Comparing The Efficiencies of Hohmann Transfer and Bi-elliptical Transfers in Sending Satellites to the Graveyard Orbit

Project code: PH017

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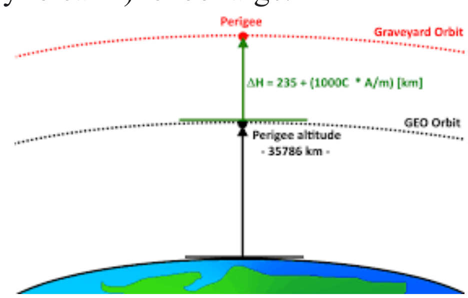
Introduction

Low Earth orbits (LEO) are the most commonly used satellite orbit with about 90% of all satellites used being found in this orbit (Roberts, 2020). LEO has a range of between 160 km or 1000km from the Earth's surface. Medium Earth Orbits (MEO) have the largest range of between 2000 km and 35786 km the common use of this orbit is for global positioning systems (GPS) due to the MEOs predictable orbit. (Types of orbits, 2020)

Satellites have a limited lifetime, after which they only obstruct the regions with active satellites where they become space debris, increasing the probability of them colliding with operational satellites (Rooney, 2021). To reduce this probability, they are often sent to graveyard orbits when no longer operating. A Graveyard orbit is an orbit that lies away from common operational orbits, usually at a radius of around 36,000km. One significant graveyard orbit is a supersynchronous orbit which has an altitude of about 36,100 km, well beyond geosynchronous orbit (Ashish, 2022). A graveyard orbit is used when the change in velocity required to perform a deorbit maneuver (i.e. reentry to earth) is too large.



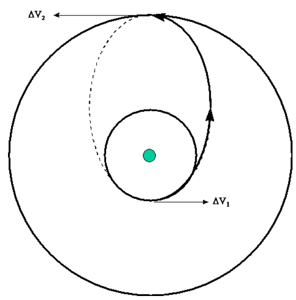
(Figure 1)

Note. From The Collisional Evolution Of Orbital Debris In Geopotential Wells And Disposal Orbits, By Benjamin Polzine, March 2017, p. 6, Copyright 2017 by Benjamin Polzine

As more satellites are being sent to space due to launch of new programmes such as Starlink, it has become an increasing concern on how to dispose of the satellites properly. Currently, most satellites in the LEO are left to crash back onto the Earth's surface. About one satellite crashes onto the earth's surface every week as it is the most energy efficient way to dispose of a decommissioned satellite (Williams, 2018). For example, with the first generation of starlink launched, it is predicted that 2.2 tonnes of dead satellites will be reentering earth's atmosphere daily (Pultarova, 2021). However, the reentry of decommissioned satellites contributes to the depletion of the ozone layer, which is an environmental concern. Currently, the graveyard orbit is mostly used for satellites in Geostationary orbits as they are the furthest away from the earth and thus would be relatively cheaper to send out to space than to crash back to earth. However, we believe that more satellites in LEO and MEO should be sent to the graveyard orbit in order to reduce the environmental damage on the Earth and thus ensure the sustainability of the satellites . Hence we conducted research on which method would be most suitable to send satellites from these orbits into the graveyard orbit.

To send satellites into their graveyard orbits, orbital maneuvering (movement of a satellite's orbit) is necessary where the total energy of the satellite is increased through impulses, to move it into higher orbit. There are 2 main forms of orbital maneuvering, the Hohmann transfer and bi-ellipitcal transfer.

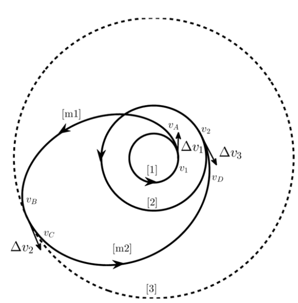
The Hohmann transfer uses two engine impulses (Hohmann transfer orbit, 2021), one to move a satellite into the transfer orbit and a second to move into the target orbit, the graveyard orbit.



(Figure 2)

Note. From the Exercise: Hohmann Transfer -- Technical Notes. (2016, September). [Illustration]. https://help.agi.com/stk/11.0.1/Content/training/tq-hohmann.htm

Bi-elliptical transfer uses three impulses (Bi-elliptic transfer, 2021), by sending the satellite into a larger intermediate orbit before sending it to the target radius.



(Figure 3)

Note. From the Galperin, Alexander & Gurfil, Pini. (2015). Closed-form Solutions for Optimal Orbital Transfers Around Oblate Planets. The Journal of the Astronautical Sciences. 61. 10.1007/s40295-015-0043-3.

Hypothesis

Our hypothesis is that the Hohmann transfer is the most energy efficient orbital maneuver to send any satellite to a graveyard orbit. Since it only requires 2 impulses and the satellite has to travel a shorter distance, it usually uses less fuel and thus expends less energy compared to the bi-elliptical transfer which requires 3 impulses and requires the satellite to travel a longer distance. Multiple research studies have reiterated this.

Materials and Methods

The energy of the satellite in orbit can be calculated via the vis-viva equation. (Vis-viva Equation, 2021)

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

The energy required for each impulse can easily be found by finding the difference in values of the energy of the system before and after the impulse. Using this, we formed an equation to find the energy each transfer requires. Simplifying the Hohmann transfer equation gives us:

$$\Delta v = \sqrt{\frac{\mu}{r1}}\left(\sqrt{\frac{2(r2)}{r1 + r2}} - 1\right) + \sqrt{\frac{\mu}{r2}}\left(1 - \sqrt{\frac{2r1}{r1 + r2}}\right)$$

We can find the equation for the Bi-elliptical transfer in a similar method.

$$\Delta v = \sqrt{\frac{2\mu}{r1} - \frac{\mu}{a1}} - \sqrt{\frac{\mu}{r1}} + \sqrt{\frac{2\mu}{rb} - \frac{\mu}{a2}} - \sqrt{\frac{\mu}{rb} - \frac{\mu}{a1}} + \sqrt{\frac{2\mu}{r2} - \frac{\mu}{a2}} - \sqrt{\frac{\mu}{r2}}$$

For this research, we have a few important assumptions:

1. Gravitational effect of other bodies (e.g. other planets) in the system on the satellite is negligible, so the satellite only experiences the gravitational field of the Earth.
2. The maneuvers take place on the same plane.
3. There is no space debris that may be in the way of our orbit.
4. The satellite has a constant mass throughout the experiment.

We used the same equation for both transfers, but the intermediate radius ratio for the Hohmann transfer is set to 1 which removes the intermediate radius, making the value of the 3rd impulse will be zero

Freeflyer

To investigate energy efficiency, we coded the Hohmann transfer and Bi-Elliptical transfer equations which was mentioned in the previous section into the Freeflyer software and also coded the transfer so to simulate and observe which transfer is more energy efficient. The software would then run the simulation and provide us with the Δv required. (Refer to appendix 1 for the code)

For the LEO, the ParkingSMA was to be set as 2000km with the target SMA to be 36100km, with the final to initial semi-major axis ratio of 18.1. However, the software could not run the simulation at a very low radius, hence we maintained the ratio and changed the values to 7000km and 126700km.

For MEO, we set the ParkingSMA as 8000km and the targetSMA as 36100km similar to the satellites created by O3b, which gives a final to initial semi-major axis ratio of 4.51.

We also varied the initial to intermediate radius ratio to see what its effect was on the Δv. Our keyed in the values as 2,3 and 5, as Hohmann Transfer only has 2 impulses it does not have an intermediate radius and thus we can put its ratio at 1.

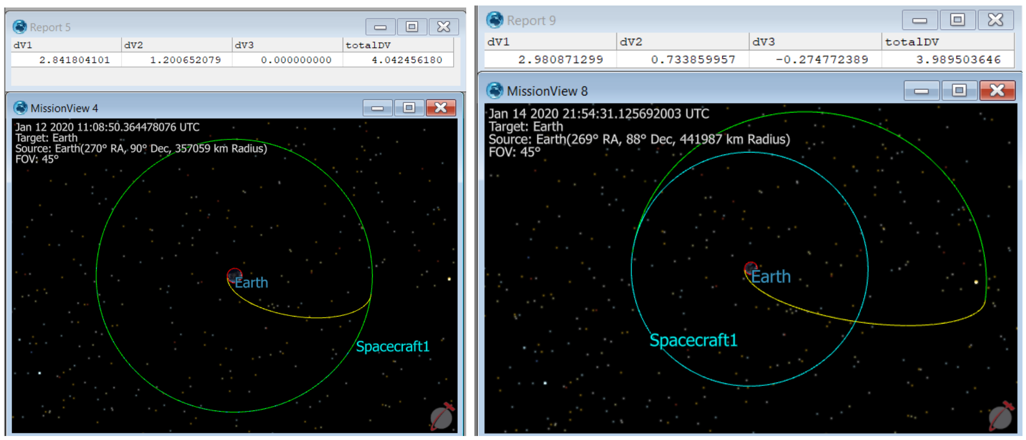
We then ran the simulations and collected the data

Results

Initial Final major semi axis ratio	Intermediate Ratio			
	Hohmann Transfer	Bi-elliptical Transfer		
	1	2	3	5
18.1(LEO)	4.0423	3.9889	3.9567	3.9240
4.51 (MEO)	3.5213	4.0578	4.2400	4.3850

Table 1.1 Δv values for each maneuver

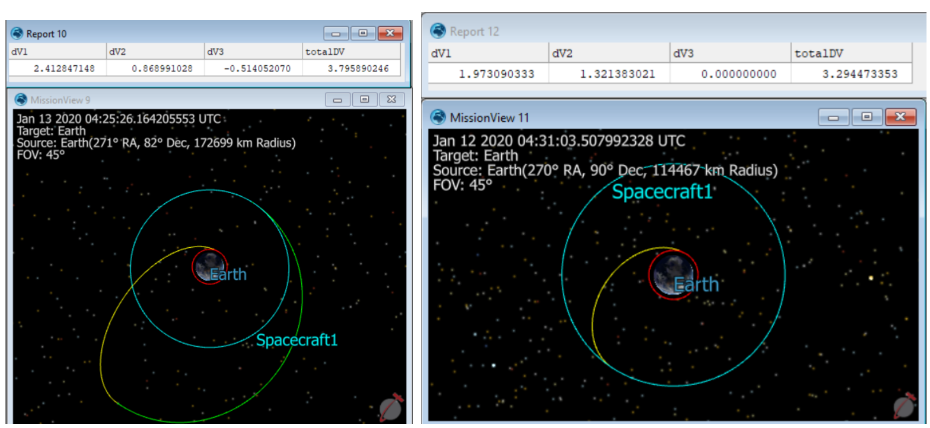
Low-Earth orbit satellite



(Figure 4: Visualisation of the transfers for LEO)

Carrying out the bi-elliptical transfer would give rise to a lower Δv value of 3.99 (right) compared to that of the hohmann transfer with a value 4.04.

Medium-earth orbit satellite



(Figure 5:Visualisation of the transfers for MEO)

However, the Hohmann transfer gives rise to a lower Δv value of 3.29 (right) compared to that of the bi-elliptical transfer of 3.79 (left).

Discussions

For LEOs, since the Δv value for the Bi-elliptical transfer is smaller than that of the Hohmann transfer, the former is more fuel and thus energy efficient. However, for MEOs, Δv value when Hohmann transfer was used is smaller than that when the Bi-elliptical transfer was used, proving it is more effective to use a bi-elliptical transfer.

This disproves our hypothesis of the Hohmann transfer being the most energy efficient orbital maneuver to send any satellite to a graveyard orbit.

However, we need to take into consideration that for this experiment, we only took into account the energy required, but not the time taken for the transfer. Hence, the efficiency cannot exactly be concluded because time is one of the factors that affect efficiency. However, graveyard orbit is used when the satellite is nearing the end of its life, hence time is usually not a thing needed to be worried about hence we did not include it. However, energy is important as fuel may be limited.

We also realized that increasing the intermediate radius ratio can lead to a decrease in the Δv. However, it is not very recommended to have a high intermediate radius ratio as the larger the value, the longer the time the satellite takes to maneuver.

Moreover, as previously stated, changing some of our assumptions can drastically affect the results. However, we made the considerations as reliable as possible so that our results can be as accurate as possible.

Conclusion

From our research, we can conclude that for MEOs, the Hohmann Transfer is the more energy efficient method to send satellites to the graveyard orbit. However for LEOs, the bi-elliptical transfer is more efficient. We also know that as this ratio increases, the bi-elliptical transfer becomes a more viable option as compared to the hohmann transfer.

This will help scientists decide which method of orbital maneuver will be the most efficient method to move decommissioned satellites out of their active orbits. This can help them make more economical decisions.

For future work, we could research including plane changes and seeing how it affects the efficiency of the transfer.

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Appendix

Nomenclature

v = relative speed of the two bodies

r = distance between the two bodies

a = length of the semi-major axis

G = gravitational constant

M = mass of the central body

$\mu = GM$, which is is the standard gravitational parameter of the primary body

$r1$ = initial orbit radius

$r2$ = final orbit radius

rb = common apoapsis radius of the two transfer ellipses and is a free parameter of the maneuver

$a1$ = semimajor axes of the initial and intermediate radii

$a2$ = semimajor axes of the final and intermediate radii

Appendix 1: Code for the Hohmann transfer and Bi-Elliptical Transfer in Freeflyer

```
FreeForm: User Input
FreeForm: Calculate Transfer
While (Spacecraft1.ElapsedTime < TIMESPAN(4 hours))
    Step Spacecraft1;
    Update ViewWindow1;
End;
FreeForm: Perform Maneuver 1
FreeForm: Perform Maneuver 2
FreeForm: Perform Maneuver 3
While (Spacecraft1.ElapsedTime < TIMESPAN(6 days));
    Step Spacecraft1;
    Update ViewWindow1;
End;
```

(Figure 6)

This the page of the mission sequence (Fig 4)

FreeForm Label | User Input | Print

```
// User Input for the Bi-Elliptic Transfer
Variable parkingSMA = 2000;
Variable targetSMA = 36100;
// Magnitude of how much larger the Bi-Elliptic transfer should be in comparison to the target orbit
Variable transferMagnitude = 2;
// Assigns the Parking SMA to the Spacecraft
Spacecraft1.A = parkingSMA;
```

(Figure 7)

In this script we can change the input. ParkingSMA refers to the initial orbital radius, while targetSMA refers to the final orbital radius. TransferMagnitude refers to the intermediate to final ratio. (Fig 5)

FreeForm Label | Calculate Transfer | Print

```
// Calculations for Maneuver 1
// SMA of the first transfer orbit
Variable transfSMA1 = (targetSMA * transferMagnitude + parkingSMA)/2;
// Speed at periapsis for the first transfer orbit
Variable transfPeri1 = sqrt(Earth.Mu * ( (2/parkingSMA) - (1/transfSMA1) ) );
// Delta V for the first maneuver
Variable dV1 = transfPeri1 - Spacecraft1.VMag;

// Calculations for Maneuver 2
// Radius of Apoapsis of the first transfer orbit Apoapsis
Variable radiusApog1 = transfSMA1 * 2 - parkingSMA;

// SMA of the second transfer Orbit
Variable transfSMA2 = (2 * transfSMA1 + (targetSMA - parkingSMA))/2;

// Speed at apoapsis for the first transfer orbit
Variable transfApog1 = sqrt(Earth.Mu * ( (2/radiusApog1) -
(1/transfSMA1) ) );
// Speed at apoapsis for the second transfer orbit
Variable transfApog2 = sqrt(Earth.Mu * ( (2/radiusApog1) -
(1/transfSMA2) ) );
// Delta V for the second maneuver
Variable dV2 = transfApog2 - transfApog1;

// Calculations for Maneuver 3
// Speed at periapsis for the second transfer orbit
Variable transfPeri2 = sqrt(Earth.Mu * ( (2/targetSMA) - (1/transfSMA2) ) );
// Speed for the target circular orbit
Variable vTargetorbit = sqrt( Earth.Mu * ( (2/targetSMA) - (1/targetSMA) ) );
// Delta V for the third Maneuver
Variable dV3 = vTargetorbit - transfPeri2;

// Assigns Delta V values to each Maneuver
ImpulsiveBurn1.BurnDirection[0] = dV1;
ImpulsiveBurn2.BurnDirection[0] = dV2;
ImpulsiveBurn3.BurnDirection[0] = dV3;
```

(Figure 8)

This is the script to calculate the Δv using the vis-viva equations as we found before (Fig 6)

FreeForm Label | Perform Maneuver 1 | Print

```
// Changes the tail color for the first transfer orbit
Spacecraft1.Color = ColorTools.Yellow;
Maneuver Spacecraft1 using ImpulsiveBurn1;
// Steps the spacecraft until its orbit apoapsis
WhileStepping Spacecraft1 to (Spacecraft1.OrbitApoapsis);
Update ViewWindow1;
End;
```

(Figure 9)

FreeForm Label | Perform Maneuver 3 | Print

```
// Changes the tail color for the target orbit
Spacecraft1.Color = ColorTools.Cyan;
Maneuver Spacecraft1 using ImpulsiveBurn3;
// Calculates the total Delta V used for the Bi-Elliptical transfer
Variable totalDV = dV1 + dV2 + abs(dV3);
// Reports each Maneuver value and the total amount of Delta V used
Report dV1, dV2, dV3, totalDV;
```

(Figure 10)

FreeForm Label | Perform Maneuver 2 | Print

```
// Changes the tail color for the second transfer orbit
Spacecraft1.Color = ColorTools.Lime;
Maneuver Spacecraft1 using ImpulsiveBurn2;
// Steps the spacecraft to its periapsis
WhileStepping Spacecraft1 to (Spacecraft1.OrbitPeriapsis);
Update ViewWindow1;
End;
```

(*Figure 11)*

These next 3 scripts each execute an impulse to perform the maneuvers for the simulation.(Fig 9, 10 and 11)