

# Reduced order modeling and parameter identification of a building energy system model through an optimization routine

V.S.K.V. Harish\*, Arun Kumar

*Alternate Hydro Energy Centre, Indian Institute of Technology Roorkee, Roorkee, Uttarakhand 247667, India*

## HIGHLIGHTS

- A BES model based on 1st principles is developed and solved numerically.
- Parameters of lumped capacitance model are fitted using the proposed optimization routine.
- Validations are showed for different types of building construction elements.
- Step response excitations for outdoor air temperature and relative humidity are analyzed.

## ARTICLE INFO

### Article history:

Received 29 July 2015

Received in revised form 9 October 2015

Accepted 20 October 2015

Available online 14 November 2015

### Keywords:

Building energy systems

Non-linear time invariant constrained optimization

Numerical model

State space

Lumped capacitance model

## ABSTRACT

Different control techniques together with intelligent building technology (Building Automation Systems) are used to improve energy efficiency of buildings. In almost all control projects, it is crucial to have building energy models with high computational efficiency in order to design and tune the controllers and simulate their performance. In this paper, a set of partial differential equations are formulated accounting for energy flow within the building space. These equations are then solved as conventional finite difference equations using Crank–Nicholson scheme. Such a model of a higher order is regarded as a benchmark model. An optimization algorithm has been developed, depicted through a flowchart, which minimizes the sum squared error between the step responses of the numerical and the optimal model. Optimal model of the construction element is nothing but a RC-network model with the values of  $R_s$  and  $C_s$  estimated using the non-linear time invariant constrained optimization routine. The model is validated with comparing the step responses with other two RC-network models whose parameter values are selected based on a certain criteria. Validations are showed for different types of building construction elements viz., low, medium and heavy thermal capacity elements. Simulation results show that the optimal model closely follow the step responses of the numerical model as compared to the responses of other two models.

© 2015 Elsevier Ltd. All rights reserved.

## 1. Introduction

A number of methods have been developed to construct load models or energy consumption models that simulate a building/plant system for load prediction or cost saving estimates. Such models vary in magnitude from modeling of a single slab (or a wall) [1] to modeling of a complete building through modeling of rooms subjected to temperature variations. A three stage process for model formulation was illustrated in [2]. In the first step, the building system is converted from continuous state to a discrete state. This involves selection of nodes at the points under study, representing the homogeneous or non-homogeneous control

volumes like that of internal air mass, boundary surfaces, building fabric elements, renewable energy systems, equipment of the room, etc. Equations satisfying mass, momentum and energy conservation principles are developed in the second step for each node which is in thermodynamic contact with its surrounding nodes. Last step involves solving the equations derived in the second step for successive time steps to obtain state variables of the node for future time periods as a function of present time state variables with the boundary conditions prevailing at both times.

Models developed to simulate the building energy systems can be divided into many types. Basically, models are classified as physical, symbolic and mental models. Symbolic models are comparatively less complex and are thus frequently used. Models can be mathematical and non-mathematical models. Development of mathematical model of a system involves mapping of the physical

\* Corresponding author. Mobile: +91 8979552840.

E-mail addresses: [harishvskv.iitr@gmail.com](mailto:harishvskv.iitr@gmail.com), [rkhrvdah@iitr.ac.in](mailto:rkhrvdah@iitr.ac.in) (V.S.K.V. Harish).

## Nomenclature

|               |  |               |  |
|---------------|--|---------------|--|
| BES           | building energy systems  | $\alpha'$     | thermal diffusivity of the material ( $m^3/s$ )                              |
| BEMS          | Building Energy Management Systems   | $\omega'$     | weighing factor  |
| BAS           | Building Automation Systems  | $Fo'$         | Fourier's number   |
| BESPs         | Building Energy Simulation Programs  | $u(t)'$       | time step response function  |
| FiD Method    | (Finite difference) method   | $f_{SSE}(T)'$ | sum-squared error function   |
| 3R-2C network | lumped capacitance network with three series resistances and two parallel capacitors | $A'$          | cross-sectional area of multi-layer construction element ( $m^2$ )           |
| ANN           | Artificial Neural Network  | $Q'/q'$       | rate of heat transfer ( $kW/m^3$ )   |
| CN            | Crank–Nicholson  | $L'$          | length of a multi-layer construction element (m)                             |
| PDE           | partial differential equation  | $T'$          | temperature values (K)/temperature vector                                    |
| $'N'$         | number of layers in a multi layered wall   | $\rho'$       | density of $i$ th layer of a multi layered wall ( $kg/m^3$ )                 |
| $R_T'$        | Total Thermal Resistance (K/W)   | $C_{pi}$      | specific heat capacity of $i$ th layer of a multi layered wall ( $kJ/kg K$ ) |
| $r_{si}'$     | internal surface resistance of a wall (K/W)  | $C_T'$        | total thermal capacitance (J/K)  |
| $r_{so}'$     | external surface resistance of a wall (K/W)  | $k_i'$        | thermal conductivity of $i$ th layer of a multi layered wall ( $kW/m K$ )    |
| $d_i'$        | thickness of $i$ th layer of a multi layered wall (m)                                | $R_3'$        | inner thermal resistance (K/W)   |
| $R_1'$        | outer thermal resistance (K/W)   | $C_2'$        | inner capacitance (J/K)  |
| $C_1'$        | outer capacitance (J/K)  | $B_i'$        | Biot number  |
| $R_2'$        | wall thermal capacitance (K/W)   | LTI           | linear-time invariant  |
| HVAC          | heating, ventilation and air-conditioning  | $C_{pa}$      | specific heat capacity (J/kg K) of air in the building space                 |
| $\rho_a$      | density of air in the building space ( $kg/m^3$ )                                    | $Q_{hvac}$    | heat output from hvac system. +ve if heating and –ve if cooling (W)          |
| $Q_{casual}$  | casual heat (due to occupants) gain (W)  | $N$           | no. of air changes per hour from the hvac system (per hour)                  |
| $T_{cs}$      | temperature of the building space to be conditioned by hvac system ( $^\circ C$ )    |               |  |
| $U_{win}$     | window $U$ -value ( $W/m^2 K$ )  |               |  |
| $t'$          | time (s)   |               |  |

laws governing the dynamics of the system's process into mathematical relations using variables and constants. Due to ease in evaluation and manipulation mathematical models are the most suitable and the most widely used category of models [3]. Mathematical models can be of theoretical and experimental type. As name suggest, theoretical models involve breaking down of a larger system under study into a number of smaller and simpler subsystems. Mathematical equations constrained through physical laws are then used to relate the different subsystems. On the other hand, experimental models are developed through empirical relations i.e., through measurement of input and output signals of the system and then, evaluating the system's response. Such models don't provide any information about the mechanics or behavior of the system. Differential or difference equations along with the use of soft computing techniques like fuzzy are made use of in experimental modeling.

Models are also classified as White box, Grey box or Black box models. White box modeling of buildings involve a detailed description of the heat transfer processes occurring in the building. A thorough understanding of the system and all influential sub processes is required to efficiently describe the dynamics of a building energy system [4]. Also called as semi-physical models, Grey box models are inherited from the white box models [5] but the parameters of the model defining the system are not measured directly but estimated through various identification processes [6]. Models which do not normally contain any physical knowledge regarding the system (majorly due to lack of knowledge about the physical structure of the system) are called as black box models. Statistical methods are used to formulate the model [7] and the physical parameters are partly hidden in the discrete time parameterization. Constructing an accurate and a generic model to interpret the thermal dynamics of a building involves solving heat transfer equations of conduction, convection and radiation and mass transfer equations.

Building Energy Management Systems (BEMS)/Building Automation Systems (BAS) are employed for achieving energy efficient targets of the owners thereby, reducing operating costs

through better supervisory controls. A number of previous studies have shown potential savings for optimized controls in the range of 10–40% of costs to provide heating, cooling & ventilation [8,9]. The primary objective of the building energy managers is ensuring occupant comfort at minimum operational cost. Considerable adaptive and predictive control strategies, based on accurate energy modeling & monitoring, are developed. Building Energy Simulation Programs (BESPs) such as TRNSYS, Energy Plus, and Design Builder, are utilized for such purposes. However, in order to achieve cost-effective implementations, the associated control design and implementation must be automated for deployment in a scalable manner.

Such BESPs are high fidelity tools as they involve solving a large number of energy balance equations for every time step to evaluate the performance of the BES under study. Moreover, direct usage of BESPs for evaluating the energy performance indices for a BES under study is complex and can sometimes, lead to an over or under estimation of building energy consumption. In order to investigate developed control strategies to BES, a sufficiently accurate energy model with reduced order is necessary.

Literature articles [7,10–13] are available which have developed model order reduction techniques in state space domain using optimization techniques of genetic algorithms, solving Kuhn–Tucker equations using sequential quadratic programming, etc. Such works involve either empirical modeling [5,7,14,15] of the building space under study through the use of sensors and energy monitoring software or using a BESP to develop a benchmark model and then, adopting an order reduction technique.

Each BES is unique in terms of its application, operation, occupancy pattern and environmental/location aspects and thus, require different engineered solutions for enhancing the energy performance of the building. Constructing an accurate and a generic model to interpret the thermal dynamics of a building involves solving the heat transfer equations of conduction, convection and radiation, mass transfer equations and energy balance equations. In this paper a generalized BES model is developed with the parameters estimated through an optimization routine.

A standard white box BES model is developed using “first principles of building physics” and is regarded as an accurate model. The conduction equations of thermal energy transfer through the building envelope are formulated as finite difference equations and are solved using Crank–Nicolson scheme.

In order to reduce the model order and hence, its complexity a non-linear time invariant optimization algorithm is developed. The algorithm assists in estimating the parameters of the reduced order state space model. Step responses of both the standard FID (Finite difference) model and reduced order (3R-2C) model are compared and sum-squared error of both the responses are minimized to estimate the numerical values of the parameters of the 3R-2C model. Proposed methodology and model of building energy systems shall lead to development of a simple and computationally efficient mapping of building physics. Overall outcome of the proposed research will lead to development of a computationally efficient (an accurate and suitably fast simulated) BES model and thus, an energy efficient analysis of a building's energy systems.

The remainder of the paper is structured as follows. Section 2 introduces the applicability of optimization techniques for estimating the parameters of a building energy model. Notable literature works which form the basis of the proposed technique is presented. Section 3 gives a brief description of building energy system modelling. Section 4 introduces lumped capacitance modelling of BES and state space analysis for a 3R-2C network model is presented. Section 5 describes the finite difference approximation of BES modelling and solves the same using CN-scheme. Section 6 presents the optimization algorithm depicted through a flowchart and describes the developed objective function to be optimized. Section 7 validates the developed routine and Section 8 presents conclusions with a few notable points for future study.

## 2. Application of optimization techniques for parameter estimation of BES models

As per [16], Optimization theory encompasses the quantitative study of optima and methods for finding them. A general mathematical description of a simpler optimization problem [17] can be given as:

$$\begin{aligned} \text{Minimize } & F(x_1, x_2, \dots, x_m) \\ \text{Subject to : } & G(x_1, x_2, \dots, x_m) \geq 0 \\ & X_i \in S_i \end{aligned}$$

The problem above can be of many types and variables. For example a multi variable – multi objective optimization problem can be formulated as:

$$\text{Minimize } [F_1(x_1, x_2, \dots, x_m), F_2(x_1, x_2, \dots, x_m), \dots, F_3(x_1, x_2, \dots, x_m)]$$

Function,  $G$  represent for a number of constraints that are by convention greater than or equal to zero. Each design variable  $x_i$  is constrained to certain values  $S_i$ , defined either as discrete values or by boundary values [18].

The best optimization procedure is problem dependent [19]. Optimization techniques such as genetic algorithms, sequential quadratic programming, fuzzy logic and ANNs have been used to estimate the parameters of the building energy model by minimizing an error function. Usually, root mean square error [20–22,12], sum squared error [23], mean absolute error/mean absolute percentage error [24], cumulative variation of root mean square error [25,26] between the estimated and measured values of the parameters are used as objective functions in the optimization problem. Optimization methods have been made use of in building energy systems research in applications of parameter estimation [27], reduced order modeling [23], building design [18,28], building

performance analysis [29], etc. In [30] used a subspace trust region solver based on the interior-reflective Newton method [31,32] to estimate the parameters of their black box model.

Wang and Xu [33–36] developed a method to identify the parameters of a simplified building model based on frequency characteristic analysis. The developed methodology involved computing the theoretical frequency characteristic of heat transfer of the building envelope, followed by deducing the frequency characteristic of the simplified model and then developing an objective function of minimizing the amplitude and phase lag of the theoretical and the simplified model. McKinley and Alleyne [30] developed a process to determine a building's time invariant parameters and time-varying source terms. Lu and Viljanen [37] developed a mathematical procedure to solve a one dimensional heat transfer equations analytically and validated against numerical solution.

Chantrelle et al. [38] analyzed renovation options to optimize cost, energy use and comfort by varying constructions and control options. The optimization was performed using a Genetic Algorithm and the simulation was with TRNSYS and COMIS. The increase in computational requirements of using three objectives were not explicitly assessed, but were stated to be not excessive. A descriptive list of notable research articles from reputed journals have been listed out in Table 1.

### 2.1. Parameters estimated in building energy modeling

The parameters and thus variables defining them affecting cooling load calculations are numerous (see Table 2); often difficult to define precisely, and always intricately interrelated. Many energy consuming components vary widely in magnitude, and possibly direction, during a 24 h period. Because these cyclic changes in load components often are not in phase with each other, each component must be analyzed to establish the maximum energy load for a building or zone. A zoned system (i.e., one serving several independent areas, each with its own temperature control) needs to provide no greater total cooling load capacity than the largest hourly sum of simultaneous zone loads throughout a design day; however, it must handle the peak cooling load for each zone at its individual peak hour. At some times of day during heating or intermediate seasons, some zones may require heating while others require cooling. The zones' ventilation, humidification, or dehumidification needs must also be considered.

## 3. Building energy systems (BES) modeling

Building energy systems (BES) can be defined as those which are responsible for consumption of energy in buildings [27]. These can be any physical equipment or machinery or can be a process or a combination of them.

All the three modes of heat transfer processes viz., conduction, convection and radiation take place within the room space. Sensible heat transfer takes place within the conditioned room space through conduction, convection, and/or radiation whereas latent heat transfer occurs due to transfer of moisture (emission of water vapour by in room equipment and occupants) in and out of the room space [43]. The building elements of a perimeter room, responsible for heat and mass transfer processes are given in the Table 3.

Fig. 1 shows a simplified representation of the developed building energy model.

## 4. Lumped capacitance/RC-network modeling for BES

LTI system are usually analyzed when the model is excited under impulse, step and/or ramp/pulse responses. Step response

**Table 1**

Summary of works pertaining to parameter estimation of control relevant BES models.

| Work source | Subsystem modeled   | Model type               | Parameters used for modeling  | Experimental realization | Occupancy | Optimization  | Objective function   | Simulation period | Simulation platform     | Validation                             |
|-------------|---|--------------------------|---|--------------------------|-----------|---|--|-------------------|-------------------------|--|
| [10]        | Building space (two external walls, an internal floor, internal ceiling and two partitions) |                          | Outdoor temperature and solar radiation                                     | No                       | No        | Non linear constrained optimization   | Parameter estimation of 3R2C model                           | 465 h             | MATLAB                  | Analytical                             |
| [30]        | Building space (a room)   | Lumped capacitance       | Thermo physical properties, solar loads on outer surface of walls           | No                       | No        | Unconstrained Hill Climbing algorithm   | Root mean squared error for temperature and humidity         | 22 h              | Energy plus, MATLAB 7.1 | Validated against model generated data |
| [39]        | Single zone building room   | Lumped capacitance model | Indoor air and surface temperature, outdoor surface and ambient temperature | Yes                      | No        | Unconstrained optimization  | Sum squared error minimization                               | 9 days            | MATLAB/Simulink         | Experimental                           |
| [40]        | Conduction heat flow through walls, Convection flow through zonal air mass                  | Black box                | Zonal temperature, zonal humidity ratio, supply air flow rate               | No                       | No        | Unconstrained optimization  | Minimization of prediction cost function                     | 50 h              | MATLAB                  | On site measurement validation         |
| [41]        | Room (wall, window, roof, floor), heater and ac plant                                       |                          | Room geometry, room zone, radiation data, Predicted Mean Vote index         | No                       | Yes       | Generalized predictive control, dynamic matrix control, Multi objective optimization, Fuzzy logic | Minimization of cost function, min-max of energy and comfort | 3 h               | MATLAB/Simulink         | None                                   |
| [42]        | Thermal comfort, visual comfort, indoor air quality   |                          | Illumination level, carbon dioxide concentration, room temperature          | No                       | Yes       | Multi objective particle swap optimization  | Minimize energy & maximize comfort                           | Not mentioned     | MATLAB                  | None                                   |

describes relationship between an input that changes from zero to a constant value abruptly and the corresponding model response (output). Although the step response of a building construction element is not directly applicable to realize practical problems [43], analyzing BES model using step response is useful as its time derivative is the response of the wall to an impulse excitation. Impulse response forms the basis for determining the response of BES model to an arbitrary excitation.

Application of analytical or numerical methods to model conduction heat transfers through multi-layer construction elements account for high accuracies especially when applied for larger time horizons. Step response analysis of a BES model enables characterizing the performance of the system and can be applied to models simulated for short or long time horizons. An important point to be kept in mind, is to analyze and check for the stability of the BES model under step excitation when applied for long and short time horizons. For transient or short time horizon simulations numerical models of building energy systems exhibit high computational demands in terms of speed and computer memory [43].

Also, to analyze and evaluate control system responses & optimization a lower model bearing sufficient accuracy is essential.

Such models of lower computation demands can be constructed by treating each building element as one or a small number of 'lumps' in which uniform thermal response is assumed.

In order to evaluate energy saving/conservation measures, simplified lumped parameter building energy models have been incorporated since the late 1970s. Moreover, lumped parameter energy models have enabled fast simulation times with appreciable accuracies. However, inadequacies such as erroneous treatment of ventilation &/or infiltration due to identical treatment of convective & radiative heat gains; poor dynamic performance for high thermal capacity constructions and approximate modelling of inter-zonal conduction persisted with the lumped parameter models. Various adjustments have been made to the RC-network configurations and their values to put the accuracy of the simulated building energy model in accordance to that obtained from analytical/numerical methods. One such approach using non-linear time invariant optimization technique has been proposed in our works.

A multi-layer construction element can be modelled as a continuous network comprising of thermal resistances and capacitances.

**Table 2**

List of the significant parameters used in development of BES models.

| S. no. | Name  | Description  | Nature         | Application <sup>a</sup> |
|--------|---|--|----------------|--------------------------|
| 1      | Solar radiation                                     | Incident solar radiation is the major thermal load at the building envelope's exterior   | Uncontrollable | R/NR                     |
| 2      | Sol-air temperature/ outside air temperature        | Sol-air temperature is the outdoor air temperature that, in the absence of all radiation changes gives the same rate of heat entry into the surface as would the combination of incident solar radiation, radiant energy exchange with the sky and other outdoor surroundings, and convective heat exchange with outdoor air [5] | Uncontrollable | NR/R                     |
| 3      | Room air temperature                                | Indoor temperature variations depend on the purpose and occupation of the building   | Controllable   | R/NR                     |
| 4      | Thermo-physical properties of construction elements | Thickness, density, specific heat/ thermal capacity, thermal conductivity  | Uncontrollable | R/NR                     |
| 5      | Wind characteristics & precipitation                | Wind speed, wind direction, and terrain roughness  | Uncontrollable | NR                       |
| 6      | Internal heat gains                                 | Internal heat gains from people, lights, motors, appliances, and equipment can contribute the majority of the cooling load in a modern building  | Controllable   | NR                       |
| 7      | Sky or cloud conditions                             | Shading, cloudiness of the outdoor weather conditions  | Uncontrollable | NR/R                     |
| 8      | Ventilation rate                                    | Flow rate due to intentional introduction of air from the outdoors into a building   | Controllable   | NR                       |
| 9      | Building location (global information)              | Information about latitude, longitude, time zone, month, day of month, directional orientation of the zone, and zone height (floor to floor)   | Uncontrollable | NR/R                     |

<sup>a</sup> R: Residential buildings; NR: Non-residential (commercial and industrial) building.

For an 'N' layered wall:

Total Thermal Resistance per unit area is given by:

$$R_T = r_{si} + r_{so} + \sum_{i=1}^N \frac{d_i}{k_i} \quad (\text{m}^2 \text{ K/W}) \quad (1)$$

Total thermal capacitance per unit area is given by:

$$C_T = \sum_{i=1}^N d_i \rho_i C_i \quad (\text{J/m}^2 \text{ K}) \quad (2)$$

#### 4.1. State space analysis of the lumped capacitance model network

The optimization approach being developed for estimating and identifying the RC-parameters of the desired lumped capacitance model of the building construction elements involves the use of state-space approach of modeling with due considerations to one-dimensional transient conduction of heat through walls, transmission of solar radiation through windows, linearized radiation exchange of heat within the internal surfaces of the construction elements, and also, the ability to separate individual walls into sub-sections and to specify multiple coupled zones (beyond scope of the paper). The linearized state-space representation of the building energy system model can be written in vector form as:

$$\dot{T} = A\tilde{T} + B\tilde{U} \quad (3)$$

where  $T$  is the temperature vector comprising of all discretized wall nodal temperatures and also, local zonal temperatures in contact with interior wall surfaces,  $U$  is the input vector with temperature and load parameters acting directly on the zonal temperatures (e.g., convective internal gains, HVAC load value).

Now, depending on the type of parameter to be controlled, output of the state space BES model is written as:

$$Y = C\tilde{T} + D\tilde{U} \quad (4)$$

where  $Y$  is the output vector with the outputs of interest (e.g., room air temperature, HVAC load, etc.).  $A$ ,  $B$ ,  $C$  and  $D$  are the coefficient matrices expressing relationship between the state vector outputs and inputs.

For a second order RC network system with nodal temperatures of  $T_x$ ,  $T_y$  (as shown in Fig. 2), and the state equations for the BES model is given as:

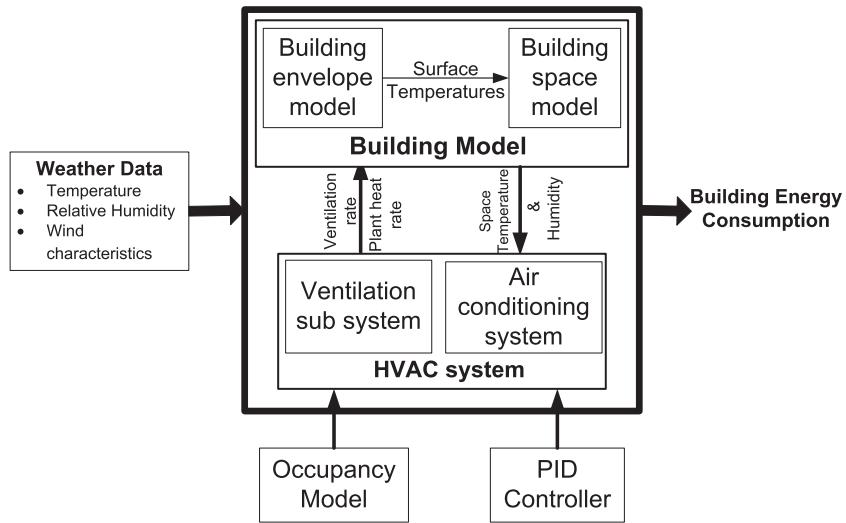
$$\begin{pmatrix} \dot{T}_x \\ \dot{T}_y \end{pmatrix} = \begin{pmatrix} -\left\{\frac{1}{R_1} + \frac{1}{R_2}\right\} \frac{1}{C_1} & \frac{1}{R_2 C_1} \\ \frac{1}{R_2 C_2} & -\left\{\frac{1}{R_2} + \frac{1}{R_3}\right\} \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} T_x \\ T_y \end{pmatrix} + \begin{pmatrix} \frac{1}{R_1 C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & \frac{1}{R_3 C_2} \end{pmatrix} \begin{pmatrix} T_{out} \\ Q_{hvac} \\ T_{bs} \end{pmatrix} \quad (5)$$

#### 5. Development of the standard BES model for benchmarking

The proposed modeling methodology involves developing a standard model for benchmarking against which the response of the target model (with reduced order; 3R2C 2nd order RC network) will be compared and the values of the target model will be identified by using a non linear optimization algorithm. The Fourier's heat conduction equation is solved using a finite difference algorithm for each of the construction element. Methodology for development of the benchmark model is described below.

**Table 3**  
Building elements and modes of heat and mass transfer processes.

| Heat and mass transfer processes                                   | Building elements  |
|--|--|
| Conduction and/or radiation heat transfer                          | External wall, roof, ceiling & floor slabs and internal partition wall, doors, skylights |
| Conduction heat transfer and solar radiation transmission          | Window glazing   |
| Conduction and/or radiation heat transfer and moisture dissipation | Occupants, lights, and other equipment   |
| Convection heat and mass transfer                                  | Infiltration from outside and adjoin rooms/lobby   |



**Fig. 1.** Overall structure of Building Energy System (BES) model.

Over several years, researchers round the world have developed a number of techniques to treat conduction heat transfer for a multilayer construction element which acts as a main classifier of a building energy model. Both analytical (using laplace transforms, response factor method, transfer function method) and numerical (finite element method, finite difference method (FiDM), etc.) methods have used to solve the heat balance equation. In the present study, finite difference method of Crank–Nicholson scheme has been used to solve the one dimensional PDE heat transfer equation.

Finite difference equations are derived using the Crank–Nicholson method which involves dividing the spatial and time domain into a large number of sub-divisional spatial planes/segments/ slices and time steps, respectively. The heat balance equation for a multi-layer construction element is a one dimensional (in space) partial differential equation.

For a 'L' length multi-layer construction element, the FiDM involves division of the complete element into a large number of equally spaced sub-divisional planes. Such planes are developed in both time and space domain.

### 5.1. Assumptions

- (1) Heat transfer is the only energy interaction and there is no heat generation. There is no generation or accumulation of the heat within the building construction elements. The temperature of each surface or surface segment is uniform within its cross section.
- (2) The wall surface is nearly isothermal. Transfer of thermal energy takes place in a direction from high temperature to lower temperature regions of the system under study and the heat transfer takes place continuously to and from the outdoor environment through the building construction element depending on summer and winter days. No transfer of heat takes place when there is no change in temperature i.e., under zero or negligible temperature gradients. There is considerable temperature gradient available between inner and outer surfaces of the building construction element and empirically, through measurements it has been found that temperatures at bottom and top as well as right and left end of the slab are almost same [44].

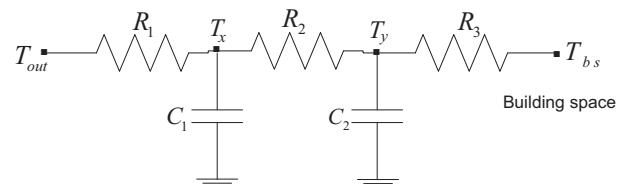
- (3) Air of building space under study is well mixed. This implies that the contamination concentration is directly proportional to the number of air changes in the building space. Such an assumption stands valid for the proposed research as the diffusion and slow mixing ensures uniform concentration or the contaminant is brought in only by inflow air [45,46].
- (4) Each wall emits or reflects diffusely and is gray and opaque.
- (5) Air is a non-participating media with respect to radiation. Outdoor space and atmospheric air flowing over short spaces are considered as non-participating medium for radiative heat exchanges i.e., the photons travel almost unimpeded from the outer surface of the building construction element to the inner surfaces [47].
- (6) Heat transfer is one-dimensional i.e., thermal energy transfer that takes place along the direction of the thickness of the building construction element (along x-axis) is considered for modeling. Energy transfer along y- and z-directions have been ignored. No significant transfer takes place in other directions.
- (7) Conduction between the window and window frame is neglected (1-D assumption). Heat transfer through conduction is majorly through the walls only because of greater temperature gradient as compared to other elements.

### 5.2. Energy dynamics

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} = 0 \quad (6)$$

That implies, the rate of heat transfer through a sub-divisional plane is constant i.e.,

$$\dot{Q}_{cond,wall} = K$$



**Fig. 2.** 3R-2C network modeling of a building construction element.

where

$$K \in \text{constant}$$

The Fourier's law of heat conduction transfer can be expressed for the multi-layer construction element under study as:

$$\dot{Q}_{\text{cond,wall}} = -kA \frac{dT}{dx} \quad (\text{W}) \quad (7)$$

For a particular sub-divisional plane, the cross-sectional area and rate of heat transfer through conduction are constant and thus, the temperature through the sub-divisional plane of the multi-layer construction element shall vary linearly with its thickness i.e., along  $x$  as shown in Fig. 3.

The temperature at each point over the thickness of the wall can be defined as a space- and time-dependent function,  $T = T(x, t)$ , where  $x$  is the space variable, and  $t$  the time variable. For an infinitesimally small element of volume  $\Delta V$ , the heat flux incident on one surface and that incident on the opposite surface is as shown in Fig. 3

The governing partial differential heat conduction equation for a temperature,  $T$  is given as:

$$\rho C \frac{\partial T}{\partial t} - \nabla \cdot k \nabla T - \dot{Q} = 0 \quad (8)$$

where

$\rho$ : density ( $\text{kg}/\text{m}^3$ )

$C$ : specific heat capacity ( $\text{kJ}/\text{kg K}$ )

$K$ : thermal conductivity ( $\text{kW}/\text{m K}$ )

$T$ : temperature (K)

$t$ : time (s)

$\dot{Q}$ : internal heat gain rate within the material ( $\text{kW}/\text{m}^3$ )

Eq. (8) takes into account all the factors of thermal energy transfer processes occurring within a multi-layer construction element. For thermal energy modeling of building construction elements following set assumptions stand valid.

- (1) Thermal energy balance is considered along the direction in which the thickness of the building construction element is infinitesimally large, i.e., along  $x$ . Thermal energy flow along  $y$ - and  $z$ -directions are negligible and thus, ignored.
- (2) All the imaginary internal planes parallel to the surfaces of the construction element are isothermal. This approximation neglects surface temperature gradients and edge effects.
- (3) Thermal energy transfer within the material is isotropic, in nature.
- (4) The thermo-physical properties of the material (such as density, specific heat & thermal conductivity) are constants and independent of temperature changes.

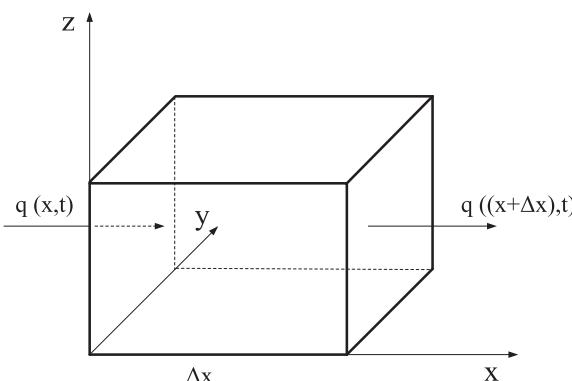


Fig. 3. Volume element for conduction heat flow.

- (5) There is only transfer of thermal energy and no source or sink is present within the construction element.

The benefits of one-dimensional heat transfer offsets the inaccuracies introduced in the results by the isothermal approximation. Applying such assumptions to Eq. (8) i.e.,  $\Delta x \rightarrow 0$  and  $\Delta t \rightarrow 0$ , we have

$$\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \Rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (9)$$

where  $\alpha$  is the thermal diffusivity of the material ( $\text{m}^2/\text{s}$ ).

$$\alpha = \frac{k}{\rho C}$$

Integrating Eq. (2) and re-arranging yields

$$\dot{Q}_{\text{cond,wall}} = kA \frac{T_i - T_{i-1}}{L} \quad (\text{W}) \quad (10)$$

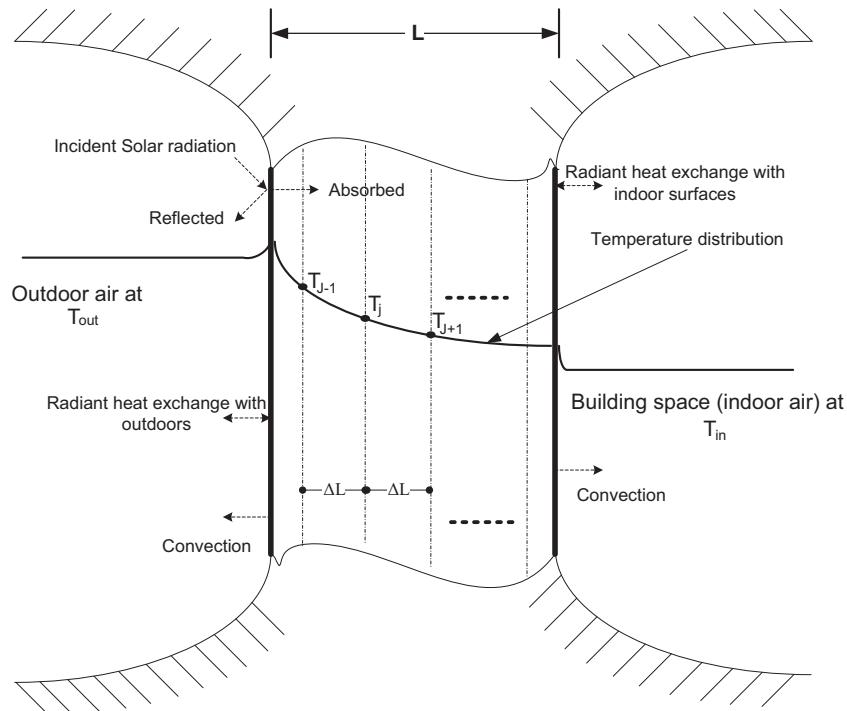
As can be seen from the Fig. 4, the multi-layer construction element is subjected to radiation and convection heat transfer on both sides (Outdoor & indoor side), while heat transfer through the multi-layer construction element is by conduction. Such is the case while considering homogeneous transfer of thermal energy through the construction element. If outside and inside conditions do not vary with time, then the heat transfer through the multi-layer construction element is steady, and using lumped capacitance method one can construct a heat transfer network considering various heat transfer resistances and capacitances.

Due to absorbed solar radiation, thermal energy is transferred from external/outdoor to the building space (indoor) through the multi-layer construction element. An equivalent or effective external temperature value is used to account for such transfer of heat that combines the effects of convection and radiation. A similar parameter, called 'environmental temperature', is used to account for the combined effects of the convective heat transfer from the internal surface to the room air and the radiant energy gain at the surface.

Energy balance equation for building space zone under study which is separated from the outside environment through the construction elements is as follows:

$$\begin{aligned} \rho_a C_{p_a} V_{bs} \frac{dT_{bs}}{dt} = & Q_{\text{causal}} + Q_{\text{hvac}} - \frac{10 \times A_{ow} \times (T_{cs} - T_{ows})}{R_{ow}} \\ & - \frac{10 \times A_{adjw} \times (T_{cs} - T_{adjws})}{R_{adjw}} \\ & - \frac{10 \times A_{adjw1} \times (T_{cs} - T_{adjw1s})}{R_{adjw1}} \\ & - \frac{10 \times A_{roof} \times (T_{cs} - T_{roofs})}{R_{roof}} \\ & - \frac{10 \times A_{floor} \times (T_{cs} - T_{floor})}{R_{floor}} - A_{win} \times U_{win} \\ & \times (T_{cs} - T_{out}) - 0.33 \times N \times V_{cs} \times (T_{cs} - T_{out}) \end{aligned} \quad (11)$$

Eq. (11) represents sensible heat transfers that take place within a building space. But there are also sources of moisture within a building space that act as thermal loads on the hvac system and are responsible for occupant comfort factors of odors, respiration, etc. In order to obtain a fairly accurate building energy model, modeling of mass transfer movements along with the thermal energy is essential. Thus, the effects of various moisture sources on the energy content of the space air should also be determined [48]. Unlike the sensible heat gains, the latent heat gains of a building



**Fig. 4.** Temperature profile and thermal energy transfer processes within a building construction element.

space instantaneously become thermal load on the hvac system. The most influential sources of moisture include the occupants, certain materials/processes and equipment or appliances that would generate moisture (such as cooking, foodstuffs, and liquid water exposed to the air) and moisture carried by the outdoor air into the room through infiltration or the ventilation system. The hygroscopic building fabric elements, and the furniture, would also exchange moisture with the indoor air. Furthermore, the hvac system may add or remove moisture from the indoor air through provision of humidification or dehumidification, which may either be under active control or be processes associated with the provision of cooling [48]. To mathematically model such transfers of energy, all the factors are cumulatively modeled as follows.

Outdoor space vapour pressure is calculated using weather data parameter of relative humidity as:

$$p_{vap} = p_{sat} \times (RH_{out}/100) \quad (12)$$

of which,  $p_{sat}$  is a function of outdoor air temperature and is given as:

$$p_{sat} = 3.376 \times \exp \left( 15.463 - \frac{7284}{1.8 \times T_{out} + 424} \right) \quad (13)$$

Energy balance equation for latent heat transfers within a building space is given as:

$$\begin{aligned} \frac{dp_{cs}}{dt} &= p_o \times N/3600 - p_{cs} \times N/3600 \\ &+ \left( \frac{Q_{lat,cas} + Q_{lat,hvac}}{1.2 \times V_{cs} \times 2.5 \times 10^6} \right) \end{aligned} \quad (14)$$

where

$$p_0 = 0.62198 \times \frac{p_{vap}}{101.325 \times p_{vap}}$$

Now, relative humidity for the conditioned space can be calculated as:

$$RH_{cs} (\text{in \%}) = \frac{100 \times 101.325 \times p_{cs}}{p_{sat,cs} \times (p_{cs} + 0.62198)} \quad (15)$$

In order to solve the one-dimensional thermal energy balance equation, multi-layer wall is divided into a number of spatial sub segments as shown in the Fig. 4. Now, solving a one-dimensional parabolic differential equation numerically, aspects of stability & convergence are important. When solving the thermal energy balance equation using forward difference (explicit) scheme, it was observed that the solution became unstable with increasing time and for a larger time span the mesh size needed to be changed. Compared with the explicit method, each time step required more computation when solving the thermal energy balance equation using backward difference (implicit) scheme, though implicit scheme was stable. But in order to ensure a steady decay of the entire solution (i.e., for convergence) time separation of the grid has to be very finely tuned which increased the number of time steps and hence, the computation time. In order to accommodate the advantages of both the schemes in terms of speed of convergence & stability, the authors employed central difference scheme using Crank–Nicholson (CN) method for solving the thermal energy balance equation of a multilayer construction element numerically. Simplified equation is a one-dimensional (spatial) partial differential equation governing the transient heat conduction in a homogeneous slab with no internal heat source or sink.

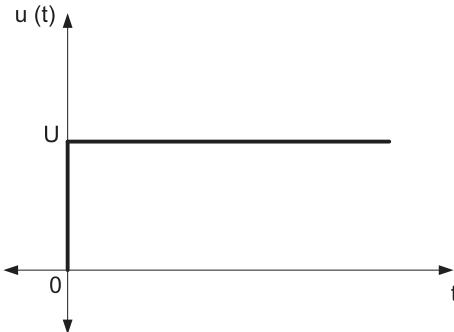
To allow the change in the temperature at an interior node from the  $k_{th}$  to  $(k+1)_{th}$  time step to be evaluated from the temperature of the node and its adjacent nodes.

To develop a finite difference solution we divide the spatial domain into  $k$  sections, each of length  $h$ , so that  $h = L/k$ , and consider as many time steps as required, each time step of duration  $k$ . For the interior nodes ( $j \neq 1, N$ ) of the multi-layer construction element the heat balance equation can be approximated using CN method as:

**Table 4**

Thermo-physical properties of the external walls under study.

| Description  | Thermo-physical properties |                              |                              |                        |
|--|----------------------------|------------------------------|------------------------------|------------------------|
|  | Length (mts.)              | Density (kg/m <sup>3</sup> ) | Thermal conductivity (W/m K) | Heat capacity (J/kg K) |
| <i>Light construction wall (90 kg/m<sup>2</sup>)</i>   |                            |                              |                              |                        |
| Stucco   | 0.025                      | 1858                         | 0.692                        | 840                    |
| Insulation   | 0.125                      | 91                           | 0.043                        | 840                    |
| Plaster/gypsum   | 0.020                      | 1602                         | 0.727                        | 840                    |
| <i>Medium construction wall (409 kg/m<sup>2</sup>)</i> |                            |                              |                              |                        |
| Brickwork  | 0.105                      | 1700                         | 0.840                        | 800                    |
| <i>Cavity</i>  |                            |                              |                              |                        |
| Concrete (heavy weight)                                | 0.100                      | 2300                         | 1.630                        | 1000                   |
| <i>Heavy construction wall (917 kg/m<sup>2</sup>)</i>  |                            |                              |                              |                        |
| Face brick   | 0.100                      | 2002                         | 1.333                        | 920                    |
| Insulation   | 0.125                      | 91                           | 0.043                        | 840                    |
| High density concrete                                  | 0.300                      | 2243                         | 1.731                        | 840                    |
| Plaster/gypsum   | 0.020                      | 1602                         | 0.727                        | 840                    |

**Fig. 5.** Unit step function.

$$\begin{aligned} T_j^{k+1} - T_j^k &= F_o \left[ \omega(T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1}) + (1-\omega)(T_{j-1}^k - 2T_j^k + T_{j+1}^k) \right] \\ &\Rightarrow -\omega F_o T_{j-1}^{k+1} + (2\omega F_o + 1) T_j^{k+1} - \omega F_o T_{j+1}^{k+1} \\ &= (1-\omega) F_o T_{j-1}^k - (2F_o(1-\omega) - 1) T_j^k + (1-\omega) F_o T_{j+1}^k \quad (16) \end{aligned}$$

where

$\omega$ : the weighing factor  
 $F_o$ : Fourier's number

The use of weighting factor makes CN method of finite difference to be stable regardless of the value of the coefficient of the nodal temperatures under study. Without giving concern to the stability limit for the time step, a weighted average of the changes in nodal temperature predicted by the explicit and by the implicit numerical schemes can be used, thus achieving appreciably higher accuracy.

In a similar way, the finite difference equations for all the sub-segment nodes making up a ' $N$ ' multi-layer construction element can be developed as follows.

For outer surface node (i.e.,  $j = 1$ ), the finite difference equations can be written as:

$$\begin{aligned} (2\omega F_o B_i + 2\omega F_o + 1) T_1^{k+1} - 2\omega F_o T_2^{k+1} \\ = 2F_o B_i ((1-\omega) T_{out}^k + \omega T_{out}^{k+1}) - (2F_o(1-\omega)(1+B_i) - 1) T_1^k \\ + 2F_o(1-\omega) T_2^k \quad (17) \end{aligned}$$

$T_{out}$ ,  $T_1$  &  $T_2$  denotes sol-air and nodal temperatures for sub-segments 1 & 2, respectively.

Here,  $B_i$  denotes Biot's number which can be written as:

$$B_i = \frac{h_0 \Delta x}{k}$$

For inner surface node (i.e.,  $j = N$ ), the finite difference equations can be written as:

$$\begin{aligned} (-2\omega F_o) T_{N+1}^{k+1} + (2\omega F_o(B_i + 1) + 1) T_N^{k+1} \\ = 2F_o(1-\omega) T_{N-1}^k - (2F_o(1-\omega)(1+B_i) - 1) T_N^k \\ + 2F_o B_i (1-\omega) T_{out}^k + \omega T_{out}^{k+1} \quad (18) \end{aligned}$$

For other sub-segment nodes (intermediate) between layers ' $p$ ' & ' $q$ ', the finite difference equations can be written as:

$$\begin{aligned} -2\omega F_{o,p} T_{j-1}^{k+1} + (2\omega F_{o,p} + 2\omega F_{o,q} + 1) T_j^{k+1} - 2\omega F_{o,q} T_{j+1}^{k+1} \\ = 2F_{o,p}(1-\omega) T_{j-1}^k - (2(1-\omega)(F_{o,p} + F_{o,q}) - 1) T_j^k + F_{o,q}(1-\omega) T_{j+1}^k \quad (19) \end{aligned}$$

where

$$F_{o,p} = \frac{\kappa_p \Delta t / \Delta L_p}{\rho_p C_p \Delta L_p + \rho_q C_q \Delta L_q}; \quad \text{Fourier number for layer } p$$

$$F_{o,q} = \frac{\kappa_q \Delta t / \Delta L_q}{\rho_p C_p \Delta L_p + \rho_q C_q \Delta L_q}; \quad \text{Fourier number for layer } q$$

Energy balance equations for all the construction elements whose thermo-physical properties are given in Table 4 are solved dividing the spatial domain into 20 layers i.e.,  $N = 20$  [43]. The finite difference equations for the same can be expressed in matrix form as:

$$\begin{aligned} \begin{pmatrix} a_{11} & a_{12} & 0 & \dots & 0 \\ a_{21} & a_{22} & a_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & a_{N-1,N-2} & a_{N-1,N-1} & a_{N-1,N} \\ 0 & \dots & 0 & a_{N,N-1} & a_{NN} \end{pmatrix} \begin{pmatrix} T_1^{k+1} \\ T_2^{k+1} \\ \dots \\ T_{N-1}^{k+1} \\ T_N^{k+1} \end{pmatrix} \\ = \begin{pmatrix} b_{11} & b_{12} & 0 & \dots & 0 \\ b_{21} & b_{22} & b_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & b_{N-1,N-2} & b_{N-1,N-1} & b_{N-1,N} \\ 0 & \dots & 0 & b_{N,N-1} & b_{NN} \end{pmatrix} \begin{pmatrix} T_1^k \\ T_2^k \\ \dots \\ T_{N-1}^k \\ T_N^k \end{pmatrix} + \begin{pmatrix} f_1 \\ 0 \\ \dots \\ 0 \\ f_N \end{pmatrix} \quad (20) \end{aligned}$$

In the present study,  $\omega = 0.5$ . Now, the Crank–Nicholson finite difference algorithm involves evaluation of the temperatures at the next time step for various nodes. This involves solving a set of linear system of equations simultaneously. And the coefficient matrix in the equations above is a tri-diagonal matrix which is solved using a tri-diagonal algorithm available in literature.

As is evident from the above equation, homogeneous thermal energy transfer (transient heat conduction) process through a multi-layer construction element is a one-dimensional (in space) partial differential equation (PDE) with no internal heat sources or sink.

Initial conditions

$$T(x, t = 0) = 0 \quad \forall x \in [0, L] \quad (21)$$

Boundary Conditions

$$T(x = 0, t) = u(t) = \begin{cases} 0 & \forall t < 0 \\ U & \forall t \geq 0 \end{cases} \quad (22)$$

$$T(L, t) = 0 \quad \forall t > 0 \quad (23)$$

where  $u(t)$  is the time step response function as shown in Fig. 5.

## 6. Parameter estimation through optimization routine

The main aim of the present study is to identify the parameters of the 3R2C model (Fig. 6) of each construction element of a building room. Unit step responses for unit step outdoor temperature for the benchmark model is compared to that of the 3R2C model for each construction element and a non linear constrained optimization algorithm is used to minimize the sum squared error between the two. Such a methodology is simple and identification of the model parameters is done by an optimization algorithm. This technique is used with the lumped capacitance models, neural network models, and linear parametric models.

A single zonal room is considered with the construction elements (one exterior wall, one interior wall, one roof and floor, one window) consisting specifications given in Table 4.

Thus, the five parameters of the 3R2C target model (Fig. 6) are identified by the application of a nonlinear constrained optimization technique with mathematical formulations as follows.

$$\text{minimize: } f_{\text{SSE}}(T) \quad (24)$$

$$\text{subject to: } x_1 + x_2 + x_3 = 1; \quad x_1, x_2, x_3 \geq 0$$

$$y_1 + y_2 = 1; \quad y_1, y_2 \geq 0$$

Upper boundary conditions :  $C_1, C_2 = C_{th}/100$ ;  $R_1, R_2, R_3 = R_{th}/100$ .

Lower boundary conditions :  $C_1, C_2 = 0.99C_{th}$ ;  $R_1, R_2, R_3 = 0.99R_{th}$ .

where  $f_{\text{SSE}}(T)$  is the sum-squared error function given as:

$$f_{\text{SSE}}(T) = \sum_1^3 (T_{\text{FiDM}} - T_{\text{3R2C}})^2 \quad (25)$$

where

$T_{\text{FiDM}}$  = Temperature step response of the FiDM model

$T_{\text{3R2C}}$  = Temperature step response of the target model

Parameters of the lumped 3R2C model are identified using a nonlinear time invariant constrained optimization algorithm as shown in Fig. 7.

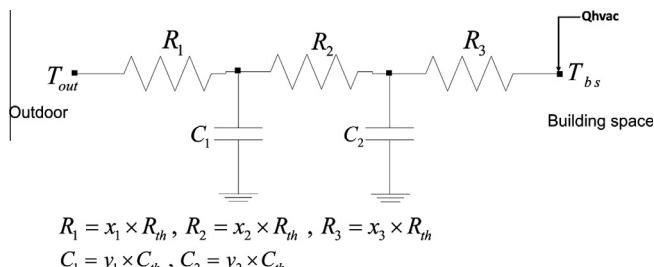


Fig. 6. RC-network model of a wall separating outdoor with the building space conditioned by a HVAC system.

## 7. Validation and simulation results

In the previous section, a nonlinear time invariant optimization approach to identify and estimate the parameter values of the 3R2C model has been presented. Accuracy of the reduced order model with the parameters identified and estimated by using a non-linear time invariant constrained optimization will be validated by comparing the step responses of the reduced order model with that of the models of other parameter configurations.

### 7.1. Different configurations of the developed BES models

In order to validate the developed reduced order 3R2C model using optimization approach, the step responses of the model are compared with those of the numerical model and two simplified 3R2C models with typical configurations normally used in application [49]. These three models are named as numerical (FiDM) model, model-I and model-II, respectively.

The multi-layer slabs studied are external walls subjected to similar sets of temperature, solar radiation and relative humidity inputs. Thermo-physical properties of the walls are taken from the ASHRAE Handbook of fundamentals [48] for light, medium and heavy thermal capacity constructions. Thermo-physical properties of the multi-layer construction wall for different types of constructions are given in Table 4. Light construction wall is mainly an insulation layer with overall density of about 90 kg/m<sup>2</sup>. Optimal model for the multi-layer construction is obtained by applying the developed optimization routine.

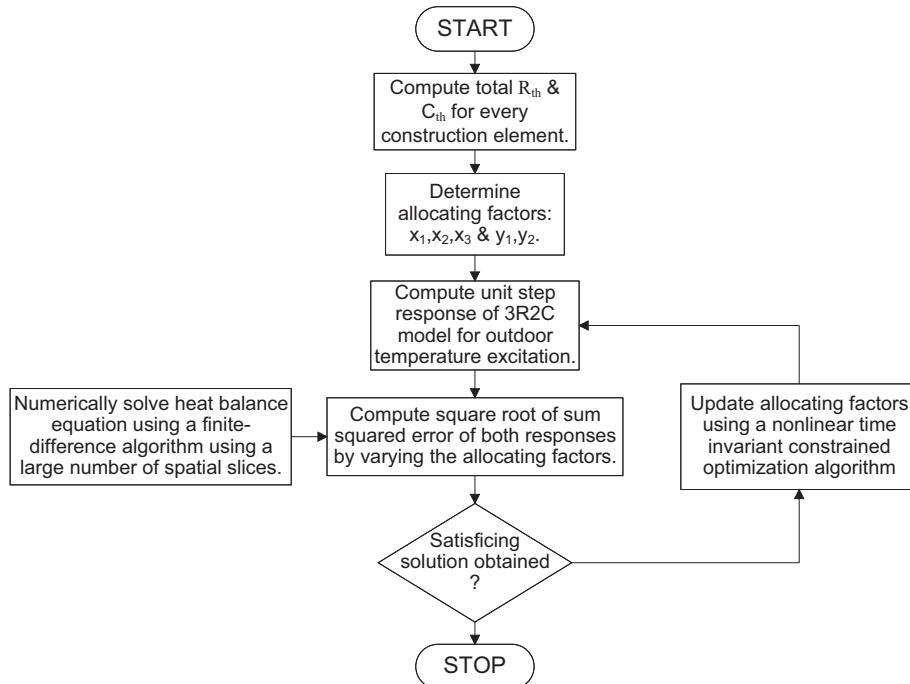
Table 5 gives the description of the different configurations of who's the step response analysis is carried out in the present study. The results of the step response analysis of different configurations of the developed BES models is presented.

Parameter values of the multilayer construction walls whose properties are depicted in Table 4 are given in Table 6.

### 7.2. Simulation results

The developed 3R2C model for a particular construction element of a building was subjected to step response inputs of outdoor temperature and HVAC plant load. BES model is simulated for light, medium and heavy construction elements. A factitive building space model has been developed for both sensible and latent heat transfer processes, based on Eqs. (11)–(15). The building space under study is separated from other thermal zones by the construction elements of walls, roofs, etc. Casual heat gains accounting for heat emitted from occupants and other equipment of the building space are also modelled. The building space is conditioned by a hvac plant whose heat gain rate is considered in modeling of the building space, as in Eq. (11). Modeling of the hvac plant is beyond the scope of this research work. The developed BES model is fed with inputs of outdoor air temperature (°C), outdoor relative humidity (%) and solar radiation (W/m<sup>2</sup>). Results of the building space ( $T_{BS}$ ) responses to a step change in outdoor temperature ( $T_{out}$ ) is shown in Fig. 8.

As is evident from Fig. 8, for all the three types of configurations (low, medium & heavy construction building elements) the step responses of the building space temperature closely follow with that of the numerical model response when the parameters of the BES model are estimated using the optimization technique. For low construction building elements with thermal capacities around 100 kg/m<sup>2</sup>, optimal model closely approximates the numerical model response. Model I response overly estimates and reaches its steady state value at a time very lesser than that of the numerical model as is evident from the settling times



**Fig. 7.** Flowchart of the non-linear constrained optimization algorithm.

**Table 5**  
Different configurations of the developed BES models.

|                           |  |
|---------------------------|--|
| Numerical<br>(FiDM) model | BES model whose energy equations are solved numerically using finite difference method   |
| Model-I                   | 3R2C model with the three resistances being outside conductive resistance, wall conduction resistance and inside conductive resistance, respectively and the two capacitances being equal to half of the total thermal capacitance of the multi-layer slab |
| Model-II                  | 3R2C model with the three resistances and the two capacitances distributed equally and evenly  |
| Optimal model             | 3R2C model with the values of resistances and capacitances obtained by applying the optimization algorithm   |

depicted in Fig. 8. During the simulation period, model II does not reach its steady state value and hence, the settling time is not shown. This applies, that for the said simulation period under study model II is unstable.

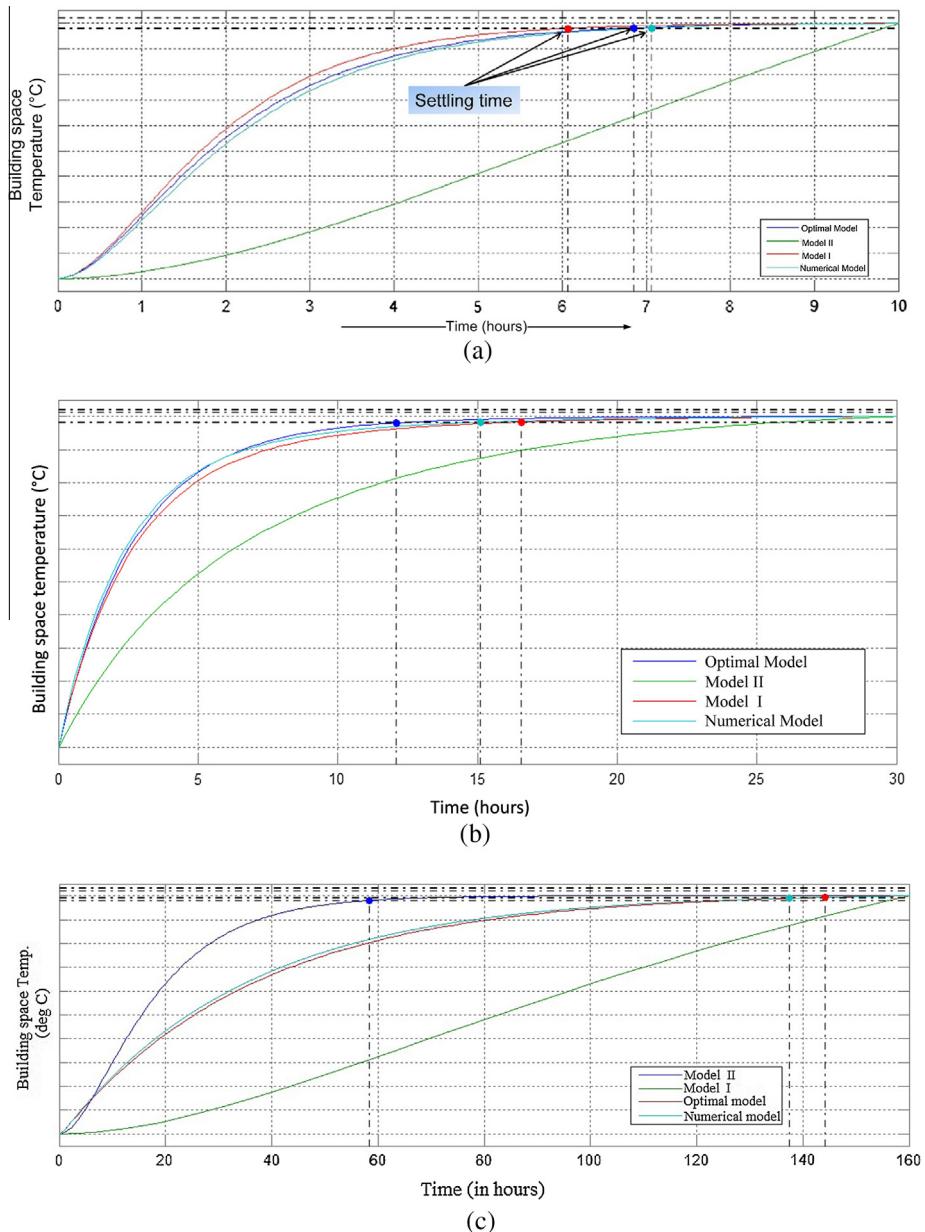
For medium construction elements of medieval thermal capacity values, optimal model, model I closely follow the response of the benchmark numerical model. Both the optimal and model I under estimate the response of the numerical model with the optimal model error profile given in Fig. 8. Model II for the simulation period under study does not reach its steady state value and hence, is unstable. Settling times of all the models are shown in Fig. 8. Even for a heavy construction element, optimal model closely follows the response of the numerical model.

Results of the building space ( $T_{BS}$ ) responses to a step change in outdoor humidity ( $RH_{out}$ ) in Fig. 9.

For all the three types of configurations (low, medium & heavy construction building elements) the step responses of the building space temperature for outdoor relative humidity almost closely follow with that of the numerical model response when the parameters of the BES model are estimated using the optimization technique. As can be noticed from Fig. 9(a), optimal model response over predict the response of the numerical model. Response of

**Table 6**  
Resistance and capacitance values for different configurations of the 3R2C model.

| Model  | Parameter values; resistances, $R$ ( $\text{m}^2/\text{K W}$ ); capacitances, $C$ ( $\text{J}/\text{m}^2 \text{ K}$ ) |        |        |         |         |        |         |
|--|---|--------|--------|---------|---------|--------|---------|
|  | $R_1$   | $R_2$  | $R_3$  | $C_1$   | $C_2$   | $R_T$  | $C_T$   |
| <i>Light construction wall (90 kg/m<sup>2</sup>)</i>   |   |        |        |         |         |        |         |
| Numerical model  | –   | –      | –      | –       | –       | 3.1498 | 75,487  |
| Model-I  | 0.0586  | 2.9706 | 0.1206 | 37,743  | 37,743  | 3.1498 | 75,487  |
| Model-II   | 1.0499  | 1.0499 | 1.0499 | 37,743  | 37,743  | 3.1498 | 75,487  |
| Optimized model  | 0.0724  | 2.9401 | 0.1373 | 42,206  | 33,281  | 3.1498 | 75,487  |
| <i>Medium construction wall (409 kg/m<sup>2</sup>)</i> |   |        |        |         |         |        |         |
| Numerical model  | –   | –      | –      | –       | –       | 0.5455 | 372,800 |
| Model-I  | 0.0586  | 0.3663 | 0.1206 | 186,400 | 186,400 | 0.5455 | 372,800 |
| Model-II   | 0.1818  | 0.1818 | 0.1818 | 186,400 | 186,400 | 0.5455 | 372,800 |
| Optimal model  | 0.1004  | 0.307  | 0.1381 | 122,398 | 250,402 | 0.5455 | 372,800 |
| <i>Heavy construction wall (917 kg/m<sup>2</sup>)</i>  |   |        |        |         |         |        |         |
| Numerical model  | –   | –      | –      | –       | –       | 3.362  | 785,889 |
| Model-I  | 0.0586  | 3.1828 | 0.1206 | 392,945 | 392,945 | 3.362  | 785,889 |
| Model-II   | 1.1207  | 1.1207 | 1.1207 | 392,945 | 392,945 | 3.362  | 785,889 |
| Optimal model  | 0.0797  | 3.0997 | 0.1826 | 20,045  | 765,844 | 3.362  | 785,889 |



**Fig. 8.** Building space temperature responses for different configurations with step change in outdoor air temperature (a) light, (b) medium, (c) heavy construction element of the building under study.

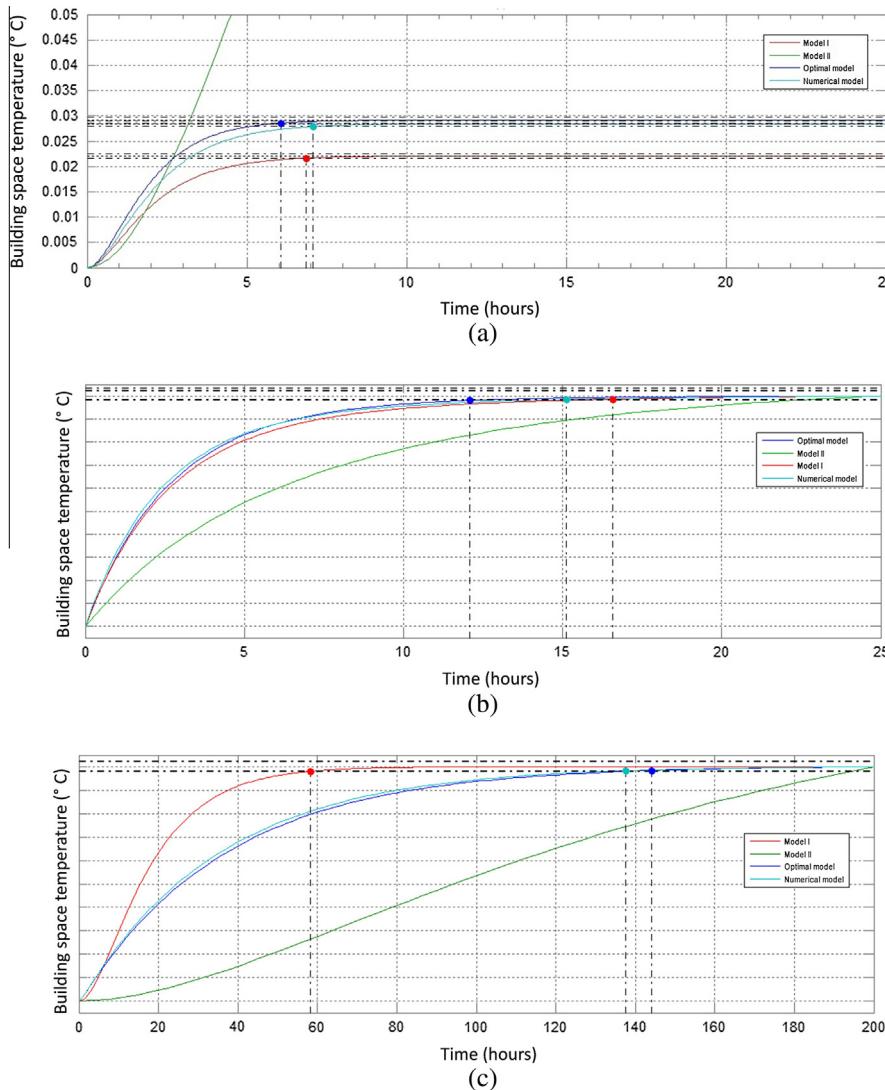
the reduced order BES model parameters estimated through the developed optimization routine closely follows the response of the numerical model for medium and heavy thermal capacity construction elements.

### 7.3. Limitations of applying an optimization algorithm for parameter estimation of a BES model

In order to develop an effective BES model which is less computationally demanding but at the same time sufficiently accurate, suitable techniques are needed which could preserve as much of the information as is available in an intrinsically higher order system. Now, a large number of parameters are needed as inputs for simulating a BES model. Monitoring and measuring of all the parameters for study and analysis of BES models is complex and also consumes time. Such an exercise of collecting data can also be expensive and cumbersome in some conditions such as

collecting the thermal characteristics of room furniture, and other domestic materials. Any BES model parameter estimation technique should facilitate selection of significant parameters and their optimization in relation to some specified criteria.

The developed optimization algorithm works towards minimizing an error index between a fairly decided accurate model (based on 1st principles of building physics) and a target model of lower order. For the developed algorithm to be computationally efficient and cost effective, suitable assumptions are necessary. Also, to initialize the optimization routine, adequate guess values of the parameters to be estimated is necessary. Unsuitable initial guess of the estimable parameters can the BES models deviate, the responses may converge after a long time span (or in some cases may not converge at all). However, if given due care in estimating the initial guess values to the parameters, the developed optimization routine has proved to be computationally efficient and can be cost effective when performed under practical circumstances.



**Fig. 9.** Building space temperature responses for different configurations with step change in outdoor relative humidity (a) light, (b) medium, (c) heavy construction element of the building under study.

It is also, to be kept in mind that according to the need or application of developing a BES model, indefinite parameters or variables should be defined as constants. For example, when applied for large number of buildings where monthly utility billing data and average daily temperature of the building space are available, the BES model developer have to decide on which of the parameters to treat as variables and which to be treated as constants. Such an exercise can be easily performed when the developer shall have a list of the influential parameters, as is given in Table 2.

Also, for multi zone modeling of the BESs, each building construction element shall have to be modeled and its parameters will have to be uniquely fitted while applying the developed optimization routine. However, such a limitation would not be problematic as once developed, the optimization routine/code can be applied repetitively to every construction element being modeled.

In case of BES models possessing dynamic eigen values, iterative solutions of the LTI systems must be obtained which shall lead to high computational effort. The developed method, thus, can be applied in time domain and is dependent on the choice of the error index.

## 8. Conclusion and future works

Energy requirements for heating and cooling of residential, commercial and industrial spaces constitute a major fraction of end-use energy consumption. Conservation of energy is required in alleviating the inefficient use of electricity by keeping a proper trade off with the users comfort levels. Different control techniques together with intelligent building technology (Building Automation Systems) are used to improve energy efficiency of buildings. Most significant part in design of control engineering is modelling. In almost all control projects, it is crucial to have good precise models with high computational efficiency of the building energy systems in order to design and tune the controllers and to simulate their performance.

Lumped capacitance modeling of the building construction elements like walls and also, building spaces has been popular since last decade because of its simplicity and easy simulations. However, there has always been debate over the allocation of number of Rs and Cs with their appropriate values so as the developed RC-network model maps sufficient thermal (and sometimes, moisture) energy transfer dynamics within the building space under study.

Researchers have developed white, black and grey box models and majorly conduct experiments using a test bed to estimate the parameter values of the RC-network model. Every building has its own story to tell and thus, technique used in estimation of parameters using experiments conducted in one location and type of building shall not work well in other cases.

In this paper, a standard model for a building construction element has been developed numerically using finite difference algorithm with 20 spaced layers within the element's thickness. This model is regarded as the benchmark model in terms of accuracy. An optimization algorithm has been developed, depicted through a flowchart, which minimizes the sum squared error between the step responses of the numerical and the optimal model. Optimal model of the construction element is nothing but a RC-network model with the values of Rs and Cs estimated using the nonlinear time invariant constrained optimization routine.

The model is validated with comparing the step responses with other two RC-network models whose parameter values are selected based on a certain criteria as given in the table. Validations are showed for different types of building construction elements viz., low, medium and heavy thermal capacity elements. Simulation results under un-occupied conditions, show that the optimal model closely follow the step responses of the numerical model as compared to the responses of other two models.

Having a validated computationally efficient model, one can now carry out stability, controllability and observability analysis to check and analyze the feasibility of the BES model to any desired control strategy. Similar optimization routine can be applied to other construction elements of BES and simulate the model under different seasonal conditions to obtain a multi-zonal BES model.

## References

- [1] Bruckmayer F. The equivalent brick wall. *Gesundheits-Ingenieur* 1940;63:61–5.
- [2] Clarke JA. Energy simulation in building design. 2nd ed. Oxford, UK: Butterworth Heinemann; 2001.
- [3] Škrjanc I, Zupančič B, Furlan B, Krainer A. Theoretical and experimental FUZZY modelling of building thermal dynamic response. *Build Environ* November 2001;36:1023–38.
- [4] Kopecky P. Experimental validation of two simplified thermal zone models. In: Proc. 9th nordic symposium on building physcis, Tampere, Finland; 2011.
- [5] Mustafaraj G, Chen J, Lowry G. Development of room temperature and relative humidity linear parametric models for an open office using BMS data. *Energy Build* 2010;42:348–56.
- [6] Jimenez M, Madsen H, Andersen K. Identification of the main thermal characteristics of building components using Matlab. *Build Environ* 2008;43:170–80.
- [7] Mustafaraj G, Lowry G, Chen J. Prediction of room temperature and relative humidity by autoregressive linear and nonlinear neural network models for an open office. *Energy Build* 2011;43:1452–60.
- [8] Harish VSKV, Kumar A. Demand side management in India: action plan, policies and regulations. *Renew Sustain Energy Rev* 2014;33:613–24.
- [9] Harish VSKV, Kumar A. Planning and implementation strategy of Demand Side Management in India. In: 1st international conference on automation, control, energy and systems. Kolkata, India; 2014. p. 1–6.
- [10] Gouda M, Danaher S, Underwood CP. Building thermal model reduction using nonlinear constrained optimization. *Build Environ* 2002;37(12):1255–65.
- [11] Wang S, Xu X. Parameter estimation of internal thermal mass of building dynamic models using genetic algorithm. *Energy Convers Manage* 2006;47:1927–41.
- [12] Wang S, Xu X. Simplified building model for transient thermal performance estimation using GA-based parameter identification. *Int J Therm Sci* 2006;45:419–32.
- [13] Balan R et al. Parameter identification and model based predictive control of temperature inside a house. *Energy Build* 2011;43:748–58.
- [14] Kramer Rick, van Schijndel Jos, Schellen Henk. Inverse modeling of simplified hygrothermal building models to predict and characterize indoor climates. *Build Environ* October 2013;68:87–99.
- [15] Schijndel Av, Schellen H. Inverse modeling of the indoor climate using a 2-state 5-parameters model in Matlab. Visby, s.n.; 2011. p. 1–12.
- [16] Wilde Douglass J, Beightler Charles S. Foundations of optimization, vol. 159. Englewood Cliffs, N.J.: Prentice-Hall; 1967. no. 3821.
- [17] Wang W, Rivard H, Zmeureanu R. An object-oriented framework for simulation-based green building design optimization with genetic algorithms. *Adv Eng Informatics* 2005.
- [18] Evins Ralph. A review of computational optimisation methods applied to sustainable building design. *Renew Sustain Energy Rev* 2013;22:230–45. ISSN 1364-0321.
- [19] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. New York, NY: Addison-Wesley; 1989.
- [20] Dewson T, Day B, Irving AD. Least squares parameter estimation of a reduced order thermal model of an experimental building. *Build Environ* 1993;28 (2):127–37.
- [21] Lee K, Braun JE. Development and application of an inverse building model for demand response in small commercial buildings. In: Proceedings of SimBuild 2004. Boulder, CO: IBPSA-USA National Conference; 2004.
- [22] Lee K, Braun JE. Reduced peak cooling loads through model-based control of zone temperature set-points. In: Proceedings of the 2007 American controls conference. New York, NY: IEEE; 2007. p. 5070–5075.
- [23] Gouda MM et al. Building thermal model reduction using nonlinear constrained optimization. *Build Environ* 2002;37:1255–65.
- [24] Seo Dong-yeon, Koo Choongwan, Hong Taehoon. A Lagrangian finite element model for estimating the heating and cooling demand of a residential building with a different envelope design. *Appl Energy* March 2015;142(15):66–79.
- [25] Royapoor Mohammad, Roskilly Tony. Building model calibration using energy and environmental data. *Energy Build* May 2015;94(1):109–20.
- [26] Yang Zheng, Becerik-Gerber Burcin. A model calibration framework for simultaneous multi-level building energy simulation. *Appl Energy* July 2015;149(1):415–31.
- [27] Harish VSKV, Kumar Arun. Techniques used to construct an energy model for attaining energy efficiency in building: a review. In: Control, Instrumentation, Energy and Communication (CIEC), 2014 international conference on, vol. no.; January 31 2014–February 2 2014. p. 366–370.
- [28] Echenagucia Tomás Méndez, Capozzoli Alfonso, Cascone Ylenia, Sassone Mario. The early design stage of a building envelope: multi-objective search through heating, cooling and lighting energy performance analysis. *Appl Energy* 2015;154:577–91. ISSN 0306-2619.
- [29] Nguyen AT, Reiter S, Rigo P. A review on simulation-based optimization methods applied to building performance analysis. *Appl Energy* 2014;113:1043–58.
- [30] McKinley Thomas L, Alleyne Andrew G. Identification of building model parameters and loads using on-site data logs. *Governing* 2008;10.
- [31] Coleman TF, Li Y. An interior trust region approach for nonlinear minimization subject to bounds. *SIAM J Optim* 1996;6:418–45.
- [32] Cormen TH, Leiserson CE, Rivest RL, Stein C. Introduction to algorithms. 2nd ed. Cambridge, MA: MIT Press; 2001.
- [33] Xu X, Wang S. Optimal simplified thermal models of building envelope based on frequency domain regression using genetic algorithm. *Energy Build* 2007;39(5):525–36.
- [34] Xu X, Wang S. A simple time domain calculation method for transient heat transfer models. *Energy Build* 2008;40(9):1682–90.
- [35] Xu X, Wang S, Chen Y. An improvement to frequency-domain regression method for calculating conduction transfer functions of building walls. *Appl Therm Eng* 2008;28(7):661–7.
- [36] Wang S, Chen Y. Transient heat flow calculation for multilayer constructions using a frequency-domain regression method. *Build Environ* 2003;38 (1):45–61. ISSN 0360-1323.
- [37] Lu Xiaoshu, Viljanen Martti. Analytical model for predicting whole building heat transfer. *World Acad Sci, Eng Technol* 2011;76:2011.
- [38] Chantrelle FP, Lahmidi H, Keilholz W, Mankibi ME, Michel P. Development of a multicriteria tool for optimizing the renovation of buildings. *Appl Energy* 2011;88(4):1386–94.
- [39] Park H, et al. Thermal parameter identification of simplified building model with electric appliance. In: Electrical power quality and utilisation (EPQU), 11th international conference on, vol. no.; 17–19, October 2011. p. 1–6.
- [40] Goyal Siddharth, Barooh Prabir. A method for model-reduction of non-linear thermal dynamics of multi-zone buildings. *Energy Build* April 2012;47:332–40.
- [41] Nowak M, Urbanik A. Utilization of intelligent control algorithms for thermal comfort optimization and energy saving. In: Carpathian control conference (ICCC), 2011 12th international, vol. no.; 25–28, May 2011. p. 270–74.
- [42] Yang Zhenyu, Li Xiaoli, Bowers CP, Schnier T, Tang Ke, Yao Xin. An efficient evolutionary approach to parameter identification in a building thermal model. *Syst, Man, Cybernetics, Part C: Appl Rev, IEEE Trans* 2012;42 (6):957–69.
- [43] Underwood CP, Yik FWH. Modelling methods for energy in buildings. Blackwell – Science; 2004.
- [44] Cengel. Heat and mass transfer. Tata McGraw-Hill Education; 2011. p. 902. ISBN 0071077863, 9780071077866.
- [45] Heinsohn Robert Jennings, Cimbala John M. Indoor air quality engineering: environmental health and control of indoor pollutants. CRC Press; 2003. p. 920. ISBN 0203911695, 9780203911693.
- [46] Wang Liangzhu. Coupling of multizone and CFD programs for building airflow and contaminant transport simulations. ProQuest; 2007.
- [47] Mills AF, Ganeshan V. Heat transfer. Pearson Education India; 2009. p. 900. ISBN: 8131727130, 9788131727133.
- [48] ASHRAE. ASHRAE handbook of fundamentals. American Society of Heating, Refrigerating, and Air Conditioning Engineering Inc, 2013.
- [49] Xu X. Model-based building performance evaluation and diagnosis (Order No. 3195410). Available from ProQuest Dissertations & Theses Global. (305394552). <<http://search.proquest.com/docview/305394552/accountid=27544>>; 2005.