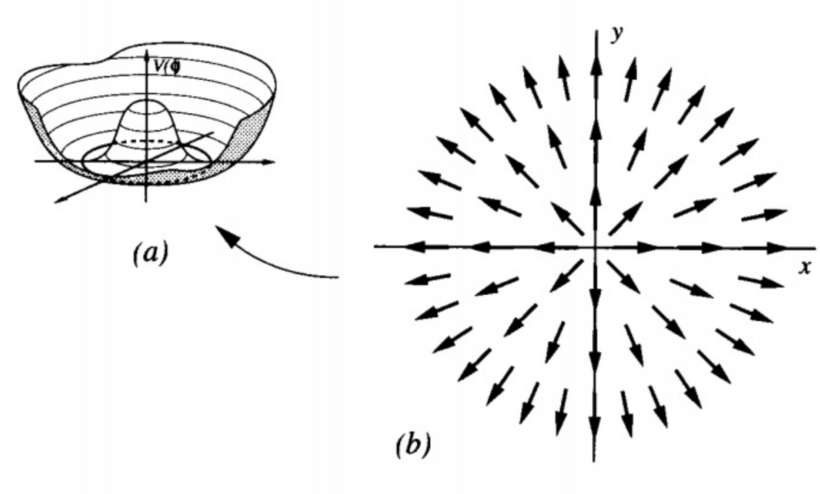

Figure 2.6: Plot of the symmetry breaking potential and its vacuum manifold (a) and of the field solution which wraps around a single point (b). The figure is taken from [12].

independent of the self-coupling constant λ. The relationship above has important implications for cosmology, which we will see later when studying cosmic string networks.

The formation of cosmic strings is ensured by the Kibble mechanism. To understand these mechanics, we will describe the behavior of the correlation length ξ as a function of the temperature without digging too much into the mathematical details. At high temperatures, the correlation length is small and all portions of the system are seemingly uncorrelated. As the temperature of the universe falls below the critical temperature T_c, ξ becomes larger, but remains bound by the causal horizon t:

$$\xi \leq t\,. \tag{2.51}$$

As a result, multiple correlated patches can meet within one Hubble radius. At the junction of these patches a string can form. Hence, a universe with cosmic string solution will contain multiple cosmic strings within a Hubble radius.

The Kibble mechanism implies that a network of strings with spatial separation of order $\xi(t_G)$ will form at the phase transition. It's possible to obtain a scaling solution for $\xi(t)$ using the following arguments. First, if the string network is much larger than the Hubble radius t, the damping term in the equations of motion will dominate and the network will be frozen in the comoving coordinates. Therefore, the correlation length $\xi(t)$ will scale as the scaling factor $a(t)$ at time t in the radiation dominated Friedmann-Robertson-Walker epoch. In other words, we have

$$\xi(t) \sim a(t) \sim t^{1/2}, \tag{2.52}$$

which implies the Hubble radius ($\sim t$) will catch up to it. However, if $\xi(t) \ll t$, the tension term in the equations of motion dominate and the strings self-intersect frequently leading to rapid loop production. Therefore, we estimate that $\xi(t) \sim t$ is the most plausible scaling solution.

This scaling solution gives us interesting results when considering the energy density of the string network. In the radiation dominated epoch, we have

$$\rho_c = H^2 \frac{3}{8\pi G} \sim G^{-1} t^{-2}. \tag{2.53}$$

Conversely, the energy density $\rho_\infty(t)$ in long strings is

$$\rho_\infty(t) \sim \mu \xi(t)^{-2} \sim \mu t^{-2}. \tag{2.54}$$

Therefore, we have

$$\frac{\rho_\infty(t)}{\rho_c(t)} \sim G\mu, \tag{2.55}$$

implying that the statistical properties of the network are time-independent of all the distances are scaled to the horizon distance. In cosmology, we wish to constrain the dimensionless number $G\mu$ since it can, by itself, describe the entire behavior of the string network. By constraining $G\mu$, we can constrain the particle physics models which have symmetry breaking scale at $\eta \sim \mu^{-1/2}$. Hence, cosmic strings are a great tool to constrain physics beyond the standard model.

2.4.2 Evolution of cosmic string networks

Soon after a network of cosmic strings form, infinite strings segments in the network will start interacting with each other and produce loops. In the present section, we describe the dynamics of both the infinite strings and the loops using scaling solutions. We will also describe more subtle effects such as cusp formation and annihilation, which will be important for our study in chapter 4.

When two string segments meet, there are three possible outcomes. The strings can either exchange ends, pass through each other or split off a loop. The first case occurs for string crossing each other at sufficiently low velocities. When the relative velocity of the strings reach the speed of light ($v > 0.9$), the strings go through each other without interacting. The third case occurs when string segments intersect in two points, splitting off a loop. This is one of two mechanisms by which a cosmic string can form a loop. The second process occurs when strings self-intersect, splitting a loop in the process (See Figure 2.7).

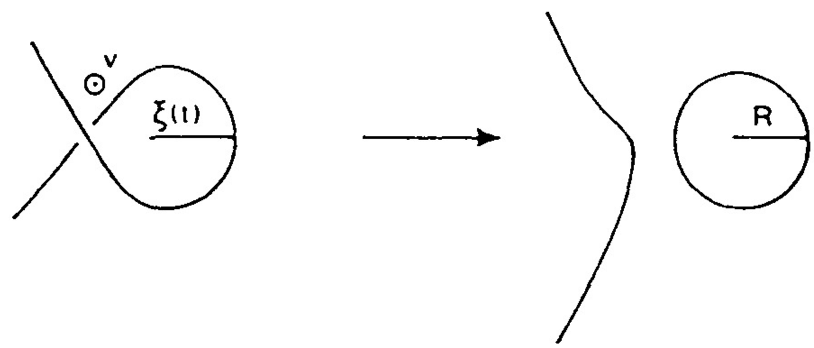

Figure 2.7: Formation of loops by self-intersection of infinite strings. Loops form with radius R determined by the instantaneous correlation length of the string network. The figure is taken from [11].

Using the scaling solution, one can estimate how many infinite cosmic string segments cross every Hubble volume. Let $\tilde{\nu}(t)$ be the mean number of infinite strings segment in a Hubble volume. The energy density of infinite strings is

$$\rho_\infty(t) = \mu\tilde{\nu}(t)t^{-2} \tag{2.56}$$

with a number $n(t)$ of loops produced per unit volume given by

$$\frac{dn(t)}{dt} = c\tilde{\nu}(t)^2 t^{-4}. \tag{2.57}$$

where c is a constant of order 1. Conservation of energy gives

$$\frac{d\rho_\infty}{dt} + \frac{3}{2t}\rho_\infty(t) = -c'\mu t\frac{dn}{dt} = -cc'\mu\tilde{\nu}^2 t^{-3} \tag{2.58}$$

or, equivalently,

$$\dot{\tilde{\nu}} - \frac{\tilde{\nu}}{2t} = -cc'\tilde{\nu}^2 t^{-1}. \tag{2.59}$$

The above equation implies that if $\tilde{\nu} \gg 1$, then $\dot{\tilde{\nu}} < 0$. Conversely, if $\tilde{\nu} \ll 1$, then $\dot{\tilde{\nu}} > 0$. Therefore, there must exist a stable solution with $\tilde{\nu} \sim 1$. Some numerical analysis works show that $10 < \nu < 20$ [35], while others have $\nu \sim 100$ [36].

From the scaling solution of infinite strings, it is also possible to find a scaling solution for loops. We start by defining a distribution of loops radius $R_i(t)$ at the time of formation and assume that the distribution is monochromatic with

$$R_i(t)/t \sim \alpha\,, \tag{2.60}$$

where α is some dimensionless constant of order one. We will also define another dimensionless constant β which determines the mean length of l in a loop of radius R

$$l = \beta R\,. \tag{2.61}$$

Since about one loop of radius $R_i(t)$ is produced by expansion time, the number density at the time of formation is given by

$$n(R_i(t), t) = ct^{-4}\,, \tag{2.62}$$

where we have defined the constant

$$c = \frac{1}{2}\beta^{-1}\alpha^{-3}\tilde{\nu}\,. \tag{2.63}$$

This initial distribution is redshifted at later times, which yields

$$n(R,t) = \left(\frac{z(t)}{z(t_f(R))} n(R, t_f(R))\right)\,, \tag{2.64}$$

where $t_f(R)$ is the time at which the loops are formed. Isolating R yields

$$n(R,t) \sim R^{-4} z(R)^{-3}\,, \tag{2.65}$$

where $z(R)$ is the redshift at time $t = R$. The redshift $z(R)$ takes different values depending on the time period of the universe. Hence, the loop density takes different values in the following cases

$$n(R,t) \sim R^{-5/2} t^{-3/2} \qquad \text{if } t < t_{eq} \tag{2.66}$$

$$n(R,t) \sim R^{-5/2} t_{eq}^{1/2} t^{-2} \qquad \text{if } t > t_{eq}, t_f(R) < t_{eq} \tag{2.67}$$

$$n(R,t) \sim R^{-2} t^{-2} \qquad \text{if if } t > t_{eq}, t_f(R) > t_{eq}\,. \tag{2.68}$$

In the equations above, t_{eq} is the time of equal matter and radiation.

So far, we neglected gravitational radiation to show how a loop distribution evolves under the expansion of the universe. In reality, loops slowly decay by emitting gravitational waves, which leads to a lower cutoff in $n(R,t)$. The power radiated by loops can be estimated using the quadrupole formula

$$P_G = \frac{1}{5} G \langle \dddot{Q} \dddot{Q} \rangle\,, \tag{2.69}$$

where Q is the quadrupole moment $Q \sim MR^2$. Since the frequency of oscillation of the loops is $\omega = R^{-1}$, the power can be estimated as

$$P_G = G(MR^2)^2 \omega^6 \sim (G\mu)\mu\,, \tag{2.70}$$

and gives a good order of magnitude approximation of the power of gravitational radiation.

String loops can also decay through the annihilation of cusps, which are spike-shaped regions of the string that form when the string double backs on itself. As derived in [37], typical cusps have a length

$$l_c \sim R^{1/2} w^{1/2}\,, \tag{2.71}$$

where $w \sim \mu^{-1/2}$ is the with of the string. From the Nambu-Goto evolution of a loop, it follows [25] that for each string loop with radius R, there will be at least one cusp per loop oscillation time R^{-1}. Hence, the power radiated by loops can be estimated as

$$P_C = \frac{\mu l_c}{R} \sim \mu^{3/4} R^{-1/2} \sim \mu^{3/4} R^{-1/2}\,. \tag{2.72}$$

The power radiated by loops can be used to estimate the mean lifetime of loops

$$\tau \sim \frac{\mu R}{P}\,, \tag{2.73}$$

where P is the power related to the dominant decay mechanism. When $R > \mu(G\mu)^{-2}$, gravitational radiation is the dominant decay process so we choose $P = P_G$. In the opposite limit, cusp annihilation dominates so we choose $P = P_C$.

The argument above assumes that loops are formed with non-circular shape and/or have initial transverse oscillations. In the unlikely case where the string is formed in a perfectly circular shape devoid of transverse oscillations, the string will collapse under its own tension and form a black hole. Eventually, all cosmic string loops decay or collapse leaving only a few infinite string segments behind. Hence, we don't expect cosmic strings to be observed in our current era. The strings may have had important implications in the early universe, where they were more numerous. This will be the topic of chapter 4.

Chapter 3

A brief review of string theory and the Swampland

String theory relies on fundamental one-dimensional objects, the ”strings”, which vibrational modes give rise to a wide variety of particles. Because of the universal nature of string theory, it will be impossible for us to review all aspects of the theory. (In fact, very few books or collections of books truly do it justice.) Instead, we will review aspects of the theory relevant to the paper ”String gases and the Swampland”, which will be presented later in the thesis. These aspects are the mass spectrum of bosonic string theory, low energy gravitational actions, and compactification.

3.1 The spectrum of bosonic strings

Bosonic strings are one of the simplest examples of fundamental objects which are studied in string theory. These strings are one-dimensional relativistic objects which trace two-dimensional world sheets in space-time. The motion of the string is determined by extremizing the area of the word sheet given by the Nambu-Goto action

$$S = -T \int d^2x \sqrt{-h}\,. \tag{3.1}$$

Here, h is the determinant of the world-sheet metric, and T is the string tension defined as

$$T = \frac{1}{2\pi\alpha'} \tag{3.2}$$

where α' is the Regge slope parameter[1]. The Nambu-Goto action can be rewritten in a more useful form called the Polyakov action. This action takes the form

$$S = -\frac{T}{2}\int \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu \,, \tag{3.3}$$

where γ^{ab} is related to the world-sheet metric by

$$h_{ab} = \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd} \,, \tag{3.4}$$

and X^μ is a coordinate in the target-space where the string lives. Because world-sheets are two-dimensional objects, they benefit from conformal flatness. This means the action is invariant under the Weyl transformations

$$\gamma_{ab} \to \Omega(x)\gamma_{ab} \,, \tag{3.5}$$

and that we can choose the flat gauge $\gamma_{ab} = \eta_{ab}$ to describe the motion of the string. In this gauge, the time-like parameter τ plays the role of time while the space-like parameter σplays the role of a parameter along the string. Using these parameters, the Polyakov action reads

$$S = \frac{T}{2}\int d\tau d\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2\right] \,. \tag{3.6}$$

It is convenient to go the light-clone coordinates

$$\xi^\pm = \tau \pm \sigma \,, \partial_\pm = \partial_\tau \pm \partial_\sigma \tag{3.7}$$

where the action reads

$$S = T\int d\xi^+ d\xi^- \partial_+ X^\mu \partial_- X_\mu \,. \tag{3.8}$$

The equations of motion in these coordinates read

$$\partial_+\partial_- X^\mu \,, \tag{3.9}$$

[1] Cosmic strings also obey the same dynamics as bosonic strings. However, the two strings should not be confused with each other. In the literature, bosonic strings are usually considered as fundamental objects, while cosmic strings are solitonic objects that can form in effective field theories. It is possible that bosonic strings form in the effective field theory of an even more fundamental theory than string theory, but this possibility won't be considered here.

and the solutions can be written as a sum of right-moving and left-moving waves along the strings

$$X^\mu = X^\mu_R(\xi^+) + X^\mu_L(\xi^-)\,. \tag{3.10}$$

So far, we found general solutions that apply to the strings, but for our analysis we will only be interested in closed strings which have periodic boundary conditions $X^\mu(\tau, \sigma = 0) = X^\mu(\tau, \sigma = 2\pi)$. The most general solution given these conditions read

$$X^\mu_R(\xi^+) = \frac{1}{2}(x^\mu + c^\mu) + \frac{1}{2}\alpha' p^\mu_R \xi^- + i\sqrt{\frac{\alpha'}{2}} \sum_{n \notin Z, n \neq 0} \frac{1}{n}\alpha^\mu_n e^{-in\xi^-} \tag{3.11}$$

$$X^\mu_L(\xi^-) = \frac{1}{2}(x^\mu - c^\mu) + \frac{1}{2}\alpha' p^\mu_L \xi^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n \notin Z, n \neq 0} \frac{1}{n}\tilde{\alpha}^\mu_n e^{-in\xi^+}\,. \tag{3.12}$$

Here, x^μ is the center of mass position of the string, c^μ is some constant, p^μ is the target space momentum and α^μ_n are oscillator modes. The periodicity of σ implies

$$p^\mu_L = p^\mu_R = p^\mu\,. \tag{3.13}$$

By averaging X^μ over the string length, we obtain the canonical position q^μ of the string defined by

$$q^\mu = \frac{1}{2\pi}\int_0^{2\pi} d\sigma X^\mu = x^\mu + \alpha' p^\mu \tau\,. \tag{3.14}$$

Now that we have worked out the basics, we can quantize the string by finding the canonical commutation relation relations and using the usual quantization procedure. The commutation relations are given by

$$[X^\mu(\tau,\sigma), \Pi^\mu(\tau,\sigma')] = i\eta^{\mu\nu}\delta(\sigma - \sigma')\,, \tag{3.15}$$

where $\Pi^\mu(\tau,\sigma) = \frac{\partial L}{\partial \dot{X}^\mu}$ is the usual expression for the canonical momentum Using the canonical variables of the system, we obtain

$$[q^\mu, p_\nu] = i\delta^\mu_\nu \tag{3.16}$$

$$[\alpha^\mu_n, \alpha^\nu_m] = in\eta^{\mu\nu}\delta_{n+m,0} \tag{3.17}$$

$$[\tilde{\alpha}^\mu_n, \tilde{\alpha}^\nu_m] = in\eta^{\mu\nu}\delta_{n+m,0}\,. \tag{3.18}$$

What remains to do is to promote the canonical variable to operators, and do the usual quantum analysis. Doing this, we notice that the spatial components of each oscillator modes satisfy the harmonic oscillator algebra with non-standard normalization

$$\alpha_n^i \sim n^{1/2} a\,, \quad \alpha_{-n}^i \sim n^{1/2} a^\dagger\,, \quad n > 0\,, \tag{3.19}$$

where $[a, a^\dagger] = 1$. This relationship allows us to quantize the oscillatory modes of the theory.

Assuming the formalism above, the mass operator of the string gives

$$M^2 = -p^\mu p_\mu = \frac{2}{\alpha}\left(N + \tilde{N} - 2\right)\,, \tag{3.20}$$

where N and $\tilde{N}$ are number operators defined by

$$N = \sum_{i=1}^{D-2} \sum_{n>0} \alpha_{-n}^i \alpha_n^i \quad , \quad \tilde{N} = \sum_{i=1}^{D-2} \sum_{n>0} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i\,. \tag{3.21}$$

For closed string, the translation symmetry along σ implies the level matching condition $N = \tilde{N}$, and the mass reads

$$M^2 = -p^\mu p_\mu = \frac{4}{\alpha}(N-1)\,. \tag{3.22}$$

The quantity above is the mass of a particle related to the oscillations of the strings. The eigenstates of the mass operator are the momentum eigenstates $|N, p\rangle$, which can be built by acting on the vacuum state $|0, p\rangle$ using rising operators α_{-n}^i. For the first few values of N, we obtain the following particle spectrum.

N = 0

When $N = 0$, we obtain a scalar tachyon. While this imaginary mass particle may seem disastrous for the theory, it is merely a consequence of us choosing the wrong vacuum state. This issue can be resolved by choosing a supersymmetric vacuum, which requires more rigorous treatment. In our case, we will simply ignore the tachyonic state and be interested in the massless state of the theory.

N = 1

When $N = 1$, we obtain a variety of massless particle states. These states have the form

$$\xi_{ij}\tilde{\alpha}^i_{-1}\alpha^j_{-1}|0,p\rangle \, , \; i,j = 1,...,24 \, . \tag{3.23}$$

where the tensor ξ_{ij} can be expanded into the irreducible representations of $SO(24)$

$$\xi_{ij} = g_{(ij)} + B_{[ij]} + \Phi \, . \tag{3.24}$$

Here, g_{ij} is traceless symmetric, B_{ij} is anti-symmetric and Φ is a scalar (corresponding to the trace). The states belong respectively to the graviton, the Kalb-Rammond fields, and the dilaton, and will be studied with more detail in the next section.

N > 1

The states with $N > 1$ are massive states which are crucial for showing the finiteness of the string amplitudes. However, they are not important for our thesis, so we will not discuss them further.

3.2 The low energy effective action

So far, we studied string dynamics in a flat target space. We will now consider the case where the string lives in a background made of gravitons $G_{\mu\nu}$, Kalb-Rammond fields $B_{\mu\nu}$ and dilatons Φ. In this background, the Polyakov action is modified to

$$S = -\frac{T}{2}\int d^2x\sqrt{-\gamma}\left[G_{\mu\nu}(X)\partial_a X^\mu \partial^a X^\nu + i\epsilon^{ab}B_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \alpha'\Phi(X)R^{(2)}\right] \, . \tag{3.25}$$

Here, ϵ^{ab} is the antisymmetric unit tensor and $R^{(2)}$ is the two-dimensional Ricci scalar. The action above is manifestly not invariant under conformal transformations. We will try to restore this symmetry by treating the fields $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ as running coupling constants. To first order, the respective β-functions of the couplings read

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha'\nabla_{\mu\nu}\nabla_\nu\Phi - \frac{\alpha'}{4}H_{\mu\lambda\kappa}H_\nu^{\lambda\kappa} + \mathcal{O}(\alpha'^2) \tag{3.26}$$

$$\beta^B_{\mu\nu} = -\frac{\alpha'}{2}\nabla^\gamma H_{\gamma\mu\nu} + \alpha'\nabla^\gamma\Phi H_{\gamma\mu\nu} + \mathcal{O}(\alpha'^2) \tag{3.27}$$

$$\beta^\Phi = +\mathcal{O}(\alpha'^2) \tag{3.28}$$

Here, H is defined as

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}\,, \tag{3.29}$$

where the square brackets denote anti-symmetrization of the indices. To ensure that conformal symmetry is preserved, we must impose that are the β-functions vanish. This gives us three respective equations of motion for $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ. The action related to there equations of motion is given by

$$S_D = \frac{1}{2\kappa_0^2}\int d^D x \sqrt{-G} e^{-2\Phi}\left[R - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + 4\partial_\mu \Phi \partial^\mu \Phi\right]\,, \tag{3.30}$$

where κ_0 is some normalization constant. Notice that the action above describes Einstein gravity with additional components. Hence, imposing conformal invariance imposes that general relativity must hold at low energies.

3.3 Compactification

While we didn't discuss it very much, the results we derived for bosonic strings are only valid in 26 dimensions, and its superstring counterpart lives in 10 or 11 dimensions. To use the theory in a four-dimensional world, we must compactify the extra dimensions. This is done, for example, by imposing periodic boundary conditions on the extra dimensions

$$X^a = X^a + 2\pi R\,, \tag{3.31}$$

such that space-time has the topology of an open manifold M^d plus a p-torus T^p. Here, R denotes the coordinate radius of the torus while it's physical size depends on the metric $G_{\mu\nu}$. To study the dimensionally reduced theory, we choose the metric ansatz,

$$ds^2 = G_{MN}x^M x^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{ab}(x)(dy^a + A^a_\mu(x)dy^\mu)(dy^b + A^b_\nu(x)dy^\nu)\,. \tag{3.32}$$

which differentiates the components related to the noncompact dimensions (greek indices) to the ones related to the compact dimensions (Latin indices). For instructive purposes, we added gauge fields A^b_ν which will play an interesting role in the theory. Using the metric ansatz above, we can expand the low energy effective action 3.30 to obtain

$$\begin{aligned} S_d = \frac{1}{2\kappa_0^2}\int dx^d X\sqrt{-g}e^{-2\Phi_d}\Big[&R^d + 4\partial_\mu\Phi_d\partial^\mu\Phi_d - \frac{1}{4}\gamma^{ab}\gamma^{cd}\left(\partial_\mu\gamma_{ac}\partial^\mu\gamma_{bd} + \right.\\ &\left.\partial_\mu B_{ac}\partial^\mu B_{bd}\right) - \frac{1}{4}\gamma_{ab}F^a_{\mu\nu}F^{b\mu\nu} - \frac{1}{4}\gamma^{ab}H_{a\mu\nu}H^{\mu\nu}_b - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\Big]\,, \end{aligned} \tag{3.33}$$

where $\Phi_d = \Phi - \frac{1}{4}\ln G$ is the effective dilaton in d dimensions and $F^a_{\mu\nu}$ is the field strength tensor associated to A^a_μ. Notice that the action of a gauge theory naturally arises from the expansion. For one compactified dimension, we recover electromagnetism for example. Compactifying on manifolds other than a torus will give rise to other non-trivial gauge theories.

Compactifying extra dimensions will also change the string spectrum. In the presence of compactified dimensions, each string momentum in the periodic dimensions becomes quantized, with $p^a = n/R$ where n is an integer. This quantization can be understood by considering a scalar field ψ in a space with one dimension compactified on a circle. Because of the compactified dimension, the field can be expanded as

$$\Psi(x^M) = \sum_{n=-\infty}^{\infty} \psi_n(x^\mu) e^{inx^d/R}\,, \tag{3.34}$$

where x^M are global coordinates, x^μ are non-compact coordinates and x^d span the compact dimension. The Klein-Gordon equation of the field reads

$$\partial_M \partial^M \Psi = \left(\partial_\mu \partial^\mu - \left(\frac{n}{R}\right)^2\right)\psi_n\,. \tag{3.35}$$

Hence, the field acquires massive states which depend on the radius of extra dimensions. These states, named Kaluza-Klein modes or momentum modes, give a contribution

$$M_n \sim \frac{n}{R} \tag{3.36}$$

to the string mass for each compactified dimension.

Closed strings can also wind along the compact directions such that the translational symmetry

$$X^a(\tau,\sigma) = X^a(\tau,\sigma) + w^a R\sigma \tag{3.37}$$

is satisfied. Here, w^a is the winding number of the string and τ and σ are the world-sheet time and length parameters. Substituting the expression above in the world-sheet action 3.25 gives

$$L = \frac{1}{2\alpha'}\gamma_{ab}\left(\dot{X}^a\dot{X}^b + w^a w^b R^2\right) - \frac{i}{\alpha'} B_{ab}\dot{X}^a w^b R\,, \tag{3.38}$$

where the dot indicates the derivative with respect to τ. Using the Lagrangian above, we obtain the canonical momenta

$$p_a = \frac{\partial L}{\partial \dot{X}^a} = \frac{1}{\alpha'}\left(\gamma_{ab}\dot{X}^b - iB_{ab}w^b R\right) \tag{3.39}$$

which again is quantized under the Kaluza-Klein modes $p_a = n_a/R$. Putting everything together, we can compute the mass of the string to obtain

$$M^2_{\vec{n},\vec{m},N} = \frac{1}{R^2}\gamma^{ab}n_a n_b + \frac{R^2}{\alpha'}\gamma_{ab}w^a w^b + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right). \tag{3.40}$$

where the level matching condition reads

$$\tilde{N} = N + n_a w^a \,. \tag{3.41}$$

Notice that there are three contributions to the string mass: one from the momentum modes, one from the winding modes and one from the oscillatory modes. Furthermore, the string mass is invariant under the transformation

$$R \leftrightarrow \frac{\sqrt{\alpha'}}{R} \quad , \quad n^a \leftrightarrow \gamma^{ab}w_b \,. \tag{3.42}$$

This symmetry, named T-duality, has important implications in string gas cosmology and will be important for our discussion of the Swampland and our paper on the subject later in the thesis.

3.4 The Swampland

The Swampland is a recent program that aims to distinguish low energy effective field theories that are consistent with string theory from those that aren't. The program is based around a set of conjectures called the "Swampland conjectures". Theories that satisfy the conjectures are said to belong to the "landscape" of theories which can be UV completed into string theory, while those which violated the conjectures are said to belong to the Swampland (See Figure 3.1). The Swampland conjectures are rather numerous. Hence, for our thesis, we will review the three most relevant ones: the distance conjecture, the weak gravity conjecture, and the de Sitter conjecture.

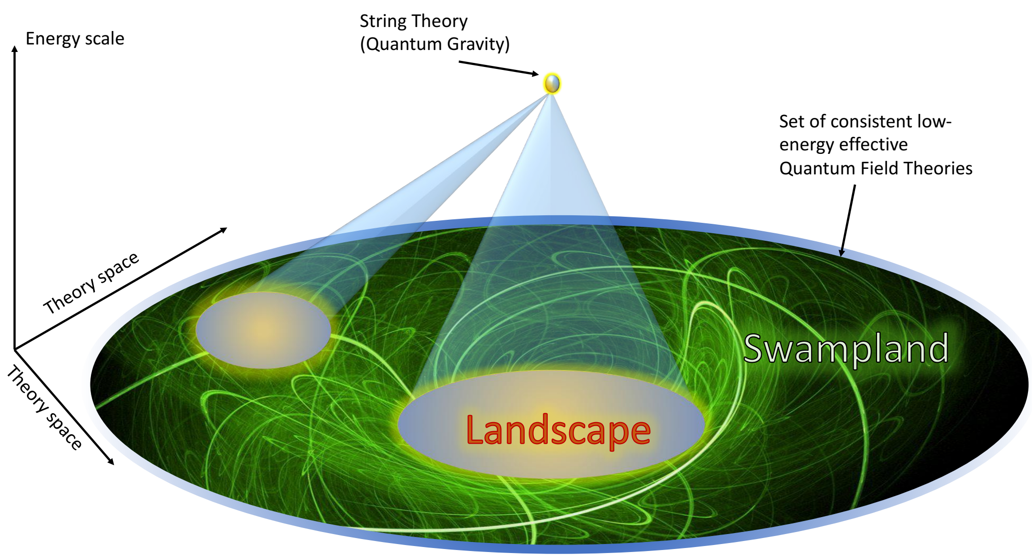

Figure 3.1: Schematic representation of the Swampland. Theories that are part of the landscape can be UV completed into string theory, while the others belong to the Swampland. The figure is taken from [15].

3.4.1 The distance conjecture

To illustrate the distance conjecture, let us consider bosonic string compactified on one dimension with the metric ansatz

$$ds^2 = G_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}dy^2\,, \tag{3.43}$$

where σ is some scalar. We will claim that σ is part of a field space $\mathcal{M}_\sigma$, which in this case has one infinite dimension. Using the ansatz above, the mass spectrum of the string reads

$$M^2_{n,m,N} = n^2 e^{2\sigma} + \frac{w^2 e^{-2\sigma}}{\alpha'} + \frac{2}{\alpha'}\left(N + \tilde{N} - 2\right)\,. \tag{3.44}$$

where we set $R = 1$ for convenience. Notice that the mass of the momentum modes $M_{KK} = M_{n,0,0}$) and the winding modes $M_w = M_{0,w,0}$ increase or decrease exponentially with respect to σ:

$$M_{KK} \sim e^{\sigma} \quad M_w \sim e^{-\sigma}\,. \tag{3.45}$$

Hence, M_{KK} decreases exponentially when σ becomes small, and M_w decreases exponentially when σ becomes large. From this fact, we make the following observation. For every interval

$\Delta\sigma$, there exists an associated mass scale M, which becomes light at an exponential rate $\Delta\sigma$

$$M(\sigma + \Delta\sigma) \sim M(\sigma) e^{-2|\Delta\sigma|} \,. \tag{3.46}$$

This behavior is very string theoretic since it emerges as a consequence of T-duality. If a massive tower of states shows up in the R limit, then it will show up in the opposite limit as imposed by the symmetry

$$R \leftrightarrow \frac{\sqrt{\alpha'}}{R} \quad , \quad n \leftrightarrow w \,. \tag{3.47}$$

With this kind of reasoning, we can generalize the statement to higher dimensions, which leads us to the statement of the distance conjecture.

Swampland Distance Conjecture [a] [38]

- *Consider a theory, coupled to gravity, with a moduli space $\mathcal{M}$ which is parametrized by the expectation values of some field ϕ^i which have no potential. Starting from any point $P \in \mathcal{M}$ there exists another point $Q \in \mathcal{M}$ such that the geodesic distance between P and Q, denoted $d(P, Q)$, is infinite.*

- *There exists an infinite tower of states, with an associated mass scale M, such that*

$$M(Q) \sim M(P) \, e^{-\alpha d(P,Q)} \,, \tag{3.48}$$

where α is some positive constant.

[a] This textbox comes from [15], and is based on the original conjecture in [38].

Here, $\mathcal{M}$ is defined as the space of all possible values of the moduli fields ϕ^i[2]. The distance $d(P, Q)$ can be computed using the moduli space metric

$$g_{IJ} = \frac{\partial \gamma_{ab}(\sigma^I)}{\partial \sigma^I} \frac{\partial \gamma^{ab}(\sigma^J)}{\partial \sigma^J} \,, \tag{3.49}$$

[2] Here, our definition of $\mathcal{M}$ includes unstable points in the moduli space, not just the points where the moduli fields stabilize. Hence, one can use the distance conjecture away from the stable points, as we did in Chapter 5.

which is found from the kinetic term of the moduli fields

$$\begin{aligned} S_d &= \frac{1}{2\kappa^2} \int dx^d X \sqrt{-g} \left[R^d - \partial_\mu \gamma_{ab}(\sigma^I) \partial^\mu \gamma^{ab}(\sigma^J) \right] \\ &= \frac{1}{2\kappa^2} \int dx^d X \sqrt{-g} \left[R^d - \frac{\partial \gamma_{ab}(\sigma^I)}{\partial \sigma^I} \frac{\partial \gamma^{ab}(\sigma^J)}{\partial \sigma^J} \partial_\mu \sigma^I \partial^\mu \sigma^J \right] . \end{aligned} \tag{3.50}$$

The geodesic distance is then given by

$$d(P,Q) \equiv \int_\gamma ds \sqrt{g_{IJ} \frac{\partial \sigma^I}{\partial s} \frac{\partial \sigma^J}{\partial s}} \,, \tag{3.51}$$

where γ is the shortest path between P and Q.

The distance conjecture has deep implications for effective field theories. In deriving the low energy action, we avoided a massive tower of states and only considered the zero modes of the moduli fields. The distance conjecture tells us that if moduli fields move by a distance greater than a Planckian distance $d(P,Q) > 1$, we must consider an exponentially massless tower of states in the theory. Hence, the effective field theory description breaks down for sufficiently large values of $d(P,Q)$.

3.4.2 The weak gravity conjecture

One can make a statement similar to the distance conjecture by considering the compactification of gauge fields A_μ. In this case, we use the metric

$$ds^2 = G_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + e^{2\sigma} (dy + A_\mu dx^\mu)^2 \,, \tag{3.52}$$

and the relevant part of the effective action is given by

$$S = \frac{1}{2\kappa^2} \int dx^d \sqrt{-g} \left[R^d - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4} e^{-2\sigma} F^d_{\mu\nu} F^{\mu\nu d} - \frac{1}{4} e^{2\sigma} H_{\mu\nu d} H^{\mu\nu}_d \right] . \tag{3.53}$$

Notice that the gauge coupling g_A in the theory depends exclusively on σ with

$$g_A = e^\sigma \,. \tag{3.54}$$

Also, the Kaluza-Klein modes σ_n in this theory are charged under the $U(1)$ gauge fields A_μ. Their charge is

$$q_n = 2\pi n \,. \tag{3.55}$$

We find there is an obvious relationship between the Kaluza-Klein mass $M_{n,0,0}$, q_n and g_A which scales as

$$M_{n,0,0} \sim g_A q^n \,. \tag{3.56}$$

A similar relationship can be found for a second gauge field V_μ arising from the Kalb-Rammond B-field

$$V_\mu = B_{[\mu d]} \,. \tag{3.57}$$

In this case, the gauge coupling is given by

$$g_V = e^{-\sigma} \,, \tag{3.58}$$

and we find a similar relationship between the winding mass $M_{0,w,0}$, g_V and the quantized charge q_w

$$M_{0,w,0} \sim g_V q_w \,. \tag{3.59}$$

Given the above results, we can assume there exist a mass scale $M \sim g$ which acts as an energy cutoff beyond which we have to consider the effects of a massive tower of states in the theory. This observation is at the heart of the weak gravity conjecture which, given a more rigorous treatment of the theory, can be described as follows.

Weak Gravity Conjecture (d-dimensions) [a] [39]

Consider a theory, coupled to gravity, with a $U(1)$ gauge symmetry with gauge coupling g

$$S = \int d^d X \sqrt{-g} \left[\left(M_p^d\right)^{d-2} \frac{R^d}{2} - \frac{1}{4g^2} F^2 + ... \right] . \tag{3.60}$$

- *(Electric WGC) There exists a particle in the theory with mass m and charge q satisfying the inequality*

$$m \leq \sqrt{\frac{d-2}{d-3}} g q \left(M_p^d\right)^{\frac{d-2}{2}} . \tag{3.61}$$

- *(Magnetic WGC) The cutoff scale Λ of the effective theory is bounded from above approximately by the gauge coupling*

$$\Lambda \lesssim g \left(M_p^d\right)^{\frac{d-2}{2}} . \tag{3.62}$$

[a] This textbox comes from [15], and is based on the original conjecture in [39].

The statement above implies there exists a particle on which gravity acts as the weakest force. Consider for example the Einstein-Maxwell theory in 4 dimensions with charged particles. The forces due to gravity F_G and the electromagnetic force F_{EM} between two such particles are given by

$$F_G = \frac{m^2}{8\pi M_p^2 r^2} \quad , \quad F_{EM} = \frac{(gq)^2}{4\pi r^2} . \tag{3.63}$$

The electric weak gravity conjecture is therefore equivalent to the inequality $F_G \leq F_{EM}$, or in other words that gravity acts as be the weakest force between the two particles.

3.4.3 The de Sitter Conjecture

Previously we saw how inflation can solve the Flatness and Horizon problem by introducing a phase of exponential expansion in the early universe. This phase of exponential expansion is notoriously hard to realize in string theory, where many no-go theorems (e.g. Madacena-Nunez theorem for type II string theories [26]) forbid the existence of a de Sitter vacua. The hardships encountered in string theory when constructing a de Sitter vacua inspired the

Swampland de Sitter conjecture, which can be stated as follows.

The de Sitter Conjecture [a] [28]

The scalar potential of a theory coupled to gravity must satisfy a bound on its derivatives with respect to scalar fields

$$|\nabla V| \geq \frac{c}{M_p} V \,, \tag{3.64}$$

where $c > 0$ is a constant of order one.

[a]This textbox comes from [15], and is based on the original conjecture in [28].

Here, the constant c is loosely considered to be of order one. Hence, constants of order 0.01 are not automatically excluded. Current observations impose $c < 0.6$ which, without being in full disagreement with the conjecture, poses some tensions with the possibility of realizing de Sitter in string theory.

The conjecture is closely tied to the distance conjecture. Consider for example slow-roll inflation with the Lagrangian in Equation 2.25 and the quadratic potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \,. \tag{3.65}$$

For slow-roll inflation to take place, the condition

$$\epsilon = \frac{M_p^2}{2} \left(\frac{V'}{V}\right)^2 = 2\frac{M_{pl}^2}{\phi^2} < 1 \qquad \Longrightarrow \qquad |\phi| > \sqrt{2} M_p \,. \tag{3.66}$$

This means by the time inflation ends, when ϕ reaches the bottom of the potential, ϕ will have been displaced by more than a Planckian distance. Hence, by the distance conjecture, we expect a massive tower of state to become important in the theory, and the effective theory must break down. To avoid a massive tower of state to become important in the theory, one would have to impose the condition

$$\frac{|V'|}{V} = \frac{2}{\phi} \geq \frac{2}{M_p} \,, \tag{3.67}$$

such that the field excursion satisfies $|\phi| \leq M_p$. In this case, we have $c = 2$ the system is consistent with the statement of the de Sitter conjecture. Given the condition above, inflation cannot take place since the slow-roll parameter ϵ is greater than unity.

Interlude

We have now reviewed some essential knowledge about cosmology, cosmic strings, string theory, and the Swampland, which are useful to understand the following parts of the thesis. In the next chapters, we present two original papers that present advances in the field of cosmic string phenomenology and string cosmology. In chapter 4, we show new bounds on the cosmic string tension and primordial magnetic fields found from ionization in the dark ages. Then, in chapter 5, we show that the late time scenario of string gas cosmology is consistent with the recent Swampland conjectures.

Part II

Publications

Chapter 4

Ionisation from Cosmic Strings At Cosmic Dawn

Abstract

Cosmic strings produce charged particles which, by emitting electromagnetic radiation, partially ionize neutral hydrogen during the dark ages. Corrections to the ionization fraction of neutral hydrogen induced by cosmic strings lead to new constraints on the string tension around $G\mu \sim 10^{-16} - 10^{-20}$ for values of the primordial magnetic field greater than $B_0 \sim 10^{-11}$ Gauss.

4.1 Introduction

Cosmic strings [1] are linear topological defects which may have formed in the early universe. In particle physics models which admits cosmic strings, a network of strings will inevitably form during a symmetry-breaking phase transition [2] and persist to late times. The string network consists of a "scaling system" of "long" strings (strings with curvature radius larger than the Hubble horizon) and a distribution of string loops with radii smaller than the Hubble horizon. By "scaling system" it is meant that the statistical properties of the network are independent of time if all lengths are scaled to the Hubble horizon. In fact, the mean separation of the long strings can be shown to be comparable to the horizon. This can be seen both using analytical arguments [1] and numerical simulations [3]. The scaling network of long strings is maintained by the strings intersecting each other and producing loops. The typical formation radius $R_i(t)$ at time t of a string loop is again comparable to the horizon, $R_i(t) = \alpha t$, α being a constant of order unity. The loops oscillate and gradually decay. For large loops gravitational wave emission [4] is the dominant decay channel, for small loops it is the process of "cusp annihilation" [5] producing elementary particles which is more important.

Cosmic strings form lines of trapped energy density. If the energy scale of the phase transition which leads to cosmic string formation is η, then the string tension (which equals the mass per unit length) is

$$\mu \simeq \eta^2 \,. \tag{4.1}$$

The dynamics of cosmic string networks can be described by the dimensionless number $G\mu$, where G is Newton's gravitational constant.

The trapped energy of the strings gravitates and leads to specific signatures which can be searched for in cosmological observations (see e.g. [6] for a review). The signatures of the long string network increase in strength proportional to μ. Hence, searching for cosmic string signatures in the cosmological observations is a way to probe particle physics beyond the "Standard Model" "from top down". Not observing signals which strings predict can be used to rule out classes of particle physics models [7]. The current robust upper bound on the cosmic string tension is $G\mu < 1.5 \times 10^{-7}$ which comes from measurements of the angular power spectrum of the cosmic microwave background [8]. If the distribution of string loops also achieves a scaling solution (which Nambu-Goto simulations of string evolution [9] indicate, but not all simulations based on solving the classical field theory equations [10]), then a stronger bound of $G\mu < 10^{-10}$ can be derived from pulsar timing constraints on the amplitude of the stochastic gravitational wave background [11].

According to the one-scale model of the string loop distribution, the number density of string loops per unit radius R at a time t after the time t_{eq} of equal matter and radiation is given by [1]

$$n(R,t) \sim R^{-5/2} t_{eq}^{1/2} t^{-2} \tag{4.2}$$

for loops produced before t_{eq}. This distribution is valid down to a lower cutoff radius $R_c(t) \sim t$

which in the case of graviational radiation dominating the string loop decay is

$$R_c(t) \,=\, \gamma\beta^{-1}(G\mu)t\,, \tag{4.3}$$

where β and γ are constants which will be introduced later. Hence, the loops dominate the energy density in cosmic strings, and dominate the string decay emission products.

The initial interest in the role of cosmic strings in cosmology was sparked by the idea that string loops could be the seeds for galaxies and galaxy clusters [12]. However, since a distribution of strings as the main source of cosmological perturbations does not produce acoustic oscillations in the angular power spectrum of CMB anisotropies [13], the discovery of these oscillations [14] killed this idea. Nevertheless, since there is good evidence from particle physics that cosmic strings might form in the early universe, it is of great interest to look for their signatures, and in particular for effects of string loops. In fact, it was shown that string loops might yield the seeds about which high redshift supermassive black holes form [15], and may play a role in globular cluster formation [16].

An important feature of cosmic string loops is that any loop (of invariant length R) will experience a cusp at least once per oscillation time $R/2$ [17] [1]. A cusp is a region of length [18]

$$l_c(R) \,\sim\, R^{1/2}w^{1/2} \tag{4.4}$$

(where $w \sim \mu^{-1/2}$ is the width of the string) about a point where the two sides of the string overlap. Such a region is unstable to annihilation into a burst of particles [5]. The primary decay products are the scalar and gauge field quanta associated with the string, but these rapidly decay into jets of long-lived elementary particles such as neutrinos, pions and electrons, and also high energy photons. The resulting flux of ultra-high energy photons and neutrinos was studied in [19] and [20]. The consequences for fast radio bursts were recently analyzed in [21], and the implications for the global 21cm signal were studied in [22]. The effects of strings typically increase in amplitude as $G\mu$ decreases, until the value of $G\mu$ for which gravitational radiation ceases to be the dominant decay channel. Thus, one can hope that searching for signals of string loops can provide constraints on lower tension strings.

[1] In the computations below, we take the oscillation time to be R. This will underestimate our effect. Another conservative assumption we make in the analysis is that there is only one cusp per oscillation period. There could be two or even more.

In our analysis, we are assuming that after a cusp annihilates, another cusp of a similar size will form after another loop oscillation period. However, it is not clear that this will in fact happen. After a cusp annihilation event, the remaining string will feature kinks that propagate away in both directions along the string from the cusp site. The presence of kinks on a string invalidates one of the continuity assumptions which goes into the argument of [17]. Kinks may also be present on the string when the loop forms (as a consequence of the process by which the string loop is produced). It is reasonable to assume that back-reaction effects will smooth out the kinks right after they form and that the argument of [17] for the cusp formation can then be used independently in each oscillation time. If this is not the case, however, then the number of cusp events on older loops may be lower than we assume here, or that the cusps would be smaller in length than what we are assuming. See [23] for interesting field theory simulations relevant to this question.

Since string loops are present and undergo cusp annihilation at all times, they will contribute to early reionization of the universe. The effect of superconducting cosmic strings, which have a direct decay channel into photons, was studied in [24]. In this work, we will study the corresponding signal for ordinary (i.e. non-superconducting) strings.

After recombination, most of the hydrogen in the universe is neutral. It is only after non-linear structure forms that hydrogen can be fully ionized again. By analyzing the spectra of most distant quasars, one can conclude that most of the hydrogen was reionized at redshift $z \sim 6$ [25]. However, if cosmic strings [26] were formed in the early universe, they could ionize the hydrogen at earlier times. We will study how this effect depends on the string tension and on the amplitude of the primordial magnetic fields.

Throughout this paper, we use parameters for a flat ΛCDM model: The Hubble expansion rate is taken to be $H_0 = h \times 100$ km/s/Mpc with $h = 0.7$, the fractional contribution of baryons to the total energy density being $\Omega_b = 0.05$ and that of matter $\Omega_m = 0.26$. The cosmological redshift $z(t)$ is related to time via $1 + z(t) = (1 + z_{eq})\sqrt{t'/t}$ in the radiation dominated epoch, and $1 + z(t) = (1 + z_{\rm eq})(t_{\rm eq}/t)^{2/3}$ in the matter dominant epoch, where $t_{eq} = (2\sqrt{\Omega_r}H_0)^{-1}$ is the time of equal matter and radiation and $z_{\rm eq} = \Omega_m/\Omega_r$ with $h^2\Omega_r = 4.18 \times 10^{-5}$. Ω_r is the current fractional contribution of radiation to the current energy density. We also adopt natural units, $\hbar = c = 1$.

4.2 Cusp Annihilation Electron Spectrum

In this section, we compute the flux of high energy particles from cusp annihilation of non-superconducting cosmic strings. In the following section, we will then use this flux to compute the contribution of cosmic strings to ionization of the universe after the time of recombination.

A cosmic string cusp initially decays by emitting quanta of the Higgs and gauge fields which make up the string. These particles, in turn, decay into a jet of Standard Model particles, in particular charged and neutral pions, electrons and neutrinos. The energy distribution of particles in such jets has been well studied by particle physicists, with the result that the spectrum of charged pions produced by a cusp annihilation event is given by [19]

$$\frac{dN}{dE} = \frac{15}{24}\frac{\mu l_c}{Q_f^2}\left(\frac{Q_f}{E}\right)^{3/2} , \tag{4.5}$$

where Q_f is the energy of the primary decay quanta, which is of the order of η. In the following we will take $Q_f \sim \eta$, leading to

$$\frac{dN}{dE} \sim \frac{15}{24}\mu^{1/2}R^{1/2}E^{-3/2} . \tag{4.6}$$

The spectrum of electrons produced by the cusp-induced jets has the same scaling.

The energy distribution at time t' (number per unit energy per unit time) of electrons emitted by cusps is found by integrating over the loop distribution at a time t' from the lower cutoff radius $R_c(t')$ to the upper cutoff $R = \alpha t'$ where ($\alpha \sim 1$)

$$\frac{d^2 n_e(t')}{dE(t')dt'} = \int_{R_c(t')}^{\alpha t'} n(R,t')\frac{dN}{dE}\frac{1}{R}dR \tag{4.7}$$

where $n(R,t')$ is the number density of loops of radius R per unit R interval at time t', and the factor $1/R$ comes from the fact that there is one cusp event per oscillation time R.

The lower cutoff radius $R_c(t)$ is related to decay time scale

$$\tau = \frac{\mu L}{P} = \frac{\mu\beta R}{P} \tag{4.8}$$

of loops. Here, L and R are, respectively, the string length and the loop radius and $\beta \sim 10$ is a parameter that measures how circular loops are on average (perfectly circular loops have

$\beta = 2\pi$). Usually, P is the power radiated away by the dominant decay process for loops. For example, if $G\mu > 10^{-18}$, gravitational radiation dominates during the entire time interval between recombination and the present time, so we use

$$P_g \, = \, \gamma G \mu^2, \tag{4.9}$$

where $\gamma \sim 100$ is dimensionless constant [4]. If $G\mu < 10^{-18}$, then cusp annihilation dominates as a decay mechanism in the time interval of interest, and we must use [18]

$$P_c \, \sim \, \mu l_c / R \sim \mu^{3/4} R^{-1/2} \ . \tag{4.10}$$

Indeed, comparing (4.9) and (4.10) we see that the power emission from cusp decay decreases less fast than that of gravitational radiation. Hence, for fixed time there will always be a value of $G\mu$ below which cusp decay starts to dominate.

From (4.7) it is clear that it is loops at the lower cutoff of the integration range which dominate the energy injected from string cusps. The reason is that the loop distribution for loops formed before equal matter and radiation scales as $R^{-5/2}$. Also, considering the $G\mu$ dependence of the electron flux from cusp annihilation, we see that the flux increases as $G\mu$ decreases as long as R_c is determined by gravitational radiation. We will be interested in the values of $G\mu$ which give the largest flux. Hence, as discussed in [27], both gravitational radiation and cusp annihilation are important decay mechanisms for the relevant values of $G\mu$. Therefore, in order to obtain an accurate description of the cusp annihilation spectrum when $G\mu \sim 10^{-18}$, we will use $P = P_g + P_c$. This yields the following expression for the cutoff radius:

$$R_c \, = \, \left(\gamma G \mu + \mu^{-1/4} R_c^{-1/2} \right) \beta^{-1} t \tag{4.11}$$

Analytic solutions to this expression can be found in the limits of large and small $G\mu$. In general, one should solve the equation above using numerical methods.

In order to evaluate equation 4.7, we use the loop distribution function (4.2) which holds if $R > R_c(t)$ and $t > t_{eq}$. Since the integral in (4.7) is dominated by the lower cutoff, the energy distribution of electrons produced at time t' can be written as

$$\frac{d^2 n_e(t')}{dE(t')dt'} \approx \frac{5}{16} \nu \mu^{1/2} t_{eq}^{1/2} t'^{-2} R_c(t')^{-2} E^{-3/2} \ . \tag{4.12}$$

We now wish to determine the energy distribution of electrons at some time t produced by cusp decays between the time of recombination and t. This is done by integrating equation (4.12) with respect to t'. The integration requires a Jacobian transformation between energies at different times to take into account the redshifting of the number density and of the energy between times t' and t:

$$E^{3/2}(t')\frac{dn_e(t')}{dE(t')} = E^{3/2}(t)\frac{dn_e(t)}{dE(t)}\left(\frac{t}{t'}\right)^{1/3}\left(\frac{t}{t'}\right)^{2} . \tag{4.13}$$

Integrating with respect to t', we obtain the following electron spectrum:

$$\frac{dn_e(t)}{dE(t)} \approx \frac{5}{16}\nu\mu^{1/2}t_{eq}^{1/2}E(t)^{-3/2}t^{-7/3}\int_{t_{rec}}^{t} dt't'^{1/3}R_c(t')^{-2} . \tag{4.14}$$

For large values of $G\mu$ when gravitational radiation dominates string loop decay, this expression scales as $(G\mu)^{-3/2}$, for small values of $G\mu$ when cusp decay is more important and when (see (4.11))

$$R_c(t) \sim (G\mu)^{-1/6} \tag{4.15}$$

the scaling of the electron energy flux is proportional to $(G\mu)^{1/6}$. Thus, the highest flux of electrons occurs in the range of $G\mu$ values when gravitational radiation of a string loop is comparable to cusp annihilation.

4.3 Ionization Fraction of Strings

To estimate the correction to the ionization fraction of hydrogen due to cosmic strings, will follow the approach taken in [24]. Assuming that one photon in the frequency interval between ω_i and ω_f emitted from the string loop ionizes one hydrogen atom, we can derive the following expression for the ionization fraction due to cosmic strings:

$$x(z) = \frac{1}{\alpha_r n_H(z)^2}\int_{\omega_i}^{\omega_f} d\omega\frac{d^2n_\gamma}{d\omega dt} . \tag{4.16}$$

where $(d^2n_\gamma/(d\omega dt)$ is the number density of photons per unit frequency interval per unit time due to the string loop cusp annihilations. Here, $\alpha_r = 2.6\times10^{-13}$ cm^3/s is a recombination coefficient, $n_H(z) = 8.42\times10^{-6}(1+z)^3\Omega_B h^2$ cm^{-3} is the number aboundance of neutral hydrogen at redshift z, Photon with frequency below the Lyman-α frequency $\omega_i = 13.6$eV do not have enough energy to ionize neutral hydrogen. Those with frequency above a cutoff frequency $\omega_f = 10^4$eV have too small a ionization cross section.

In the case of superconducting strings, there is direct photon emission from the string loops. In our case, the photons in the relevant low energy regime are mainly produced by electrons from cusp annihilation via Bremsstrahlung and Synchroton emission. There is also direct photon emission from cusps, but the corresponding spectrum is only known at high energies, i.e. for photon energies larger than the pion mass [19]. Hence, we focus on the more robust mechanisms of photon production in the frequency range relevant to ionization, mechanism which reply on electrons producing photons during their propagation. There are two main mechanisms - Synchrotron emission and Bremsstrahlung. Synchrotron emission depends on the strength of the primordial magnetic fields in which the string loops live, whereas Bremsstrahlung is a more general phenomenon. In the following, we will study both channels.

4.3.1 Bremsstrahlung Ionization

Charged particles emitted by a string are slowed down by the surrounding medium, which creates Bremsstrahlung radiation. The strength of emission depends on both the flux of the charged particles produced by cusp radiation and on the density of the surrounding medium. According to [27,28], the spectrum of photons emitted via Bremsstrahlung per unit time per unit frequency for a cusp at a given time is given by

$$\frac{d^2 n_\gamma(t)}{d\omega dt} \approx \left(\frac{8}{3} m_\pi^{-1/2}\right) \alpha_{EM} r_0^2 K_e(t) E(t)^{-1} \left(\sum_s n_s(t)\tilde{\phi}_w\right) . \tag{4.17}$$

Here, m_π is the mass of the pion, α_{EM} is the electromagnetic fine structure constant, $r_0 = m_e^{-2}$ is the Compton wavelength of the electron, $E(t)$ is the energy of the induced photons, $n_s(t)$ is the number density of charged particles the electrons interact with and $\tilde{\phi}_\omega \sim 300$ is a dimensionless weak shielding coefficient. The dependence on the flux of charged particles enters through the quantity $K_e(t)$ which is the coefficient of the power law

$$dn_e(t)/dE(t) \equiv K_e(t) E(t)^{-3/2} . \tag{4.18}$$

By inspection with equation 4.14, we conclude that $K_e(t)$ takes the value

$$K_e(t) = \frac{5}{16}\nu\mu^{1/2}t_{eq}^{1/2}t^{-7/3}\int_{t_{rec}}^{t} dt' t'^{1/3} R_c(t')^{-2} \,. \tag{4.19}$$

Note that we are only considering the effect of charged particles produced after the time t_{rec} of recombination. Those produced before t_{rec} interact with the charged plasma and rapidly lose their energy, and we will neglect their contribution. Also, the weak shielding approximation breaks down when neutral hydrogen becomes fully ionized at reionization. Therefore, our solution only holds for times between t_{rec} and the time of reionization.

To estimate the ionization fraction $x_b(z)$ for Bremsstrahlung, we use equation (4.16) to obtain

$$\begin{aligned} x_b(z) = & \frac{1}{\alpha_r n_H(z)^2}\left(\frac{8}{3}m_\pi^{-1/2}\right)\alpha_{EM} r_0^2 K_e(t(z)) \\ & \times \left(\sum_s n_s(t(z))\tilde{\phi}_w\right)\log\left(\frac{\omega_f}{\omega_i}\right) . \end{aligned} \tag{4.20}$$

The ionization fraction for Bremsstrahlung is plotted in Figure 4.1, where we have used a single type of species which the electrons interact with, namely the neutral hydrogen atoms. The dependence on $G\mu$ in the low and high $G\mu$ limits, respectively, comes from the dependence of the charged particle density on $G\mu$ which was discussed at the end of the previous section. In the high $G\mu$ limit the induced ionization fraction scales as $(G\mu)^{-3/2}$, in the small $G\mu$ limit the scaling is as $(G\mu)^{1/6}$. In the intermediate regime, the result has to be determined by numerically solving for $R_c(t)$. The main message to draw from this calculation is the Bremsstrahlung contribution to re-ionization is negligible.

4.3.2 Synchrotron Ionization

Charged particles emitted from a string can be accelerated radially in the presence of primordial magnetic fields. The resulting acceleration of the charged particles creates synchrotron radiation which can partially ionize the surrounding medium. The spectrum of photons emitted from cosmic strings via synchrotron radiation is computed as

$$\frac{d^2 n_\gamma(t)}{d\omega dt} = \frac{2D_e(t)e^3}{m_e}B(t)^{5/4}\left(\frac{3e}{2m_e}\right)^{1/4} a(3/2)E(t)^{-5/4} \,. \tag{4.21}$$

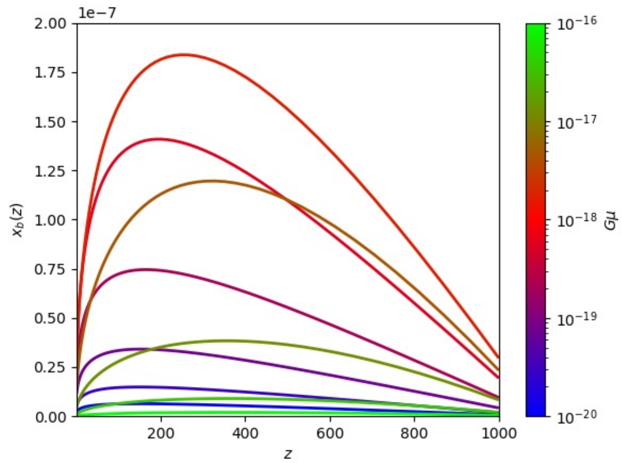

Figure 4.1: Ionization fraction from Bremsstrahlung at different redshifts. The values of $G\mu$ run from 10^{-19} to 10^{-17}.

Here, e is the electron charge, m_e is the mass of the electron, $B(t)$ is the magnetic field acting on the cusp at time t, $a(3/2)$ is a dimensionless constant of order 10^{-1}. The function $D_e(t)$ comes from the flux of charged particles and is defined by

$$\frac{dn_e(t)}{d\Gamma} \equiv \frac{D_e(t)}{4\pi}\Gamma^{-3/2} , \tag{4.22}$$

where $\Gamma = E(t)/m_e$.

We can find $D_e(t)$ using equation 4.14 and obtain

$$D_e(t) = \frac{5}{4}\pi m_e^{-1/2}\nu\mu^{1/2}t_{eq}^{1/2}t^{-7/3}\int_{t_{rec}}^{t} dt' t'^{1/3}R_c(t')^{-2} . \tag{4.23}$$

We then get the following expression for the ionization fraction $x_s(z)$ of synchrotron radiation:

$$\begin{aligned} x_s(z) &= \frac{1}{\alpha_r n_H(z)^2}\frac{8D_e(t(z))e^3}{m_e}B(t(z))^{5/4} \\ &\times \left(\frac{3e}{2m_e}\right)^{1/4} a(3/2)\left(\omega_i^{-1/4} - \omega_f^{-1/4}\right) . \end{aligned} \tag{4.24}$$

The ionization fraction for synchrotron radiation is plotted in Figure 4.2 as a function of redshift assuming the following scaling of the primordial magnetic field

$$B(t(z)) = B_0(1+z)^2 . \tag{4.25}$$

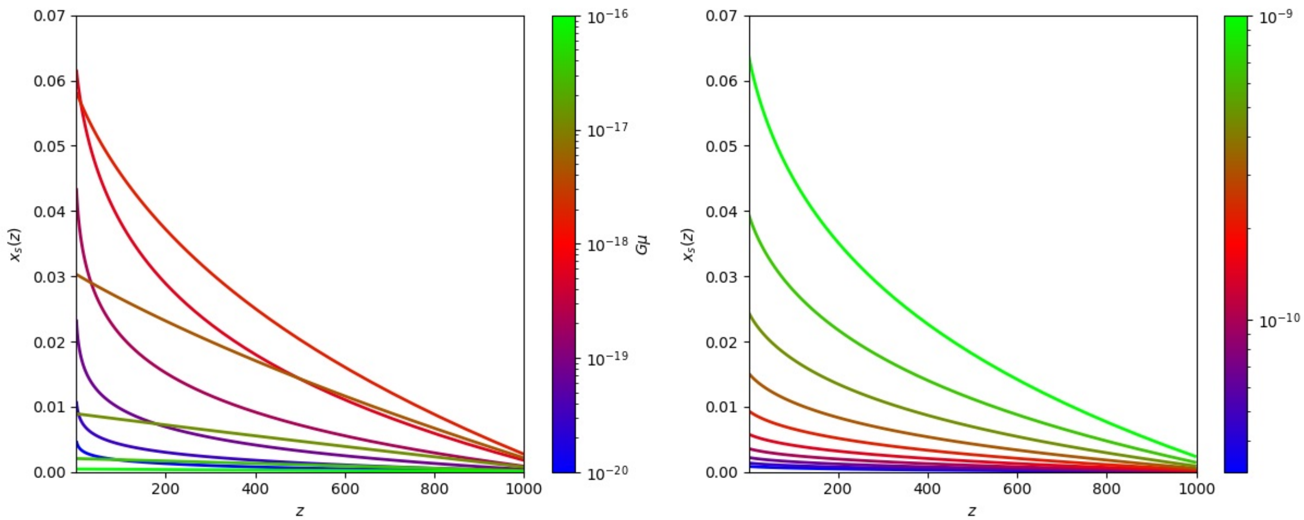

Figure 4.2: Ionization fraction from synchrotron radiation at different redshifts. On the left, the value of B_0 is fixed at 1 nG while the values of $G\mu$ run from 10^{-24} to 10^{-18}. On the right, the value of $G\mu$ is fixed at 10^{-18} while the values of B_0 run from 10^{-12} Gauss to 10^{-9} Gauss.

In each subplot of Figure 4.2, we explore the parameter space of the ionization fraction by fixing $G\mu$ and B_0 one at a time. We see that, for a large area of parameter space, the ionization fraction due to synchrotron radiation is much larger than that due to Bremsstrahlung. As we will see in the next section, it can also dominate over the ionization fraction in the standard ΛCDM model at redshifts between recombination and reionization.

4.4 Effect on Reionization History and Constraints

If the ionization produced after the time of recombination is important compared to what is produced in the standard ΛCDM model of cosmology, measurable effects on cosmological observables are possible. Possible effects include an increase in the optical depth and a modification of the spectrum of CMB anisotropies. In the present section, we will discuss the constraints on cosmic strings from the optical depth alone, leaving the study of other observables to future work.

Using the CAMB code [29, 30] and the best fit cosmological parameters of the ΛCDM

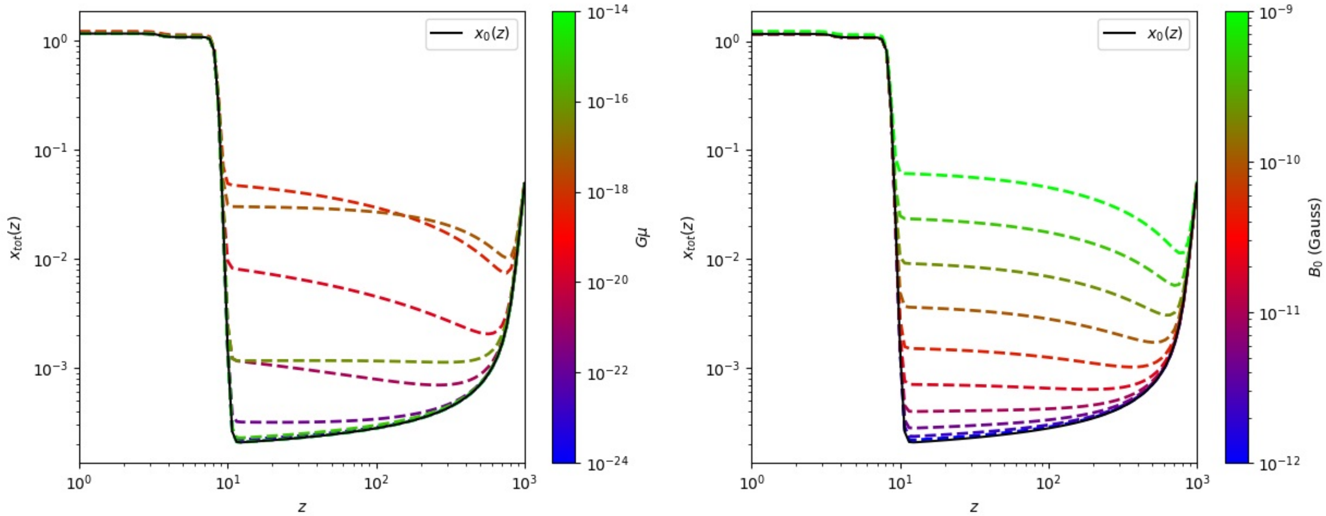

Figure 4.3: Ionization histories which take into account the ionization fraction from strings (dashed lines) are compared to a standard ΛCDM ionization history (black curve). On the left, the value of B_0 is fixed at 1 nG while the values of $G\mu$ run from 10^{-24} to 10^{-18}. On the right, the value of $G\mu$ is fixed at 10^{-18} while the values of B_0 run from 10^{-12} Gauss to 10^{-9} Gauss.

model mentioned in the introduction, we can compute the ΛCDM total background ionization $x_0(z)$ from recombination to our current time. We will assume $\tau = 0.06$ to generate the cosmological data required for our analysis. This value of the optical depth is consistant with the contraint $\tau = 0.066 \pm 0.016$ (see e.g. [31], and [32] from observation. Given that cosmic strings emit charged particles which contribute to reionization, they contribute to the total ionization by an amount $x_s(z)$. Here, we neglect Bremsstrahlung radiation since $x_g(z) \ll x_s(z)$ for the region of parameter space which interest us. The total ionization fraction

$$x_{tot}(z) = x_0(z) + x_s(z) \tag{4.26}$$

takes the values described in Figure 4.3.

Since cosmic strings contribute to the ionization fraction of hydrogen, they affect the optical depth and we can use the current bounds on the optical depth to constrain the relevant parameters for cosmic strings. The relevant parameters in our case are the dimensionless number $G\mu$ and the primordial magnetic field strength B_0. To determine the bounds on

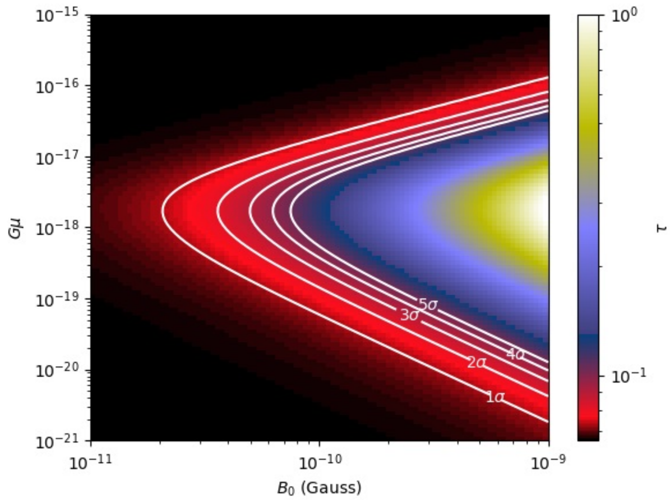

Figure 4.4: The total optical depth $\tau = \tau_0 + \tau_s$ between recombination and reionization is computed for various values of the string tension $G\mu$ and for various values of the magnetic field B_0. The white curves show the different confidence bounds for the optical depth. In the black region, τ is approximately τ_0. From left to right, each curve marks an increase $\Delta\tau = 0.008$ (1σ deviation) from τ_0.

these parameters, we will compute the optical depth for different values of $G\mu$ and B_0. The expression for the optical depth is

$$\tau = \int_0^\infty dz n_e^f(z) \sigma_T \left| \frac{dt}{dz} \right| , \tag{4.27}$$

where σ_T is the Thomson cross section and $n_e^f(z) = x_H(z) n_H(z)$ is the number density of free electrons. The expression for τ is linear for linear corrections to $x_H(z)$. For example, if $x_0(z)$ yields an optical depth τ_0, then a correction $x_s(z)$ due to cosmic string will lead to an optical depth $\tau = \tau_0 + \tau_s$ where τ_s is the correction to the optical depth from cosmic strings. This allows us to choose $\tau_0 = 0.066$, the current value from observation, and compare the correction τ_s to the 2σ confidence bound $\Delta\tau_{2\sigma} = 0.016$. This is done in Figure 4.4, where we compute the total optical depth τ for different values of $G\mu$ and B_0 and compare it to the confidence bound from observation. We find that a triangular region in the range $G\mu \sim 10^{-20} - 10^{-16}$ and $B_0 > 10^{-11}$ Gauss violates the 2σ confidence bound and should be excluded from the space of possible parameters in the theory.

4.5 Conclusion

We have computed the contributions of Bremsstrahlung and synchrotron radiation from cosmic string cusp annihilation to the total ionization fraction of the universe in the dark ages. Whereas the contribution of Bremsstrahlung is negligible, that of synchrotron radiation can be important, depending on the values of the string tension $G\mu$ and of the primordial magnetic field B_0. Under the assumption that after one cusp annihilation, a cusp of similar size on the loop will reform a loop oscillation time later, we have identified the range of values in the $G\mu$ vs. B_0 parameter space where the cosmic string contribution to the optical depth is significantly larger than what is predicted in the standard ΛCDM model and therefore constrained by observation. In this work, we have considered non-superconducting strings. Superconducting strings lead to a larger effect, as studied in [24].

In work in progress, we are calculating the effects of the string-induced extra ionization of the spectrum of microwave anisotropies, with the goal of determining a larger exclusion region in the $G\mu$ vs. B_0 parameter space.

Acknowledgements

We would like to thank Bryce Cyr, Jonathan Sievers and Disrael da Cunha for their valuable input to the project. RB thanks the Institute for Theoretical Physics of the ETH Zuerich for hospitality. SL wishes to thank NSERC for a graduate scholarship. The research at McGill is supported in part by funds from NSERC and from the Canada Research Chair program.

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Chapter 5

String gases and the Swampland

Abstract

In this paper, we study some aspects of moduli stabilization using string gases in the context of the Swampland. In the framework which we derive, the matter Lagrangian for string gases yields a potential for the size moduli which satisfies the de Sitter conjecture with the condition $\frac{|\nabla V|}{V} \geq \frac{1}{\sqrt{p}} \frac{1}{M_p}$, where p is the number of compactified dimensions. Moreover, the moduli find themselves stabilized at the self-dual radius, and gravity naturally emerges as the weakest force.

5.1 Introduction

There has recently been a lot of discussion concerning what types of effective field theories for gravity coupled to matter can be consistent with superstring theory. Those that do belong to the *landscape*, while those that do not are said to be in the *swampland*. Specifically, [1, 2] (see also [3]) suggested that all scalar fields arising from string theory must have a potential which satisfies

$$\frac{|\nabla V|}{V} \geq \frac{c}{M_p} , \tag{5.1}$$

or

$$\frac{\min(\nabla_i \nabla_j V)}{V} \leq -\frac{c'}{M_p} , \tag{5.2}$$

where c, c' are universal constants of order 1 and M_p is the Planck mass (see e.g. [4, 5] for recent reviews). This bound, called the *de Sitter conjecture*, excludes stable and meta-stable de Sitter vacua in string theory and is motivated by two key arguments. First, the conjecture is supported by multiple examples in string theory [1]. Second, we expect that when the de Sitter conjecture is violated, a massive tower of states becomes massless leading to a breakdown of the effective field theory. This second argument was first demonstrated in [2] using Bousso's [6] covariant entropy bound in an accelerating universe. It was shown that for field values larger than the Planck scale, an exponentially large number of string states become massless (see also [7]), saturate the entropy bound and force the scalar field to satisfy the de Sitter criteria.

The de Sitter conjecture is known to severely constrain inflationary scenarios. Hence, it becomes relevant to ask if other cosmological scenarios such as *matter bounce* [8] or *string gas cosmology* [9] satisfy the Swampland conjectures. In the next sections, we will be interested in the case of string gas cosmology, and more particularly moduli stabilization using string gases.

Our work is structured as follows. In section 5.2, we introduce a framework for string gases in the context of effective field theories and show how string gases give a potential that stabilizes extra dimensions at their self-dual radius. In section 5.3, we show how moduli stabilization by string gases implies that gravity must be the weakest force at late times in the universe. Finally, in section 5.4, we show how the string gas potential satisfies the Swampland de Sitter conjecture.

5.2 Moduli sabilisation using string gases

We start with the action of a closed string in a D-dimensional space-time where d dimensions are non-compact and p are compact. In the "adiabatic" approximation, the Nambu-Goto action can be approximated as the energy of a particle with a mass corresponding to the oscillations of the closed string. The reader is referred to [10] [11] (see also [12]) for more information on the adiabatic approximation. A large number of closed strings with the same mass and momentum give a string gas whose action is given by [11]

$$S = -\int dx^D \sqrt{-G} n_{D-1} \sqrt{\vec{p}^2 + M^2_{\vec{n},\vec{w},N}} \ , \tag{5.3}$$

where $\vec{p}$ is the momentum of the center of mass of the string in the non-compact dimensions, n_{D-1} is the number density of strings in the $D-1$ spatial dimensions and $M_{\vec{n},\vec{w},\vec{N}}$ is the mass of the string, which depends on quantum numbers $\vec{n}$,$\vec{w}$ and $\vec{N}$. The quantum numbers are, respectively, related to the momentum modes, the winding modes, and the oscillatory modes. The mass spectrum of the string depends on the choice of compactification. The simplest example is to consider a torus with a metric γ_{ab} which only has diagonal elements. In this case, the string mass reads

$$M^2_{\vec{n},\vec{n},N} = \frac{1}{R^2}\gamma^{ab} n_a n_b + \frac{R^2}{\alpha'}\gamma_{ab} w^a w^b + \frac{2}{\alpha'}\left(2N + n^a w_a - 2\right) . \tag{5.4}$$

The variable R in the expression for the mass describes the coordinate interval for each cycle of the torus. In other words, coordinates $x^d = \theta^d R$ parametrise a circle with $0 \leq \theta^d \leq 2\pi$. We will use this compactification example repeatedly throughout the paper. For simplicity, we will also set R to unity, with the physical length of the cycles hence being described by the metric γ_{ab}. We now wish to find an action that describes a string gas with all possible momentum states and all possible masses in the theory. We start with a string gas made of one string by choosing the number density

$$n_{D-1} = \frac{\delta^{D-1}(x^\tau - x^\tau_{cm})}{\sqrt{G_s}} \ , \tag{5.5}$$

where x^τ_{cm} is the center of mass of the particle and G_s is the determinant of the spatial part of the metric. We will work in a regime where $M_{\vec{n},\vec{w},N}$ does not depend on the coordinates of the compactified dimensions. Therefore, we can integrate over the p internal dimensions to obtain

$$S = -\int dx^d \sqrt{-g_{00}} \sqrt{\vec{p}^2 + M^2_{\vec{n},\vec{w},N}} \delta^{d-1}(x^\tau - x^\tau_{cm}) \ . \tag{5.6}$$

Here, g_{00} is the zeroth component of the d-dimensional metric. To obtain the desired string gas, we sum over a large number of one string "string gas" actions and overall possible massive states to obtain

$$S = -\int dx^d \sqrt{-g_{00}} \sum_{\vec{n},\vec{m},N} \sum_i \sqrt{\vec{p_i}^2 + M^2_{\vec{n},\vec{w},N}} \delta^{d-1}(x^\tau - (x^\tau_{cm})_i) \ . \tag{5.7}$$

In the thermodynamic limit, the sum in the action above can be viewed as the thermally averaged energy of a relativistic gas with particles of massive states given by $M_{\vec{n},\vec{w},N}$. The string gas action then reads

$$S_{sg} = \int dx^d \sqrt{-g} n_{d-1} \langle E_1 \rangle \ , \tag{5.8}$$

where $\langle E_1 \rangle$ is the thermal average of the energy of a single string and n_{d-1} is the number density of strings in the $d-1$ non-compact spatial dimensions. The spatial determinant of the metric $\sqrt{g_s}$ is factored in the action by defining $n_{d-1} = (n_{d-1})_0/\sqrt{g_s}$ where $(n_{d-1})_0$ is the comoving number density. The thermal average $\langle E_1 \rangle$ is computed as

$$\langle E_1 \rangle = -\frac{\partial}{\partial \beta} \ln Z_1, \tag{5.9}$$

where β is the inverse temperature, and Z_1 is the finite temperature partition function of a relativistic string with mulitiple massive states given by $M_{\vec{n},\vec{n},N}$:

$$Z_1 = \sum_{\vec{n},\vec{m},N} V^d \int \frac{d^{d-1}\vec{p}}{(2\pi)^{d-1}} e^{-\beta\sqrt{p^2 + M^2_{\vec{n},\vec{m},N}}} \ . \tag{5.10}$$

Note that we do not include in the sum the tachyonic states of our theory since we expect them to vanish in a supersymmetric generalization of our model.

For our analysis, we will rewrite the partition function as a sum of Kelvin functions K_a [13]. The result reads

$$Z_1 = V^d \, (2\beta)^{-(d-2)/2} \sum_{\vec{n},\vec{m},N} \left(\frac{M_{\vec{n},\vec{m},N}}{\pi} \right)^{d/2} K_{d/2}(\beta M_{\vec{n},\vec{m},N}) \ . \tag{5.11}$$

To account for all the massive states in the theory, we should sum over all the quantum numbers in the equation above. However, if $\beta \gg M_s^{-1}$, only the states which are massless to first order at the self-dual radius contribute significantly, and the other ones are windowed out by the Boltzman factors. This "massless modes" approximation was introduced in [11], and can be used to significantly simplify the partition function. Using equation 5.4, we can show that the modes which are massless at the self-dual radius $\gamma_{ab} = \alpha' \delta_{ab}$ obey

$$(n^a + w^a)(n_a + w_a) = 4(N-1) \tag{5.12}$$

$$N + w^a n_a \geq 0 \, . \tag{5.13}$$

Among the states which satisfy the condition above, only those which have $N = 1$ and $n^a = -w_a = \pm 1$ are massless to first order. All modes which are massless to first order are degenerate, so the sum in equation (5.11) is easily evaluated and yields

$$Z_1 = \tilde{N}\,(2\beta)^{-(d-2)/2}\left(\frac{M_{1,-1,1}}{\pi}\right)^{d/2} K_{d/2}(\beta M_{1,-1,1})\;. \tag{5.14}$$

Here, $\tilde{N}$ is the number of states which are massless to first order. Using the partition function above, the thermal average $\langle E_1\rangle$ is easily evaluated and yields

$$\langle E_1\rangle = M_{1,-1,1}\frac{K_{(d-2)/2}(\beta M_{1,-1,1})}{K_{d/2}(\beta M_{1,-1,1})} + \frac{d-1}{\beta}\;. \tag{5.15}$$

So far, we derived an expression for the string energy which holds when the temperature of the universe is below the string mass ($\beta \gg M_s^{-1}$). This condition provides an operational definition for 'late times' in the universe. For simplicity, we will take the limit where the temperature goes to zero ($\beta \to \infty$) and only consider very late times in the universe. In this limit, β^{-1} is very small and the last term can be neglected, and we are left with a term proportional to the string mass. It is worth noting that the system reaches the late time limit exponentially fast, and that $\beta \leq 10 M_s^{-1}$ is enough to suppress most modes which are not massive to first order. We warn the reader to be careful with the high-temperature limit since the massless mode approximation may not apply in this case, or only apply to certain regions of parameter space.

The expression for the string energy can be simplified further by approximating the Kelvin function in the appropriate limit:

$$K_n(\beta M_{1,-1,1}) \approx \left(\frac{\pi}{2\beta M_{1,-1,1}}\right)^{1/2} e^{-\beta M_{1,-1,1}} \qquad\qquad \text{if } \beta M_{1,-1,1} \gg 1\,. \tag{5.16}$$

Using the expression above, the thermal energy of the string is easily expressed as the string mass

$$\langle E_1\rangle \approx M_{1,-1,1}\,. \tag{5.17}$$

The result above was previously used to show how size and shape moduli stabilize in the presence of string gases [15]. In the latter case, fluxes are required to be present. Making use of a gaugino condensation mechanism it can then be shown that also the dilaton can be stabilized [16], without destabilizing the size and shape moduli. In our case, expressing the string gas Langragian as the energy/mass density of the strings will prove useful to show why the extra dimension must stabilize at the self-dual radius, and also satisfy the de Sitter conjecture.

5.2.1 Moduli stabilisation on the torus

It is known that string gases can stabilize the size of extra dimensions [10–12] (see also [14] for a review), even in the absence of fluxes. In this section, we will show how the size of extra dimensions is stabilized when string gases are included as matter in effective field theories. For simplicity, we will start with toroidal compactifications, then try to extend our results to more general compactifications.

We start with a simple dimensional reduction ansatz for the metric of a space-time topology $\mathbb{R}^d \times T^p$, were T^p denotes a p-dimensional torus. The metric reads

$$ds^2 = G_{MN}dx^M dx^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{ab}(x)dy^a dy^b \ , \tag{5.18}$$

where $g_{\mu\nu}$ is the metric of the non-compact dimensions, and γ_{ab} in the metric of the torus. We will enforce that γ_{ab} only has diagonal elements for the mass spectrum of the string to be given by (5.4). With our ansatz, the dimensionally reduced low energy action of bosonic string theory with a string gas reads

$$S = \frac{1}{2\kappa_0^2}\int d^dX\sqrt{-g}e^{-2\Phi_d}\left[R^d + 4\partial_\mu\Phi_d\partial^\mu\Phi_d - \frac{1}{4}\partial_\mu\gamma_{ac}\partial^\mu\gamma^{ab} - 2\kappa^2 e^{2\Phi_d}n\langle E_1\rangle + ...\right] \ . \tag{5.19}$$

Here, we will ignore the $B_{\mu\nu}$ fluxes, which play no role in our analysis. We will also assume that the dilaton Φ_d is stabilized and that we can consider it to be a constant.

The metric γ_{ab} will generally involve various scalar fields ϕ^I called moduli fields. The kinetic term of the moduli fields reads

$$-\frac{1}{4}\partial_\mu\gamma_{ac}\partial^\mu\gamma^{ab} = -g_{IJ}\partial_\mu\phi^I\partial_\mu\phi^J \ . \tag{5.20}$$

Therefore, a good parametrization of the moduli fields on the torus is $\gamma_{ab} = e^{2\phi^a}\delta_{ab}$. With this parametrization, the kinetic term of the moduli fields is Euclidean and the low energy action reads

$$S = \frac{1}{2\kappa_0^2}\int d^dX\sqrt{-g}e^{-2\Phi_d}\left[R^d - \sum_a \partial_\mu\phi^a\partial^\mu\phi^a - 2\kappa_0^2 e^{2\Phi_d} n\langle E_1(\phi^a)\rangle + ...\right] . \tag{5.21}$$

It's worth noting that the string gas Lagrangian depends explicitly on the metric γ_{ab}, and therefore on the moduli fields ϕ^a. Thus, the string gas lagrangian yields a potential for the moduli fields given by

$$V(\phi^a) = 2\kappa_0^2 e^{2\Phi_d} n\langle E_1(\phi^a)\rangle \, . \tag{5.22}$$

The moduli will naturally stabilize at the minimum of this potential. To find this minimum, we study the thermal average $\langle E_1\rangle$ in the late time limit derived in section 5.2. We obtain

$$V(\phi^a) \approx 2\kappa_0^2 e^{2\Phi_d} n M_{1,-1,1} \tag{5.23}$$

with $M_{1,-1,1}$ satisfying

$$M_{1,-1,1}^2 = \sum_a e^{-2\phi^a} + \sum_a \frac{1}{\alpha'^2} e^{2\phi^a} - \frac{2p}{\alpha'} \, . \tag{5.24}$$

The expression for $M_{1,-1,1}^2$ is obtained using equation (5.4) with $\gamma_{ab} = e^{2\phi^a}\delta_{ab}$, $N = 1$ and $n^a = -w_a = \pm 1$. Notice that the masses of the momentum modes become exponentially large as $\phi^a \to -\infty$ and that the winding modes become exponentially heavy as $\phi^a \to \infty$. As a result, the moduli ϕ^a will be driven towards the self-dual radius of the torus where

$$e^{2\phi^a} = \alpha'. \tag{5.25}$$

Therefore, all moduli will stabilize at the self-dual point $\phi^a = 1/2 \ln \alpha'$ where the gradient of the potential vanishes ($\partial_a V = 0$).

5.2.2 Moduli stabilisation on other compact manifolds

So far, we went over a simple example of how moduli stabilize in toroidal compactifications. This stabilization process can be conceptually explained for arbitrary compactifications. The key aspect of the process is that the moduli potential $V(\phi^I)$ is directly proportional to the string mass M, and that the string mass is composed of a momentum mode component and a winding modes component. When the size of the extra dimensions shrinks, the mass of the momentum mode starts to dominate and becomes exponentially massive below the self-dual radius. As a result, the moduli potential increases. The potential also increases when the size of the extra dimensions increases above the self-dual radius and the winding mode becomes exponentially massive. As a result, we find that the moduli potential is always minimized at the self-dual radius. This behavior of the momentum and winding modes is a direct consequence of T-duality, which states that the mass spectrum of string theory is invariant under

$$R \leftrightarrow \frac{\alpha'}{R} \qquad\qquad n \leftrightarrow m \; . \tag{5.26}$$

Therefore, we expect that the size of extra dimensions will always stabilize at the self-dual radius regardless of the choice of compactification, as long as the mass spectrum of string states obeys the T-duality symmetry.

5.3 String gases and gravity as the weakest force

We saw that string gases can stabilize the size of extra dimensions to their self-dual radius. We expect Standard Model physics to emerge when the moduli are stabilized at the minimum of their potential. As a result, gravity must be the weakest force. We will show that for string gases, this is always the case to low order in α'. We start with a simple Kaluza-Klein dimensional reduction ansatz:

$$ds^2 = G_{MN}x^M x^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{ab}(x)(dy^a + A^a_\mu(x)dy^\mu)(dy^b + A^b_\nu(x)dy^\nu) \; . \tag{5.27}$$

For simplicity, we will continue to assume γ_{ab} only has diagonal non-zero elements. The low energy action of bosonic string theory in the presence of string gases then reads

$$S = \frac{1}{2\kappa_0^2}\int d^dX\sqrt{-g}e^{-2\Phi_d}\left[R^d - \frac{1}{4}\partial_\mu\gamma_{ab}\partial^\mu\gamma^{ab} + 2\kappa_0^2 e^{2\Phi_d}n\langle E_1\rangle - \frac{1}{4}\gamma_{ab}F^a_{\mu\nu}F^{b\mu\nu} + ...\right] \; . \tag{5.28}$$

Again, we will assume Φ_d to be constant for simplicity. If we choose $\gamma_{ab} = e^{2\phi^a}\delta_{ab}$, then we saw earlier that the moduli ϕ^a stabilize at the self-dual dimensions of the torus where

$$e^{2\phi^a} = \alpha' \; . \tag{5.29}$$

In the self-dual configuration, the gravitational coupling and the gauge couplings are respectively given by

$$\kappa^2 = \kappa_0^2 e^{2\Phi_d} \qquad \text{and} \qquad (g_A^a)^2 = \frac{2}{\alpha'}\kappa^2 \ . \tag{5.30}$$

Therefore, in the limit where $\alpha' \ll 1$ (the limit in which string perturbation theory is trustworthy), the electromagnetic force will naturally be stronger than gravity, in agreement with the *weak gravity conjecture* [17].

5.4 String gases and the de Sitter conjecture

To see how the de Sitter conjecture is satisfied by the string gas, let us use the ansatz used in Section 5.2.1 for moduli stabilization. We will also work with $d = 4$ where the string coupling can be written as $\kappa^2 = M_p^{-2}$. The dimensionally reduced low energy action of bosonic string theory with a string gas then reads

$$S = \int d^dX\sqrt{-g}\left[\frac{M_p^2}{2}R^d - \sum_a \frac{M_p^2}{2}\partial_\mu\phi^a\partial^\mu\phi^a - n\langle E_1(\phi^a)\rangle + ...\right] \ . \tag{5.31}$$

Notice that so far, our definition of the moduli fields ϕ^a has made them dimensionless. However, in 4 dimensions, scalar fields have mass dimension 1. We will restore the right units to the moduli fields by defining the scalar fields $\tilde{\phi}^a = M_p\phi^a$. These scalar fields have the right mass dimensions and their kinetic therms are canonically normalized in equation (5.31). The potential of the scalar field is given by

$$V(\tilde{\phi}^a) = n\langle E_1(\tilde{\phi}^a/M_p)\rangle \ , \tag{5.32}$$

where $\langle E_1(\tilde{\phi}^a/M_p)\rangle$ can be approximated in the same limits as in Section 5.2.1. For the purpose of our analysis, we will use the field redefinition $\tilde{\phi}^a = \tilde{\phi}_0^a + \Delta\tilde{\phi}^a$ where $\tilde{\phi}_0^a = (2M_p)^{-1}\ln\alpha'$ is the value of $\tilde{\phi}^a$ at the self-dual point. With this field redefinition, the mass $M_{1,-1,1}^2$ can conveniently written as

$$M_{1,-1,1}^2 = M_s^2\left(\sum_a e^{2\frac{\Delta\tilde{\phi}^a}{M_p}} + \sum_a e^{-2\frac{\Delta\tilde{\phi}^a}{M_p}} - 2p\right) \ . \tag{5.33}$$

We now use the massless approximation of section 5.2 ($\langle E_1\rangle \approx M_{1,-1,1}$) to study the potential of our system and compute the quantity $|\partial_a V|/V$, which is of constrained by the de Sitter conjecture. When $|\Delta\phi^a/M_p| \ll 1$, the potential reaches zero exponentially and $|\nabla V|/V$ blows up to infinity. In the opposite limit, the potential quickly becomes dominated by either the momentum or winding modes and can be approximated as

$$V(\Delta\tilde{\phi}^a) = \sum_a nM_s e^{\frac{|\Delta\tilde{\phi}^a|}{M_p}} . \tag{5.34}$$

The potential increases exponentially at a rate on the order of M_p^{-1} no matter which direction you go in the moduli space away from the self-dual point. Therefore, far from the self-dual point, the potential obeys the de Sitter criterion

$$\frac{|\partial_a V|}{V} \sim \frac{1}{M_p} \geq \frac{c}{M_p} . \tag{5.35}$$

What remains to do now is to derive the exact value of the constant c in our case. We know that $|\partial_a V|/V$ has the mirror symmetry $\Delta\tilde{\phi}^a = -\Delta\tilde{\phi}^a$ and asympotically reaches a constant of order M_p^{-1} far from the self-dual point. Therefore, let us assume $\Delta\tilde{\phi}^a \gg 0$. In this case, $|\partial_a V|/V$ is written as

$$\frac{|\partial_a V|}{V} = \frac{1}{M_p}\frac{\sqrt{\sum_a e^{\frac{2\Delta\tilde{\phi}^a}{M_p}}}}{\sum_a e^{\frac{\Delta\tilde{\phi}^a}{M_p}}} . \tag{5.36}$$

It's easy to show that the gradient of the equation above is zero when all $\Delta\tilde{\phi}^a$'s are the same ($\Delta\tilde{\phi}^a = \Delta\tilde{\phi}^b$). In this case, equation (5.36) reads

$$\frac{|\partial_a V|}{V} = \frac{1}{\sqrt{p}}\frac{1}{M_p} . \tag{5.37}$$

Note at away from the line $\Delta\tilde{\phi}^a = \Delta\tilde{\phi}^b$, $|\partial_a V|/V$ takes values which are very close to $1/M_p$ except near certain lines passing trough the self-dual point. For example, when $\Delta\tilde{\phi}^a = \Delta\tilde{\phi}^b$ but one of the $\Delta\tilde{\phi}^a$'s is zero, $|\partial_a V|/V$ will reach a value close to $\frac{1}{\sqrt{p-1}}\frac{1}{M_p}$ and so on. Aside from these other local minima, the absolute minium remains $\frac{1}{\sqrt{p}}\frac{1}{M_p}$.

5.5 Conclusion and Discussion

We studied some aspects of moduli stabilization using string gases in the context of the Swampland. When included as matter in the effective field theory context, the string gas yields as a potential that stabilizes the moduli (the scalar fields which emerge from string theory) at the self-dual radius of the extra dimensions. As a result, gravity emerges as the weakest force (in agreement with the *weak gravity conjecture*) and the *de Sitter conjecture* on the slope of the scalar field potential is naturally satisfied. Our analysis yields the value of the numerical coefficient c which arises in the de Sitter conjecture. Its value is $1/\sqrt{p}$. [1]

It would be interesting to apply similar arguments to string gases using other compactifications (e.g. Calabi-Yau manifolds). Based on our qualitative arguments, we expect the main conclusions to be unchanged, but it would be interesting to study the value of the constant c in the *de Sitter conjecture* bound which emerges. It would also be interesting to study the constraints on the potentials for other moduli fields. Based on the analysis in [15], we expect that similar conclusions can be derived for shape moduli fields.

Our analysis supports the view that it will be very difficult to embed cosmological inflation into string theory: the scalar field potentials which emerge are too steep to support inflation with sufficiently slow rolling. The challenge for primordial inflation has recently been strengthened by the *Trans-Planckian Censorship Conjecture* (TCC) [18] which shows that [19] that a phase of inflation which can explain the origin of structure in the universe is only possible if the scale of inflation is smaller than about 10^{10} GeV. Demanding the correct power spectrum for cosmological perturbations leads to further constraints which, in particular, yield an upper bound on the tensor to scalar ratio r of about $r < 10^{-30}$ [2]. String theory may hence favor an alternative early universe scenario such as *string gas cosmology* [9], a scenario which assumes an initial high-temperature state of strings in thermal equilibrium, and yields an alternative to cosmological inflation for producing a spectrum of nearly scale-invariant cosmological perturbations [21] with a slight red tilt, and predicts a nearly scale-invariant spectrum of gravitational waves with a slight blue tilt [22], a prediction with which the predictions of string gas cosmology can be distinguished from those of

[1] It is important to mention a caveat when comparing our results to the de Sitter conjecture (we thank C. Vafa for stressing this point): the de Sitter conjecture concerns properties of the bare potential. The potential we are studying, on the other hand, is an effective potential which assumes the presence of a gas of string modes with momenta and windings about the compactified dimensions. On the other hand, this potential plays the same role in stabilizing the string moduli as the potentials usually considered in string inflation models. Hence, it is fair to compare our potential to the ones which are usually studied.

[2] These conclusions can be mitigated by adding new whistles to the inflationary scenario [20].

canonical inflation models (see e.g. [23] for a review).

Acknowledgements

This research was supported in part by funds from NSERC and the Canada Research Chair program. RB thanks C. Vafa for discussions and comments on the draft. SL thanks Heliudson Bernardo for useful discussions, NSERC for a graduate scholarship, the MSI for a graduate award, and Eran Palti and Miguel Montero for useful lectures on the Swampland at SIFTS 2019.

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Chapter 6

Conclusion

Let us conclude by highlighting some important thoughts and results we presented in this thesis.

In chapter 2, we reviewed some aspects of standard big bang cosmology, cosmic strings, string theory and the Swampland relevant to the studies presented in chapter 4 and 5. Some main points to take away are that cosmology can be used to study physics beyond the standard model (e.g. cosmic strings), but still suffers from important problems such as the horizon, flatness and singularity problem. These problems can be solved by invoking scenarios which modify the early time of the universe. For example, inflation solves the horizon and flatness problem by invoking a phase of exponential expansion at the beginning of the universe. Alternatively, string gas cosmology solves the singularity problem by using new symmetries from string theory. String theory is the leading candidate to explain early universe physics. Therefore, we expect the late-time evolution of the universe to be consistent with the Swampland conjectures.

In chapter 4, we showed how cosmic strings emit charged particles that ionize neutral hydrogen in the early universe. The charged particles emit electromagnetic radiation through Bremmstrahlung and synchrotron emission. While Bremmstrahlung emission is negligible, synchrotron emission from the charged particles can become strong enough to impose a constraint on cosmic strings from observations. By using the current bounds on the ionization fraction of the universe after recombination, we find that string tensions around $G\mu \sim 10^{-16} - 10^{-20}$ must be excluded for values of the primordial magnetic field greater than

$B_0 \sim 10^{11}$ Gauss.

In chapter 5, we derived a moduli potential inspired by the string gas cosmology scenario. We showed that this potential stabilizes extra dimensions to their self-dual radius and that at the minimum of the potential gravity emerges as the weakest force. The potential also satisfies the Swampland de Sitter conjecture, which suggests that late time string gas cosmology is described by an effective field theory consistent with string theory. For string gases, we find $c = 1/\sqrt{p}$ for the constant in the de Sitter conjecture.

6.1 Some future avenues of research

Now that we highlighted some of the main results of our thesis, let us discuss some avenues for future research.

Cosmic strings and reionization : In chapter 4, we constrained the string tension $G\mu$ and the primordial magnetic field B_0 by using the current bounds on the ionization fraction. In a current project, we are trying to improve this bound by doing a full Bayesian analysis of standard big bang cosmology plus added effects from cosmic strings. This analysis is done numerically using Monte Carlo programs such as CosmoMC [40], which find the best set of cosmological parameters related to the CMB data. We expect added effects from cosmic strings to change the large scale portion of the CMB power spectrum. Hence, using a modified version of CosmoMC with cosmic string parameters is guaranteed to bring new constraints.

String gas cosmology and the Swampland: In chapter, 5, we found that string gas cosmology satisfies the de Sitter conjecture with $c = 1/\sqrt{p}$ given that we compactify on the torus. We expect the constant c to depend on the choice of compactification. Hence, it would be interesting to redo our analysis using more realistic compact manifolds such as the Calabi-Yau manifolds. Assuming c remains of the order of unity, it would further confirm the consistency of string gas cosmology with string theory. It would also be interesting to study compactification in the presence of Kalb-Rammond fluxes $B_{\mu\nu}$. In our analysis, we assumed them to be stabilized at $B_{\mu\nu} = 0$. String gases are known to stabilize B-fluxes, hence adding such fluxes may slightly change our results.

6.2 Final words

In light of our results, we believe that cosmology is at an interesting stage. Current observations have the power to tighten constraints on new physics like never before. For example, we expect the parameter space of cosmic strings to become further constrained in the future as new data is made available. In addition, the recent Swampland conjectures are casting doubt on the existence of an inflationary phase at the beginning of the universe. Inflation is the current paradigm of early universe cosmology. Hence, we believe progress in the study of the Swampland conjecture may inspire the study of alternatives to inflation such as string gas cosmology or bouncing cosmologies [41]. In any case, we should learn a great deal about cosmological models consistent with string theory in the next years.

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