Problem Set 1, Q1:

poisson distribution: $P(n) = \frac{e^{-\lambda} \lambda^{n}}{n!}$

define: $x = n - \lambda$ deviation from mean

$\rightarrow n = x + \lambda$

Substitute for n:

$$p(x) = \frac{e^{-\lambda} \lambda^{x+\lambda}}{(x+\lambda)!}$$

$$\ln(p(x)) = \ln(e^{-\lambda}) + (x+\lambda)\ln\lambda - \ln(x+\lambda)!$$

Since λ is larg, we can use Stirling's approximation:

$$\ln(n!) = n\ln n - n$$

$$\ln(p(x)) = -\cancel{\lambda} + (x+\lambda)\ln\lambda - (x+\lambda)\ln(x+\lambda) + x + \cancel{\lambda}$$

$$\ln(p(x)) = \cancel{(x+\lambda)\ln\lambda} - \cancel{(x+\lambda)\ln\lambda} - (x+\lambda)\ln\left(1+\frac{x}{\lambda}\right) + x$$

$$x \ll \lambda \rightarrow \ln\left(1+\frac{x}{\lambda}\right) \sim \frac{x}{\lambda} - \frac{1}{2}\frac{x^2}{\lambda^2}$$

$$\ln(p(x)) = x - (x+\lambda)\left(\frac{x}{\lambda} - \frac{1}{2}\frac{x^2}{\lambda^2}\right)$$

$$\ln(p(x)) = \cancel{x} - \frac{x^2}{\lambda} + \frac{1}{2}\frac{x^3}{\lambda^2} - \cancel{x} + \frac{1}{2}\frac{x^2}{\lambda}$$

$$\ln(p(x)) = \frac{-1}{2}\frac{x^2}{\lambda} + \frac{1}{2}\frac{x^3}{\lambda^2}$$

Since λ is larg, we can safely ignore the second term, thus only leaving:

$$p(x) = e^{-\frac{1}{2} x^2/\lambda}$$

which is in the form of a gaussian.