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All you need to know about model predictive control for buildings

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ABSTRACT

It has been proven that advanced building control, like model predictive control (MPC), can notably reduce the energy use and mitigate greenhouse gas emissions. However, despite intensive research efforts, the practical applications are still in the early stages. There is a growing need for multidisciplinary education on advanced control methods in the built environment to be accessible for a broad range of researchers and practitioners with different engineering backgrounds. This paper provides a unified framework for model predictive building control technology with focus on the real-world applications. From a theoretical point of view, this paper presents an overview of MPC formulations for building control, modeling paradigms and model types, together with algorithms necessary for real-life implementation. The paper categorizes the most notable MPC problem classes, links them with corresponding solution techniques, and provides an overview of methods for mitigation of the uncertainties for increased performance and robustness of MPC. From a practical point of view, this paper delivers an elaborate classification of the most important modeling, co-simulation, optimal control design, and optimization techniques, tools, and solvers suitable to tackle the MPC problems in the context of building climate control. On top of this, the paper presents the essential components of a practical implementation of MPC such as different control architectures and nuances of communication infrastructures within supervisory control and data acquisition (SCADA) systems. The paper draws practical guidelines with a generic workflow for implementation of MPC in real buildings aimed for contemporary adopters of this technology. Finally, the importance of standardized performance assessment and methodology for comparison of different building control algorithms is discussed.

1. Introduction

Buildings today contribute to roughly 40% of the global energy use (approx. 64 PWh), of which a large portion is used for heating, cooling, ventilation, and air-conditioning (HVAC) (IEA International Energy Agency & International Partnership for Energy Efficiency Cooperation, 2015). Energy savings thus become a priority in the design and operation of modern HVAC systems. Numerous studies reported that advanced HVAC control can notably reduce energy use and mitigate greenhouse gas emissions with average energy savings of 13% to 28% (Gyalistras et al., 2010; del Mar, Álvarez, de A., & Berenguel, 2014; Roth, Westphalen, Dieckmann, Hamilton, & Goetzler, 2002). This means that in the ideal case of full employment of this technology, annual final energy savings of roughly 8PM h to 18PM h can be projected. Based on

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this potential, recently revised EU policy on the energy performance of buildings states that large buildings should be equipped with building automation and control systems by 2025 if economically and technically feasible (EUp, 2018).

However, the majority of buildings today still adopt simple rule-based control (RBC) techniques with only limited energy saving capabilities (Aghemo et al., 2013; Mechri, Capozzoli, & Corrado, 2010). The promise of a digital age comes with decreasing costs in computation and sensing, which is paving the way for the adoption of advanced control strategies, like model predictive control (MPC). In the last decade, MPC has become a dominant control strategy in research on intelligent building operation. The main benefit of MPC is a systematic thermal comfort improvement with simultaneous energy savings spanning from 15% up to 50% demonstrated on numerous simulation and several pilot case studies (Ma et al., 2012; Oldewurtel et al., 2012; Sturzenegger, Gyalistras, Morari, & Smith, 2016; Široký, Oldewurtel, Cigler, & Prívara, 2011), as well as grid flexibility services via price-responsiveness and active demand response capabilities (Bianchini, Casini, Pepe, Vicino, & Zanvettor, 2017; Borsche, Oldewurtel, & Andersson, 2014; Cutsem, Kayal, Blum, & Pritoni, 2019a; Esther & Kumar, 2016). The strength of MPC lies in the use of a mathematical model of the building to predict its future behavior. By using these predictions, MPC can optimally choose the control actions based on a given objective while taking into account the comfort and technological constraints, and weather forecasts in a systematic and flexible way.

Despite the abundance of research papers and several pilot installations, the transfer of this technology to the building market is still in its early stages. The difficulty of the building sector stems from the fact that building management systems (BMS) engineers do not have advanced education in modern optimal control methods and tools, as control engineers do in other fields that have utilized MPC successfully, such as the process industry. Moreover, in contrast to the production of cars or user electronics, design and production of building and their HVAC systems are not standardized. Every building is a unique system which requires tailored modeling and control design, hence imposing increased engineering time and cost, particularly for advanced control strategies. All of this emphasizes the requirement for extending the theoretical education and practical skill set of the building control practitioners to enable the installation, maintenance, and operation of advanced MPC applications. An additional limiting factor is the poor ICT infrastructure in pre-existing buildings. One of the emerging advanced building control solutions is cloud-based control as a service platform. Although, significant privacy and cyber-security challenges are linked with these remote control architectures. Based on the observations described above and reflections presented in Cigler, Gyalistras, Široký, Tiet, and Ferkl (2013a); Prívara et al. (2013), six main challenges for wide-spread application of MPC to buildings are defined:

1. Availability of appropriate hardware and software infrastructure with compatible communication interfaces.
2. User-friendly, control-oriented, accurate, and computationally efficient building modeling.
3. Automated design, tuning, and deployment of MPC.
4. Plug-and-play implementation, and robust operation of MPC.
5. Privacy and cyber-security issues and the user trust.
6. Trained personnel to handle commissioning, and maintenance of MPC in practice.

The first challenge does not fall in the scope of research anymore because it lies in the domain of market adaptation. To address the second challenge, a methodology for the automatic synthesis of building models based on Building Information Models (BIM) has been proposed (Andriamamonjy, 2018). Different attempts in reducing the model development effort via available templates in Modelica libraries like Buildings (Wetter, Zuo, Nouidui, & Pang, 2014) and IDEAS (Baetens et al., 2015), or via physically inspired reduced-order automated system identification toolchains (De Coninck, Magnusson, Åkesson, & Helsen, 2016). Dedicated tools are also emerging for automated MPC design for buildings (Blum & Wetter, 2017; Drgoňa, 2019; Jorissen, Boydens, & Helsen, 2018a). Computationally lightweight approximations of MPC control laws (Drgoňa, Picard, Kvasnica, & Helsen, 2018), and rule extraction algorithms based on machine learning (Domahidi, Ullmann, Morari, & Jones, 2014), or toolchains for generation of optimized C-code (Jorissen et al., 2018a) aim to tackle the fourth challenge of easy installation and robust operation. The privacy issues could be solved in two ways, first by employing local control solutions without the need for real-time remote communication, and second by the adoption of advanced cybersecurity measures (Cybersecurity in smart buildings inaction is not an option anymore, 2015).

The ambition of this paper is to deliver a comprehensive summary on the topic of MPC for buildings, which could help to tackle the last challenge from the list. The necessary theoretical base on MPC is first supported by a literature review of the most recent advances in the field. Then, an extensive overview and conceptual comparison of dedicated software tools is given, followed by practical guidelines for implementation and performance assessment of MPC in real buildings.

1.1. Previous reviews considering MPC for buildings

We would like to acknowledge a first attempt to provide a unified MPC framework, which was given in Serale, Fiorentini, Capozzoli, Bernardini, and Bemporad (2018). This review paper aims to build a bridge between control and building engineers with a common dictionary and taxonomy of classes to enhance the professional relationship between these two originally distinct engineering areas. The most recent review on MPC for buildings with the focus on demand-side flexibility compares the pros and cons of the current technology and highlights the requirement of expert knowledge as the main bottleneck (Zong et al., 2019). An overview on three major research topics in building control, in particular semantic interoperability, fault detection, and MPC, was presented in Benndorf, Wystrcil, and Réhault (2018). A paper with in-depth literature review and classification of building control methods with particular focus on MPC has been published by (Afram & Janabi-Sharifi, 2014b).

More specific reviews of the MPC technology focusing on particular aspects of building control are as follows. One of the earliest short reflections on MPC technology for buildings was given by (Henze, 2013) envisioning a large impact of MPC technology on intelligent building operation. A review paper focused on artificial neural network based MPC was given in Afram, Janabi-Sharifi, Fung, and Raahemifar (2017). A review on an important aspect of occupancy behavior focused MPC was introduced in Mirakhorli and Dong (2016), concluding that using occupancy measurement and models in combination with MPC can improve the comfort and decrease the energy use in contrast to a standard schedule based control strategy. Reviews (Hilliard, Kavgic, & Swan, 2015; Rockett & Hathway, 2017) focus on challenges, aspects and future trends of MPC for commercial buildings. Paper (Hilliard et al., 2015) provides recommendations for selecting a building response model, simulation timestep, prediction horizon, forecast resolution, and optimization algorithm, while (Rockett & Hathway, 2017) stresses the urgent need for research on the automated creation and updating of predictive models for MPC. Authors in Killian and Kozek (2016) ask ten questions about MPC for buildings and provide critical analysis of challenges, future trends, and potential of MPC for the general building market. The identified challenges are high modeling and parametrization effort, shortage of modeling and optimal control experts active in the building automation domain, and lack of commercial tools for expert-free building modeling. in Kavgic, Hilliard, and Swan (2015), the authors discussed the opportunities for implementation of MPC in commercial buildings together with the identification of specific building characteristics indicating increased potential for MPC, like large thermal mass, high solar gains, discrete occupancy periods, and the

Table 1
Nomenclature of terms and acronyms used in the paper.

Notation	Meaning	Notation	Meaning
Control terminology			
PID	proportional-integral-derivative	RBC	rule-based control
MPC	model predictive control	LMPC	linear MPC
NMPC	nonlinear MPC	HMPC	hybrid MPC
eMPC	explicit MPC	OSF-MPC	offset-free MPC
RMPC	robust MPC	SMPC	stochastic MPC
LBMPC	learning-absed MPC	RHC	reciding horizon control
DPC	data predictive control	OCP	optimal control problem
SSM	state-space model	TF	transfer function
KF	Kalman Filter	MHE	moving horizon estimation
UKF	Unscented Kalman Filter	EKF	extended Kalman Filter
TVKF	time-varying Kalman Filter	SKF	stationary Kalman Filter
ADP	approximate dynamic programming	DP	dynamic programming
HJB	Hamilton-Jacobi-Bellman equation	RL	reinforcement learning
Optimization terminology			
OP	optimization problem	ADMM	alternating direction method of multiliers
LP	linear programming	QP	quadratic programming
NLP	nonlinear programming	SQP	sequential quadratic programming
MIP	mixed integer programming	MINLP	mixed integer nonlinear programming
MILP	mixed integer linear programming	MIQP	mixed integer quadratic programming
GDP	generalized disjunctive programming	mpP	multi parametric programming
mpQP	multi parametric quadratic programming	mpLP	multi parametric linear programming
LMI	linear matrix inequality	CC	chance constraints
SDP	semidefinite programming	SOCP	second order cone programming
Modeling terminology			
ODE	ordinary differential equations	DAE	differential algebraic equations
AR	auto regressive	ARMA	auto regressive moving average
BJ	Box-Jenkins	ARMAX	auto regressive moving average with exogenous inputs
ANN	artificial neural network	DT	decision tree
SVM	support vector machines	RF	random forests
kNN	k-nearest neighbors	GP	gaussian processes
4SID	subspace state space system identification	OE	output error
MBE	mean biased error	RMSE	root mean square error
EEP	expected error percentage	CV	coefficient of variation
PRBS	pseudo random binary signal	CRPS	continuous ranked probability score
Building domain terminology			
HVAC	heating, ventilation, and air conditioning	AHU	air handling unit
VAV	variable air flow	BES	building energy simulation
FMI	functional mockup interface	BIM	building information modeling
PIR	passive infrared sensor	BaU	business as usual
iCRTF	inverse comprehensive room transfer functions	CFD	computational fluid dynamics
SCADA	supervisory control and data acquisition	BMS	building management system
HMI	human machine interface	CM	number of comfort violations minimization
CT	comfort tracking	PMV	predicted mean vote

opportunity to vary temperature setpoints.

Dounis and Caraiscos (2009); Naidu and Rieger (2011); Wang and Ma (2008) are more general building control reviews and classification studies covering advanced intelligent and optimal building control strategies. A detailed review in Shaikh, Nor, Nallagownden, Elamvazuthi, and Ibrahim (2014) summarizes the impact of smart control strategies on energy and comfort management in buildings focusing on aspects such as building sector, optimization objectives, energy source, control algorithm and simulation tools used. Optimal operation of energy management systems with a weather forecast is reviewed in Lazos, Sproul, and Kay (2014) concluding that weather has a significant influence on building energy operation and that the minimization of forecast uncertainty can lead to increased energy savings in the range of 15% to 30%. A most recent analysis of optimization-based building automation and control systems focusing on performance gap mitigation and uncertainty evaluation was given in Aste, Manfren, and Marenzi (2017). Different optimization methods applied to different energy domain areas are reviewed in Baños et al. (2011). A review of multi-criteria decision analysis (MCDA), which could be used to aid the selection of the objectives for MPC for buildings, was presented in Wang, Jing, Zhang, and Zhao (2009). Finally, deeper insights into MPC technology, in general, can be found, e.g. in Bemporad (2006); Mayne (2014).

1.2. Contributions and structure of the paper

The presented paper aims to provide a comprehensive up to date overview of MPC technology applied to buildings. Although there are several reviews on general intelligent building operation strategies and MPC, to the authors' best knowledge, a unifying overview integrating both theoretical and practical aspects is still missing in this field. The ambition of this paper is thus to fill this gap and provide the reader with a single publication capable of guiding the whole process of implementation of MPC in a real building. The paper is also aimed to act as a detailed introduction to the topic for control and mechanical engineers and researchers, facilitating the information exchange in the multidisciplinary domain of building control. In comparison to referenced literature reviews in Section 1.1, the purpose of this paper is not to redefine, but refine and extend given MPC frameworks from previous literature overviews with a particular focus on providing a detailed list of software resources for increased accessibility of the technology.

The first part of the paper emphasizes a theoretical perspective. Section 2 defines the general MPC framework with standard notation. Section 3 elaborates on building modeling. Section 4 gives a brief summary of algorithmic principles behind MPC. Sections 5 and 6 summarize different MPC problem classes and corresponding solution approaches, respectively. Section 7 compactly reviews methods for dealing with uncertainties in MPC for buildings. Everything is supported by a comprehensive up-to-date literature review reporting successful real-world applications or in-depth simulation case studies of the presented concepts.

In the second part of the paper, the emphasis lies on the practical aspects of the technology. Section 8 provides a comprehensive list of available software tools for modeling, analysis, and solution of MPC problems. Section 9 delves deeper into practical aspects of MPC implementation, such as control configuration, communication infrastructure, and SCADA architecture, together with practical guidelines for implementation in real buildings. Section 10 introduces the need and methodology for performance assessment and comparison of MPC strategies for buildings. Finally, Section 11 concludes the paper.

1.3. Nomenclature

Table 1 summarizes the terminology and acronyms used in the paper with domain-specific classification.

2. Model predictive building control

The purpose of the following section is to compactly define and summarize the general MPC framework for building applications. We present here the fundamental building blocks and corresponding concepts of MPC, different problem formulations, and a notation based on standards used in the control engineering community. The general MPC framework compatible with the structure of this paper is presented in Fig. 1. The presented framework is the extension of the MPC framework given in Serale et al. (2018).

2.1. Model predictive control basics

MPC is a constrained optimal control strategy that calculates the optimal control inputs by minimizing a given objective function over a finite prediction horizon. The mathematical model of the system together with the current state measurements and weather forecast are used to predict and optimize the future behavior of the building.

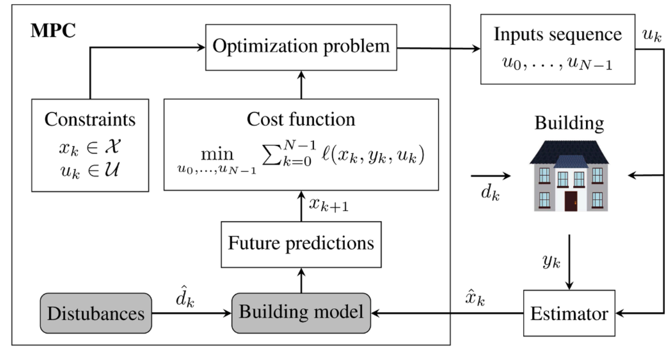


Fig. 2. Schematic representation of the standard closed-loop system for building control with MPC and state estimator.

2.1.1. Standard MPC scheme

Fig. 2 illustrates a typical abstract closed-loop MPC scheme which can describe most of the building control applications. The control loop consists of the building affected by disturbances d (e.g., weather conditions), predicted by weather forecast $\widehat{d}$, the state estimator providing the state estimates $\widehat{x}$ and the MPC controller which optimally manipulates the control actions u (e.g., heat flows, valves opening, pump powers), e.g., such that it minimizes used energy and keeps the output vector y (e.g., room temperatures) within the given comfort bounds.

2.1.2. General MPC formulation

The general MPC formulation for buildings can be represented as the following optimal control problem (OCP) in discrete time:

$$\min_{u_0,\ldots,u_{N-1}} \ell_N(x_N) + \sum_{k=0}^{N-1} \ell_k(x_k, y_k, r_k, u_k, s_k) \tag{1a}$$

$$\text{s.t. } x_{k+1} = f(x_k, u_k, d_k), k \in \mathbb{N}_0^{N-1} \tag{1b}$$

$$y_k = g(x_k, u_k, d_k), k \in \mathbb{N}_0^{N-1} \tag{1c}$$

$$u_k = f_{\text{HVAC}}(x_k, a_k, m_k), k \in \mathbb{N}_0^{N-1} \tag{1d}$$

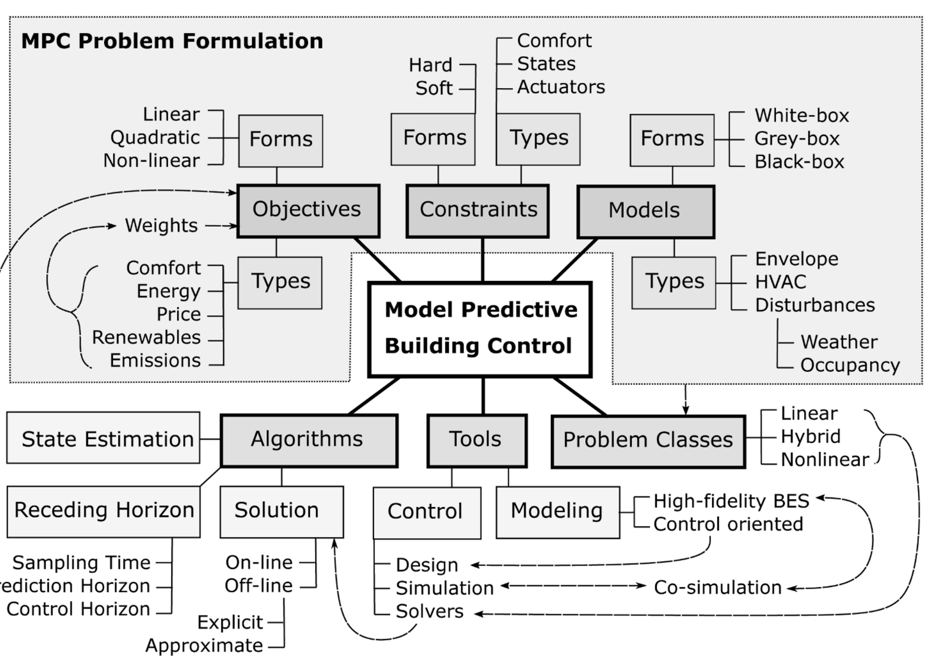


Fig. 1. Structure of the general MPC framework for building control applications compatible with the structure of this paper. Solid lines represent the sub-categories, while dashed lines with arrows depict causal dependencies and information flow during the design process.

$$s_k = h(x_k, y_k, u_k, r_k), k \in \mathbb{N}_0^{N-1} \tag{1e}$$

$$x_k \in \mathcal{X},\ u_k \in \mathcal{U},\ a_k \in \mathcal{A},\ s_k \in \mathcal{S}, k \in \mathbb{N}_0^{N-1} \tag{1f}$$

$$d_k = d(t + kT_s), k \in \mathbb{N}_0^{N-1} \tag{1g}$$

$$r_k = r(t + kT_s), k \in \mathbb{N}_0^{N-1} \tag{1h}$$

$$x_0 = \hat{x}(t), \tag{1i}$$

where $x_k \in \mathbb{R}^{n_x}$ denotes the values of states, $y_k \in \mathbb{R}^{n_y}$ the outputs, $u_k \in \mathbb{R}^{n_u}$ the building envelope inputs, $a_k \in \mathbb{R}^{n_a}$ the HVAC actuators, $m_k \in \mathbb{R}^{n_m}$ the additional measured variables, $d_k \in \mathbb{R}^{n_d}$ the disturbances, $r_k \in \mathbb{R}^{n_r}$ the reference signals, and $s_k \in \mathbb{R}^{n_s}$ denote the slack variables, at the kth step of the prediction horizon N with a sampling time T_s, where $n_\star$ denotes the dimensionality of associated variable $\star$.

The objective function is given by (1a), where $\ell_N(x_N)$ represents the terminal penalty used to ensure the stability and convergence of the control. For most of the building control applications the terminal penalty is omitted. $\ell(r_k, y_k, u_k, s_k)$ is called a stage cost and its purpose is to assign a cost to a particular choice of x_k, y_k, r_k, u_k and s_k.

The predictions of the state values are obtained from the state update Eq. (1b), while the values of the predicted outputs are given by the output Eq. (1c). The building envelope inputs u_k are subject to the HVAC dynamics (1d). Slack variables usually represent the violations of additional algebraic constraints (1e), such as comfort zones. States, envelope inputs, actuators, and slack variables are often subject to bounding constraints (1f). The initial conditions of the state variables are given by (1i) which are either measured or estimated. A forecasts of the disturbances and reference signals are given by (1g) and (1h), respectively. For building control applications, disturbances usually represent weather conditions and occupancy behavioral patterns, while reference signals span from tracking a single reference signal to more common comfort ranges on controlled variables. For the sake of generality we denote by ξ the vector that encapsulates all time-varying parameters of (1), i.e., the current state estimates $\hat{x}(t)$, current and future disturbances $d(t), \ldots, d(t+(N-1)T_s)$, and reference signals $r(t), \ldots, r(t+(N-1)T_s)$. Compression of all parameters into single vector ξ is convenient for compact representation of MPC feedback law $A = f_{MPC}(\xi)$, where $A = [a_0, a_1, \ldots, a_{N-1}]$ is the vector of computed optimal control actions.

2.1.3. Standard MPC notation

Table 2 summarizes the standard notation and meaning of the variables used in the control community together with most common physical representations in buildings.

Table 2
Standard notation and most common physical representation of the variables used in MPC for buildings.

Notation	Controller	Building	Units
x	states	building structure temperatures	[K]
y	outputs	room operative temperatures	[K]
u	inputs	heat flows to the zones	[W]
a	actuators	valve and pump modulations	[%]
m	measurements	HVAC states	[K, W, %]
d	disturbances	ambient temperatures, solar radiation,	[K, W]
		and internal heat gains	[W]
r	references	comfort zones, setpoints	[K]
s	slack variables	discomfort measures	[K]
ξ	parameters	aggregate of the building states, references, and disturbances	[K] [K,W]
Q	weighting factors	importance of particular objective	[−]
N	prediction horizon	predicted future time window	[−]
N_c	control horizon	optimized future time window	[−]

2.2. Objectives in building control

The objective, or also called cost function, represents the performance target to be minimized. When two or more targets are set, the problem is referred to as a multi-objective optimization. In such cases, the terms of the objective function are often conflicting and a trade-off among them has to be found. Common approaches for multi-objective optimization include *goal attainment, minimax*, and *Pareto front*.

Goal attainment In building control, the vast majority of MPC problems are using *goal attainment* formulations aiming to find a balance between weighted goals, such as energy use and thermal comfort of the occupants. This balance is typically adjusted by means of weighting terms to give priority to one of the targets. For example, Eq. (1a) can be re-written as (2). Where $\| Q_s s_k \|_2^2$ represents an arbitrary discomfort term in the form of the weighted squared 2-norm of the slack variables, and $Q_u u_k$ stands for the weighted linear energy term. The matrices Q_s and Q_u here represent the weighting factors, and κ_k is the time-varying factor representing, e.g. the weight associated with price or emission profiles. In human perspective, these weighting factors represent the "price" that the user is willing to pay to have more or less comfort. Besides standard weighting techniques, other methods to select the preferred objective have also been tested, such as lexicographic formulations which assume that the objectives can be ranked in order of importance (O'Dwyer, De Tommasi, Kouramas, Cychowski, & Lightbody, 2017).

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} \left(\|Q_s s_k\|_2^2 + Q_u \kappa_k u_k \right) \tag{2}$$

Minimax Also called *Min-Max* formulations aim to minimize the worst-case values of a set of multivariate functions. *Minimax* objective functions are typically being used for finding conservative solutions to the optimization problems in the presence of uncertainties. More details on this class problems are provided in Section 7.2 dedicated to robust MPC.

Pareto front Finds trade-off solutions in which an improvement in one objective requires a degradation in another. A generic review on MPC and PID design with Pareto front objectives was provided in Gambier (2008). Authors in Zhao, Shen, Li, and Bentsman (2017) demonstrated how to formulate and solve the preference adjustable multi-objective MPC for constrained nonlinear systems. The advantage of multi-objective MPC in the context of building control is that the resulting Pareto front solution space allows the user to choose the outcome according to his comfort preferences and economic constraints (Arendt et al., 2019; 2016; Ascione, Bianco, De Stasio, Mauro, & Vanoli, 2016; Ascione, Bianco, Mauro, Napolitano, & Vanoli, 2019; Li & Malkawi, 2016; Liu et al., 2013).

The formulation of the objective function is influenced by several factors, like building dynamics, type of the HVAC system, the level of detail of the controller model, observability and controllability of the system and user preference. For example, if only the building envelope is modeled, a classic approach is to minimize its heat inputs from the different heating and cooling systems, with each system having an associated cost (Picard & Helsen, 2018). In other approaches where the HVAC is explicitly modeled, setpoints of the components are usually manipulated to minimize the energy use (Jorissen, 2018). Although energy use and user comfort are the most frequently used objectives, it is possible and for some cases desired to have also different objectives like minimization of monetary costs, or greenhouse gases (GHG) emissions, maximization of the share of renewable energy sources (RES), and more. The following subsections elaborate more on different objectives used in the building control sector. Earlier reviews on MPC objective functions for building control can be found in Cigler, Široký, Korda, and Jones (2013b); Cupeiro Figueroa, Cigler, and Helsen (2018).

2.2.1. Comfort satisfaction

The main purpose of heating, cooling, and ventilation systems in buildings is to maximize the thermal comfort and indoor environmental quality (IEQ) for the occupants. Enhanced IEQ can improve occupants' productivity by 5 to 10% Olesen (2005), or satisfy the specific requirements of more demanding occupants like elderly people who in general prefer warmer thermal conditions (Schellen, van Marken Lichtenbelt, Loomans, Toftum, & De Wit, 2010).

In general, the main constituent of the IEQ is thermal comfort. The standard way to achieve thermal comfort is to maintain the zone temperatures of the building within a given temperature range or so-called comfort zone, e.g., as defined by the international standard ISO7730 (International Organization for Standardization, 2005). An advanced metric used to assess thermal comfort is the Predicted Mean Vote (PMV) indicator of Fanger (Fanger, 1973). PMV is used not only in the thermal comfort model of ISO7730 (International Organization for Standardization, 2005) but also in other standards like ASHRAE55 (American Society of Heating Refrigerating & Air Conditioning Engineers, 2013), EN15251 (Comite'Europe'en de Normalisation, 2007), and ISSO74 (van der Linden, Boerstra, Raue, Kurvers, & de Dear, 2006). PMV is a nonlinear model, which depends on various parameters like the metabolic rate, the clothing insulation, the indoor air temperature, the radiant temperature, the air velocity, the relative humidity, and on the outdoor meteorological conditions. However, its nonlinear nature makes it computationally more expensive for MPC applications (Castilla, Álvarez, Normey-Rico, & Rodriguez, 2014; Castilla et al., 2011), leading to the use of approximated versions of this model (Cigler, Prívara, Váňa, Žáčeková, & Ferkl, 2012; Klaučo & Kvasnica, 2014; Yang et al., 2018). The PMV value is moreover complicated to calculate in such a way that it fits the real observed mean vote (Humphreys & Nicol, 2002). On the other hand, some studies recommend an adaptive thermal model that involves acclimation of people, which could improve people's health by increasing their thermo-neutral zone (van Marken Lichtenbelt & Kingma, 2013). Standards including adaptive comfort bounds are defined by the thermal models in EN15251 (Comite'Europe'en de Normalisation, 2007), ASHRAE55 (American Society of Heating Refrigerating & Air Conditioning Engineers, 2013), and ISSO74 (van der Linden et al., 2006). A comprehensive comparison of adaptive thermal comfort models defined by different standards can be found in Sourbron and Helsen (2011). The main disadvantage of these personalized comfort metrics is that their parameters need to be properly measured or estimated, which often increases their cost and limits their applicability in control practice. For a more comprehensive overview of thermal comfort models, we refer the reader to Enescu (2017). Table 3 presents a compact summary of most common thermal comfort models used in MPC.

However, thermal comfort constitutes only a part of IEQ since it also depends on additional factors, such as indoor air quality (IAQ), lighting quality, visual and acoustic comfort. For example, evidence exists that mechanical ventilation systems lead to an overall improvement of the IAQ and reduction of reported comfort and health-related problems (Kephalopoulos, Geiss, Barrero-Moreno, D'Agostino, & Paci, 2016). To predict the air quality an occupancy model needs to be developed, e.g., based on statistical data or available measurements (Jorissen, Boydens, & Helsen, 2017). The occupancy models can also be used to predict the thermal loads and thus improve thermal comfort, and when correctly implemented they can further save up to 30% of energy (Mirakhorli & Dong, 2016). Furthermore, ventilation units can have some degree of freedom with respect to the relative humidity of the supplied air, and therefore they can also be straightforwardly incorporated into MPC formulations via humidity models or additional constraints on temperatures (Freire, Oliveira, & Mendes, 2005). The lighting quality can be improved by utilizing blind control and electric lighting power control (Oldewurtel, Sturzenegger, & Morari, 2013). In general, based on the available sensors, the output vector y can include not only the temperature measurements but also CO_2 concentrations, humidities, illuminance, and others.

Table 3
Selective summary of thermal comfort models used in MPC formulations.

Reference	Static model	Adaptive model	PMV	Others
Sturzenegger et al. (2013)	•	–	–	–
Oldewurtel et al. (2013)	•	–	–	•
Feng, Chuang, Borrelli, and Bauman (2015)	–	•	–	–
Maasoumy et al. (2014)	–	•	–	–
Castilla et al. (2014, 2011)	–	–	•	–
Freire, Oliveira, and Mendes (2008)	–	–	•	•

2.2.2. Minimization of cost

The minimization of the energy use in a building does not necessarily result in the minimization of the related operational costs. If for example, the energy prices are volatile, as it is the case for electricity, it may be worth to shift the load and store thermal energy during cheap periods for its later use when the energy prices are higher. This thermal energy can be stored in buffer tanks, geothermal borefields or by using the building's own thermal inertia. An economic objective can be formulated by transforming the energy use into monetary cost by means of a variable cost factor, (i.e., the term κ_k in Eq. (2)) which can be considered as a forecasted disturbance to the model.

The variability fuel prices (gas, oil, and wood) can be neglected because their dynamics is relatively slow, making the cost factor quasi-constant over the prediction horizon. These cost factors could be updated offline in the formulation when the price has a substantial change. Nonetheless, times are changing for electricity prices. The minimization of the monetary cost is equal to the minimization of energy in the cases where only electricity-based systems are used and the user has contracted a flat tariff. However, today, a wider variety of tariffs are being implemented with higher variability in both energy and peak demand prices. With the implementation of smart meters, even for the residential sector, it would be possible to access, e.g., hourly prices. Subsequently, using an economic objective has major potential if electricity-based supply systems such as heat pumps and chillers are used. The advantage of these objectives has been widely studied in the context of demand-response problems with real-time pricing (Avci, Erkoc, Rahmani, & Asfour, 2013; Bianchini, Casini, Vicino, & Zarrilli, 2016a). It has been shown that economic optimization could be used to reduce the peak electricity demand (Oldewurtel, Ulbig, Parisio, Andersson, & Morari, 2010b), or increase the stability, flexibility, and sustainability of the energy system, particularly in the face of growing intermittent renewable generation (Patteeuw, Henze, & Helsen, 2016; Qureshi & Jones, 2018). Examples of such a pricing-formulation are given by Bianchini, Casini, Vicino, and Zarrilli (2016b); Oldewurtel, Ulbig, Parisio, Andersson, and Morari (2010c); Vrettos, Lai, Oldewurtel, and Andersson (2013). A simulation study of different economic MPC formulations under commercial time-of-use tariffs concluded that multiple MPC formulations could offer the same value for the user (in terms of utility bill cost) but different grid service capabilities such as load shifting (Cutsem, Kayal, Blum, & Pritoni, 2019b).

2.2.3. Minimization of greenhouse gas emissions

This objective can be chosen if the user is motivated to reduce the carbon footprint of the building HVAC system. In contrast to the economic objective, the cost factor is replaced by an emission factor on the used energy amount. The minimization of GHG is equal to the minimization of energy in the cases where only conventional fossil energy sources are being used. The emissions for gas and oil boilers are proportional to the amount of combustible used. When electricity is supplied by a distributor who guarantees that it comes from the renewable electricity pool, the direct GHG emissions are zero. In this case, minimization of GHG emissions is not possible. The emission factor differs from the cost factor when electricity-based components take the energy from a standard electricity supplier. The cost profiles usually do not coincide with the GHG emissions profile. The GHG emission factor varies with the distribution of the different generation system types active at the considered moment. These emission factors can be provided or estimated through generation schedules by the grid operators. Cases where this objective function is used, can be found in Knudsen and Petersen (2016); Vogler-Finck, Wisniewski, and Popovski (2018).

2.2.4. Maximization of the share of renewable energy use

In cases where the building has local RES, these terms can typically be added to the above formulations with a negative cost/emission factor, which would lead to their maximum usage. The formulation that maximizes the share of RES (or minimizes the share of fossil fuels) uses different weighting factors on different available energy sources. Moreover, when sufficiently large thermal energy storage capacity and accurate controller models are available, the MPC can harness the power of the predictions to maximize the use of intermittent renewable systems by storing the energy for later use into thermal mass or batteries. The abstract factor $\kappa_k = 1 - R_k$ in Eq. (2) is used as the time-varying factor, where R_k represents the share of renewable energy in the load at the moment k. Some examples that use this objective function are treated in Vandermeulen, Vandeplas, Patteeuw, Sourbron, and Helsen (2017); Vogler-Finck, Pedersen, Popovski, and Wisniewski (2017)

2.2.5. Optimization of multiple generation and storage components

Another prominent set of multi-term objectives is optimizing the use of multiple energy generation (eg., PV cells) and energy storage components. The objective here is to increase the energy efficiency and flexibility of the building stock by load shifting, the energy exchange between multiple buildings or storage units, and by prioritizing the use of cheapest, cleanest, or most efficient energy sources. For instance, MPC formulation increasing the flexibility of a commercial building with thermal energy storage (TES) in demand-side management (DSM) programs was evaluated in Cao, Du, and Soleymanzadeh (2019). Authors in Tarragona, Fernández, and de Gracia (2020) apply MPC in a heating system with TES, PV panels, and electricity grid supply and study the impact of different MPC settings on the energy cost performance. MPC formulation for extremely large central cooling systems with TES was introduced in Shan, Fan, and Wang (2019). Kuboth, Heberle, König-Haagen, and Brüggemann (2019) formulate economic MPC for a residential building with a coupling of thermal and electric supply by an air source heat pump. A simulation study of MPC formulation for a residential house leveraging local PV microgeneration was presented in Godina, Rodrigues, Pouresmaeil, and Catalão (2018). MPC formulation optimizing the coefficient of performance (COP) of a hybrid geothermal system with a borefield heat exchanger was presented in. Cupeiro Figueroa, Picard, and Helsen (2020). in Zhao, Lu, Yan, and Wang (2015), an MPC formulation with multiple energy generation and storage components was tested on a real building.

2.2.6. Optimization of large-scale systems

In the case of large-scale commercial HVAC systems, the implementation of MPC as a single monolithic optimization problem is not practical nor desirable given real-time operating requirements (Rawlings et al., 2018). In these situations, decomposing the problem into a set of smaller problems presents a viable and practical alternative. A hierarchical decomposition of economic MPC in large-scale HVAC systems with district heating/cooling networks was applied and tested on a 500-zone campus in Rawlings et al. (2018). The energy hub concept allows optimizing a set of buildings in a cooperative manner, providing opportunities for load shifting, and sharing of costly energy generation and storage components, such as heat pumps, boilers, batteries (Darivianakis, Georghiou, Smith, & Lygeros, 2015).

2.2.7. Long-term objectives

In general, it is difficult to incorporate the long-term dynamic effects of the system which exceed the defined prediction horizon N. Such problems arise, for example, in MPC applications with seasonal energy storage units, like underground thermal energy storage (UTES), large-scale storage tanks, etc. However, to avoid thermal depletion of these storage systems, a thermal balance should be ensured on the long-term. To this end, some authors (Jorissen, 2018; Verhelst, 2012) have included a long-term cost in the objective function, that penalizes the use of the seasonal storage system at specific moments, thereby inviting the system to use the secondary production unit. However, this long-term cost could move from penalization objectives within the horizon towards shadow costs over a longer horizon. Since the accuracy of the predictions would decay over time, historical data may be needed to fit the predictions over longer horizons.

2.2.8. Design and tuning factors

In MPC there are several important setup and tuning factors, which can be considered as hyperparameters with a strong influence on the overall performance of the system. We summarize them in the following list:

- Weighting factors Q: give the preferences to the multiple objectives to be penalized in the objective function.
- Sampling period T_s: is the time interval in which the computed control actions remain constant, and the choice of it depends on the time constant of the controlled system.
- Prediction horizon N: N are here the number of time steps and hence $N\,T_s$ defines the length of a time window for which MPC computes the predictions given by model (1b) and enforces the system behavior desired by the objective function.
- Control horizon N_c: $N_c \le N$ represents the number of time steps for which MPC computes the optimal control actions which minimize the given objective function. Hence the length of an optimized time window is given by $N_c\,T_s$.

Fig. 3 provides a conceptual example of the characteristic MPC behavior for a building with a highlighted summary of design and tuning factors.

Weighting factors are usually determined based on magnitudes of the penalized signals, while the other parameters should be set up based on the dynamics of the controlled system. The practical rule is that T_s should be large enough for computing, communicating, and implementing the next control signal, though small enough to control the system in a stable way. The general rule in control theory is to choose T_s such that there are at least 10 to 20 samples in the rise time T_{90} of the process step response. Buildings are in principle slow dynamic systems with T_s usually spanning from 15min to 180min. In control theory, N should be large enough to cover the settling time of the process step response. N for building control applications usually spans between 5h to 48h (Afram & Janabi-Sharifi, 2014b). Typically, the control horizon is chosen such that $N_c \le N$ and $N_c \ge 2$. For many practical applications, the rule of thumb is to set N_c roughly to 20% of N. The advantage of $N_c < N$ lies in reduced computational demands by having fewer decision variables in the resulting optimization problem (Cagienard, Grieder, Kerrigan, & Morari, 2004). The reason why $N_c \le N$ is often used in practice is that the effect of the computed control actions decreases with each step in the future, which means that only the first few computed control actions have a major impact on the trajectory of the controlled variables. Eq. (3) serves as an example of such an objective function where $N_c < N$. The number of optimized variables is decreased from $n_u N$ to $n_u N_c$ which can significantly reduce the computational burden especially for problems with many control inputs.

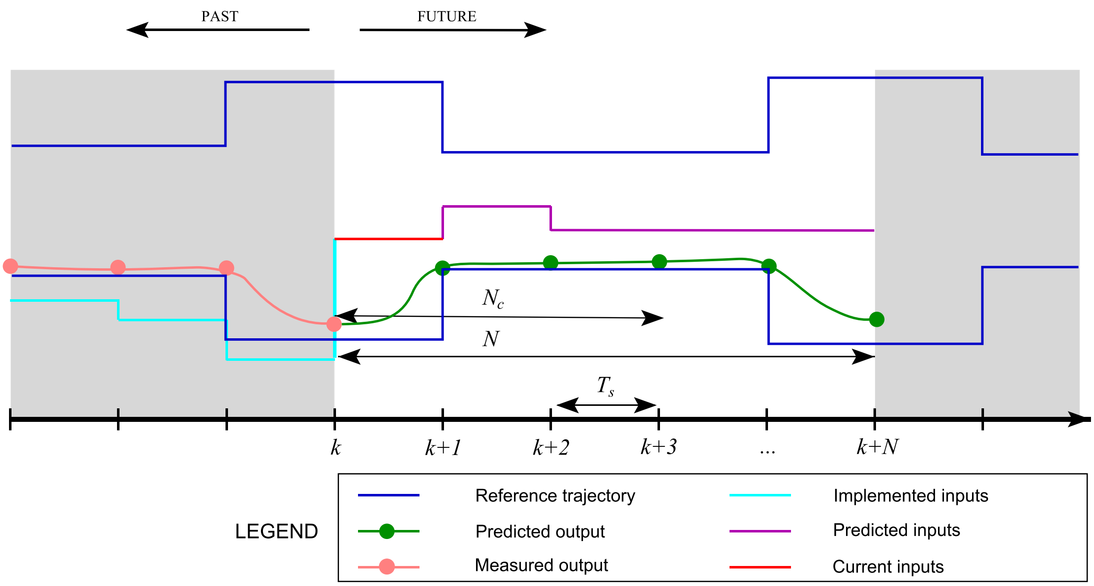


Fig. 3. Characteristic features and illustrated behavior of MPC for building temperature control.

$$\min_{u_0,\ldots,u_{N_c-1}} \sum_{k=0}^{N-1} \| Q_s s_k \|_2^2 + \sum_{k=0}^{N_c-1} Q_u \kappa_k u_k \tag{3}$$

A common practical problem that can appear in poorly tuned MPC is an oscillatory behavior. If the weights are unbalanced and control constraints are not tight enough, the control actions can result in bang-bang control profiles, i.e., either idle (no energy) or deadbeat (full power) control actions. These oscillations can be corrected by properly balancing the weighting terms, e.g., based on the magnitudes and ranges of the penalized variables, or by introducing the rate of change or slew rate constraints on control inputs (Cigler et al., 2013b). Definitions and a discussion about different types of constraints that can be used for tuning the performance of MPC are given in Section 2.3.

Further reading with detailed overviews, comparisons, and strategies for selection of an appropriate objective function and tuning parameters for MPC-based building control can be found, e.g. in Afram and Janabi-Sharifi (2014b); O'Dwyer et al. (2017); Rincón, Santoro, and Mendoza (2016); Serale et al. (2018). Please note that different mathematical formulations of the objective function can lead to different MPC problem classes with varying solution complexity and computational demands, which is further discussed in Section 5.

2.3. Constraints used in building control

MPC can handle a wide variety of constraints on state, input or output variables (Maciejowski, 2002). In general, there are two types of constraints: inequality (control inputs range, comfort zones, etc.) and equality (building model dynamics, rate limits, etc.) constraints. *Hard constraints* are those for which satisfaction is mandatory. An example of such constraints is the state update equation given by the equality constraint (1b), or control action bounds (4), which need to be satisfied at every time instant for the whole prediction horizon.

$$\underline{u} \le u_k \le \overline{u} \tag{4}$$

Soft constraints, on the other hand, are those for which violations can occur. They are usually relaxed by slack variables s_k that are added to and penalized in the objective function (1a). Soft constraints commonly used in building control are thermal comfort zone inequality constraints given by (5), defined by upper $\overline{y}_k$ and lower bounds $\underline{y}_k$ on the controlled variable y_k. For these types of constraints, the softening may be necessary to avoid infeasibility of the optimization problem during the time periods when the comfort constraints will be violated, as can happen in practical implementation. In general, soft constraints are preferable due to numerical reasons that guarantee the feasibility of the resulting optimization problem.

$$\underline{y}_k - s_k \le y_k \le \overline{y}_k + s_k \tag{5}$$

Another type of constraints consist of *time-varying constraints*, which in contrast to constant constraints, vary in time. Eq. (5) represents such constraints because comfort boundaries are defined as time-varying parameters $\overline{y}_k$ and $\underline{y}_k$. *Slew rate constraints* penalize the rate of change of certain variables, for example Eq. (6) limits the one-step difference of the control variable u_k. This type of constraint is useful for avoiding overshooting and peak behavior.

$$\Delta u_k = u_k - u_{k-1} \tag{6a}$$

$$\underline{\Delta u} \le \Delta u_k \le \overline{\Delta u} \tag{6b}$$

Move blocking constraints represent a formulation strategy for decreasing the computational burden by reducing the number of decision variables of the resulting optimization problem, as discussed in the Section 2.2.8. The basic idea is based on reducing the degrees of freedom by fixing the control variables or its derivatives to be constant over a defined time-period (Cagienard et al., 2004). See for example Eq. (7).

$$u_k = \begin{cases} u_k & \text{if } k \le N_c \\ u_{N_c} & \text{otherwise} \end{cases}, \qquad k \in \mathbb{N}_0^{N-1} \tag{7}$$

Terminal constraints penalize the last predicted state to stay within a given terminal region: $x_N \in \mathcal{X}_N$. They are usually used for enforcing the

Table 4
Summary of constraints types used in MPC for buildings, inspired by Serale et al. (2018).

Form	Violations	Time	Math	Variables	Meaning
Inequality	Soft	Varying	Linear	States	Model dynamics
Equality	Hard	Constant	Nonlinear	Outputs	Ranges
			Mixed-integer	Inputs	Slew rate
					Move blocking
					Terminal

stability and recursive feasibility of the OP (1) with respect to the controller model.

From a practical perspective in building control applications, the constraints are most commonly used to enforce selected variables to stay within given ranges, e.g., heat fluxes and room temperatures (Picard, Drgoňa, Kvasnica, & Helsen, 2017), supply air temperatures (Rehrl & Horn, 2011), airflow rates (Huang, 2011), and other HVAC variables (Afram & Janabi-Sharifi, 2014b), or for tuning the MPC performance via, e.g., slew rate constraints on control variables (Cigler et al., 2013b). From a mathematical point of view, the constraints can be further classified as linear, nonlinear or mixed-integer. The latter two can lead to better performance but also result in an increased complexity of the optimization problem. Table 4 compactly summarizes the constraint types discussed in this section. The influence of the constraints on the type and complexity of the OP is discussed in Section 5 in more detail.

3. Building models for control

The main bottleneck in practical implementation of MPC is the controller model development (Cigler et al., 2013a). Naturally, the quality of the MPC solution relies on the model accuracy, but also the overall MPC implementation is affected by the chosen modeling approach in a number of ways. Efficient optimization algorithms utilize specific model characteristics, like linearity, continuity, or known derivatives. However, the phenomena and processes occurring in buildings are often nonlinear and discontinuous, and complex physical models or advanced data-driven models are required to model such processes accurately. On the other hand, more complex models increase the overall computational demand of MPC, not only by increased simulation time but also because they are not suitable for efficient optimization algorithms and gradient-free algorithms have to be used instead. Therefore, a sound trade-off between the model accuracy and simplicity is required. This section provides an overview of the building model types, three modeling paradigms used in building modeling, as well as practical aspects of building modeling.

3.1. Building model types

This section elaborates on individual components of a generic building model structure, as shown in Fig. 4, and summarized by hybrid differential algebraic equations (DAE) with continuous and discrete time dynamics (1b)–(1d). These components are the building envelope, HVAC system, sources of disturbances such as weather and occupancy, and the peripherals represented by sensors and actuators.

3.1.1. Envelope

A building envelope consists of the external an internal walls, roofs, windows, ground floors, and other partitions separating the indoor environment from the outdoor environment, or two indoor thermal zones. In general, the building envelope model should take into account the heat transfer through conduction, radiation (especially solar gains), and convection (especially air infiltration). The conductive heat transfer depends on the thermal resistance of the partition and on its thermal mass. Heavier materials, e.g. brick or concrete, have higher thermal mass (inertia) and can absorb more energy, effectively working as a thermal buffer between the indoor and outdoor environments. This buffer can be utilized in MPC to shift energy use. Lighter materials, e.g. wood or thermal insulations, have low thermal mass resulting in a lower potential for accumulating energy. On the other hand, lighter materials have lower conductivity and therefore increase the thermal resistance of the partition. The radiative heat transfer from solar gains has to be taken into account in the case of transparent partitions (windows, curtain walls), but is often considered also for opaque partitions (heating building thermal mass). The transparent partitions have low thermal mass and are often modeled using the steady state heat equation.

The conduction in building envelopes is typically modeled using the 1D transient heat equation (Clarke, 2001; Hensen & Lamberts, 2019), converted to a system of ordinary differential equations using for example the method-of-lines, whereas the radiation and convection modeling approaches vary extensively. For example the radiative heat transfer can be modeled with anything from a simple solar heat gain coefficient to a complex dynamic shading model. In contrast, data-driven models typically do not consider the building envelope directly. Instead, they model indoor environment as a function of indoor and outdoor disturbances, and therefore the effect of building envelope is taken into account implicitly (Arendt, Jradi, Shaker, & Veje, 2018a).

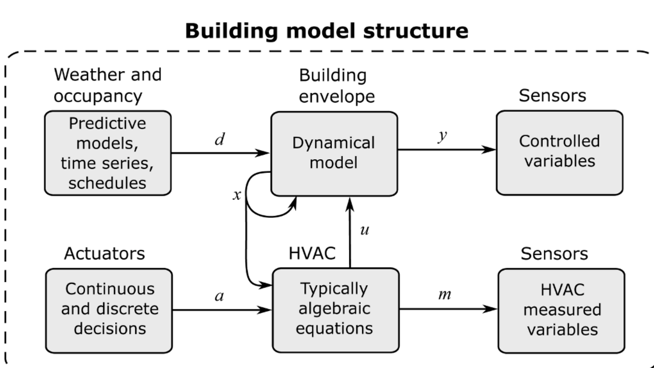


Fig. 4. Generic structure of a building model.

3.1.2. HVAC

Building HVAC systems vary greatly in designs, however some of the commonly used components are as follows: boilers, heat pumps, chillers, fans, filters, pumps, dampers, valves, heat exchangers, diffusers, ducts, and pipes. There is a vast spectrum of controls regulating the fluid flow, supply temperatures, and indoor air conditions.

HVAC components and controls coupled to building envelope models are challenging to simulate while maintaining reasonable computational demands (Jorissen, Wetter, & Helsen, 2018d). Fans and pumps have nonlinear characteristics (Wetter, 2013), which are coupled to nonlinear relations of mass flow rates and pressure differences in the system caused by active components such as valves and dampers and static components such as ducts and pipes. Excluding computationally expensive modeling approaches such as Computational Fluid Dynamics (CFD) that may be prohibitive from an MPC point of view, the final model structure highly depends on phenomena and processes the model has to cover. For example, in some cases MPC does not control all HVAC components directly and instead controls high-level setpoints. In such cases the model may have to include some embedded controls, which also can be nonlinear or even discrete (e.g. on/off), or assume ideal, instantaneous control.

3.1.3. Disturbances

Disturbances refer to every non-controllable input that has an influence on the building system. Some examples are weather conditions (e.g. outdoor temperature, solar radiation), internal heat gains (e.g. occupancy, equipment), and electricity prices. The weather conditions are simply inputs to the building simulation and so just an accurate forecast is required (no feedback). The easiest way to obtain it is from some online weather forecast service (many free and commercial are available), typically providing forecasts based on advanced climate models. A potential drawback of this approach is that the online weather forecast is typically generated based on data from climate stations which can be far from the considered building, and may not represent the actual weather conditions for the building. Alternatively, machine learning models can be employed and trained on the data collected from the building site, if available. The machine learning models can be especially accurate for short-term predictions, up to several hours ahead (Wollsen & Jørgensen, 2015), which is the range typically relevant for

MPC in buildings.

The most straightforward approach for including weather forecast in the prediction model of MPC is based on a data-driven linear dynamics model of the weather variables (Oldewurtel et al., 2012; Prívara, Široký, Ferkl, & Cigler, 2011), representing a cost-effective alternative to sophisticated simulation models or costly weather stations (Hedegaard, Pedersen, Knudsen, & Petersen, 2018). However, in some cases, linear dynamics might not be sufficiently accurate and can result in hampering the performance of the predictive controller (Kim, Witmer, Brownson, & Braun, 2014). In case of inaccurate disturbance models, stochastic (Drgoňa, Kvasnica, Klaučo, & Fikar, 2013; Parisio, Fabietti, Molinari, Varagnolo, & Johansson, 2014) or adaptive (Mazar & Rezaeizadeh, 2020) data-driven methods have been applied for mitigating the uncertainties associated with the weather forecast errors. For instance, (Liu, Paritosh, Awalgaonkar, Bilionis, & Karava, 2018) use a probabilistic time-series autoregressive model to quantify solar irradiance uncertainty. However, the disadvantage of data-driven correction methods is that the underlying disturbance distributions are often poorly represented based purely on historical data. Authors in Darivianakis, Georghiou, Smith, and Lygeros (2019) address this issue by exploiting the historical data to construct families of distributions based on real weather data, and construct a first-order model for weather prediction error.

The indoor occupancy can be modeled either as the heat gain profile, presence (room empty vs. at least one person in the room), occupant count, or occupant count and behavior. The latter approach is the most accurate, since building occupants not only generate heat, but also interact with the building, sometimes taking actions to adjust the indoor environment (window opening, overriding default setpoints). However currently, the state-of-the-art occupancy behavior models (e.g. obFMU Hong, Sun, Chen, Taylor-Lange, & Yan, 2016 or StROBe Baetens & Saelens, 2016) are computationally too expensive to be included in MPC. Therefore, less computationally demanding approaches are typically adopted in the context of MPC, for example models based on heuristics (e.g. anticipated schedules) or machine learning. Reviews in Balvedi, Ghisi, and Lamberts (2018); Yan et al. (2015) provide in-depth discussion on current methods of monitoring and modeling occupant behavior suitable for real-time control applications. For more comprehensive and systematic literature review of models for occupant behavior we refer the reader to (Carlucci et al., 2020). One of the popular data-drive models for the occupancy behavior are Markov chains processes, providing systematic framework for evaluating accurate scenarios for human-building interaction suitable for integration in scenario-based MPC formulations (Johnson, Starke, Abdelaziz, Jackson, & Tolbert, 2014). Sangogboye et al. (2017) presented data-driven occupancy prediction methods with average errors of 7% and 3% for passive infrared (PIR) sensor and stereovision camera training data, respectively. Peng, Rysanek, Nagy, and Schlüter (2018) incorporated data-driven occupancy models to optimize rule-based control in a real building and reported 7–52% energy savings, depending on the room type. Capozzoli, Piscitelli, Gorrino, Ballarini, and Corrado (2017) reported 14% energy savings through a pattern-recognition analysis of occupants' displacement. The most accurate occupancy predictions are yielded by models trained on dedicated-sensor data (PIR, cameras), however occupancy can be also predicted from other sensors, such as CO_2 or plug power (De Coninck & Helsen, 2016; Jorissen et al., 2017; Sangogboye et al., 2017).

Purely theoretical studies of the building dynamics using detailed white-box models can often include dozens sometimes up to a hundred of disturbance signals (Picard et al., 2017). However, for most practical applications, it is sufficient to take into account only a small subset of dynamically dominant disturbances. Authors in Drgoňa et al. (2018) used feature extraction based on principal component analysis (PCA) to select the most significant disturbances for residential building control, selecting the ambient temperature and solar irradiation. Similarly, (Lambrichts, 2020) studied the impact of the weather and occupancy uncertainties on MPC's performance, finding that uncertainties associated with the forecast of ambient temperature, solar irradiance, and internal heat gains have the largest impact on the performance of the predictive controller. Some intuition on the selection of dynamically dominant disturbances in specific cases can be derived from the studies above. However, systematic theoretical studies and practical guidelines for selecting dominant disturbances in a wide range of building model types, climate zones, and types of use are lacking in the current literature.

3.2. Modeling paradigms

This section provides an introduction to the three modeling paradigms used in building modeling and discusses their applicability to MPC. For a more extensive review on the modeling techniques used in HVAC control we refer to Afroz, Shafiullah, Urmee, and Higgins (2018). Additionally, a broad comparative study between the different modeling paradigms can be found in Boodi, Beddiar, Benamour, Amirat, and Benbouzid (2018).

3.2.1. White-box

White-box models describe the building dynamics from physical knowledge. They are based on the principles of heat transfer and conservation of energy and mass. The parameters of these models are physically meaningful and are obtained from the building technical documentation regarding geometry, material properties, and equipment specifications. For a detailed description of the equations that are considered in this modeling approach we refer to Jorissen et al. (2018c).

The main challenge in white-box modeling is the significant effort required to describe the building properties. Despite the advances in Building Information Modeling (BIM), this process is still largely manual and tedious (Gao, Koch, & Wu, 2019). The resulting model typically includes hundreds or, more likely, thousands of parameters. Hence, there are many potential sources of model inaccuracy, making the process of parameter setup difficult. With available measurement data, calibration may be used to tune the selected parameters. However, a model calibrated based on the overall yearly energy use might be still inaccurate for predicting performance on the individual zone level (Arendt et al., 2018a) or at smaller timescales. Moreover, the large number of equations and their nonlinear nature makes the implementation of white-box MPC more difficult.

On the other hand, when the parameters of the white-box models are accurate, their physical properties endorse them with highly reliable results. They can also track the evolution of physically meaningful variables. As a consequence, they are often considered suitable for fault detection (Henze, 2013) and building monitoring systems (Jradi et al., 2018). In addition, detailed building envelope and HVAC models can also enable control of the building at a more granular level, since the optimization variables may have a direct translation into the signals used in the actuators. This direct control skips the development of any sub-controller, which can be a cumbersome task, and it also increases the overall MPC performance due to direct assessment of HVAC efficiencies. For these reasons, research has been conducted to facilitate the implementation of these models into optimal control. As a result, toolchains have been developed to define white-box models for buildings and couple them with or translate them into an optimization problem. Coupling has traditionally taken the form of using a building energy simulation program within iterations of a numerical optimization technique, such as a scheme that couples EnergyPlus and a particle swarm optimization algorithm (Corbin, Henze, & May-Ostendorp, 2012) or one that couples TRNSYS and a genetic algorithm (Coffey, Haghighat, Morofsky, & Kutrowski, 2010). However, these schemes can be computationally expensive, especially as the number of optimization variables grows and complexity of the model increases, and prone to convergence issues (Wetter, 2004; Wetter & Wright, 2004). Wetter, Bonvini, and Nouidui (2016) argued that equation-based languages,

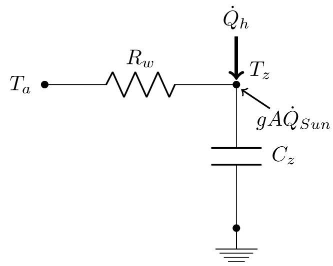


Fig. 5. Example of an RC building envelope model.

such as Modelica, can address some of the limitations of traditional building energy modeling software tools, such as EnergyPlus, specifically by (a) enabling symbolical manipulation of model equations and by (b) separating the model definition from the numerical solver. For instance, some of the most prominent Modelica libraries for building modeling are Buildings (Wetter et al., 2014), IDEAS (Baetens et al., 2015), AixLib Müller, Lauster, Constantin, Fuchs, and Remmen (2016), and BuildingSystems (Nytsch-Geusen et al., 2016). Jorissen et al. (2018a) implemented and validated an automated toolbox for automatically parsing white-box models written in Modelica into MPC, showing the feasibility of this approach. A detailed overview of the available software tools is presented in Section 8.1.1.

3.2.2. Gray-box

The gray-box category represents a wide spectrum of models encompassing simplified physical relationships, but also requiring parameter estimation based on measured data. Usually, the physics in gray-box models is simplified by means of state space dimensionality reduction or linearization. A typical concept in gray-box modeling is the RC analogy that defines any model by its affinity with a resistor-capacitor electrical circuit as the one shown in Fig. 5. This very simple example represents the model of a building envelope where C_z is the thermal capacitance of the zone which can be seen as the capacity of a zone to store thermal energy. The thermal resistor R_w represents the building walls that separate the ambient temperature T_a from the zone temperature state T_z. Finally, $\dot{Q}_h$ and $gA\dot{Q}_{Sun}$ represent the thermal power from the building heating system and the solar irradiation, respectively. From this scheme it is possible to derive the equations that define the simplified physics of the system. For a one state RC model as the one shown in the example, the only equation defining the model is shown in Eq. 8. In this way, the model can be represented using state-space matrices by carefully grouping the parameters in the elements of the matrices for the specified inputs and outputs. An alternative formulation, called inverse comprehensive room transfer functions (iCRTF), is derived from discretization of the state space formulation and creation of linear transfer functions, whose coefficients can be identified based on regression (Armstrong, Leeb, & Norford, 2006). Such an approach has been used in simulation (Blum, Xu, & Norford, 2016; Zakula, Armstrong, & Norford, 2014) and experimental (Gayeski, Armstrong, & Norford, 2012) studies.

$$C_z\frac{dT_z}{dt} = \frac{T_a - T_z}{R_w} + \dot{Q}_h + gA\dot{Q}_{Sun} \tag{8}$$

For buildings, model order reduction has proven to be able to maintain the same level of accuracy even when strong simplifications are carried out (Picard et al., 2017). This enables the use of more suitable models for optimization without any expected loss of controller model performance. It is often argued that the gray-box approach can address the limitations of both white- and black-box models. First, since some knowledge about the modeled system is already *hardcoded* in the model equations, gray-box models are more likely to stay reliable outside the calibration range than black-box models (Afroz et al., 2018), they require less data for calibration (Arendt et al., 2018a), and there is a lower risk of overfitting than in black-box models. Second, the equations used in a gray-box model can be more easily adapted to the needs of MPC solvers, e.g. by ensuring continuity, linearity or differentiability. Finally, gray-box models are found to be easily portable between similar systems. For instance, (Reynders, Diriken, & Saelens, 2014) argued that only few model types are required to represent the majority of buildings. Verhelst (2012) showed low-order models provide similar accuracy to higher order models for both building and borehole heat exchanger modeling. It was concluded that the quality of the measured data has higher impact on the accuracy of the model than the model structure itself. A direct comparison of the gray- and white-box approaches for their application in MPC can be found in Picard et al. (2016). In this case, the white-box MPC resulted in a better thermal comfort and used only half of the energy used by the gray-box MPC.

The main challenge related to gray-box modeling is the need for a robust parameter estimation method. The approaches can be divided into batch and online estimation. The batch estimation is an offline process in which model parameters are found by minimization of the model error over a specific time period. The estimation can be performed only once or the models can be periodically recalibrated based on more recent data. Typically, the batch estimation leads to a non-convex optimization problem with many local and flat optima as shown by Arendt, Jradi, Wetter, and Veje (2018b). The complexity of the objective function can also bring the parameters to the constraint boundaries. Therefore, there is a need for a global optimization strategy, either by using evolutionary methods as in Arendt et al. (2018b) or multi-start methods as in De Coninck et al. (2016). In addition, an expert involvement and cross-validation of the parameter estimation results is advised (Verhelst, 2012). The online estimation is usually based on methods related to recursive Bayesian estimation, such as sequential Monte Carlo Rouchier, Jiménez, and Castaño (2019) or Kalman filtering (Shi & O'Brien, 2019). Online parameter estimation forms the basis of indirect adaptive MPC approaches, which are covered in more detail in Section 7.4.

Finally, unlike many data-driven models which usually perform better when trained on more data, gray-box models often require special care regarding the data chosen for training. For example, parameter estimation in an RC (resistor-capacitor) thermal network may lead to an overestimated thermal mass if the training period is too long and the gray-box model cannot find a good fit for the entire period (Arendt et al., 2018a). Blum et al. (2019b) found that the optimal training period length depends on the MPC horizon, suggesting that a periodic re-callibration is necessary.

3.2.3. Black-box

Black-box models learn the dynamics of the buildings from the measured data without making any prior assumptions regarding any physical relationships. The main advantages of the black-box approach compared to gray- and white-box are that they usually lead to lower development cost and that any signal can be used as an input or output, as there are no physics involved. On the other hand, black-box models require more training data than gray-box models (Afroz et al., 2018) and are not reliable outside the training range (Afram & Janabi-Sharifi, 2014a).

Linear models The simplest and most intuitive black-box models are the parametric linear models. The forecasts of these models are linear in the observations and the uncertainty increases with the prediction horizon. The models that belong to this group are Auto Regressive (AR), Auto Regressive with eXogenous inputs (ARX), Auto Regressive with Moving average or Box-Jenkins (ARMA or BJ), Auto Regressive with Moving Average and eXogenous inputs (ARMAX) and Output Error (OE). All these models can be transformed into the general state space formulation. A common alternative for estimating the state space parameters is the Subspace-based State Space System IDentification (4SID) (Van Overschee & De Moor, 1996). With this methodology, the state sequence and its order are first calculated, and later the state space matrices are estimated just by solving a least squares problem. A comparison between subspace identification and an ARMAX model for their use within the MPC of a large building was made in Ferkl and Široký (2010). The authors concluded that subspace identification is faster, easier to implement and more accurate. This conclusion is corroborated by Prívara et al. (2011). Finally, a number of MPC relevant identification methods exist, which aim to minimize multi-step ahead prediction error in relation to the MPC optimization horizon, such as the MRI+PLS method introduced by Prívara, Cigler, Váňa, Oldewurtel, and Žáčceková (2013b).

Parametric nonlinear models The parametric nonlinear models provide a nonlinear relation between the inputs and outputs of the model and have a non-monotonous increase of their uncertainty over the prediction period. Artificial Neural Networks (ANN) (Hagan, Demuth, & Beale, 1996; Jiang & Wang, 1999; Siegelmann & Sontag, 1995) are probably the most renowned models of this type. Some building implementation examples of these models can be found in Dodier and Henze (2004); Huang, Chen, and Hu (2015); Kusiak and Xu (2012); Ruano, Crispim, Conceicao, and Lucio (2006); Tang and Wang (2001). Some researchers have shown that these models perform better and more accurately than physical models (Arendt et al., 2018a; Ruano et al., 2006) and other forms of statistical models (Mustafaraj, Lowry, & Chen, 2011). However, Huang, Chen, and Hu (2014) state that the application of ANN for model predictive control on real commercial buildings is still challenging because it has a complicated structure, which results in non-convex optimization problems that are hard to solve. The latest advances in convexification of neural network modeling may provide a remedy. The use of convex ANN in optimal control of the building HVAC system demonstrated a performance improvement compared with classical linear models (Chen, Shi, & Zhang, 2018).

Nonparametric nonlinear models The nonparametric models, like k-Nearest Neighbors (kNN), Support Vector Machines (SVM), Decision trees (DT), and Random Forest (RF), do not make strong assumptions about the model structure. Therefore, these models can learn generic function mapping between inputs and outputs. The main drawbacks of these modeling approaches are the larger data requirements and the higher risk of overfitting. Control-oriented building models based on regression trees and random forests have been presented in Jain, Behl, and Mangharam (2017a); Jain, Smarra, and Mangharam (2017b); Smarra et al. (2018).

Gaussian Processes (GP) are particularly powerful nonparametric stochastic models, which has been recently used to model building dynamics. They capture the model uncertainty directly, providing a distribution of the predictions of the model, and enable the use of prior knowledge in the system identification process. Moreover, a comparison of four data-driven methods for building energy prediction concluded that GP are accurate and highly flexible (Zhang, O'Neill, Dong, & Augenbroe, 2015). Short- and long- term building energy consumption forecasts using GP were investigated in Noh and Rajagopal (2013). More examples of GP-based models in the building modeling context can be found in Abdel-Aziz and Koutsoukos (2017); Ahn, Kim, Kim, Park, and Kim (2015); Jain, Nghiem, Morari, and Mangharam (2018); Nghiem and Jones (2017).

3.3. Practical aspects of building modeling

3.3.1. Data acquisition and processing

Special care should be taken with data sets used for training data-driven models because poor data may not capture the main dynamics of the system. The data can be obtained from a detailed model or from actual measurements. The first approach is interesting for research purposes since different types of excitation signals can be applied at no cost. The drawback is that a reliable model is required. The second approach is more suitable for real applications. However, when using real measurements, the input excitations for obtaining rich training data are limited by the technical and operational constraints of the available HVAC systems.

Design of Experiments assesses which excitations provide the most useful data. When the objective is to build a model suitable for control, the generated inputs do not need to cover the entire frequency domain, but rather some control-relevant selection. Therefore, the sampling time should be chosen based on the time constants of the building, with a typical range for building systems between 5min to 60min. In system identification of building systems, usually Pseudo Random Binary Signals (PRBS) and normal operation (business as usual) signals are used to generate the training data sets. The former is probably the most appropriate signal to provide rich data (Ljung, 1999), while the latter is used to avoid the extra costs of the experiments, as well as the risks of discomfort and the need for technical support. Case studies of building system identification using PRBS as input signals are (Bacher & Madsen, 2011; Hazyuk, Ghiaus, & Penhouet, 2012; Madsen & Holst, 1995; Prívara et al., 2011; Royer, Thil, Talbert, & Polit, 2014), while examples of cases that used normal operation are (Berthou, Stabat, Salvazet, & Marchio, 2014; Ferkl & Široký, 2010; Reynders et al., 2014; Verhelst, 2012). Although a lot of system identification studies have already used data from normal operation, this data is usually insufficiently informative to reliably estimate a model (Prívara et al., 2013). This is because during normal operation only a small part of the possible HVAC range is used. Consequently, the other operating conditions remain unexplored in the data and cannot be learned. Jain et al. (2018) proposed a method for optimal experiment design based on maximizing information gain or variance with a faster learning rate than using uniform random sampling or PRBS. This method reduced the required training period up to 50%, but was tailored for black-box Gaussian Processes.

There exist different indicators to check the quality of the obtained data. The most commonly used are the signal-to-noise ratio. This ratio is proportional to the amplitude of the response of the output to the excited input, and inversely proportional to the amplitude of the response to modeled disturbances and to measurement noise. The measurement length is also important and it should be at least larger than the largest time constant of the system. The minimum sampling time period should be defined by the Nyquist criterion, but in practice, a smaller sampling time is advised. Obviously, missing data-points should be avoided, although it is a common issue in building management systems. Filtering and re-sampling the data can not only overcome this threat, but can also help in the modeling process by smoothing the data to get rid of the measurement errors and other fast dynamics that may be blurring the main dynamics.

3.3.2. Model validation

The main purpose of the validation process is to ensure that the identified model is reliable not only within the training conditions, but also beyond. For this purpose, the data is normally divided into two sets: 1) a training set and 2) a validation or test set. The training set is used to tune the parameters of the model, while the test set simulates the trained model to check whether it captures the real behavior of the building when using different data than that used in the training.

There exist different statistical tests to validate a model. One example is the analysis of the residuals which are defined as the differences between the measurements and the outputs of the model given as $e_k = y_k - m_k$. Here, e_k, y_k and m_k are the residual, the model output and the measurement at time k, respectively. These residuals should be white-noise in the training data to ensure that all systematic dynamics are captured within the model. Any correlation in the residuals would indicate that the model can be improved further. Another option to test the performance of a model are the typical t-tests for checking the significance of the parameters, and the maximum likelihood tests for comparing the goodness of fit of two statistical models.

Many statistical indicators exist, such as the n-step ahead prediction error, the Root Mean Square Error (RMSE), the Continuous Ranked Probability Score (CRPS), the Expected Error Percentage (EEP), the Coefficient of Variation (CV), the Mean Biased Error (MBE) or the R^2

(also called coefficient of determination or fit). However, these indicators do not provide information about the control performance of the model, but instead about the simulation errors. Therefore, their interpretation has to be taken carefully. The statistical indicator choice depends on the desired highlights to put forward when analyzing the model. The RMSE, for instance, provides a symmetric and absolute score for model error over a period of time facilitating the comparison of different models. The CRPS is used for stochastic models and is defined as the mean root squared value of the difference between the cumulative distribution function of the forecast and the cumulative distribution function of the observations. The CRPS in probabilistic forecasting is the analogous key performance indicator to the RMSE in deterministic forecasting. In some cases, mainly for tuning purposes, it may be interesting to investigate the direction of the bias of the model. In such cases, metrics that indicate the direction of the bias like the MBE should be used. Alternatively, a box-plot with the n-step ahead prediction error can be used. Finally, the CV, EEP and R-squared indicators show relative values for the evaluation of the residuals.

3.4. Concluding remarks of building modeling

Modeling is one of the main bottlenecks for implementing MPC in buildings. White-, gray- and black-box modeling are three different paradigms used in practice. The choice of a particular paradigm mainly depends on the available resources and possibly on additional requirements, such as transferability between buildings and systems, high accuracy, smoothness (required by some optimization solvers), or reliability (generalization capabilities) (Fig. 6). If detailed technical documentation and physics-based modeling expertise are available, then it may be preferable to follow the white-box approach, as it leads to reliable and interpretable models with little requirements on the sensor data amount and quality (Afroz et al., 2018). On the other hand, if extensive reliable measurement data is available, the black-box approach provides models which are often more accurate and easily transferable to different buildings and systems, reducing the implementation time (Afroz et al., 2018). In industry, there is a trend towards data-driven modeling approaches as they can be more easily automated. Finally, if information about the building and HVAC design is available as well as some historical measurements, the gray-box approach may be the most convenient, as it shares many features of white- and black-box paradigms (Afroz et al., 2018). In any case, it is strongly recommended to carry out an exhaustive model validation to ensure good MPC performance.

Table 5 shows some examples of building modeling applications for optimal control that have been classified by the building system size, real implementation, modeling paradigm, the excitation input signal in the training data, the training data period, the modeling tool used to estimate the model parameters, and the model complexity regarding the number of thermal zones and the number of states in the model. BaU stands for business as usual and refers to the standard operation of the building without any additional excitation. Finally, the hyphens indicate that the attribute does not apply to that type of model or that such characteristic is not specified in the reference. Notice that a more elaborate list of modeling tools is provided in Section 8.

4. MPC algorithms and methods

After building modeling and MPC problem formulation, designing and tuning the algorithmic implementation is the next step to take on the path towards real life operation of the building. This section sumarizes key algorithmic principles and methodologies which are being used to implement and solve MPC problems in practice.

4.1. Receding horizon control

Typically, the MPC algorithms are being implemented in closed-loop using the principle of *receding horizon control* (RHC), defined by Algorithm 1. Here, the prediction horizon N keeps being shifted forward, with the controller implementing only the first step of the computed control strategy and discarding the rest, as described in Step 3. The algorithm introduces feedback into the system in the first step of Algorithm 1, where, at each time step, it corrects the deviations of the prediction from reality by updating the initial conditions of the system with measurements or estimates of the system parameters.

4.2. State estimation

Successful application of MPC relies on accurate information about the state variables to be used by the controller model for predictions. However, in most of the building control applications, measuring all state variables is not possible and state estimation algorithms need to be used as an integral part of the MPC system. By definition, the state estimator is an algorithm that provides an estimate of the internal states of a real system, from the measurements of its inputs and outputs. There are many distinct state estimation algorithms. The suitability and performance of each depends on the type of the observed system, nature of the disturbances, and availability and accuracy of the prediction model. A comprehensive review of the different state estimators in the context of process control can be found in Ali, Hoang, Hussain, and Dochain (2015).

The nature of the building's dynamics allows us to use several assumptions to simplify the selection and design of the appropriate state estimator. First, the building envelope model can be accurately described by the linear dynamics (9):

$$x_{k+1} = Ax_k + Bu_k + Ed_k + w_k, \tag{9a}$$

$$y_k = Cx_k + Du_k + v_k, \tag{9b}$$

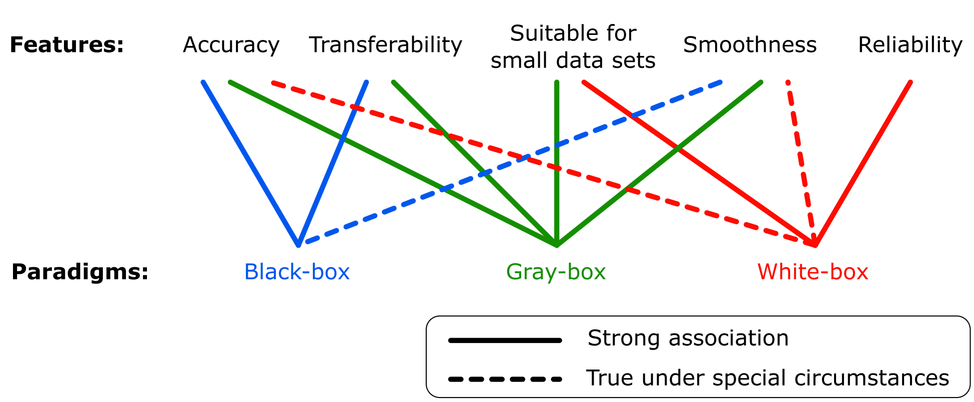


Fig. 6. Summary of the often cited features of the three modeling paradigms (based on Afram and Janabi-Sharifi (2014a); Afroz et al. (2018)).

Table 5
Sample of building modeling applications for optimal control categorized by building size, real implementation, modeling paradigm, modeling tool, input data, training period, number of zones and number of states.

Ref.	Building size [m^2]	Real impl.	Modeling paradigm	Modeling tool	Input data	Training period [days]	#Zones	#States
May-Ostendorp, Henze, Corbin, Rajagopalan, and Felsmann (2011)	1750	–	White	EnergyPlus (Crawley et al., 2001)	–	–	11	–
Corbin et al. (2012)	46,320	–	White	EnergyPlus (Crawley et al., 2001)	–	–	15	–
Drgoňa et al. (2020)	3760	–	White	Modelica Lin. (Picard et al., 2015)	–	–	12	700
Picard and Helsen (2018)	10,135	–	White	Modelica Lin. (Picard et al., 2015)	–	–	10–20	941
Jorissen and Helsen (2019)	150	–	White	Modelica (Baetens et al., 2015; Wetter et al., 2014)	–	–	9	330
Jorissen et al. (2018b)	2232	–	White	Modelica (Baetens et al., 2015; Wetter et al., 2014)	–	–	27	1262
Jorissen (2018)	10,000	–	White	Modelica (Baetens et al., 2015; Wetter et al., 2014)	–	–	32	1151
Li et al. (2015)	6982	–	White	TRNSYS (Beckman et al., 1994)	–	–	10	–
Bengea et al. (2011)	–	–	Gray	RLS MATLAB (Jiménez et al., 2008)	Monte-Carlo	2	5	15
Sourbron et al. (2013b)	24	–	Gray	TRNSYS (Beckman et al., 1994)	Step, BaU[a]	4	1	2–4
Bacher and Madsen (2011)	120	–	Gray	CTSM-R (Kristensen et al., 2004a)	PRBS	6	1	2–4
Reynders et al. (2014)	136	–	Gray	CTSM-R (Kristensen et al., 2004a)	BaU[a]	7–28	1	3–5
Madsen and Holst (1995)	60	–	Gray	CTSM-R (Kristensen et al., 2004a)	PRBS	4	1	2
De Coninck and Helsen (2016)	960	•	Gray	Modelica (Baetens et al., 2015), GB tbx (De Coninck et al., 2016)	BaU	18	1	4
Arroyo, van der Heijde, Spiessens, and Helsen (2018)	109	–	Gray	Modelica (Wetter et al., 2014), GB tbx (De Coninck et al., 2016)	BaU[a]	14	9	23
Blum and Wetter (2017)	37	–	Gray	Modelica (Wetter et al., 2014), MPCpy (Blum & Wetter, 2017)	BaU[a]	3	3	10
Blum et al. (2019b)	48	–	Gray	Modelica (Wetter et al., 2014), MPCpy (Blum & Wetter, 2017), ModestPy (Arendt et al., 2018b)	BaU[a]	1–21	1	1–4
Blum et al. (2016)	4982	–	Gray	MATLAB (The MathWorks, 2000)	Pulses[a]	5	18	72
Li et al. (2015)	6982	•	Black	MATLAB (The MathWorks, 2000)	BaU[a]	2	10	
Hilliard, Swan, Kavgic, Qin, and Lingras (2016)	27,000	•	Black	Rand forest R (Liaw & Wiener, 2001)	BaU[a]	6570	32	–
Hilliard, Swan, and Qin (2017)	10,000	•	Black	Rand forest R (Liaw & Wiener, 2001)	Pulses, BaU[a]	–	40	–
Ma, Qin, and Salsbury (2014)	–	•	Black	MATLAB (The MathWorks, 2000)	PRBS[a]	9	5	–
Royer et al. (2014)	515	–	Black	MATLAB (The MathWorks, 2000)	PRBS[a]	24	5	–
Kusiak and Xu (2012)	–	•	Black	Neural network	–	22	4	–
Mustafaraj et al. (2011)	260	–	Black	MATLAB (The MathWorks, 2000)	BaU	5	1	–
Smarra et al. (2018)	210	–	Black	Rand forest MATLAB (The MathWorks, 2000)	BaU	46	4	–

[a] Simulation model used to generate training data.

where x_k, u_k and d_k are states, inputs and disturbances at the k-th time step, respectively. The model is subject to uncertainties, where model uncertainty is represented by the process noise variable w_k and measurement uncertainty is defined by the measurement noise v_k. Second, the HVAC dynamics can be decoupled from the building envelope model. Third, the statistical properties of the measurement noise v_k can be induced from the data, and the nature of model uncertainty described by the process noise w_k can be induced from the model accuracy by verifying it with real data. Therefore, we focus only on the class of Bayesian estimators. They use the accurate mathematical model of the building and update its predictions by measurements in a feedback fashion with estimator gain L or by solving an online optimization problem. The probabilistic distributions of the process and measurement noise act as tuning factors, similarly to the weighting factors in the MPC objective function. The linear dynamics of the prediction model make linear Bayesian estimators the most straightforward choice. In particular, the Kalman filter (KF) family, for which, based on the nature of the estimator gain L, computation can come in stationary (SKF) or time-varying (TVKF) form. A more advanced optimization-based algorithm is moving horizon estimation (MHE), which is an extension of the KF framework capable of handling constraints over an arbitrary estimation horizon. When the prediction model is nonlinear, classical linear estimators do not guarantee satisfactory performance. In such a case, one can use nonlinear estimators, most notably extended (EKF) or unscented (UKF) Kalman filters.

For the complete picture, we provide here the equations for SKF as a most straightforward example. In general, a KF consists of two stages executed at every sampling instant: update and prediction. The prediction stage, represented by Eq. (10b), predicts the state at the next time step $k+1$ based on the current state and the mathematical model of the building. In the update stage represented by Eq. (10a), the measurement y_k^{m} is used to refine the predicted state estimate from the previous time step by introducing feedback into the system.

$$\widehat{x}_{k|k} = \widehat{x}_{k|k-1} + L\left(y_k^{\mathrm{m}} - \widehat{y}_{k|k-1}\right) \tag{10a}$$

$$\widehat{x}_{k+1|k} = A\widehat{x}_{k|k} + Bu_k + Ed_k \tag{10b}$$

A compact overview of selected works with a focus on state estimators applied to building control can be found in Table 6. For more technical details and performance comparison of the linear estimators using white-box building models, we refer the reader to Cupeiro, Drgoňa, Abdollahpouri, Picard, and Helsen (2018).

4.3. Optimal control solution methods

In general, optimal control problems (OCP) are traditionally solved via numerical methods which can be classified into three categories. For

1. At time t, measure, estimate, or forecast the plant's parameters ξ, i.e. states $\hat{x}(t)$, references $r(t), \ldots, r(t+(N-1)T_s)$ and disturbances $d(t), \ldots, d(t+(N-1)T_s)$.
2. Compute the optimal sequence of control inputs $U^*_{N_c}(\xi) = \{u^*_0, \ldots, u^*_{N_c}\}$ by solving the problem (1).
3. Select only the first element of the control signals sequence, i.e., $u^*(t) = u^*_0$.
4. Implement the selected control signal over a pre-defined time interval, called sampling time T_s.
5. Time advances to the next interval $t + T_s$, and the procedure is repeated from step 1 with updated parameters ξ, using values of $\hat{x}(t+T_s)$, $r(t+T_s), \ldots, r(t+NT_s)$ and $d(t+T_s), \ldots, d(t+NT_s)$.

Algorithm 1. Receding horizon control.

more details, see (Binder et al., 2001; Kelly, 2017; Rao, 2019): *Direct methods* These approaches are based on the translation of the OCP (1) to the corresponding optimization problem (OP) and solution via optimization algorithms. Their efficiency and versatility make direct methods most popular for the solution of the OCP in practice today. They are discussed into more detail in Section 4.4. *Indirect methods* These approaches are based on the calculus of variations and Pontryagin's maximum (minimum) principle. Here, the OCP (1) is reformulated as a boundary value problem and the optimal solution is obtained by maximization (minimization) of the control Hamiltonian, which is the function incorporating the stage cost and costate equations. This problem can be solved by several types of numerical methods, namely, gradient-based, multiple shooting and collocation methods. Indirect methods carry, however, several practical drawbacks: difficult formulation of the problem in a numerically suitable way, problems with handling the active constraints, the need for an accurate initial guess, and difficulties with including changes in the problem formulation, such as re-parameterization of the constraints. *Dynamic programming (DP) methods* These approaches provide a globally optimal control policy via recursive solution of the Hamilton-Jacobi-Bellman (HJB) equations as a single step optimization problem. The main disadvantage of this approach is the so-called *curse of dimensionality*, which restricts the solutions to very small state dimensions. However, this disadvantage is reduced with the concept of *approximate dynamic programming* (ADP), which is also known as *reinforcement learning* (RL) in the machine learning community. These algorithms are based on simple principles of reward and punishment to facilitate the learning of approximate control policies and/or value functions by interacting with the controlled system. Advances in RL research in recent years may provide an interesting framework for solution of the building climate control tasks (Liu & Henze, 2007).

Recently, there has been given a substantial research effort into new optimal control (OC) solution methods emerging from various fields. To give the reader a broad overview of the complexity and various possibilities in solving OCP, we refer to Fig. 7, which captures an approximate taxonomy of the classical and alternative OC solution methods relevant for the field of building control and MPC in general.

Due to the before-mentioned claims on the dominance of direct methods in today's practice, the scope of this paper will focus on direct methods only. Further, in Section 5, we define basic MPC problem classes which differ in the type of the resulting OP, and in Section 6, we define thee solution paradigms based on direct methods.

4.4. Direct methods

Direct methods are based on translation of the OCP into an OP and obtain its solution via numeric optimization methods. In general, there are two distinct strategies for the translation (Binder et al., 2001):

Sequential simulation and optimization: In every time step, the model equations (1b) are solved via numerical integration for the current control variables.

Simultaneous simulation and optimization: The model equations (1b) are represented in the OP as equality constraints that can be violated during the optimization process and need to be satisfied at the solution.

The particular methods are: *Single shooting* This method is also called *dense formulation*, or *state condensing* method. It is a sequential approach, which solves a boundary value problem by reducing it to the solution of an initial value problem. It 'shoots' the candidate trajectories in different directions until it finds the one which satisfies the boundary conditions. The OCP is rewritten into a smaller, but denser OP form, eliminating the states from the vector of optimization variables. This approach is recommended for systems with computationally cheap numerical integration, such as linear systems. The underlying principle of this strategy is illustrated in Fig. 8a. *Multiple shooting* This method is also called *sparse formulation*. It is a hybrid method because it divides the solution interval into smaller intervals, for each of which an initial value problem is being solved with additional conditions that match the solution on the whole interval. In this formulation, each input u_k and each state x_k are considered as optimization variables, forming a large, but sparse OP form. The efficiency of many advanced optimization solvers tailored to solve OCP is based on exploiting the sparsity of the problem. This approach is usually faster than single shooting for systems with nonlinear dynamics. The underlying principle of this strategy is illustrated in Fig. 8b. *Collocation* This method is a simultaneous approach, which selects a finite-dimensional space of candidate solutions and set of collocation points in the parametric domain, and chooses the solution which satisfies the given equations at these collocation points. In this formulation, the set of optimization variables consists of all inputs u_k, states x_k, and collocation points $x^c_{k,j}$, where index j corresponds to the j-th collocation point for each state x_k. Therefore, the resulting OP is even larger, but also sparser, than in multiple shooting approach. Collocation may bring improved speed and performance for systems with highly nonlinear dynamics. The underlying principle of this strategy is illustrated in Fig. 8c.

5. MPC problem classes

In this section, we recall the most notable MPC problem classes which differ in the type and structure of the corresponding optimization problem to be solved via *direct methods*.

5.1. Linear MPC

We speak about linear MPC (LMPC) when the objective function (1a) is either linear or quadratic and the prediction model (1b) is linear as given by Eq. (9). Then, the OCP (1) can be translated to a Linear Programming (LP) or Quadratic Programming (QP) problem, depending on whether the objective function is linear or quadratic. The main advantage of linear systems is that they can be integrated in a straightforward manner via dense formulation by recursive substitution of consecutive state variables. The complexity of such dense LP becomes $\mathcal{O}(N^3 n_u^3)$, with N the control horizon, and n_u the number of inputs. Usually, because of a large number of states, the condensing method is appropriate for linear building control applications. On the other hand, the computation cost of sparse LP is $\mathcal{O}(N^3(n_x + n_u)^3)$, where n_x is the number of states (Frison

Table 6
Selective summary of state estimators applied to building control.

Reference	SKF	TVKF	EKF	UKF	MHE
Picard et al. (2017)	•	–	–	–	–
Zong et al. (2017)	•	–	–	–	–
Cupeiro et al. (2018)	•	•	–	–	•
Li, O'Neill, and Braun (2013); Li et al. (2015)	–	•	–	–	–
Chandan and Alleyne (2014)	–	•	–	–	–
O'Neill, Narayanan, and Brahme (2010)	–	–	•	–	–
Fux et al. (2014)	–	–	•	–	–
Chen, Wang, and Srebric (2015)	–	–	•	–	–
Maasoumy et al. (2013, 2014)	–	–	•	•	–
Baldi, Yuan, Endel, and Holub (2016)	–	–	•	•	–
Radecki and Hencey (2012)	–	–	–	•	–
Bonvini, Sohn, Granderson, Wetter, and Piette (2014)	–	–	–	•	–
Ferhatbegović, Zucker, and Palensky (2012)	–	–	–	•	–
Fielsch, Grunert, Stursberg, and Kummert (2017)	–	–	–	•	–
Vande Cavey et al. (2014)	–	–	–	•	•

& Jorgensen, 2013). If the solver makes use of the sparsity of the problem, the complexity of the problem becomes $\mathcal{O}(N(n_x+n_u)^3)$.

Today, LMPC is a well-studied and established technology in many industries, with efficient online implementation scalable even to problems with hundreds of thousands of parameters and optimization variables (Muske & Rawlings, 1993). Due to this fact, and also because the thermal dynamics of the building envelope can be linearized with high accuracy (Picard, Jorissen, & Helsen, 2015b), LMPC is considered to be a mature technique for building climate control (Rehrl & Horn, 2011; Sourbron, Verhelst, & Helsen, 2013b).

5.2. Nonlinear MPC

Nonlinear MPC (NMPC) emerges when either the objective function (1a) or the prediction model (1b) is nonlinear. Then, the translation of the OCP (1) yields a Nonlinear Programming (NLP) problem. In general, for nonlinear dynamic equations, multiple shooting and direct collocation methods are numerically more efficient. This is due to the available solvers' capabilities of exploiting the sparsity of the corresponding NLP. However, in general, nonlinearities in building models can be decoupled from the linear dynamics and represented by Hammerstein-Wiener models. These models are composed of linear dynamical equations representing the building envelope, and nonlinear static algebraic equations representing the HVAC and effects of disturbances. In this case, single shooting is more efficient than multiple shooting and collocation due to cheaper numerical integration of linear dynamic equations.

NLPs can be efficiently solved even on larger scales by using algorithms such as sequential quadratic programming algorithms (SQP) (Gill, Murray, & Saunders, 2005b), or newton-based methods (Wächter & Biegler, 2006). A more detailed discussion about solutions for NMPC can be found in Binder et al. (2001). NMPC has a large potential in the building sector due to more accurate predictions of nonlinear models (HVAC models in particular) and higher flexibility in the formulation of the OCP (1). Several studies and real applications of NMPC for buildings have already been reported (Castilla et al., 2014; Jorissen, Picard, Cupeiro Figueroa, Boydens, & Helsen, 2018b; Santos, Zong, Sousa, Mendonca, & Thavlov, 2016; Touretzky & Baldea, 2014), and we can expect more of them to come in the years to come.

5.3. Hybrid MPC

When the dynamical model of the system (1b) employs switching dynamics, binary or integer control variables, logic states or constraints, then we speak about hybrid MPC (HMPC). If the hybrid dynamic model is piecewise linear

$$x_{k+1} = A_i x_k + B_i u_k + E_i d_k, \quad \text{if} \quad (x_k, u_k, d_k) \in \mathcal{R}_i, \tag{11}$$

the corresponding optimization problem to be solved is either a Mixed-Integer Linear Programming (MILP) or Mixed-Integer Quadratic Programming (MIQP) problem, depending on the objective function being linear or quadratic. On the other hand, when the hybrid dynamical model incorporates nonlinearities, we end up with an extremely difficult Mixed-Integer Nonlinear Programming (MINLP) problem.

There exist three main frameworks for modeling of HMPC:

Mixed logical dynamical (MLD) systems: This framework incorporates both continuous and binary variables by means of mixed-integer linear equalities and inequalities and auxiliary binary variables (Bemporad & Morari, 1999a).

Big-M approach: This approach translates the hybrid model into a set of if-then-else conditions which are subsequently translated into corresponding mixed-integer equalities and inequalities by using auxiliary binary variables and large positive values of the constant parameters (Williams, 1993).

Generalized Disjunctive Programming (GDP): This method represents discrete decisions in the continuous space via logical disjunctions and uses logical propositions to denote algebraic constraints in the discrete space (Castro & Grossmann, 2012; Grossmann & Ruiz, 2012). Compared to traditional MIP, the inherent logic structure in GDP yields tighter relaxations that are exploited by the global branch and bound algorithms to improve solution quality (Bhattacharya, Ma, & Vrabie, 2020).

In general, Mixed-Integer Programming (MIP) problems are NP-complete problems and thus are hard to solve. However, there are several state-of-the-art optimization solvers capable of solving these problems even on larger scales (Bemporad, 2006). From a practical point of view, HMPC based on MIP optimization is a powerful tool for control of buildings employing discrete decision variables (e.g., shadings positions, on-off valves, etc.) (Le, Bourdais, & Gueguen, 2014), switching dynamics (e.g., operating modes of the heat pump) (Mayer, Killian, & Kozek, 2015), or for the formulation of supervisory HMPC optimizing

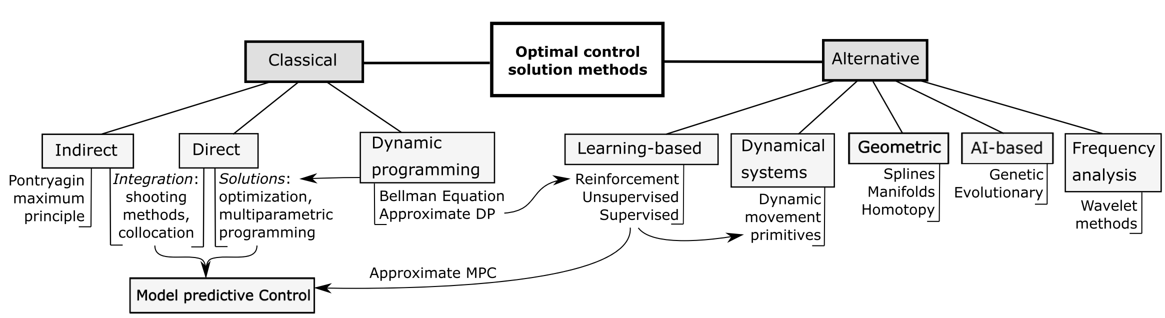
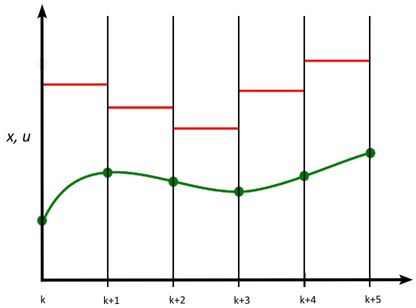
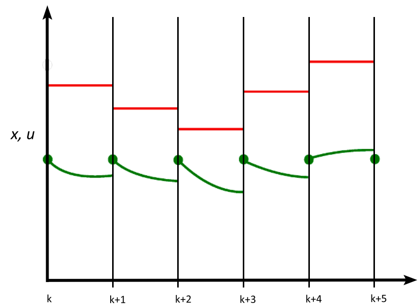


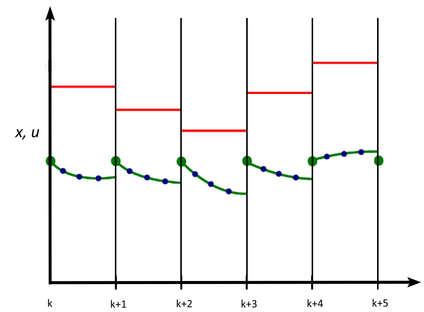
Fig. 7. Approximate taxonomy of optimal control solution methods.



(a) Single shooting.



(b) Multiple shooting.



(c) Collocation.

Fig. 8. Visual comparison of discretization principles behind different translation methods. Actions u_k (red) are discretized at each sampling interval to control the state trajectories x_k (green). Green dots represent the values of state variables, or their initial guess in the case of multiple shooting and collocation, while blue dots correspond to collocation points. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the performance of relay-based thermostats (Drgoňa, Klaučo, & Kvasnica, 2015). The first use of GDP in the context of building control and its comparison with classical MIP method was reported in (Bhattacharya et al., 2020).

6. MPC problem solutions

In this section, we recall three distinct solution paradigms based on *direct methods* which can be used to obtain solutions to the MPC problems described in the previous section.

6.1. Implicit MPC

In the case of implicit MPC, the optimal control sequence U_N^* for a particular choice of parameters ξ is obtained by solving online the corresponding optimization problem (1). Computational complexity of obtaining such a sequence depends on the type of the prediction model (1b) and the choice of the cost function (1a), as discussed in the previous section. Depending on the problem type and solver used, the solution of such OP usually requires a relatively powerful computing platform, and in practice it is performed most often via desktop or industrial computers. However, recent advances in dedicated solvers for fast MPC allow us to implement the online MPC algorithms also on embedded hardware with limited computing power and memory storage (Wang & Boyd, 2010). An overview of the most notable optimization solvers for each class of problems is provided in Section 8.3.

As mentioned in Section 2.2, buildings are inherently slow dynamic systems, which allow sufficiently large time windows for the solution of a large-scale OP. Such problems are emerging from MPC formulations with long prediction horizons and a larger number of parameters, which are typical for building control applications. Hence, there is no surprise that most of the building MPC applications reported in a survey (Afram & Janabi-Sharifi, 2014b) have been implemented in an online fashion via implicit MPC.

6.2. Explicit MPC

The development of explicit MPC was driven by the motivation to overcome the primary drawback of implicit MPC, which is the need to compute the optimal control law online at every sampling instant by solving the corresponding OP. Instead, explicit MPC employs *parametric programming* (Bemporad, M., Dua, & Pistikopoulos, 2002; Borrelli, 2003) to pre-calculate the optimal control law for *all* admissible values of parameters ξ. Hence, the explicit representation of the optimizer is constructed offline as a function of the MPC parameters given as $u = f_{\text{MPC}}(\xi)$. Then, the online identification of the optimal control action boils down to mere function evaluations for particular measurements. This significantly reduces computational requirements of the implementation. From a mathematical point of view, the problems to be solved in the case of linear MPC are multi-parametric linear programs (mpLP) or multi-parametric quadratic programs (mpQP), respectively.

The fundamental limitation of explicit MPC solution is, however, that the complexity of the computed explicit control law grows exponentially with the dimensionality of the parametric space imposed by the number of constraints of the problem, which grow with higher prediction horizon and number of parameters. Therefore, it can only be used for small-scale systems with up to 10 parameters (Mayne, 2014). Also, the memory storage capacity of the hardware should be large enough to accommodate the pre-computed explicit control law (Bemporad, 2006). One possible remedy to overcome this large memory footprint drawback is to employ the recently introduced approach of so-called *region-free explicit MPC* (Kvasnica, Takács, Holaza, & Cairano, 2015b). The solution complexity of this approach no longer depends on the number of parameters ξ, but rather on the number of optimization variables u, for instance up to 20. This still limits the problem complexity mainly with respect to the prediction horizon length and number of decision variables. In both cases, these restrictions are usually not a realistic assumption for complex building control problems with several thousands of parameters and hundreds of optimization variables. Thus, only a few applications of explicit MPC with simplified building models have been reported (Drgoňa et al., 2013; Parisio et al., 2014).

6.3. Approximate MPC

The idea behind this solution approach is to train machine learning (ML) models such that they mimic the behavior of MPC. This concept is also known as imitation learning, where MPC is acting as a teacher and generates the training data for an ML model. The training data are generated in closed-loop simulations by implicit MPC, as defined in Section 6.1. Then, the ML model represents an approximation of the MPC control law, also called control policy.

The parametric solution of problem (1) represents a mapping of the parametric space to the space of control variables, i.e. $f_{\text{MPC}}: \mathbb{R}^{n_\xi} \to \mathbb{R}^{n_u}$. For this task, state-of-the-art supervised learning algorithms can be used to approximate MPC policies with an arbitrary type of cost functions and constraints. Regression algorithms can be used for problems with continuous control variables, while classification algorithms can be used for problems with discrete control variables. Consider a set of m training data[1] $\{(\xi^{(1)}, u^{(1)}), \ldots, (\xi^{(m)}, u^{(m)})\}$, with $\xi^{(i)} \in \mathbb{R}^{n_\xi}$ and $u^{(i)} \in \mathbb{R}^{n_u}$ generated by an implicit MPC approach acting as an expert teacher for an ML algorithm. The objective is to devise a regression/classification function $f_\Theta: \mathbb{R}^{n_\xi} \to \mathbb{R}^{n_u}$, which predicts the values of control variables u (often called the *response* or *target variable*) that correspond to the parameters ξ

[1] Here, $\xi^{(i)} \in \mathbb{R}^{n_\xi}$ denotes the ith sample of a vector ξ.

(representing the *feature vector*) as accurately as possible. During online evaluation, the implicit MPC described in Section 6.1 is replaced by an approximate control policy $u = f_{\Theta}(\xi)$.

The advantage over implicit MPC is that the solution of the optimization is replaced by a computationally cheap function evaluation similar to the case of explicit MPC. The main advantage over the explicit MPC approach, however, is that the ML approach is not limited to lower-dimensional parametric spaces, which allows for construction of the approximated explicit control laws with a low memory footprint for large-scale problems with many parameters. The drawback of the ML approach is that the control policy is suboptimal with respect to the solution of the MPC problem (1), and that a larger amount of informative training data is needed to learn well-performing control policies. Additionally, in a standard imitation learning setup, the learned controller does not provide any guarantees on stability and constraints handling.

Generic approaches dealing with imitation learning of MPC control laws have been recently proposed in Chen, Wang, Atanasov, Kumar, and Morari (2019b); Hertneck, Köhler, Trimpe, and Allgöwer (2018); Lucia, Navarro, Karg, Sarnago, and Lucía (2018); Maddalena, Moraes, Waltrich, and Jones (2019); Zhang, Bujarbaruah, and Borrelli (2019). One of the first attempts to generate MPC laws for building control problems in the form of look-up tables was introduced by Coffey (2013). Other researchers used classification algorithms for extracting decision rules from hybrid MPC closed-loop behavior (Domahidi et al., 2014; Le, Bourdais, & Guéguen, 2014b; May-Ostendorp, Henze, Corbin, Rajagopalan, & Felsmann, 2011b). Approaches for the more challenging task of approximating the continuous control laws are also available. For example, they are based on a piecewise linear mixing architecture (Baldi, Michailidis, Ravanis, & Kosmatopoulos, 2015), regression trees with piecewise linear approximations (Klaučo, Drgoňa, Kvasnica, & Di Cairano, 2014), nonlinear regression (Žáčeková et al., 2015), or deep learning models (Drgoňa et al., 2018).

7. Dealing with uncertainties in MPC

The real-world implementation of the model-based control strategies suffers from the plant-model mismatch and inaccurate or corrupted measurements. This section aims to present an overview of methods used to mitigage the effect of uncertainties on the performance and safety indicators of MPC, such as constraints handling and stability guarantees. In general, we face two classes of uncertainties modeled by following parameters:

Parametric uncertainty: $q \in \mathbb{R}^{n_q}$ originates directly in neglected dynamics of the plant, so-called plant-model mismatch.
Non-parametric uncertainty: also called additive is caused by external disturbances, in particular measurement noise $v_k \in \mathbb{R}^{n_y}$, and process noise $w_k \in \mathbb{R}^{n_x}$.

Lets consider following uncertain linear system:

$$x_{k+1} = A(q)x_k + B(q)u_k + E(q)d_k + w_k, \tag{12a}$$

$$y_k = C(q)x_k + D(q)u_k + v_k. \tag{12b}$$

From the building perspective, the most common parametric uncertainties arise from the modeling errors caused by unknown parameters, inaccurate equations, or components not working according to specifications. Most common non-parametric uncertainties are associated with measurements and predictions of ambient temperature, solar irradiation, temperature sensors inaccuracy, or by a limited number of sensors, and unmeasured disturbances, such as windows opening. In principle, implementation of MPC in RHC approach implicitly reduces the plant-model mismatch due to the presence of feedback. However, for higher degrees of uncertainties, it is often not sufficient by itself and more advanced techniques need to be adopted to ensure the desired control performance.

7.1. Offset-free MPC

The purpose of this popular technique is to compensate the effect of uncertainties via prediction model augmentation by extra states p representing unmeasured disturbances (Muske & Badgwell, 2002). These disturbances p are estimated by Kalman Filters or moving horizon estimation (MHE), and their effect is subsequently compensated by the MPC via predictions. One extra state with a constant dynamic is added per each output or state of the prediction model (Pannocchia & Rawlings, 2003). This approach is also called active disturbance rejection control and allows us to consider a simpler controller model, because the modeling error is compensated for in real time (Picard et al., 2017). For a linear system, the disturbance augmented prediction model represented by matrices $\tilde{A}, \tilde{B}, \tilde{E}, \tilde{C}, \tilde{D}$ is given as:

$$\underbrace{\begin{bmatrix} \widehat{x}_{k+1} \\ \widehat{p}_{k+1} \end{bmatrix}}_{\tilde{x}_{k+1}} = \underbrace{\begin{bmatrix} A\mathbf{0} \\ \mathbf{0}I \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} \widehat{x}_k \\ \widehat{p}_k \end{bmatrix}}_{\tilde{x}_k} + \underbrace{\begin{bmatrix} B \\ \mathbf{0} \end{bmatrix}}_{\tilde{B}} u_k + \underbrace{\begin{bmatrix} E \\ \mathbf{0} \end{bmatrix}}_{\tilde{E}} d_k, \tag{13a}$$

$$\widehat{y}_k = \underbrace{\begin{bmatrix} CF \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \widehat{x}_k \\ \widehat{p}_k \end{bmatrix} + \underbrace{\begin{bmatrix} D \\ \mathbf{0} \end{bmatrix}}_{\tilde{D}} u_k. \tag{13b}$$

where the output disturbance matrix F was chosen as a full column rank identity matrix and all other matrices are the same as in Eq. 9.

Variants of OSF-MPC A linear offset-free MPC (OSF-MPC) for reference tracking formulation was studied in Maeder, Borrelli, and Morari (2009). A comprehensive overview of OSF-MPC for the linear and nonlinear discrete-time system together with economic MPC formulation was presented in Pannocchia, Gabiccini, and Artoni (2015). A disturbance modeling and estimator design were systematically studied for different formulations of state-space process models in Tatjewski (2011). The design and tuning of OSF-MPC based on the black-box ARX model was discussed in Huusom, Poulsen, Jørgensen, and Jørgensen (2010). Authors in Huang, Biegler, and Patwardhan (2010b) presented an approach for reduction of the computational burden associated with the online computation of nonlinear OSF-MPC with MHE.

OSF-MPC for Buildings In the context of building control, the OSF-MPC formulation for a white-box heat pump model developed in Modelica was given (Wallace, Mhaskar, House, & Salsbury, 2014). A multi-zone heat pump model developed in Modelica was augmented with a disturbance offset of the measured outputs for the design of centralized linear OSF-MPC (Krupa et al., 2019). A simulation study of an OSF-MPC for energy-efficient operation of the hotel's central chiller plant in a tropical climate was presented in Lara, Molina, Yanes, and Borroto (2016). Systematic analysis with varying order of the building envelope model for three variations of the residential houses showed that state augmentation can reduce the modeling errors and improve the overall control performance in terms of energy use and comfort constraints satisfaction (Picard et al., 2017).

7.2. Robust MPC

In case the impact of uncertainties significantly decreases the control performance, or even endangers the closed-loop system stability, we introduce the *robust* MPC policy, see (Bemporad & Morari, 1999b) and references therein. Robust MPC strategy is also suitable if we need to certify the designed MPC w.r.t. the impact of the bounded uncertainties. As the values of uncertain parameters vary, there are various scenarios of the future behavior of the plant. Therefore, the crucial task of robust MPC is to design a control law that guarantees the closed-loop system stability of the plant subject to all the admissible evolution scenarios of the uncertain system. As a consequence, the robust MPC strategy is usually *conservative*. This means that the robust control policy ensures the constraints satisfaction by creating an energy buffer (in the case of energy minimization) to be able to mitigate the impact of some unexpected disturbances. More generally, the robust MPC creates reserves for potentially difficult times in the future, the quantity of which is determined based on the estimates of the worst-case scenario and robust control design method.

Complexity of RMPC The robust MPC assumes the impact of the bounded disturbance. Consider a linear state-space system in (12) affected by bounded uncertainty $q \in \mathcal{Q}^{n_q}$, where $\mathcal{Q}^{n_q} \subset \mathbb{R}^{n_q}$ is the n_q-dimensional set of uncertain parameters. Consider constraints given by (1f), where $\mathcal{U}$, $\mathcal{X}$ are polytopes including origin in their strict interior. Then, the closed-loop system is robustly stable if and only if all the vertices of the constraints parametrized by uncertainty q are simultaneously stable. In other words, although the uncertainty set $\mathcal{Q}$ includes an infinite number of a possible realization of q, the system is stable within all constraints on the feasible region under bounded uncertainty q by checking 2^{n_q} the system vertices. Total number of uncertain system vertices 2^{n_q} originates in the enumeration of hyper-box vertices defined in n_q- dimensional space, e.g., see (Kothare, Balakrishnan, & Morari, 1996). The main drawback is that the number of investigated vertices increases exponentially with a prediction horizon N, i.e., $2^{n_q} \times N$. The complexity is high because the controller evaluates all scenarios w.r.t. all combinations of uncertain parameters. Therefore, the dominant term of the complexity evaluation is determined by the number of uncertain parameters n_q.

Min-max RMPC In general, robust optimal values of the manipulated variables are computed either (i) directly as a *sequence* of the control actions, or (ii) by designing the linear/affine state-feedback *control laws*, see (Langson, Chryssochoos, Raković, & Mayne, 2004). Various approaches are considered to keep the optimization problem tractable, mostly considering the *worst-case*, i.e., so-called MIN-MAX optimization (Campo & Morari, 1987). In this approach, only the worst-case scenario is evaluated and used for the robust controller design.

LMI-based RMPC Another popular approach in tackling the exponential complexity is based on linear matrix inequalities (LMIs), see (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). The advantage of LMIs lies in the possibility of transforming non-convex optimization problem into the convex form. The original problem could also have infinity many decision variables, but introducing LMIs enables to transform it into a tractable optimization problem with modest complexity. The idea of LMIs is to optimize the control performance by minimizing the eigenvalues of the matrices. For instance, the aim is to optimize the trajectories of the controlled variables, e.g., zone temperatures, subject to the influence of uncertain parameters. The solution of the optimization problem shapes the set of admissible values of the controlled variables, i.e., defines their limit values. In the case of LMIs, the resulting optimal set has the shape of *ellipsoid* ε and contains the setpoint values in its center. The volume of this set is minimized in each control step to keep the controlled variables closer to the setpoint value. This strategy is pioneered by Kothare et al. (1996) and refined by many later works, see Oravec, Pakšiová, Bakošová, and Fikar (2017); Zhang, Wang, and Wang (2014) and references therein.

SDP in RMPC From a technical point of view, the problem is transformed into the form of *semidefinite* programming (SDP) (Vandenberghe & Boyd, 1996) that has a convex (usually linear) objective function and the constraints have the form of LMIs. For the class of SDP problems, various tailored solvers are available, for instance, SeDuMi, MOSEK, to list some, while a more extensive list is provided in Table 13. The online computational complexity of SDPs can be further reduced by replacing them by QPs w.r.t. the construction of the maximal robust positive *invariant* sets (Blanchini, 1999), forward and backward *reachable* sets (Borrelli, Bemporad, & Morari, 2017). Once the system state enters the invariant set, it is trapped inside this set also in the future. As a consequence, the states will not diverge into infinity/instability. For instance, life sentence in a prison is an invariant set. Analogously, the reachable sets limit the future behavior of the states. In control theory, the reachability for a dynamical system means that a certain state is reachable from a given initial state within a given cost threshold (Allen, Clark, Starek, & Pavone, 2014). We can think of it as a formal reality check, answering questions of a type: "Can we reach the thermal comfort zone from a given room temperature within an hour by using a given amount of energy?". Therefore, these properties are crucial tools to guarantee/certify the closed-loop system stability and performance.

Explicit and tube-based RMPC The explicit solution of the robust MPC problem was proposed in Kvasnica, Takács, Holaza, and Ingole (2015). However, from a computational viewpoint, it is limited by the modest complexity of the optimization problem, i.e., a number of constraints. So-called *tube*-based robust MPC addresses the problem of the conservatives minimization of robust MPC policy by reducing the exponential evolution of the predicted states, see pioneer work (Langson et al., 2004), or more recent papers (Yadbantung & Bumroongsri, 2018; Zeilinger, Raimondo, Domahidi, Morari, & Jones, 2014), and references therein. The "tube" refers to the shape of the bounded set of admissible evolutions of the controlled variable.

RMPC for buildings The detail analysis of the sources of the uncertain parameters and the origins of the imperfect models in building energy assessment is provided in Tian et al. (2018). From building control perspective, the robust MPC based on offline precomputed LMIs for temperature control of variable-air-volume air-handling units was designed in Huang, Wang, and Xu (2010a), Xu, Wang, and Huang (2010). Simulation results show robust control performance and constraints satisfaction. A robust MPC framework based on the input disturbance feedback for building HVAC systems was proposed in Maasoumy, Razmara, Shahbakhti, and Vincentelli (2014); Maasoumy and Sangiovanni-Vincentelli (2012). A novel robust adaptive MPC strategy reducing the conservativeness of the uncertainty handling was presented in Tanaskovic, Sturzenegger, Smith, and Morari (2017). The simulation results show improved control performance in contrast to non-robust adaptive MPC. in Antonov and Helsen (2016), robustness analysis of the designed MPC was performed. Satisfaction of the robustness conditions subject to the uncertain prediction of the system states was investigated *a posteriori* to prevent evaluation of computationally demanding robust MPC design procedure. The classification of various Robust MPC approaches to building control is given in Table 7.

7.3. Stochastic MPC

Stochastic MPC (SMPC) is a framework for systems affected by probabilistic uncertainty. A key feature of SMPC are chance constraints (CC), which enable a systematic trade-off between control performance and probability of constraints violations (Heirung, Paulson, OĹeary, & Mesbah, 2018). Chance constraints, for example on state variables, are given in the form:

$$\Pr(x_k \in \mathcal{X}) \geq 1 - \alpha, \qquad k \in \mathbb{N}_0^{N-1} \tag{14}$$

where $\Pr(x_k \in \mathcal{X})$ denotes the probability of satisfaction of the constraint $x_k \in \mathcal{X}$, and $1 - \alpha$ specifies the value of that probability for $\alpha \in [0, 1]$. Unfortunately, these types of constraints are in general non-convex and extremely computationally demanding for optimization. Hence, for any practical implementation of SMPC, a computationally tractable reformulation of CC needs to be derived. For this task, there are several approaches which are based on solving convex realizations of chance constrained optimization problems.

An overview of linear SMPC with CC classifying alternative approaches in terms of the system model, the objective function, the meaning and management of the chance constraints, and their feasibility and convergence properties was given in Farina, Giulioni, and Scattolini (2016a). The connection to stochastic dynamic programming as well as Bayesian estimation of SMPC problem in the dual control paradigm was reviewed (Mesbah, 2018). Authors in Lorenzen, Müller, and Allgöwer (2017d) provide assumptions that are sufficient to establish closed-loop stability for various approximations of CC used in SMPC methods. For the purposes of this paper, we classify the alternative SMPC methods into three principal groups based on Mesbah (2016), namely scenario-based approaches, chance constraints approximations, and disturbance feedback control law parametrizations. The conceptual difference of the latter two approaches compared to scenario approaches is that no samples need to be generated. Instead, some prior knowledge of the system or the past realization of the uncertainties is exploited to derive the accurate approximations of chance constraints.

Scenario-based approaches Sampling-based techniques replace the CC with a finite number of deterministic constraints generated by the various realizations of the stochastic variables. Sampling density is chosen as a trade-off between computational demands and violation probability. A larger number of samples decreases the violations but usually leads to increased computational burden (Zhang, Schildbach, Sturzenegger, & Morari, 2013). Another concern of SMPC is safety in terms of closed-loop stability and constraint handling capabilities. Stochastic stability and recursive feasibility can be enforced through linear matrix inequality (LMI) for linear problems (Bernardini & Bemporad, 2009). An alternative approach uses an offline sampling of probabilistic constraints realizations to guarantee recursive feasibility and asymptotic stability of linear SMPC (Lorenzen, Allgöwer, Dabbene, & Tempo, 2015). Additionally, it has been shown that bounds on closed-loop constraint violations can be provided for linear SMPC formulations (Schildbach, Fagiano, Frei, & Morari, 2014). Modern approaches involve machine learning methods in the design of SMPC, for instance, using Gaussian processes GP (Bradford, Imsland, Zhang, & del Rio Chanona, 2019), or Support Vector Clustering (SVC) for learning an uncertainty set directly from available data (Shang & You, 2019).

Chance constraints approximations Sometimes also referred to as stochastic tube approaches, these approximations are based on replacing CC with deterministic constraints by tightly bounding the disturbances. A convexity of chance-constrained SMPC for linear systems was studied in Cinquemani, Agarwal, Chatterjee, and Lygeros (2011). An extension of CC-based SMPC to nonlinear dynamics was presented in Xie, Li, and Wozny (2007). Of the latest approaches, CC defined as a discounted sum of violation probabilities on an infinite horizon guarantees the recursive feasibility without an assumption of boundedness of the disturbance (Yan, Goulart, & Cannon, 2018). Authors in Lorenzen, Dabbene, Tempo, and Allgöwer (2017c) propose a constraint tightening to non-conservatively guarantee recursive feasibility and stability of CC-based SMPC.

Control law parametrizations Set of techniques directly mapping the influence of the disturbances onto control actions, for instance by expressing the feedback control law as an affine function of the past disturbances. Authors in Oldewurtel, Jones, and Morari (2008) presented a tractable approximation of CC based on affine disturbance feedback for linear systems. An alternative approach with affine parametrization of the control law capable of handling possibly unbounded stochastic disturbances via solving convex second-order cone program (SOCP) was given in Paulson, Buehler, Braatz, and Mesbah (2017).

SMPC for buildings Table 8 summarizes numerous applications of SMPC in the building control context and classifies them based on the principal method used. Please note that the domain of SMPC is far more complex and used methods are more numerous and branched as those presented here. For more detailed overviews and fundamentals on SMPC we refer the interested reader to the following publications (Farina, Giulioni, & Scattolini, 2016b; Heirung et al., 2018; Mayne, 2016; Mesbah, 2016).

7.4. Adaptive MPC

The essential idea of adaptive control is online update of the controller or the prediction model parameters, such that the systems with time-varying dynamics can be handled using the adaptive strategy, see (Åström & Wittenmark, 2008) and references therein. On the other hand, standard receding-horizon MPC addresses real-time computation of the optimal control actions subject to the fixed structure and parameters of the system model. The control law itself is static, but the control actions are parametrized by system states, references, and disturbances. *Adaptive MPC* merges the benefits of both, i.e., introduces the model updates in the context of MPC. The uncertainties are then corrected not only via feedback of the control law parameters, but also with updates of the model parameters. The parameters updates are typically obtained from autoregressive models, recursive least squares (RLS), Kalman Filters, or other maximum likelihood parameter estimation algorithms. Adaptive model updates allow the MPC to potentially cope with time-varying disturbances and correct plant-model mismatch over longer prediction horizon, as opposed to static disturbance correction terms of the offset-free MPC.

Challenges and approaches Except for the closed-loop system stability and recursive feasibility, the crucial challenges lie in (i) handling MIMO systems (Maniar, Shah, Fisher, & Muthas, 1997); (ii) design control action subject to constraints (Tanaskovic, F., Smith, & Morari, 2014); and (iii) considering the impact of the uncertain parameters (Lorenzen, Allgöwer, & Cannon, 2017; Tanaskovic et al., 2017). A compact overview of adaptive MPC challenges was given in Kim (2010). As pointed out in Qin and Badgwell (2003), only a few adaptive MPC algorithms have been used in practice, despite the strong market incentive for a self-tuning MPC controller. Moreover, due to the above-mentioned challenges, this status quo is likely to be maintained in the near future.

Adaptive MPC remains an active area of research, and it is out of the scope of this paper to provide a complete survey and classification of different approaches. Instead, we mention only a few occurring themes. For increased robustness, an adaptive MPC is often combined with stochastic and robust MPC principles such as set membership identification (Adetola, Guay, 2011; DeHaan, Adetola, & Guay, 2007; Fagiano, Schildbach, Tanaskovic, & Morari, 2015; Lorenzen, Allgöwer, & Cannon, 2017b). An adaptive strategy based on multiple linear models was introduced in Dougherty and Cooper (2003). A novel approach of *dual adaptive MPC* reformulates the original nonlinear deterministic problem into the tractable problem of convex optimization (Heirung, Ydstie, & Foss, 2017; Kumar, Heirung, Patwardhan, & Foss, 2015). The literature on simultaneous state and parameter estimation is complimentarily focused on aspects such as estimation error, and signal excitation (Kamalapurkar, 2017; Rangegowda, Valluru, Patwardhan, & Mukhopadhyay, 2018). In recent years, the principles of adaptive MPC are being revised and combined with various machine learning-based methods and are often labeled as learning-based MPC, which is covered separately in the following section.

Adaptive MPC for buildings Adaptive MPC of the HVAC system based on self-adapting building models was investigated in Herzog, Atabay, Jungwirth, and Mikulovic (2013) using simulation. The self-adaptive model for buildings enabling correction of the prediction errors of pre-defined models using a dynamic Kalman filter-bank was proposed in Killian, Leitner, Goldgruber, and Kozek (2017). Robust adaptive MPC for building climate control was proposed in Tanaskovic et al. (2017), where the uncertainty set was recursively updated based on the system identification procedure. Authors in Lauro, Longobardi, and Panzieri (2014) studied an adaptive distributed MPC scheme for multi-zone building temperature control and its comparison with a decentralized approach. Adaptive MPC based on multiple linear regression for the control of a low-temperature thermo-active building system was designed in Schmelas, Feldmann, and Bollin (2017). A self-adaptive MPC based on EKF improved the model prediction accuracy for a passive house (Fux, Ashouri, Benz, & Guzzella, 2014). An adaptive MPC mechanism proposing recursive estimation and updating approach for electronic expansion valves with adjustable setpoint for evaporator superheat minimization was addressed in Tesfay, Alsaleem, Arunasalam,

Table 7
Classification of the publications reporting Robust MPC for building control.

Reference	Robust constraints satisfaction	Min-Max approach	LMI-based approach	Offline optimization	Control law parametrization
Huang et al. (2010a); Xu et al. (2010)	•	•	•	•	- –
Tanaskovic et al. (2017)	•	–	–	–	- –
Ma et al. (2012b); Ma, Borrelli, Hencey, Packard, and Bortoff (2009)	•	–	–	–	- –
Maasoumy et al. (2014); Maasoumy and Sangiovanni-Vincentelli (2012)	•	•	–	–	•
Yang, Wan, Chen, Ng, and Zhai (2019)	•	•	–	–	•
L. Chen and Hu (2016)	•	•	–	–	- –
Antonov and Helsen (2016)	–	–	•	–	- –

and Rao (2018). An online simultaneous state and parameter estimation for building predictive control using extended and unscented Kalman Filters have been proposed in Maasoumy, Moridian, Razmara, Shahbakhti, and Sangiovanni-Vincentelli (2013); Maasoumy et al. (2014).

7.5. Learning-based MPC

In recent years the intersection of the areas of control and learning has been rapidly expanding with the emerging concept of learning-based MPC (LBMPC). However, due to the ubiquitous use, the label LBMPC has an ambiguous meaning. Moreover, LBMPC is an active area of research with rapidly emerging new concepts and applications. The most recent review (Hewing, Wabersich, Menner, & Zeilinger, 2020) classifies LBMPC approaches into three categories, (i) learning of the prediction model from data with uncertainty quantification, (ii) learning the controller design, i.e., learning the constraints and cost function terms, (iii) MPC for safe learning to derive safety guarantees for learning-based controllers. The comprehensive overview of the method is beyond the scope of this paper. Instead, we refer the interested reader to Hewing et al. (2020) and references therein.

Uncertainty-aware LBMPC The first category of LBMPC approaches is the most numerous. The case of learning a static model with uncertainty quantification is directly related to some of the gray- and black-box modeling approaches, discussed in Section 3.2.3. The concept of LBMPC in the context of robust and safe control with data-driven models and online updates was first introduced by (Aswani, Gonzalez, Sastry, & Tomlin, 2013). The main insight of LBMPC is that performance and safety can be decoupled by using reachability analysis (Asarin, Bournez, Dang, & Maler, 2000; Rakovic, Kerrigan, Mayne, & Lygeros, 2006), making the approach tractable and practical. In general, LBMPC is considered to be a generalization of robust adaptive MPC, which is typically restricted to specific types of model structures and learning algorithms. Instead, LBMPC uses statistical learning methods to improve the model of the system dynamics, while using robust MPC techniques to ensure stability and constraints handling (Aswani, Bouffard, Zhang, & Tomlin, 2014). Alternative methods in this category, include, formulation of robust MPC with state-dependent uncertainty for data-driven linear models (Soloperto, Müller, Trimpe, & Allgöwer, 2018), or an iterative model updates for linear systems with bounded additive uncertainty and robust guarantees on all feasible offsets (Bujarbaruah, Zhang, Rosolia, & Borrelli, 2018).

Learning-based controller design and updates Approaches falling in the second category are represented, e.g., by control methods updating time-varying dynamics, constraints, and stage cost based on closed-loop data for period tasks (Scianca, Rosolia, & Borrelli, 2019). An inverse optimization is a more challenging task dealing with an inference of unknown parameters of an optimization problem based on knowledge of its optimal solutions (Aswani, Shen, & Siddiq, 2015). In this context, pivotal research without performance guarantees on learning of the MPC parameters from available closed-loop data was recently proposed by differentiable MPC (Amos, Rodriguez, Sacks, Boots, & Kolter, 2018). It is important to mention that inverse optimal control approaches are closely linked with imitation learning and approximate MPC solutions discussed in Section 6.3. The difference is that approximate MPC deals with parameterizing an explicit control law based on given samples of closed-loop behavior of the expert controller, as opposed to finding parameters of a given MPC formulation matching the data.

MPC safety certificates for learning-based control The methods in the third category represent new research avenues and are primarily concerned with employing robust or stochastic MPC in conjunction with data-driven controllers for safety certificates (Muntwiler, Wabersich, Carron, & Zeilinger, 2019) or safe exploration (Koller, Berkenkamp, Turchetta, & Krause, 2018), aspects particularly important for reinforcement learning (RL) approaches.

LBMPC for buildings One of the first experimental results of LBMPC applied to the office building in the US with significant energy savings was reported in Aswani et al. (2012), where learning refers to model updates of the gray-box hybrid system model. In the building control literature there is a multitude of learning-based, data-driven, data-enabled, or data predictive approaches representing an ambiguous set of methods, which primary concern is learning of the prediction model. Those methods are often not dealing with uncertainty quantification in the sense of original LBMPC (Aswani et al., 2013). Hence some of them may not provide robust performance guarantees or uncertainty

Table 8
Classification of the publications reporting SMPC for building control based on their principal methods.

Reference	Offline optimization	Scenario-based approach	Chance constraints approximation	Control law parametrization
Oldewurtel et al. (2008); Oldewurtel, Jones, Parisio, and Morari (2014); Oldewurtel et al. (2010)	–	–	•	•
Ma, Matusko, and Borrelli (2015); Ma, Vichik, and Borrelli (2012c)	–	–	•	–
Zhang, Grammatico, Schildbach, Goulart, and Lygeros (2014); Zhang et al. (2013)	–	•	–	–
Long, Liu, Xie, and Johansson (2014)	–	•	–	–
Tanner and Henze (2014)	–	•	–	–
Garifi, Baker, Touri, and Christensen (2018)	–	•	–	–
Kumar et al. (2020)	–	•	–	–
Drgoňa et al. (2013)	•	•	–	–
Parisio et al. (2014)	•	•	–	–

quantification. Of those methods, authors in Jain et al. (2017b); Smarra et al. (2018) successfully implemented random forest and regression trees for optimal buildings control in different scenarios. However, they showed that in some cases, these models suffered from limitations due to overfitting. These so-called data-predictive controllers (DPC) can achieve comparable performance to MPC while avoiding the cost and effort associated with constructing a gray/white-box model of the building (Jain et al., 2017a). An experimental validation of the DPC method based on random forests applied to the room temperature control reported significant energy savings and thermal comfort improvement compared to baseline rule-based controller (Bünning, Huber, Heer, Aboudonia, & Lygeros, 2020). Another popular approach is the use of gaussian process (GP) models for real-time receding horizon control with probabilistic guarantees on constraint satisfaction applied to closed-loop simulations of large-scale building models (Jain et al., 2018). The authors showed how this approach could provide the desired load curtailment in the context of demand response with high confidence. Data-driven MPC based on GP models of the building's power demand compensating the uncertainty was presented in Nghiem and Jones (2017). A preliminary experimental result on the use of differentiable linear MPC trained offline on the RBC data with online reinforcement learning-based updates was presented in Chen, Cai, and Bergés (2019a).

8. Software tools for building modeling, simulation and control

This section aims to provide an extensive overview and high-level comparison of tools for the modeling, simulation, and control of buildings in the context of MPC. The inspiration and some information were obtained from online directories listing available software tools for modeling, analysis, optimization, and simulation for buildings (EUROSI, 2020; Berkeley Lab, 2020; Nghiem, 2011; US Department of Energy, 2020).

8.1. Building modeling and simulation tools

8.1.1. Building energy simulation tools

Building energy simulation (BES) programs are software tools that simulate energy, mass, and contaminant flows in buildings. This includes the interaction between the building envelope and its surroundings (i.e., weather, radiation heat losses, etc.), internal loads (i.e. occupants, lighting, equipment), and HVAC systems. A number of software modeling tools for buildings are available, which usually consider detailed models of building components. Typically, these tools are built and used for building design purposes. However, as discussed previously in Section 3.2.1, these tools may also be used to implement white-box models for MPC. In addition, these tools are often used to develop digital twins of real buildings (also called emulators), which can be used in simulation case studies to assess the performance of MPC algorithms.

BES tools can be divided into two main subgroups (Wetter et al., 2016). First, traditional imperative languages which declare the sequence of commands to be executed and are usually defined in function-based format. In this approach, the modeling is interconnected with the solver with a primary purpose of building performance evaluation. An advantage here is that the execution of the simulation can be relatively fast. However, the main disadvantage is that these programs are difficult to extend to support new use cases, such as modeling of controls, reformulation of model equations into optimal control problems or integration with electric grid simulation tools. The second group represents equation-based, object-oriented, declarative languages such as Modelica. The principal difference of this paradigm of modeling in contrast to the imperative paradigm is that instead of giving the sequence of instructions which define the way how the program should behave, they provide a higher-level abstraction in the form of hybrid differential algebraic systems of equations. These equations can then be encapsulated into graphical components and organized into hierarchical libraries in an object-oriented fashion, which makes them highly reusable and modular. In addition, this type of implementation allows for the explicit definition of state initial conditions as well as symbolic differentiation for efficient numerical integration. Finally, these equations, and their derivatives, can be used for generation of an optimal control problem for MPC, or more easily be integrated with modeling tools from other domains.

A compact summary of BES tools which have been used to replace real buildings for testing MPC algorithms using co-simulation is given in Table 9. Besides programming language paradigm type, the last column indicates whether it is possible to model the control systems with these tools directly. The mentioned programs, however, represent only a subset of all BES tools. For a more comprehensive overview of building and HVAC system modeling and simulation tools, we refer to (Clarke, 2001; Hensen & Lamberts, 2019; Trcka & Hensen, 2010; Zhou, Hong, & Yan, 2013). More comprehensive comparisons and discussions about BES programs can be found in (Nageler et al., 2018; Sousa, 2012; Wetter, 2011; Wetter et al., 2016; Wetter & Haugstetter, 2006).

8.1.2. Co-simulation tools and interfaces

BES programs are typically not directly suitable for design, synthesis, and testing of optimal controllers. To deal with this issue, middleware software and interface protocols were designed for making communication bridges between BES programs and control-oriented tools and programming languages like MATLAB or Python. Table 10 provides a summary of selected interface tools and standards relevant to building simulation and control. FMI here stands for Functional Mock-up Interface, which is an interface standard for general modeling and simulation tools not only pertaining to buildings (Blochwitz et al., 2011). For an in-depth overview and comparison of co-simulation technology we refer to (Trcka, Hensen, & Wetter, 2009).

8.1.3. Control-oriented building modeling tools

Obtaining models that are accurate enough and at the same time not too complex for optimal control represents one of the bottlenecks which prevents wider adoption of MPC in practice. The main reasons are, first, that models generated by BES programs described in previous sections are often too complex for use in the subsequent optimization problems. Second, they compute numerical approximations to cost functions that are not differentiable (Polak & Wetter, 2006; Wetter & Polak, 2004). Third, there is a substantial shortage of user-friendly and freely available tools for accurate control-oriented modeling of the buildings. Luckily, in recent years, there has been some progress in this direction, and several tools have emerged to help create the models for MPC. Table 11 provides an overview of such tools. However, it is important to note that most of the tools in this list still either require substantial multi-disciplinary expertise or are only available as a research tool.

Tools exist for the linearization of Modelica models (Picard, Jorissen, & Helsen, 2015), returning a state space formulation of the model. This allows for direct integration within the optimal control problem. The linearization methodology has proven to have a high level of accuracy. Moreover, Modelica models can be exported as a Functional Mockup Unit, which allows accessing directional derivatives as needed to solve optimal control problems (Blochwitz et al., 2011). Another white-box control-oriented modeling approach for multi-zone buildings was developed based on the Simspace library in Matlab/Simulink environment (Lapusan, Balan, Hancu, & Plesa, 2016). The emphasis lies on easy modeling with a modular framework based on a set of pre-defined blocks. The popularity of gray-box models extends to toolboxes for parameter estimation and application of the derived models into MPC. The Grey-Box Toolbox (De Coninck et al., 2016), for instance, allows parameter estimation of Modelica models using the JModelica (Modelon, 2017) platform with a front end in Python. The toolbox has been extended for the direct application of the obtained models into MPC (Vande Cavey, De Coninck, & Helsen, 2014). MPCPy (Blum & Wetter, 2017) is another toolbox using reduced order grey-box models and relying on JModelica (Modelon, 2017) for both parameter estimation and solving MPC problems, with the parameter estimation and optimization problems automatically generated based on specification of a Modelica model and high-level input parameters in Python. The modeling environment (ME) for MPC (Zakula et al., 2014) is based on TRNSYS and its coupling with MATLAB to obtain a simplified inverse thermal response model in the form of an inverse comprehensive room transfer functions (iCRTF). The Building Resistance-Capacitance Modeling (BRCM) toolbox (Sturzenegger, Gyalistras, Semeraro, Morari, & Smith, 2014) facilitates physical modeling of buildings for MPC via generation of control-oriented linear RC models from EnergyPlus models. OpenBuild (Gorecki, Qureshi, & Jones, 2015) provides an integrated simulation environment for building control in MATLAB. Like BRCM, it generates the RC models from EnergyPlus. In both tools, co-simulation of MATLAB with EnergyPlus is built on BCVTB (Wetter & Haves, 2008) and MLE+ (Bernal, Behl, Nghiem, & Mangharam, 2012). Another Matlab toolbox BLDG (Kircher & Zhang, 2016) provides users with a standalone building model based on simplified PDE equations with a small number of parameters, along with system identification and parameter estimation functionality. IDENT (Jiménez, Madsen, & Andersen, 2008) provides a graphical user interface in MATLAB to estimate the RC models of building envelopes from the measurement data. BASBenchmarks (Cauchi & Abate, 2018) represents a modular model library for building automation systems covering physical components as well as digital control strategies. The software package LORD (Gutschker, 2008) performs a combination of two different methods alternatively for parameter estimation. One is the Downhill Simplex Method, and the other is a specially adopted Monte Carlo procedure. LORD also offers a graphical user interface for creating the RC model structures based on nodes and connections. CTSM-R (Kristensen, Madsen, & Jørgensen, 2004a) and MoCaVa (Bohlin, 2003) feature maximum likelihood and maximum a posteriori estimation of stochastic grey-box models. The former is accessed through the programming language R, while the latter runs under Matlab. A comparison between MoCaVa and CTSM was studied in Kristensen, Madsen, and Jørgensen (2004b). It shows that CTSM has better performance in terms of quality of estimates for nonlinear systems with significant diffusion and in terms of reproducibility. In particular, CTSM provides more consistent estimates of the diffusion term parameters. Finally, there exist more generic tools that can be used to calibrate simulation models that do not make any assumptions regarding the model (language, paradigm) except the interface. For example, ModestPy (Arendt et al., 2018b) is a parameter estimation Python package for FMI-compliant models, mostly used with gray-box models as in Arendt et al. (2018a), while GenOpt (Wetter, 2001) is an optimization software that can be used for parameter estimation in any model that can be interfaced through text files, e.g. EnergyPlus, TRNSYS.

Table 9
Summary of the selected BES programs used to emulate the buildings for testing MPC in co-simulation.

BES Tool	Free	Equation-based	Imperative	Explicit controls modeling
ESP-r (Yahiaoui, Hensen, & Soethout, 2003)	•	–	•	•
EnergyPlus (Crawley et al., 2001)	•	–	•	–
TRNSYS (Beckman et al., 1994)	–	–	•	–
Modelica (Baetens et al., 2015; Wetter et al., 2014)	•	•	–	•

8.2. MPC design tools

Table 12 provides an overview of the most important software tools which can be used or are particularly dedicated to modeling, simulation, evaluation, deployment and code generation of MPC controllers. Most advanced and widely popular tools are based on MATLAB, Modelica or Python languages and come with a free license. These modeling languages allow for high-level modeling of the optimization problems and provide an interface to a wide variety of optimization solvers in an automated way. This reduces the engineering burden of error-prone and time-consuming manual translation of the OCP (1) to the OP form required by a particular solver.

OpenBuild (Gorecki et al., 2015) supports the design and simulation of the state observer and MPC using an RC model generated based on an EnergyPlus model. BRCM toolbox (Sturzenegger et al., 2014) offers the generation of the cost and constraint matrices for MPC based on the generated RC model from EnergyPlus. However, it does not provide the environment for simulation, tuning, and analysis of MPC. EHCM toolbox (Darivianakis, Georghiou, Smith, & Lygeros, 2020) is an extension of BRCM providing a framework for controlling the operation of the energy hub with multiple interconnected buildings in a cooperative manner. BLDG (Kircher & Zhang, 2016) provides functionality for state and parameter estimation, and MPC based on the identified simplified RC model. BeSim (Drgoňa, 2019) provides functionality for fast development, tuning, and simulation of model order reduction, state estimation and MPC based on linearized white-box building models from Modelica (Picard et al., 2015) and optimization toolbox Yalmip (Löfberg, 2004). Modeling environment (ME) (Zakula et al., 2014) is a modular simulation tool for buildings that employs MPC based on TRNSYS for virtual building modeling and Matlab for MPC implementation. TACO (Jorissen et al., 2018a) automates the process of setting up an MPC from a white-box model in Modelica. The nonlinear MPC is formulated using the CasADi (Andersson, Gillis, Horn, Rawlings, & Diehl, 2018) framework and solved with the JModelica (Modelon, 2017) optimizer.

8.3. MPC solvers

Today, dozens of optimization solvers are available, both commercially and free, for a wide variety of problems. Tables 13 and 14 provide an overview of the most significant solvers suitable to solve MPC problems on desktop and embedded platforms, respectively. The used acronyms stand for Linear Programming (LP), Quadratic Programming (QP), Mixed-Integer Linear Programming (MILP), Mixed-Integer Quadratic Programming (MIQP), Mixed-Integer Nonlinear Programming (MINLP), Nonlinear Programming (NLP), Second Order Cone Programming (SOCP), Semi Definite Programming (SDP), Multi-Parametric Linear Programming (mpLP), and Multi Parametric Quadratic Programming (mpQP), respectively.

Progress in the solution techniques and an increase in the computational power of the desktop platforms allow us to efficiently solve large-scale optimization problems with up to hundreds of thousands of variables. In the case of embedded platforms, several tools have automated and optimized code generation features supporting different languages (e.g., C, C++ or Python) for rapid development and deployment of the MPC controllers in real-world applications. These embedded applications are, however, mostly suitable for small, fast dynamic systems, which are different from the large and slow dynamics of the buildings. Nevertheless, their efficiency and cheap computational power could be harnessed in large buildings for local control loops, or small-scale residential applications of MPC.

In the case of data-driven approximate MPC, the machine learning models can be trained by solving a wide variety of optimization problems offline. The type of OP to be solved depends on the used models (e.g., neural networks, regression trees, etc.) and their specification. While dedicated algorithms also exist to train more complex and specific ML

Table 10
Summary of the co-simulation tools and interface standards to bridge BES programs with other simulation platforms and control-oriented programming languages.

Co-simulation tool or interface standard	Free	Interface for					
		ESP-r	EnergyPlus	TRNSYS	Modelica	MATLAB	Python
BCVTB (Wetter & Haves, 2008)	•	•	•	•	•	•	–
MLE+ (Bernal et al., 2012)	•	–	•	–	–	•	–
OpenBuild (Gorecki et al., 2015)	•	–	•	–	–	•	–
FMI (Broman et al., 2013; Pang et al., 2016)	•	–	•	•	•	•	•

Table 11
Summary of selected control-oriented building modeling tools. The acronyms are explained in the text.

Tool	Free	Language					Paradigm		
		Modelica	MATLAB	Python	TRNSYS	R	White	Grey	Black
Modelica Linearization (Picard et al., 2015)	•	•	–	–	–	–	•	–	–
Simscape Library (Lapusan et al., 2016)	•	–	•	–	–	–	•	–	–
ME for MPC (Zakula et al., 2014)	–	–	•	–	•	–	–	•	–
OpenBuild (Gorecki et al., 2015)	•	–	•	–	–	–	–	•	–
IDENT (Jiménez et al., 2008)	•	–	•	–	–	–	–	•	–
BRCM Toolbox (Sturzenegger et al., 2014)	•	–	•	–	–	–	–	•	–
BLDG (Kircher & Zhang, 2016)	•	–	•	–	–	–	–	•	–
BASBenchmarks (Cauchi & Abate, 2018)	•	–	•	–	–	–	–	•	–
Grey-box Toolbox (De Coninck et al., 2016)	•	•	–	•	–	–	–	•	–
MPCPy (Blum & Wetter, 2017)	•	•	–	•	–	–	–	•	–
LORD (Gutschker, 2008)	•	–	–	–	–	–	–	•	–
CTSM-R (Kristensen et al., 2004a)	•	–	–	–	–	•	–	•	–
MoCaVa (Bohlin, 2003)	–	–	•	–	–	–	–	–	•
System Identification Toolbox (Ljung, 2006)	–	–	•	–	–	–	–	–	•

models (Sra, Nowozin, & Wright, 2011), most of the problems in this domain are solved via gradient descent algorithms. However, they can also be solved by using general purpose solvers listed in Table 13.

9. Practical implementation of MPC in buildings

The ambition of this section is to provide a complete overview of the key components and nuances of practical implementation of MPC in buildings. A schematic representation of the presented framework corresponding to the structure of this section is given in Fig. 9. The three key elements are: the control configuration discussed in Section 9.1, the SCADA architecture presented in Section 9.2, and the communication infrastructure described in Section 9.3. Section 9.4 concludes the topic and provides experience-based practical guidelines for MPC

Table 12
Overview of the modeling software for optimization problems suitable for formulating and solving MPC problems.

Tool	Free	MATLAB	Python	Julia	Modelica	C/C+	Java	Tool-specific language
Yalmip (Löfberg, 2004)	•	•	–	–	–	–	–	–
CVX (Grant & Boyd, 2014)	•	•	–	–	–	–	–	–
MPC Toolbox™ (Mathworks, 2020)	–	•	–	–	–	–	–	–
MPC Tools Package (Amrit, 2008)	•	•	–	–	–	–	–	–
Hybrid Toolbox (Bemporad, 2004)	•	•	–	–	–	–	–	–
MPT3 (Herceg, Kvasnica, Jones, & Morari, 2013)	•	•	–	–	–	–	–	–
NMPC Tools (Rawlings & Amrit, 2008)	•	•	–	–	–	–	–	–
ACADO (Houska, Ferreau, & Diehl, 2011)	•	•	–	–	–	•	–	–
ACADOS (Verschueren et al., 2019)	•	•	•	–	–	•	–	–
CasADi (Andersson et al., 2018)	•	•	•	–	–	•	–	–
APMonitor (Hedengren, Shishavan, Powell, & Edgar, 2014)	•	•	•	•	–	–	–	–
HPMPC (Frison, Sørensen, Dammann, & Jørgensen, 2014)	•	–	–	–	–	•	–	–
CVXPY (Diamond & Boyd, 2016)	•	–	•	–	–	–	–	–
Pyomo (Hart et al., 2017)	•	–	•	–	–	–	–	–
Picos (Sagnol & Stahlberg, 2018)	•	–	•	–	–	–	–	–
OpenModelica (Fritzson et al., 2018)	•	–	•	–	•	•	–	–
JModelica.org (Modelon, 2017)	•	–	•	–	•	•	•	–
JuMP (Dunning, Huchette, & Lubin, 2017)	•	–	–	•	–	–	–	–
AMPL (Fourer, Gay, & Kernighan, 2002)	–	–	–	–	–	–	–	•
GAMS (Rosenthal, 1988)	–	–	–	–	–	–	–	•
Building control oriented								
OpenBuild (Gorecki et al., 2015)	•	•	–	–	–	–	–	–
BRCM toolbox (Sturzenegger et al., 2014)	•	•	–	–	–	–	–	–
EHCM toolbox (Darivianakis, 2020)	•	•	–	–	–	–	–	–
BLDG (Kircher & Zhang, 2016)	•	•	–	–	–	–	–	–
BeSim (Drgoňa, 2019)	•	•	–	–	•	–	–	–
FastSim (Arroyo et al., 2018)	•	–	•	–	•	–	–	–
MPCPy (Blum & Wetter, 2017)	•	–	•	–	•	–	–	–
GenOpt (Coffey et al., 2010)	•	–	–	–	–	–	•	–
ME for MPC (Zakula et al., 2014)	–	•	–	–	–	–	–	–
TACO (Jorissen et al., 2018a)	–	–	–	–	•	•	–	–

Table 13
Overview of the most notable optimization solvers suitable to solve MPC problems on a desktop platforms.

Solver	Free	LP	QP	MILP	MIQP	MINLP	NLP	SOCP	SDP
CPLEX (ILOG, 2007)	–	•	•	•	•	–	–	•	–
Gurobi (Gurobi Optimization, 2012)	–	•	•	•	•	–	–	•	–
MOSEK (Andersen & Andersen, 2000)	–	•	•	•	•	–	–	•	•
XPRESS (Berthold, Farmer, Heinz, & Perregaard, 2018)	–	•	•	•	•	–	–	•	–
SeDuMi (Sturm, 2003)	•	•	•	–	–	–	–	•	•
SDPT3 (Toh, Todd, & Tütüncü, 1999)	•	•	•	–	–	–	–	•	•
CVXOPT (Andersen & Vandenberghe, 2018)	•	•	•	–	–	–	–	•	•
GLPK (Makhorin, 2012)	•	•	–	•	–	–	–	–	–
IPOPT (Wächter & Biegler, 2006)	•	•	•	–	–	–	•	–	–
ALGLIB (Bochkanov, 2019)	•	•	•	–	–	–	•	–	–
Artelys Kitro (Byrd, Nocedal, & Waltz, 2006)	–	•	•	–	–	–	•	–	–
SNOPT (Gill, Murray, & Saunders, 2005a)	–	•	•	–	–	–	•	–	–
APOPT (APOPT, 2020)	–	•	•	•	•	•	•	–	–
BARON (Sahinidis, 2017)	–	•	•	•	•	•	•	–	–
Bonmin (Bonami et al., 2005)	•	•	•	•	•	•	•	–	–
WORHP (Büskens & Wassel, 2013)	•	•	•	•	•	•	•	–	–
GenOpt (Wetter, 2001)	•	–	–	–	–	•	•	–	–

implementation in real buildings.

9.1. Control configuration

The following terminology is used in this section for networked control systems. See Fig. 10 for conceptual diagrams.

Centralized control: a centralized agent (or controller) regulates an entire system.
Decentralized control: each agent controls its own subsystem without communicating with neighbors.
Distributed control: multiple agents are distributed horizontally over a whole system. There is no central agent.
Hierarchical control: multiple agents are arranged in a hierarchical tree to control an entire system.

Centralized MPC The centralized MPC scheme solves a plant-wise optimization problem in a central computer and has been the primary method in the building sector. However, for buildings which are composed of a large number of dynamic subsystems composing a complex topological network, applying centralized MPC could be challenging due to increased computational complexity and reliability issues (Jamshidi, 1996). In this case, it is favorable to decompose a large centralized optimization problem into smaller multiple subproblems, which motivates configurations of decentralized, distributed and hierarchical MPCs.

Decentralized MPC In the decentralized MPC scheme, each local controller is designed as MPC and optimizes its own performance index without considering costs and dynamic influences of the others. Therefore, overall performance could be quite poor, especially for strongly coupled systems, though the communication overhead is minimal (Rawlings & Mayne, 2009).

Distributed MPC In the distributed MPC approach, each local controller which regulates its own subsystem solves a subproblem and communicates with others in order to improve the entire system performance. The information exchange consist of predicted state or control inputs so that any local controller can predict better by considering influences of neighboring systems. The communication can occur only once at each sampling time (non-iterative algorithms), i.e. only after each local optimization problem is solved, or many times within the sampling time (iterative algorithms) (Scattolini, 2009). Iterative algorithms could show better performance in terms of convergence and closed loop stability, but have higher communication burdens, causing concerns about communication delays, overloads and transmission package losses (Camponogara, Jia, Krogh, & Talukdar, 2002).

In the literature of process control, the non-cooperative and cooperative MPCs (Rawlings & Mayne, 2009; Venkat, Rawlings, & Wright, 2005) are the most popular distributed MPC methods. Both of them optimize control inputs to minimize a global index in the form of $\sum_{k=0}^{N-1}\sum_{i=1}^{S}\ell_i(x_k^i,u_k^i)+\sum_{i=1}^{S}\ell_i(x_N^i)$ where ℓ_i is the stage cost for the ith subsystem. Note that the cost function is not separable[2] when control inputs are the only optimization variables (because of dynamic couplings between subsystems), although separable with respect to the state and control inputs. During the optimization phase, each MPC predicts the state evolution considering dynamic couplings to neighbors assuming that input profiles received from neighbors are fixed. The key difference between the non-cooperative and cooperative MPCs is that local controllers of the cooperative MPC tend to minimize the same global cost function, while those of non-cooperative MPC tend to minimize individual cost functions, i.e. the ith MPC minimizes $\sum_{k=0}^{N-1}\ell_i(x_k^i,u_k^i)+\ell_i(x_N^i)$ (Moroşan, Bourdais, Dumur, & Buisson, 2011). In cooperative control, the distributed optimization problems are equivalent to the centralized MPC problem and are solved iteratively. Therefore, the cooperative control guarantees global performance, such as constraint feasibility, convergence, optimality and closed loop stability. See (Stewart, Venkat, Rawlings, Wright, & Pannocchia, 2010) for detailed proofs.

Hierarchical MPC The hierarchical control configuration is particularly useful when coordination between local controllers is needed in order to improve overall performance, or control actions for different time scales need to be decided (Scattolini, 2009), e.g. an upper layer computes optimal temperature setpoints in an economic sense while lower layers focus on setpoint tracking.

To design a coordinator (upper-level) and local MPCs (lower-level), the dual decomposition method (Jamshidi, 1996) or the Alternating Direction Method of Multipliers (ADMM) (Boyd et al., 2011) can be employed. Both methods solve a global optimization problem indirectly by solving the Lagrangian dual problem and adopt the dual ascent method (Bazaraa, Sherali, & Shetty, 2013). The key to decomposing the primal MPC problem is that coupled dynamic equations can be separated in the Lagrangian function when the primal objective function is separable. If the dynamics are linear and the objective function is convex on a convex compact set, the dual and primal problems are equivalent (the strong duality theorem (Boyd & Vandenberghe, 2004)). In the hierarchical control scheme, the upper-level controller represents the dual ascent step, and hence vertical communication between the upper and all lower-level controllers are necessary. For the ADMM,

[2] An objective function, $f(x^1, \cdots, x^S)$, is called *separable*, if f can be expressed as a sum of functions of the individual variables of $x^1, \cdots, x^S$, i.e. $f(x^1,\cdots,x^S)=\sum_{i=1}^{S}f_i(x^i)$

Table 14
Overview of the most notable optimization software tools suitable to solve MPC problems on embedded platforms.

Solver	Free	Code generation	LP	QP	mpLP/mpQP	MILP/MIQP	NLP
OOQP (Gertz & Wright, 2003)	•	–	•	•	–	–	–
qpOASES (Ferreau, Kirches, Potschka, Bock, & Diehl, 2014)	•	–	•	•	–	–	–
ECOS (Domahidi, Chu, & Boyd, 2013)	•	–	•	•	–	–	–
CVXGEN (Mattingley & Boyd, 2012)	•	•	•	•	–	–	–
FiOrdOs (Ullmann, 2011)	•	•	•	•	–	–	–
FORCES PRO (Embotech, 2020)	–	•	•	•	–	–	–
Falcopt (Torrisi et al., 2017)	•	•	•	•	–	–	•
Toolbox							
ACADO (Houska et al., 2011)	•	•	•	•	–	–	•
Hybrid Toolbox (Bemporad, 2004)	•	•	•	•	•	•	–
MPT3 (Herceg et al., 2013)	•	•	•	•	•	•	•
MPC Toolbox™ (Mathworks, 2020)	–	•	•	•	•	•	•

communications between lower-level MPCs which represents the Gauss-Seidel algorithm is additionally required. Note that the upper-level controller is not MPC since it does not predict future behaviors.

When both short-term and long-term behaviors of a system are concerned, a hierarchical control system can be designed so that an upper layer regulator acts on lower frequencies and computes a control action concerning a long-term effect, while lower layer controller(s) act on higher frequencies and are responsible for short-term behavior(s) (Scattolini & Colaneri, 2007). This approach is related to cascade controls in which the inner and outer control loops are associated to faster and slow dynamics, respectively. One of the most significant advantages of this control approach is that it can substantially improve control performance under disturbances and nonlinearities associated with the inner loop, and that control designs can be separated when the upper layer works on a sufficiently low frequency range, say a factor of five or more in terms of inner close-loop system (Skogestad & Postlethwaite, 2007).

9.1.1. Review of applied MPC architectures for HVAC systems

Centralized MPC in buildings In the building control domain, the majority of the theoretical work and simulation-based case studies consider centralized MPC architecture. However, there are not many truly centralized MPC solutions that have been considered for application in practice. The main reason is non-standardized use of the communication protocols preventing straightforward access to the field layer of the SCADA architecture, which will be discussed in the following Section 9.2. Moreover, keeping the low-level RBC and PID loops intact may improve the operational robustness of the hierarchical MPC implementation by avoiding a single point of failure in the control system. Despite this fact, the paper (Jorissen et al., 2018b) presents an implementation strategy of centralized high-fidelity MPC for the real office building in Belgium.

Decentralized MPC in buildings The design of decentralized MPC for thermal control of buildings based on reduced order models and state observers was studied in Chandan and Alleyne (2014). A methodology determining an appropriate decentralized architectures, which provide a satisfactory trade-off between control performance and robustness for building control was proposed in Chandan and Alleyne (2013). An agent-based approach for distributed monitoring and model-based control of an office building was presented in Davidsson and Boman (2005). A graph theory-based approach and consensus protocols applied to thermal modeling of buildings was presented in Moore, Vincent, Lashhab, and Liu (2011). However, all of the aforementioned decentralized studies on building modeling and control remain in the simulation domain.

Distributed MPC in buildings An application of non-cooperative MPC can be found in Ferrarini, Mantovani, and Costanzo (2014); Moroşan, Bourdais, Dumur, and Buisson (2010) and those of cooperative MPC-like schemes[3] are found in Moroşan et al. (2011); Putta, Zhu, Kim, Hu, and Braun (2012); Putta, Kim, Cai, Hu, and Braun (2014). For those studies, the objectives are to distribute multi-zone building loads to multiple units in an optimal way. The dynamic interactions are due to thermal couplings between zones through convective or conductive heat transfer. More precisely, in Putta et al. (2012), the case study building has two coupled zones and each zone is served by a separate air handling unit (AHU). Two local MPCs were designed to control the individual AHUs targeting to reduce operating costs for the entire system. in Putta et al. (2014), a nine-zone building served by one AHU is considered. Ten local MPCs were designed, where nine of them control air flow rates of individual variable air volume boxes (VAVs) to regulate nine zone air temperatures. The remaining MPC optimizes the supply air temperature setpoint. Similarly, in Moroşan et al. (2011), four distributed MPCs were designed where three of them manipulate their own local electric heaters while the remaining one controls a central heating system in order to optimally reduce the electricity cost while meeting individual zonal heating loads. For the last two cases, the control configurations are not purely distributed MPC and modifications of the cooperative MPC were necessary since there is a global variable which influences all subsystems, i.e. the supply air temperature of the central heating unit, resulting in a different cost structure compared to that of the cooperative MPC. The proposed MPCs have two-level pyramid structures where the upper-level controller optimizes the global variable based on information from lower-level controllers, while multiple lower-level controllers solve their own problems in a cooperative-MPC approach using the optimized global variable.

Hierarchical MPC in buildings In the literature of the building control, many applications adopt traditional hierarchical control architectures as discussed in previous section. Examples of such applications can be found in Abreu, Bourdais, and Guéguen (2018); Kim and Braun (2018); Ma, Anderson, and Borrelli (2011). in Ma et al. (2011), a three-zone building served by a VAV AHU system is considered. A standard dual decomposition method (Jamshidi, 1996) for decomposing MPC problems was adopted to design a hierarchical control system where lower-level MPCs regulate individual zone air temperatures in an economic way while the upper-level optimizer coordinates possible conflicts in local decisions. in Kim and Braun (2018), the hierarchical MPC system was designed for optimal demand response for a building served by multiple on-off stage HVAC units. The upper layer MPC predicts longer-term performance (about a day) and optimizes thermostat temperature setpoints to shift building loads in response to a utility price signal, while the lower layer MPC predicts short-term performance

[3] It means that each local MPC has a shared objective function and considers influences of neighbors like the cooperative MPC, although decomposition methods and implementation details are different.

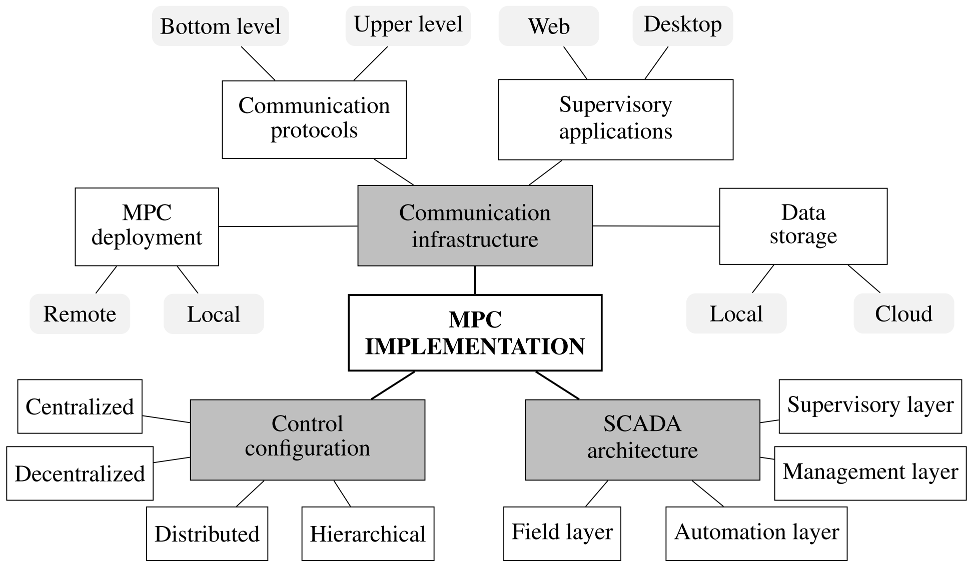


Fig. 9. General framework for the MPC implementation in buildings.

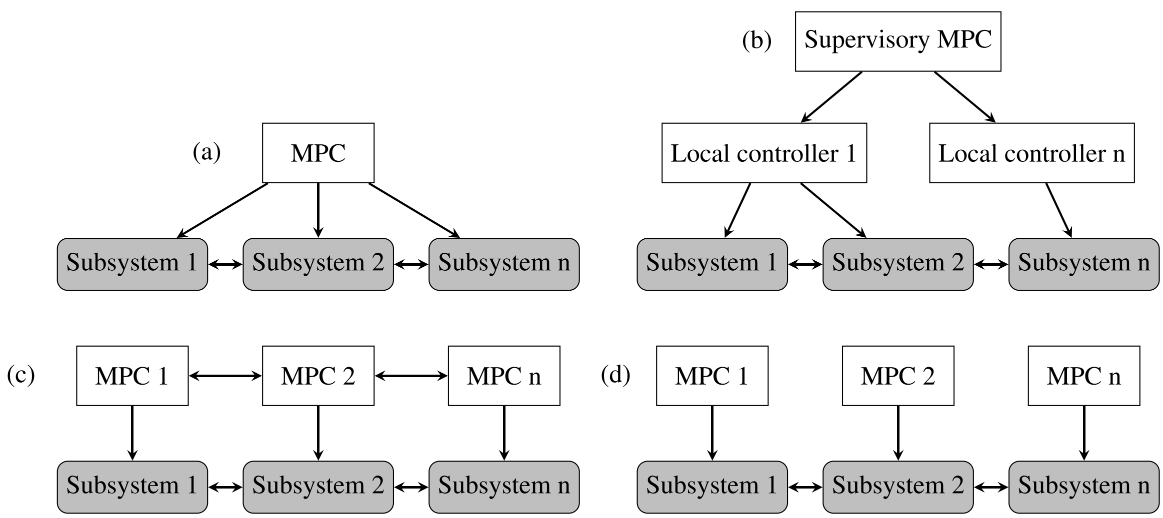


Fig. 10. Schematic of a centralized (a), hierarchical (b), distributed (c) and decentralized (d) MPC control configuration. Extension of figure given in Serale et al. (2018).

(about an hour) and supervises multiple units to prevent simultaneous unit activation during a precooling period, which could cause an unnecessarily higher demand charge. in Abreu et al. (2018), the upper layer MPC optimizes the setpoint while the lower layer MPCs track the setpoint. Recently, studies of applying ADMM to decompose MPC or general optimization problems for buildings become popular and are found in Cai, Braun, Kim, and Hu (2016a,b); Cai, Kim, Putta, Braun, and Hu (2015); Gupta, Kar, Mishra, and Wen (2015); Hou, Xiao, Cai, Hu, and Braun (2017); Moroşan et al. (2011); Xiao, Hou, Cai, and Hu (2018).

Concluding remarks on MPC architecture in buildings Despite a large number of MPC studies, distributed or hierarchical-distributed MPC schemes got relatively little attention from the building HVAC control field. This may be due to lack of practical needs of distributing computational loads. In other words, many MPC problems in building HVAC systems could be handled in a centralized way. In addition, the sufficient conditions for convergence, i.e. convex functions for objective and inequality constraints and a linear structure for equality constraints, make it difficult to use distributed algorithms for practical building controls where HVAC systems exhibit nonlinear and non-convex characteristics and constitute nonlinear equality constraints. However, because building systems need to be integrated with renewable energy resources, energy storage systems and networks (electric, thermal, gas), and because the study of convex approximations is progressing, e.g. (Atam & Helsen, 2015), in the near future it is expected that there are more opportunities of applying distributed and/or hierarchical MPCs for building controls.

9.2. SCADA architecture

Supervisory control and data acquisition (SCADA) is a standard architecture to define the different layers of hierarchical control systems. SCADA systems are widely used in various fields, such as process control, energy, and power systems operation, and have recently gained a lot of importance for the control and data acquisition of the so-called Building Automation Systems (BAS). One of the main advantages of using a SCADA configuration is that the different layers of control and communication flows can be depicted sequentially, in a much more structured and organized way. Another advantage is that other automated systems used in the building can be integrated into one single platform (i.e. HVAC, security, lighting or gas automation systems), which makes the management of the whole installation more effective (Figueiredo & Costa, 2012). A SCADA system for building control and operation typically consists of four different layers (see Fig. 11):

Supervisory layer: the highest layer of the control architecture, where MPC is normally executed. It also includes all *clients* that interact with the system for the purpose of top-management activities. For example, supervisory control or data-analysis by means of visual interfaces used to monitor the whole building's performance.
Management layer: includes one or several *servers* that allow the interaction between the higher and the lower layers of the control architecture. It is also used to conduct preliminary monitoring and preprocessing of information, as well as to store data by means of local or online databases. This layer includes all Building Management Systems (BMS) that are normally used to manage and control modern building installations.
Automation layer: integrates all local controllers that allow the execution of primary plant control by using conventional control strategies, like PID and RBC. All different modules collecting the measurements from the building process downstream are also included in this layer.
Field layer: the lowest layer of the control architecture. It includes all physical components, sensors and actuators.

It is important to outline that the division between layers of control can be apparent in software, hardware or a combination of both. This will depend on the communication infrastructure which is tackled in the next section.

9.3. Communication infrastructure

Communication is yet another crucial element of any practical control implementation. The importance can be emphasized if we would put the whole building control concept it into a human body analogy. The building envelope would then be the torso, heating/cooling capacities the digestive system, air handling units (AHU) the respiratory system, piping the blood vessels, and pumps the heart. The SCADA infrastructure would be the nervous system, control configuration the wiring of the brain, and the MPC formulation its mental program. The communication infrastructure would represent the electrochemical signals traveling throughout the pathways of the nervous system, carrying the information from the subconscious level of low-level control to the conscious level of supervisory applications, while storing the data in the memory represented by a database.

9.3.1. MPC deployment

In SCADA-based control systems, the interaction between MPC and the building is implemented in a *client-server* model. A *client* can be defined as a device or computer program that executes the MPC formulation and accesses the building by means of a *server*; which can be seen as a device or computer program that acts as a bridge of communication between MPC and the rest of the building installation. There are two main configurations and networking typologies for the implementation of MPC in a client-server model: local and remote configuration.

Local: The MPC algorithm is executed in the same building installation where the control is performed. Hence, the division between client and server is only apparent in software (Afram & Janabi-Sharifi, 2017; Skeledzija et al., 2014). Local configurations, however, lack flexibility, since MPC developers need to be present in the building during the commissioning phase. Moreover, any modifications in the controller's formulation will have to be applied locally to the building, which might result in a quite tedious and ineffective process.

Remote: The MPC algorithm is executed remotely from the building installation where the control is performed. The division between client and server is apparent in software but also in hardware, as two separate devices are normally implemented. Internet or other wireless communications are used to interact with the building, see e.g. (Gwerder, Gyalistras, Sagerschnig, Smith, & Sturzenegger, 2013; Ma, 2012). Remote configurations have several advantages, such as increased flexibility and interoperability from multiple platforms and devices. However, the disadvantage is the need for secure and stable communication channels.

Regarding MPC solvers, for practical installations, they are being deployed using several programming languages, such as C++, Python, Julia, or even JavaScript. In the research domain, however, the MPC algorithm can often run in MATLAB, with limited industrial applicability due to the associated software costs.

9.3.2. Communication protocols

In general we can differentiate between two levels of communication and their corresponding protocols, bottom and upper-level:

Bottom level: communication on the lowest layers of control, for example local controllers and HVAC actuators.
Upper level: communication on the highest layers between MPC and the local controllers of the building by means of a server. Servers can be understood as 'interpreters' that translate all inputs coming from the MPC into a language that local controllers can understand, and vice-versa (Nyvlt, 2011).

In recent years, plenty of communication protocols have been developed for the purpose of Building Automation Systems (BAS). They can be grouped into two main categories: closed and open protocols.

Closed protocols: based on proprietary communication structures developed by each manufacturer separately, usually tailored to particular applications, hence they often lack versatility and flexibility (Bovet, Ridi, & Hennebert, 2014).

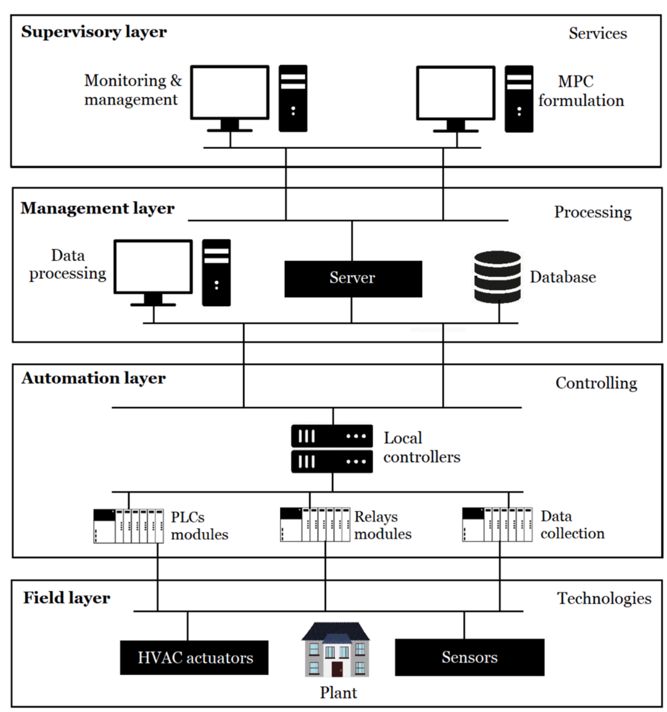


Fig. 11. SCADA-based control architecture for building control and operation using MPC.

Open protocols: based on standard specifications which leads to clear advantages, such as greater flexibility of implementation and interoperability of devices from hundreds of different vendors (Nyvlt, 2011).

Table 15 provides a compact overview and classification of selected communication protocols specifically designed or reported to be used in building control applications.

The communication challenges in real buildings proliferate with the scale, use of multi-vendor devices, different protocols, and geographical distribution of the units. In recent years, modern communication platforms for distributed sensing and control systems have been under development to mitigate those challenges. Examples of such platforms are the commercial Niagara Framework® (Tridium, 2019) developed by Tridium's Inc. belonging to the Honeywell group, or open-source Volttron™ (Akyol et al., 2016) developed by Pacific Northwest National Laboratory.

9.3.3. Supervisory applications

Supervisory applications are implemented by means of Human Machine Interfaces (HMIs), which allow monitoring the MPC performance using visual and graphical interfaces. In a SCADA-based architecture, HMIs act as clients that connect to the building server. They can be divided into two groups: desktop, and web-based applications.

Desktop applications: stand-alone software tailored to one particular computer, which only can be accessed by a restricted number of users. They offer more privacy, security, and usually also better performance than web-applications, but they lack the portability, scalability, and flexibility of implementation (Pop, 2008), which are crucial for integration with other automation systems, using the same BMS.

Web-based applications: accessed through the network by multiple clients and devices simultaneously, exploiting the use of internet and web-services. They are much more flexible because they are platform-independent and are not tailored to one specific device. Moreover, they are more scalable and can be easily integrated into the whole BMS of the building. For obvious reasons, they are more suitable in a remote configuration. Some disadvantages of using web-applications are slower performance, internet-dependency, or security risks compared to a desktop application (Pop, 2008).

9.3.4. Data storage

The storage of data has significant importance for the implementation of MPC in buildings. MPC developers make use of historical data for three main purposes: (i) to develop and calibrate the building model used by MPC; (ii) to keep track of variables that are used as parameters in the MPC formulation (i.e. weather-data, electricity prices, etc.); (iii) and to analyze the performance of the controller. Regarding their implementation, databases can be classified into local and cloud dtabased.

Local databases: stored in a dedicated device or computer and can only be accessed by a limited number of applications.

Cloud databases: make use of a web-server to store data, which is connected to the Internet and can be accessed remotely by multiple applications.

For MPC implementations cloud databases are usually preferred above local databases due to their flexibility of operation and less tedious setting-up phase. Moreover, a common practice today is to outsource the storage of data using an external server from a third-party, normally referred to as a cloud provider. As a result, cloud services provide a reduction in the creation and maintenance costs of the database, better scalability, and more safety towards losing backups (Li, Li, Vrabie, Bengea, & Mijanovic, 2014). The downsides of the cloud-based solutions are potential cyber-security issues, which may often impose more secure local implementation.

Regarding the model they implement, databases can be classified into relational and non-relational databases (Gyorodi, Gyorodi, & Sotoc, 2015).

Relational databases: are based on a Structure Query Language (SQL) to store and retrieve data from the database in a really organized way using tables. They count on rigid schemes that need to be designed before data is stored and are quite difficult to change afterward.

Non-relational databases: do not use relational management systems, hence data is not stored using tables, nor rigid schemes. They offer big advantages compared to relational databases, such as superior performance, better scalability and more flexibility of implementation.

Relational databases are widely implemented for all kinds of applications showing a pretty good performance. However, recent studies have proven that they present some limitations, especially when dealing with large amounts of data and transaction (Gyorodi et al., 2015). Thus, big-data organizations (e.g. Google, Amazon or Facebook) are starting to use non-relational databases to store their data. However, this is still yet a relatively new direction and the majority of MPC implementations reported in the literature opted for the relational databases, see. e.g. (Fabietti, 2014; Skeledzija et al., 2014). However, for the future implementation of MPC, non-relational databases seem to be a better candidate, since the controller is expected to deal with big volumes of data, especially in large-scale buildings where a central database might be used for the whole installation.

9.4. Practical guidelines

This section summarizes practical aspects discussed in detailed in previous sections and extracts step by step guidelines for developing and implementing a successful MPC application for a real building. A general methodology is systematically shown in Fig. 12 covering the high-level workflow, starting with setting up the communication infrastructure, followed by control-oriented modeling, control configuration with MPC design and tuning, finalized by MPC deployment as a supervisory application in modern SCADA systems and closing the loop with communication setup in case of necessary modifications.

A more detailed and practically oriented flowchart is presented in Fig. 13. It encompasses the necessary actions and decisions of the whole MPC workflow from scratch to implementation in a real building. The preliminary phase starts with a feasibility analysis which should be based on controllability and measurability of the building via the

Table 15
Summary and classification of selected communication protocols used in building control.

Protocol	Standard	Bottom level	Upper level	Closed	Open	BAS oriented
Nikobus		•	–	•	–	–
iNels		•	–	•	–	•
BACnet	ISO 16484-5	•	–	–	•	•
KNX	ISO/IEC 14,543	•	–	–	•	•
Modbus		•	–	–	•	•
LonWorks	ANSI/ CEA-709.1	•	–	–	•	•
M-bus	EN 13,757	•	–	–	•	•
OPC		•	•	–	•	–
TCP/IP	IETF	–	•	–	•	–
UDP	RFC 768	–	•	–	•	–
FTP	RFC 2428	–	•	–	•	–
HTTP/ HTTPS	RFC 7230	–	•	–	•	–

installed building automation system (BAS). The second step is to evaluate the economic potential for a building of interest via return on investment analysis.

The design phase starts with the third step of the flowchart by setting up the real-time communication between the BAS and the supervisory computer for automated data logging and storage, as summarized in Section 9.3. This automated communication functionality is a must for any real-time dynamic optimization scheme such as MPC, while historical data stored in databases serve for modeling and tuning purposes. Necessary data points need to be selected based on the design of the model and control architecture. However, today, such functionality still represents a bottleneck due to the large variety of used protocols, interfaces and BAS vendors with closed solutions.

The fourth step consists of modeling, as elaborated in Section 3. First, engineers need to define the objectives, constraints and key performance indicators (KPIs) for performance assessment of the models and control strategies. Subsequently, a control-oriented model needs to be developed via dedicated software tools partially listed in Section 8.1.3 and evaluated with respect to selected performance measures.

After construction of a sufficiently accurate model, a control configuration needs to be defined in the fifth step. If the selected configuration is realizable within the current communication infrastructure, then we proceed to the next step, else either control configuration is modified, or a list of available data points is extended.

In the sixth step, a control engineer initiates MPC design by formulating the optimal control problem, identifying the problem class and selecting the solution paradigm as described in Sections 2, 5, and 6, respectively. Subsequently, appropriate design tools and solvers are selected, e.g. based on the lists given in Sections 8.2, and 8.3, respectively. The implementation of MPC algorithms presented in Section 4 follows.

After tuning and performance evaluation in closed-loop simulation studies, controllers with acceptable quality are selected for deployment in the seventh step of the workflow. MPC solvers need to be installed either locally or on a remote computational platform and integrated in the SCADA system of the building (see Section 9.2). The deployment phase consists of functionality tests, and installation of the user interface and backup solutions, such as watchdog timers, alarms and automatic fallback controller for recovering operation after failures. Only after this, the operation phase can be initiated in the final eighth step. The installed applications continuously monitor MPC functionality and if error handling logic is triggered, the operation autonomously switches to the fallback control strategy, typically represented by simple RBC logic or PID loops. Each operational failure is typically accompanied with alarm messages to the building operators.

10. Comparison and performance assessment

Comparison and performance assessment of MPC approaches are important to identify the most promising approaches and guide transition of MPC strategies from research to industry. However, a number of challenges exist that make such comparisons difficult. Therefore, this section outlines these challenges, reviews the literature on studies that have compared MPC approaches, and suggests the needs of a more unifying framework for such assessment.

10.1. Challenges

An initial challenge of comparing MPC approaches is the large variation any implementation can take compared to another. As presented in this paper, there are a number of factors and methods to consider for each of the many components of the MPC, creating a very large solution space. In addition, each application, whether it be a single zone, building, campus, or neighborhood, presents its own set of design and operation characteristics that may promote the use of one method over another. These include architectural design and construction, climate, HVAC and lighting system design, occupancy and usage, system controllability, available measurements, data management, and control objective.

A second challenge of comparing MPC approaches is the relatively small number of field tests available, compared to the solution space of available approaches and applications. In such field tests reported in literature, it is common to document the performance of a single implementation for a particular application to demonstrate performance advantages over a more conventional method of control. It is important to point out that the choice and tuning of the benchmark controller has a direct influence on the improvements calculated for MPC. It is uncommon to consider and compare a range of methods. In addition, the real implementations are often not long-term studies, lasting weeks to months and not years, limiting the insight on how MPC strategies perform during all seasons, holidays, and other specialty types of days, as well as how much maintenance is required over time. Moreover, few studies report on or discuss implementation costs and payback periods.

A final challenge is defining the grounds for comparison. Common metrics are used in the literature associated with energy savings, operating costs savings, and occupant comfort improvement. However, other important bases of comparison of implementation and performance include computer hardware and software requirements, computation time, robustness to changing conditions, sensitivity to model and forecast uncertainty, data requirements, implementation effort, and installer expertise. Such a broad range of factors makes objective comparison difficult.

10.2. Literature

Studies that have compared specific MPC formulations are summarized in Table 16. All studies were performed using simulation and the baseline for comparison tended to be a centralized, linear, deterministic MPC implementation, except for one study that compared the use of two different nonlinear optimization solver algorithms. Each study utilized metrics related to energy use or cost and thermal comfort, while some other metrics included computational burden and setpoint tracking error. The results of each study were consistent with the hypotheses presented for each test implementation. For instance, stochastic and robust MPC can significantly improve the handling of disturbance or model uncertainty with respect to maintaining comfort compared to deterministic MPC, with only a small loss in energy savings potential. Another example is that distributed MPC can lessen the computational burden and communication requirements of a centralized MPC, with only small losses in energy savings potential and comfort. Differences in the studies, however, make it difficult to compare the implementations among each other and to evaluate the scalability of each technique in practice. First, each study considered a different building design, construction, climate, and HVAC system. In addition, each study considered different periods of operation, ranging from one hour to one year.

Other studies have focused on comparing various factors and techniques related to thermal envelope model development for the MPC. In (Blum et al., 2019b) seven factors affecting the accuracy of thermal envelope models were identified and their subsequent effect on MPC performance was tested, including building design, model structure, model order, identification data set, identification data quality, identification algorithm, and software tool-chain. The study showed that model order and initial parameter guesses during identification have strong influences. In (Sourbron et al., 2013b) the effects of model order and training data on final MPC performance for a concrete core activated HVAC system were studied. Studies in Picard et al. (2017, 2016) showed that linearizing detailed models, rather than building grey-box models, is a technique that works well. Other authors Harb, Boyanov, Hernandez, Streblow, and Müller (2016); Reynders et al. (2014) studied the effect of building and HVAC system type, training data, model order, noise, and measured inputs on parameter identification accuracy. Study in Reynders et al. (2014) found that a fourth order model was

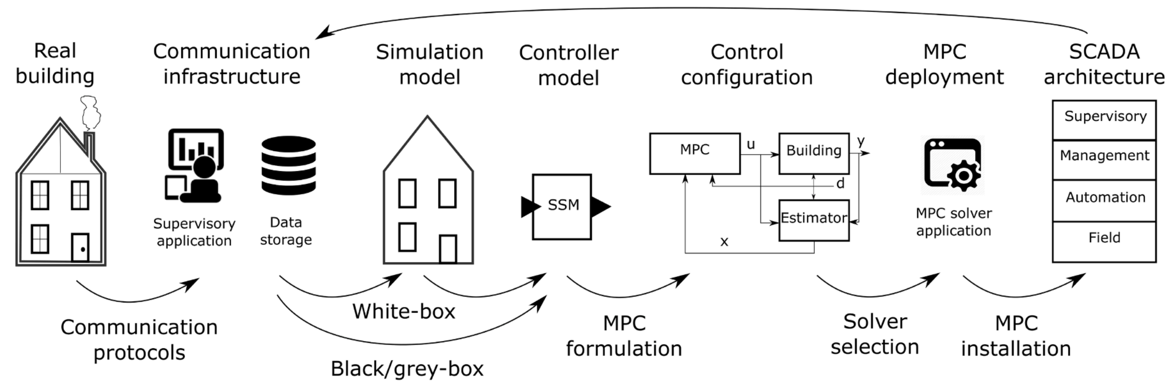


Fig. 12. A general methodology for modeling, design, and implementation of MPC in buildings based on (Drgoňa, Picard, & Helsen, 2020).

acceptable, while Harb et al. (2016) found that a second order model was acceptable, though neither tested these models in an MPC controller. Finally, (Vande Cavey et al., 2014) compared MPC performance with and without proper state estimation, showing the importance of using a well-tuned state estimator. Similar to the studies comparing specific MPC formulations, these studies suffer from not utilizing the same building cases or evaluation periods, making inter-study comparison difficult.

In addition to academic studies, Zurich (2020) and Cigler, Tomáško, and Široký (2013c) present tools developed to assess the performance of MPC. The BACTool (Zurich, 2020) represents a web-based tool that utilizes a large number of pre-calculated, yearly building energy simulation results to display performance indicators using MPC and one of two rule-based controllers. Users can build cases to compare from a number of inputs, including among them building construction, orientation, climate, HVAC system, and control type. Performance metrics that can be compared include energy use [kWh/m^2], comfort [K-h], and peak demand [W/m^2], as well as timeseries of indoor temperature, illuminance, blind position, and power demand of HVAC and lighting system components. In this way, users can evaluate the potential benefits of using MPC over rule-based control in a similar building project. Cigler et al. (2013c) presents BuildingLAB, a tool for illustrative and educational purposes related to MPC control in buildings. Users can change parameters such as prediction horizon, initial conditions, constraints, and objective function weights (e.g. of operating cost and discomfort) and execute simulations of building control using MPC, with optimal control results calculated upon execution using the given parameters. In this way, users can see the differences that result from changing parameters and gain intuition on expected performance.

10.3. Framework development

While the literature review presents a number of studies that compare two or more MPC formulation and modeling methods as well as tools that were designed to compare MPC performance among various conditions and parameter settings, performance and assessment of MPC lacks a unified framework designed to tackle the challenges outlined in the previous section. The literature studies are limited to the specific implementations and conditions under which they were compared, while the tools are limited to the building models and MPC approaches implemented by the tool designers. Instead, the framework needs to provide representative, yet bounded, testing conditions and scenarios which any control developer can use to test his/her individual approach. Such a framework is similar to the BESTEST (American Society of Heating Refrigerating & Air Conditioning Engineers, 2008), a set of building specifications and operating scenarios developed for benchmarking and comparison of building energy simulation tools. This can be implemented in the form of reference building models and simulation scenarios that represent a range of building and system types, are implemented with the necessary dynamics for controls design and testing represented, are available for use by all MPC researchers and control developers regardless of expertise in building simulation modeling, can be simulated within a controlled, yet distributable, computing environment, and are independent from the control implementation. In addition, the framework needs a reference set of performance indicators to objectively compare MPC controllers with respect to all or a chosen subset of these metrics. The metrics should include operational performance, such as energy, cost, and comfort, as well as implementation metrics, such as computational requirements and data needs. In this way, as MPC developers use such a common framework to test their implementations, true comparison and assessment can be done relative to other approaches, and development of high-performing, cost-effective MPC approaches can be accelerated.

While development of such a simulation framework presents its own set of challenges, the task is being undertaken in Blum et al. (2019a), presenting a BOPTEST framework (Building Operation Performance Test) consisting of various building types and software platform for the testing of advanced control strategies. The approach is similar to an existing platform called Alfalfa (National Renewable Energy Laboratory, 2020), which utilizes OpenStudio models for building simulation, implements a Project Haystack (2020) API to connect with potential test controllers and other data analytics platforms, and is designed to be a scalable web-service. The BOPTEST framework differs in that it utilizes FMI and Modelica for building simulation, will have an API for also providing disturbance forecasts to MPC controllers, utilizes a controller-blocked synchronization scheme rather than a real-time synchronization scheme, and also produces reports on key performance indicators. In the future, the BOPTEST framework aims to leverage the Alfalfa architecture to provide an industrial-strength tool for controls testing that provides the functionality of BOPTEST with the scalable architecture of Alfalfa.

11. Conclusions

This paper provides a complete overview and unified framework of MPC for building climate control applications.

MPC theory and problem classification The process of MPC formulation starts with the definition of control loop variables and its interconnections via constraints, objective functions, and a control-oriented building model. The theory behind this process is compactly summarized at the start of this paper. The paper presents three algorithmic principles behind MPC which are essential for real-time implementation. In particular, we talk about receding horizon control (RHC), state estimation, and optimal control solution methods. The details of the particular case, such as building model type (e.g., linear, nonlinear), comfort index (e.g., comfort zone, PMV), and other factors penalized in the objective function, together with imposed constraints are the key building blocks of the MPC formulation. Based on these features, MPC

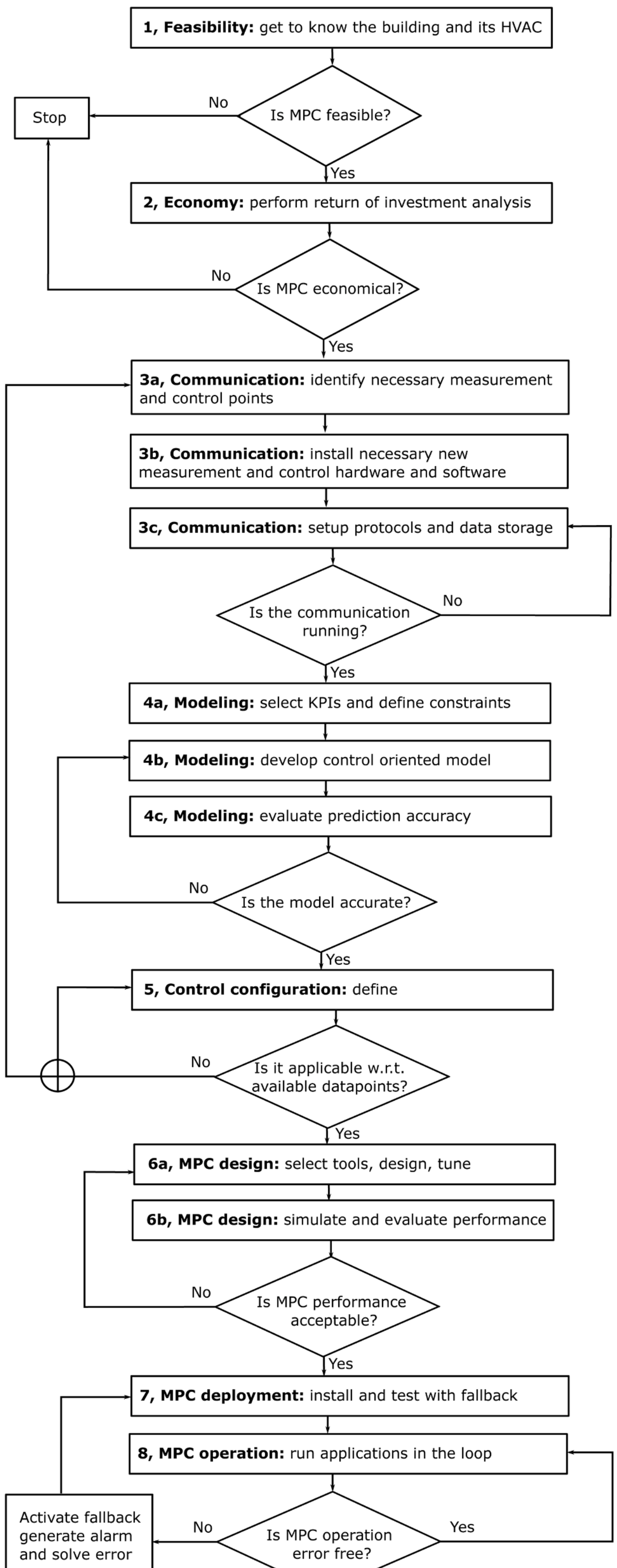


Fig. 13. Flowchart of MPC implementation in real buildings.

problems are classified into three important problem classes (linear, nonlinear, hybrid). Moreover, translation methods for direct optimal control and its use in association with each MPC class are discussed. A linear MPC formulation is computationally least demanding and thus easiest to implement. Many modeling tools support linear MPC with a wide variety of examples and tutorials. Even though it has certain limitations regarding the formulation flexibility, it is the most commonly used MPC class in the building sector, mainly because the building envelope can be accurately approximated by linear dynamics. Nonlinear MPC provides us with higher flexibility in formulation and possibly increased performance, due to the incorporation of the nonlinear HVAC model. On the other hand, this comes with the cost of more elaborate modeling and increased computational demands for implementation. Hybrid MPC is useful when one needs to deal with integer decision variables or switching dynamics like heat pump modes, etc., a situation very common in building applications. For the cost of increased computational demands, it can provide increased performance compared to the more straightforward linear case.

Algorithmic solutions of MPC Three MPC solution techniques based on direct methods, i.e., implicit, explicit, and approximate MPC, have been discussed with their pros and cons. MPC approaches have been further differentiated based on their problem class, solution approach, and dimensionality of the problem defining the computational complexity of the optimization problem, and thus determining the feasibility, as well as hardware and software requirements for real implementation. Building climate control applications have specific characteristics, such as a large number of state variables and slow dynamics resulting in longer sampling times. For these reasons, and increased availability of computation power, in recent years, MPC is most often being implemented by solving a corresponding optimization problem online in an implicit way. The drawback of this approach is the necessity of available computation power and software dependencies associated with dedicated optimization solvers. Such a method is universally applicable, with the biggest return of investment potential associated with larger tertiary buildings due to the smaller ratio on the investment cost compared to the overall construction or renovation costs. Explicit MPC has been proven to be feasible so far only for small case studies, limiting its applicability in practice in multi-zone building control problems. The potential use of this approach is within low-level control tasks or decentralized single-zone control strategies, e.g. for individual apartments within a block or small residential houses. Approximate explicit MPC solutions appear to be a promising alternative also for large-scale problems providing memory-based control policies with low computational footprints. The main strength of this approach is its numerically robust operation due to lightweight computation requirements with minimal software dependencies and its applicability even on lower-level hardware. The main drawback of such an approach, however, is the requirement of the original MPC and the need for larger training datasets, which can be computationally demanding and hence time-consuming to generate. The theoretical part of the paper is finalized by the formalism of uncertainties in the MPC problem and methods conventionally used for their mitigation. In particular, these methods are offset-free MPC via state augmentation, robust MPC, stochastic MPC, adaptive MPC, and learning-based MPC.

Software tools for building modeling and control For all types of MPC formulations and implementation approaches, a wide variety of modeling and design tools and solvers are available. The wide variety of used modeling tools reflects the lack of understanding of what model formulation and level of detail is best suited for MPC in buildings. The practical part of this paper summarizes an extensive overview and conceptual comparison of dedicated software tools used for building modeling, (co-)simulation, MPC design tools, and available optimization solvers for both desktop as well as embedded platforms. The aim of this overview is to help the reader with a selection of the most appropriate tool from the broad range of options.

Practical deployment of MPC in buildings To facilitate a faster transfer of the technology into practice, a whole section is dedicated to key building blocks and aspects of practical implementation. The underlying implementation framework is defined consisting of the MPC configuration, SCADA architecture, and communication infrastructure. Four conceptual types of MPC configuration are considered, namely

Table 16
Studies comparing two or more MPC formulations.

Ref	MPC comparison	Case	Metric(s)	Result
Oldewurtel et al. (2012)	Stochastic (SMPC) vs. Deterministic (DMPC)	Single room with six variants of HVAC system, European locations, and building construction. Simulation period is one year.	Energy use [kWh/m^2/y] and comfort violations [Kh]	SMPC had comparable energy use (slightly higher) and comfort violations to best case DMPC.
Drgoňa et al. (2013)	Stochastic (SMPC) vs. Deterministic (DMPC)	Single room with simple heating and cooling. Simulation period is nine days.	Energy use [kWh] and comfort violations [% Simulation Samples]	SMPC had comparable energy use (slightly higher) and comfort violations to best case DMPC.
Ma et al. (2015)	Stochastic (SMPC) vs. Deterministic (DMPC)	Multizone VAV HVAC system in Berkeley, CA, USA. Simulation period is 55 days.	Energy savings compared to rule-based control [%], comfort improvement compared to rule-based control [%], thermal efficiency of HVAC system [−]	SMPC had comparable energy savings (slightly less) and comfort improvement over rule-based control to best case DMPC.
Maasoumy et al. (2014)	Robust (RMPC) vs. Deterministic (DMPC)	Single room in Houghton, Michigan, USA with ground-source heat-pump heating system. Simulation period is one day.	Energy use [kWh] and comfort violations [°C-h]	For intermediate levels of model uncertainty, RMPC outperformed DMPC, while DMPC is preferred for low levels of model uncertainty. If model uncertainty is very high, rule-based control is preferred.
Scherer et al. (2014)	Distributed (DisMPC) vs. Centralized (CenMPC)	Multiple zones each served by fan coil units served by common hot and chilled water central plants. Simulation period is one hour.	Integral of squared setpoint error [$°C^2$]	DisMPC was able to have similar setpoint tracking performance to CenMPC when central plant resources are limited.
Walker, Lombardi, Lesecq, and Roshany-Yamchi (2017)	Distributed (DisMPC) vs. Centralized (CenMPC)	Three-zone open office in Cork, Ireland where each zone has radiator and window operation. Simulation period is nine hours.	Energy use [kWh], temperature and CO_2 setpoint tracking (visually in plots), and normalized computational time [−].	DisMPC had comparable energy use to CenMPC (slightly higer) and similar temperature and CO_2 tracking with less computational burden on each local controller.
Pcolka, Zacekova, Robinett, Celikovsky, and Sebek (2014)	Nonlinear (NLMPC) vs. Linear (LMPC)	One zone building with radiant ceiling HVAC system in Prague, Czech Republic. Simulation period is three months.	Energy cost [Euro], maximal comfort violation [°C], and hours of comfort violation larger than 0.2 °C [h].	NLMPC outperforms LMPC by using less energy, having less maximum comfort violation, and having less total hours of discomfort.
Putta, Zhu, Kim, Hu, and Braun (2013)	Affine Quadratic Regulator (AQR) vs. Sequential Quadratic Programming (SQP)	Single room in Indiana, USA with VAV AHU and cooling plant.	Energy cost [$/day], discomfort cost [$/occupant/day, and computational time [s/decision]	AQR saved significantly on discomfort costs compared to SQP due to SQP sensitivity to initial guesses and local minima.
Drgoňa and Kvasnica (2013)	Setpoint Tracking (ST) vs. Comfort Tracking (CT) vs. Number Comfort Violation Min (CM)	Single room with simple heating and cooling.	Energy use [kWh], energy savings compared to rule-based control [%], and comfort violations [% Simulation Samples]	ST used most energy with good comfort control. CT used less energy with worse comfort control. CM used least energy with good comfort control.

centralized, decentralized, distributed, and hierarchical configurations, and their usability is discussed. Centralized MPC controls an entire system and is currently the most commonly used configuration in building applications. Decentralized configurations with multiple local MPCs are less favorable for buildings due to the loss of dynamic coupling between controlled subsystems. Distributed MPC represents a more favorable configuration and is based on solving a decoupled problem by communicating the local solutions to other sub-controllers to improve the entire system performance. Meanwhile, hierarchical configurations improve the overall performance when controlling the system over different time scales and including the subsystems with notable differences in their time constants. Examples of such systems are demand response control or long-term behavior of a ground source heat exchanger coupled to the short-term behavior of the building. The SCADA architecture defines the standards for modern industrial hierarchical control systems with four basic levels, which are widely adopted in modern buildings. In practice, this functionality is provided by the building automation systems (BAS) via commercial vendors like Honeywell, Johnson Controls, Priva, Siemens, Schneider Electric, ABB, or Delta Controls, to name the most prominent ones. A functional, automated, and full-scale communication outside the commercial BAS appears to be currently one of the tedious tasks of real MPC implementation. Although they can be built mostly on open standards, the problem lies in a large number of used communication protocols, closed commercial BAS software solutions, and lack of standardized interfaces which make the integration of hardware components from different vendors a real challenge. With practical cases in mind, clear guidelines and a flowchart for MPC implementation are provided for researchers and early adopters of the technology. The fundamental steps of any successful application are based on preliminary feasibility and economic studies guiding the decision of whether to implement the MPC for a particular case or not. The design phase consists of setting up the communication, followed by control-oriented building modeling, control configuration selection, and MPC design and tuning. The operation phase consists of testing and deployment of the MPC algorithm with backup solutions.

Performance assessment of MPC in buildings Comparison and performance assessment of MPC in buildings plays an important role during the selection of an appropriate strategy for a particular application. However, due to the large solution space, there remain a number of challenges to be tackled on the roadmap towards generalized performance assessment methodology and tools. First initiatives are being taken to standardize this process in a scalable framework built upon next-generation building energy modeling tools that emulate the response of the building system to the MPC controller, using predefined performance indicators and application programming interfaces, all brought together in the BOPTEST.

Market potential and future of MPC in buildings The practical aspects of integration of MPC algorithms with contemporary BAS create an opportunity for startup companies to deliver customized MPC solutions backed by universal SCADA platforms with multi-protocol, multi-manufacturer compatibility. Examples of such companies are e.g. DeltaQ, IES, BuildingIQ, Feramat Cybernetics, Energocentrum with their Mervis control as a service platform, or QCoefficient, Inc. which successfully operates cloud-based real-time white box MPC based on EnergyPlus models in a number of large commercial office buildings in the US.

A big leap forward in MPC market penetration can also be made by implementing MPC applications into integrated software platforms, enabling the communication management and control of diverse systems regardless of manufacturer or protocol. The most notable of such communication platforms are the commercial Niagara Framework®, or the open-source Volttron™. It is very hard to make predictions, especially about the future. However, based on the advanced stage of basic research tackling the current bottlenecks of MPC, several pilot case studies, emerging startups, and awareness of the major companies in the field of building controls, the large-scale market penetration of MPC technology for newly built buildings can be optimistically expected to happen within the next decade.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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