Problem Set 1, Q5:

$$\chi^2 = (\underbrace{d - A(m)}_{n^T})^T N^{-1} (\underbrace{d - A(m)}_{n})$$

we can introduce an orthogonal matrix:

$$VV^T = V^TV = I$$

thus:

$$\chi^2 = n^T V^T V N^{-1} V^T V n$$

$$\chi^2 = (Vn)^T (VN^{-1}V^T)(Vn)$$

$$\chi^2 = \underbrace{(Vn)^T}_{\tilde{n}^T} \underbrace{(VN^{-1}V^T)^{-1}}_{\tilde{N}^T} \underbrace{(Vn)}_{\tilde{n}}$$

Now, $\tilde{N}$ is no longer diagonal.

We have:

$\tilde{n}_i = V^T_{:,i}\, n$ the i^{th} column of V dotted against the residual n

$$n_j^T = n^T V_{:,j}$$

$$\rightarrow \langle n_i n_j \rangle = \langle V^T_{:,i} \underbrace{n n^T}_{N} V_{:,j} \rangle = \langle V^T_{:,i} N V_{:,j} \rangle$$

as defined before: $V^T N V = \tilde{N}$

leaving us with:

$$\boxed{\langle n_i n_j \rangle = \tilde{N}_{ij}}$$