Problem 2)

a) $\xi[0,0] = 1$ $\quad\quad$ $v[0,0] = 1$

$v[1,0] = ?$ $\quad\quad$ $v[2,0] = ?$

(Just for checking: $v[5,0] = -1.05$)

v - Average of neighbors $= \xi$

and we have : $v = n \log r + m$

applying this rule to $\xi[0,0]$ and $v[0,0]$:

$$v[0,0] - \frac{1}{4}\left(v[0,1] + v[1,0] + v[-1,0] + v[0,-1]\right) = \xi[0,0]$$

(I am not sure if doing this for the origin is correct!)

$r = \sqrt{1} = 1$ $\quad$ $r = \sqrt{1} = 1$ $\quad$ $r = \sqrt{1} = 1$ $\quad$ $r = \sqrt{1} = 1$

$$\cancel{1} - \frac{1}{\cancel{4}}\left(\cancel{4}m + \cancel{4}n \log 1\right) = \cancel{1} \rightarrow m + n \log 1 = 0 \rightarrow$$

$$n \log 1 = -m \rightarrow \log 1 = \frac{-m}{n}$$

and now for the $v[1,0]$ we have:

$$v[1,0] - \frac{1}{4}\left(v[0,0] + v[1,1] + v[2,2] + v[1,-1]\right) = \xi[1,0]$$

$r = 1$ $\quad\quad$ 0

$$n \log 1 + m - \frac{1}{4}\left(1 + n \log \sqrt{2} + m + n \log 2 + m + n \log \sqrt{2} + m\right) = 0$$

$$n \log 1 + m = \frac{1}{4}\left(2n \log \sqrt{2} + 1 + 3m + n \log 2\right) = 0$$

$n \log 2$

$n \log 1 + m = \frac{1}{4}(2n\log 2 + 1 + 3m)$

from the previous calculation, we know that:

$\log 1 = -m/n \rightarrow m = -n\log 1$

So we have:

$\cancel{nx \frac{-m}{n}} \cancel{+ m} = \frac{1}{4}(2n\log 2 + 1 - 3n\log 1)$

$\rightarrow 2n\log 2 + 1 - 3n\log 1 = 0 \rightarrow$

$n(2\log 2 - 3\log 1) = -1$

$n = \dfrac{1}{-2\log 2 + 3\log 1}$

$n = -0.72 \quad , \quad m = 0$

$\rightarrow v = n\log r + m$

$v[5,0] = -0.72 \log 5 + 0 \sim -1.1587$

which is a little off

the amount that it should be $(-1, 0.5)$