CSCI 1315
Discrete Mathematics for Computer Science
Assignment 3
Proof

Welcome to your third assignment! This assignment covers our Proof module. The assignment includes 7 problems that total 43 points, as well as one Bonus question and four Extra questions if you want more practice (which are not worth marks but will be given feedback if you complete them). Please submit your work (show what you've done!) as a single PDF on Brightspace. Note that the M, C, and A given in each question represent the marks available for Method, Content, and Answers. Consider these when presenting your work.

Note that,since this assignment does not count towards your final grade, you can submit what ever you have completed on the due date even if it's not the entire assignment. However, students are strongly encouraged to complete the entire assignment so that they can get the most amount of feedback possible prior to the module test.

Problem 1 (3 points - 1M/1C/1A): Use a direct proof to show that:

If an integer x is odd, then x^3 is odd.

Problem 2 (4 points - 1M/2C/1A): Use a proof by cases to show that:

If two integers have the same parity (both even or both odd), then their sum is even.

Problem 3 (4 points - 1M/2C/1A): Use a proof involving sets to show that:

If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Problem 4 (2 points - 1M/0C/1A): Use a proof by example to show that:

There exists a number x such that $x^2 < \sqrt{x}$.

Section 3: Indirect Proofs

These questions apply the techniques we learned using indirect proofs. For each of these, do not forget to include the type of proof you are using, what you assume (the premise), and what you want to show (the conclusion).

Problem 5 (4 points - 2M/1C/1A): Use a proof by contrapositive to show that:

For all integers n, if $5 \nmid n^2$ then $5 \nmid n$, where the symbol $|$ means "divide".

Problem 6 (4 points - 2M/1C/1A): Use a proof by contradiction to show that:

The sum of a rational number and an irrational number is irrational.

Problem 7 (5 points - 1M/3C/1A): Use induction to show that:

$$\text{For any } n \in \mathbb{N}, n < 2^n.$$

BONUS: Use strong induction to show that:

An object costing \$$n$, $n \geq 8$, can be purchased using only \$3 and \$5 coins.

Extra Proofs

Prove the following statements using the method of your choice. Before you begin your proof, explain why you chose that method. If the proof ends up not working, change methods and explain why you chose the new method. Submit all of your work, including the methods that did not work.

Note: some of these proofs are tricky. Do not fall into the trap of using the result (the conclusion) inside of the proof itself. For instance, for Extra question 4, do not say that $S(n) + c = S(m) + c$ implies $S(n) = S(m)$ since this is what you are trying to show.

Extra 1 (4 points - 2M/1C/1A): If x is a positive irrational number, then $\sqrt{x}$ is also irrational.

Extra 2 (4 points - 2M/1C/1A): If $x, y, z \in \mathbb{Z}$, and the sum $x + y + z$ is odd, then at least one of x, y, and z is odd.

Extra 3 (5 points - 3M/1C/1A): If A, B and C are sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

Extra 4 (6 points - 2M/3C/1A): Let $n \in \mathbb{N}$. We define the successor of n, denoted by $S(n)$, to be $n + 1$. We assume the following axiom (which is a theorem that we define to be true, so we do not need to prove it):

$$\text{If } S(n) = S(m) \text{ for } n, m \in \mathbb{N}, \text{ then } n = m$$

which essentially says that if we have two natural numbers whose successors are the same (i.e., if we add one to both of them and we get to the same number), then those numbers had to have been equal.

Using this axiom, but **NOT** using subtraction, prove the following:

$$\text{Let } a, b, c \in \mathbb{N}. \text{ Then } a + c = b + c \text{ if and only if } a = b.$$