

Report on the week of November 18

# The search for suitable dx

Numerical Derivative:

$$f(x+dx) = f(x) + f'(x) dx + \frac{1}{2} f''(x) dx^2 + \dots$$

keeping only the first two terms:

$$f'(x) = \frac{f(x+dx) - f(x)}{dx}$$

order unity random numbers  $\rightarrow$  accuracy

$$\text{Derivative} \sim \frac{[f(x) + f'(x) dx + \frac{1}{2} f''(x) dx^2](1+g+\epsilon) - f(x)(1+g_0\epsilon)}{dx}$$

$$\epsilon \sim 10^{-7} \text{ single}, 10^{-16} \text{ double}$$

$$\text{Deriv: } \frac{f(x)}{dx} + f'(x) + \frac{1}{2} f''(x) dx (1+g_0 \epsilon) - \frac{f(x)}{dx} (1+g_0 \epsilon)$$

$$\text{Leading Order: } f'(x) + \frac{f(x)}{dx} (g_0 \epsilon) + \frac{1}{2} f''(x) dx$$

$$\frac{d}{d(dx)} = \frac{-f g_0 \epsilon}{dx^2} + \frac{1}{2} f''(x) = 0 \rightarrow \frac{f \epsilon}{dx^2} = \frac{1}{2} f''$$

$$\rightarrow dx^2 = \frac{2f\epsilon}{f''}$$

$$\rightarrow dx = \left( \frac{2f\epsilon}{f''} \right)^{1/2}$$

## Two Sided Derivative

if we do the same process with :

$$\frac{f(x+dx) - f(x-dx)}{2dx}$$

we get :

$$dx \sim \left( \frac{f\epsilon}{f'''} \right)^{1/3}$$

But this is not easy for a multivariable function

Another option:

- Divide and multiply the dx by a constant factor iteratively until you find a covariance that is reasonable
- First tried writing the algorithm for this but it was really messy
- So, I decided to do it by hand
- And I found the suitable dx:

$$dx = \text{parameter} / (4E5)$$

# Checking LM with a known ares curve

```
on iteration 8 chisq is 35.11993817510575 and lamda is 0
step [ 0.00825153 -0.00527821  0.00190958 -0.00283937]
new params [ 4.02044549 37.65625335  5.00283505  0.87302918]

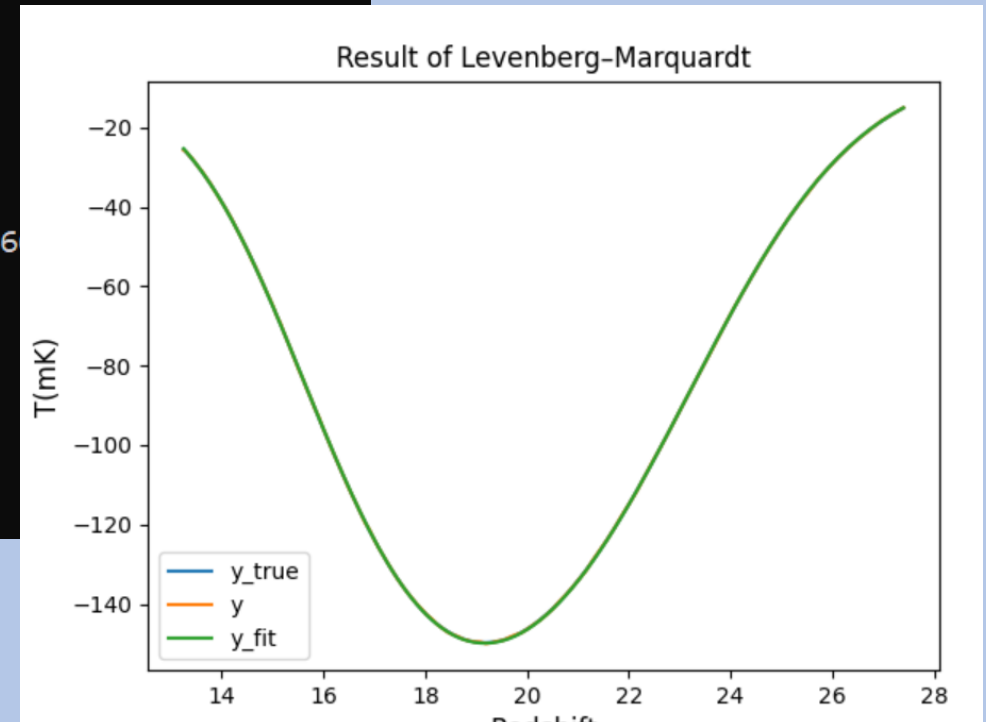
on iteration 9 chisq is 18.683737356955312 and lamda is 0
step [ 1.10105765e-02 -1.30126242e-01  1.27976633e-02  5.14061093e-05]
new params [ 4.03145606 37.52612711  5.01563271  0.87308059]

on iteration 10 chisq is 1.9524549158272093 and lamda is 0
step [-0.00409884 -0.08953417  0.00198708  0.00043634]
new params [ 4.02735722 37.43659294  5.01761979  0.87351692]

on iteration 11 chisq is 1.9524549158272093 and lamda is 1
step [ 0.00179082 -0.00457116  0.00152309  0.00407376]
new params [ 4.02914805 37.43202178  5.01914289  0.87759068]

on iteration 12 chisq is 1.8734032140742372 and lamda is 0.6666666666666666
step [ 4.90170891e-05 -2.72518052e-03  4.91323808e-05  3.04302293e-04]
new params [ 4.02740624 37.43386776  5.01766893  0.87382122]

on iteration 13 chisq is 1.8461104697760566 and lamda is 0
step [-3.17231704e-05 -2.32366875e-03  2.72045457e-05  2.91208368e-06]
new params [ 4.02737452 37.43154409  5.01769613  0.87382414]
Converged after 14 iterations of LM
```



# Other points

- Chapters of my thesis