EE208 EXPERIMENT 4

CONTROLLER DESIGN ON MATLAB PLATFORM USING DISCRETE ROOT LOCI

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Objective

To observe the closed-loop behavior of a digital transfer function with respect to the variation of open-loop zeros.

System

The given digital transfer function has three marginally stable poles, that is, certain frequencies at which we may expect sustained oscillations.

$$G_{OL}(z) = \frac{z^2 + 1.5z - 1}{(z-1)^3}$$

Task

To investigate how the frequencies of sustained oscillations may be changed in CL, by modifying the location of OL zeros.

A proportional gain $K \in (-\infty, \infty)$ may be assumed for closing the loop.

Study the problem with reference to the default values of OL zeros, which may be located within or outside the unit circle.

Keep a check on the gain by which the OLTF is closed to realize the CLTF, though the focus is only on the sustained oscillation frequencies.

MATLAB Functions Used

```
tf, rlocus, pzmap, zgrid, controlSystemDesigner
```

MATLAB Startup Code

```
z = tf('z',1); %Ts needed for ControlSystemDesigner
G_ol = (z^2 + 1.5*z +1)/(z-1)^3; %original system
g1 = 1/(z-1)^3;
```

Although we are forced to specify a sampling time, this should not affect our observations beyond a constant factor in the frequency.

The Original System

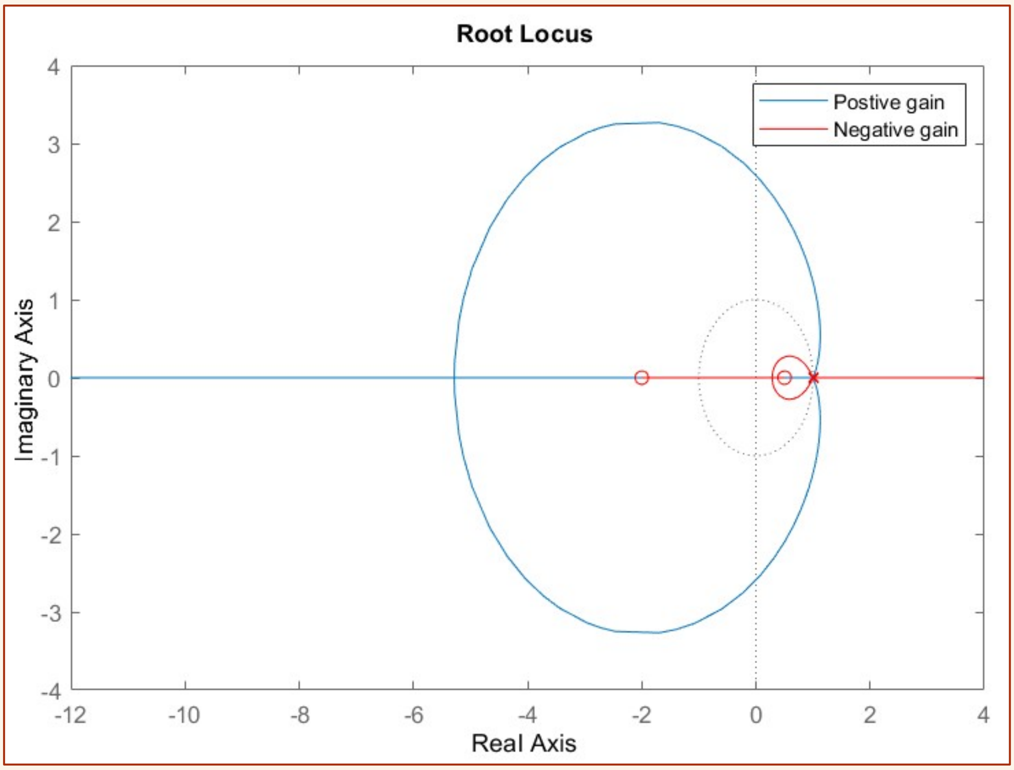
The Original OLTF is $\frac{z^2+1.5z-1}{(z-1)^3}$, which has 3 poles at $z = 1$, a zero at $z = -2$, and a zero at $z = 0.5$.

We can view the closed-loop system with the *feedback ()* command.

```
feedback(G_ol,1)
pole(feedback(G_ol,1))
zero(feedback(G_ol,1))
```

Which gives: $\frac{z^2+1.5z-1}{z^3-2z^2+4.5z-2}$. It has poles at $0.7307 + 1.783j$, $0.7307 - 1.7830j$, and 0.5387, and zeros at -2, and 0.5. We will need to specify a sampling time of 1s to use the Control System Designer tool, although this will not have much effect on our analysis.

We'll use to *rlocus* command to find a suitable gain to obtain sustained oscillations.



From the figure we can see that if we use a **positive gain**, we can only achieve sustained oscillations for $K = 0$, which is just the open-loop system.

For **negative gain**, we can find the intersection point of the locus with the unit circle using the *rlocfind* command. The corresponding gain is $\mathbf{-5.331}$ for a frequency of 3.14 rad/s.

At this gain we have an unstable pole at $z = 8.8408$.

Real Zeros

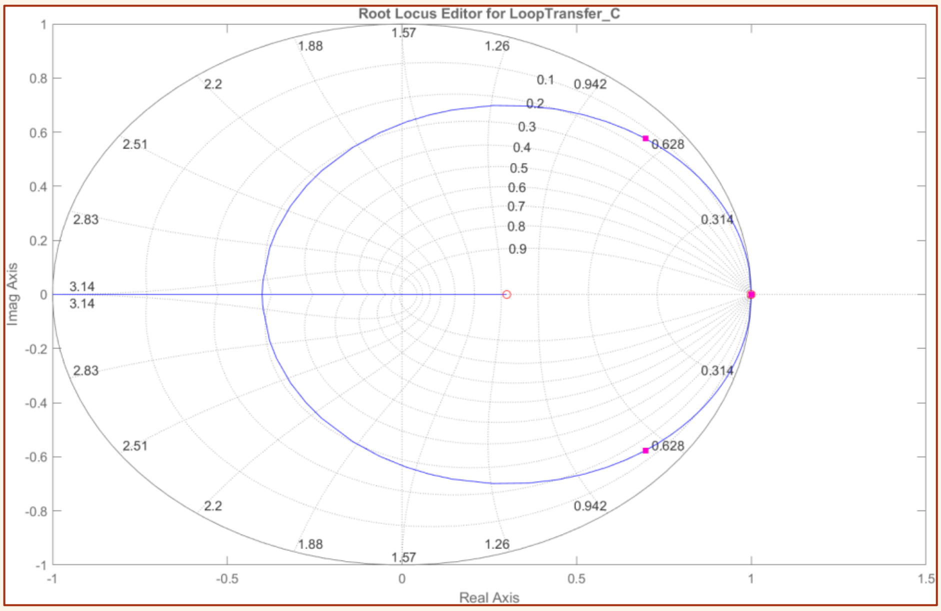
We place two zeros at various locations on the real line and find the sustained oscillation frequencies possible, along with the gain required for the same.

*We shall use **a** and **b** to refer to the position of the zeros.*

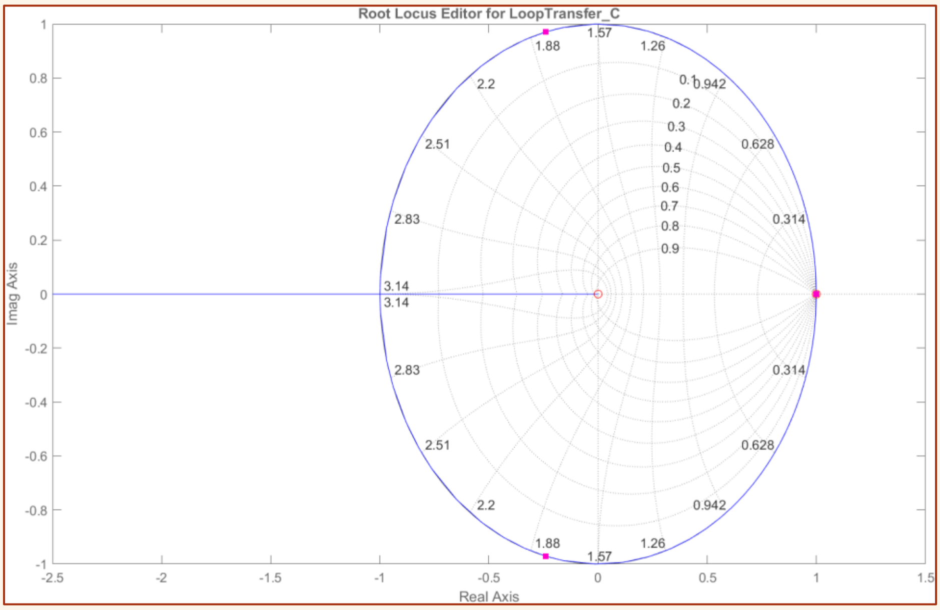
Both zeros inside the unit circle

a = 0.3, b = 1, **-ve K**

By placing one zero at the pole location, we can cancel it and obtain a root locus as shown below. In this configuration the root locus crosses the unit circle at $z = -1$, and a suitable *negative* gain can be found always for a>0 to obtain oscillations at 3.14 rad/s.



a = 0, b=1, **-ve K**



Upon placing one zero at the origin, the locus behaves as expected, with the complex branches moving along the unit circle.

There is a double pole at $z = -1$ for suitable negative gain.

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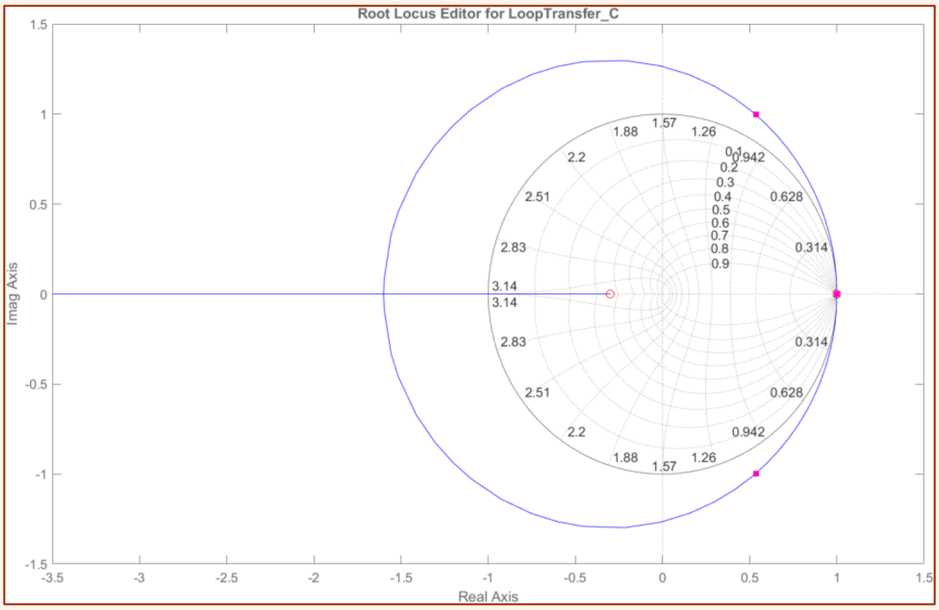
a = -0.3, b = 1, **-ve K**



Moving **a** beyond 0 moves the meeting point of the complex branches outside the unit circle. As a result, as we increase the (negative) gain to obtain sustained oscillations, we will necessarily have an unstable pole on the -ve real line.

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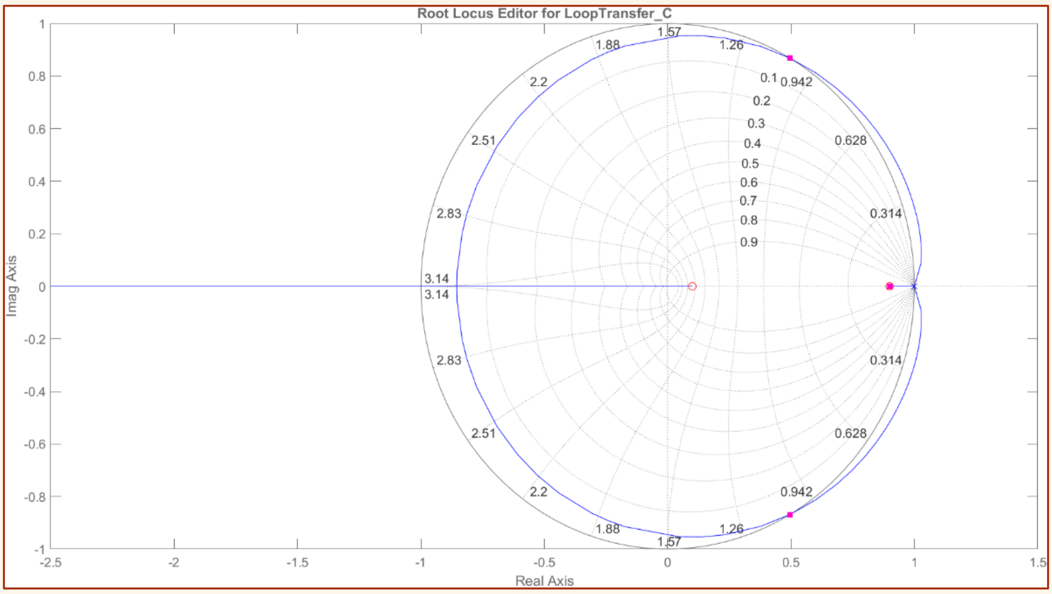
a = 0.1, b = 0.9, **+ve k**

At $\boldsymbol{k} = \mathbf{1.1053}$, we obtain poles at $0.4968 \pm 0.8675j$, with frequency being 1.05 rad/s.

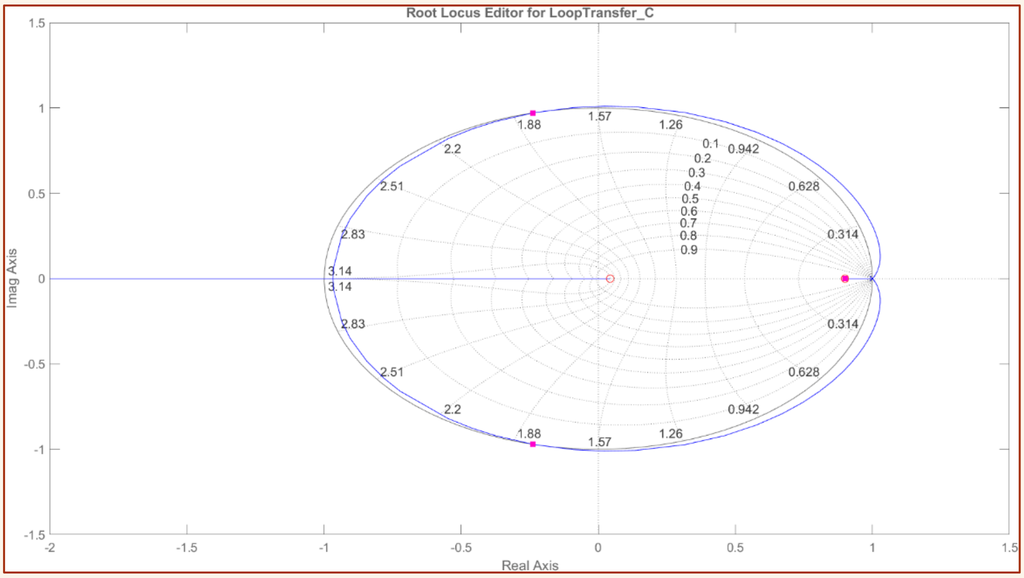


The root loci are similar to those for $\boldsymbol{b} = \mathbf{1}$, but since we have third-order dynamics in this case we can see how the root loci initially break-off at angles of 120° to each other.

a = 0.04319, b = 0.9, **+ve K**

We can drag **a** to a minimum value of 0.04319, corresponding to a frequency of 3.14 rad/s (**keeping b constant**).

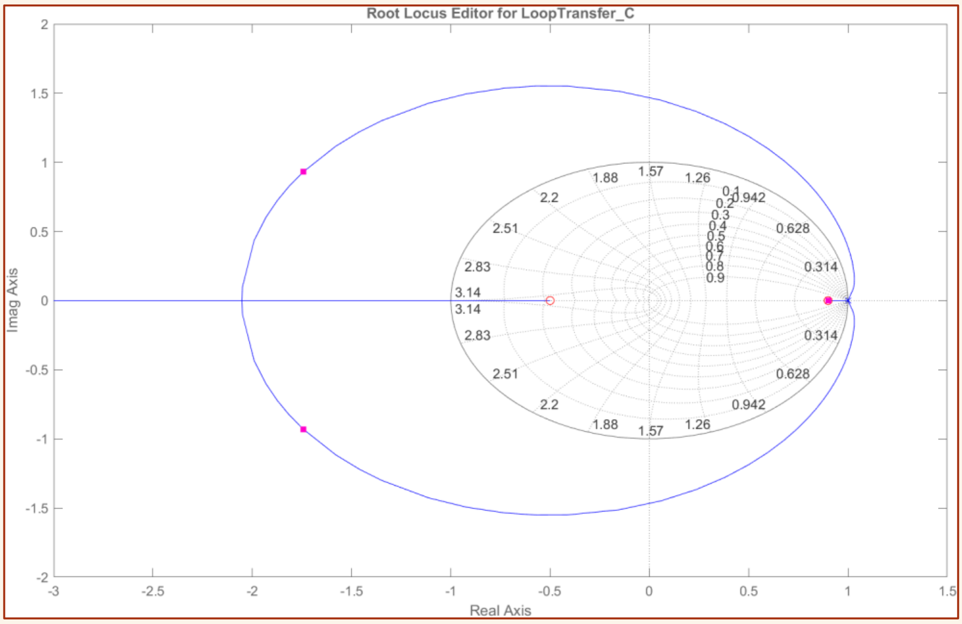
After this point the complex branches no longer cross the unit circle and we can only obtain sustained oscillations for $z = -1$.



At **k = 2.5766**, we obtain poles at $-0.239 \pm 0.971j$, of frequency 1.81 rad/s.

We can achieve various sustained oscillation frequencies by moving **a** and **b**, as per our requirements.

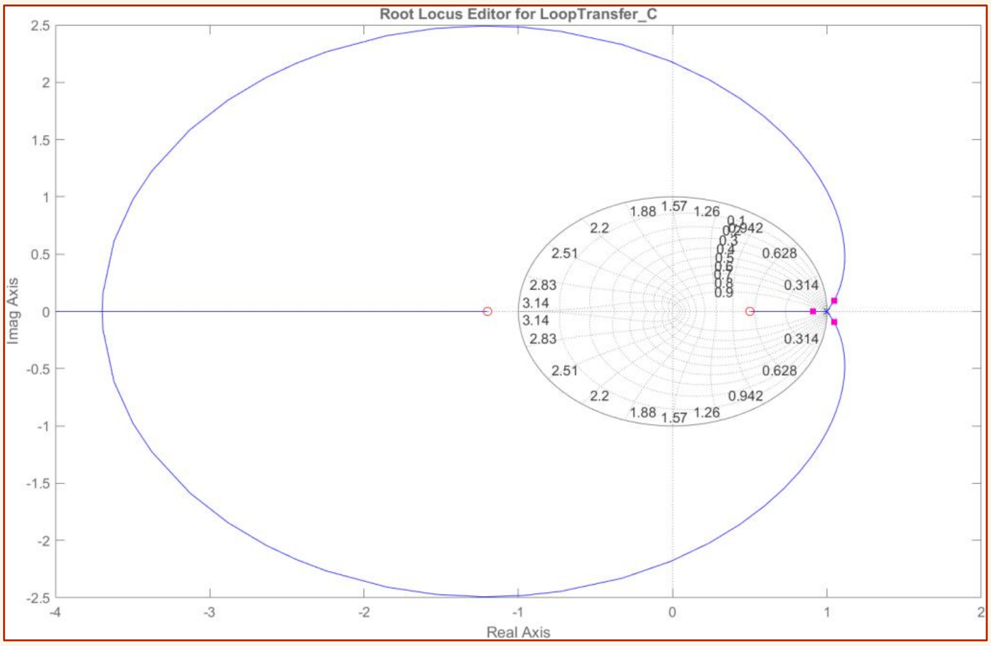
a = -0.5, b = 0.9, **+ve K**



We can see that the break-in point is shifted outside the unit circle. If we increase gain to obtain sustained oscillations at $z = -1$, we will have an unstable pole on the -ve real line.

One zero outside unit circle

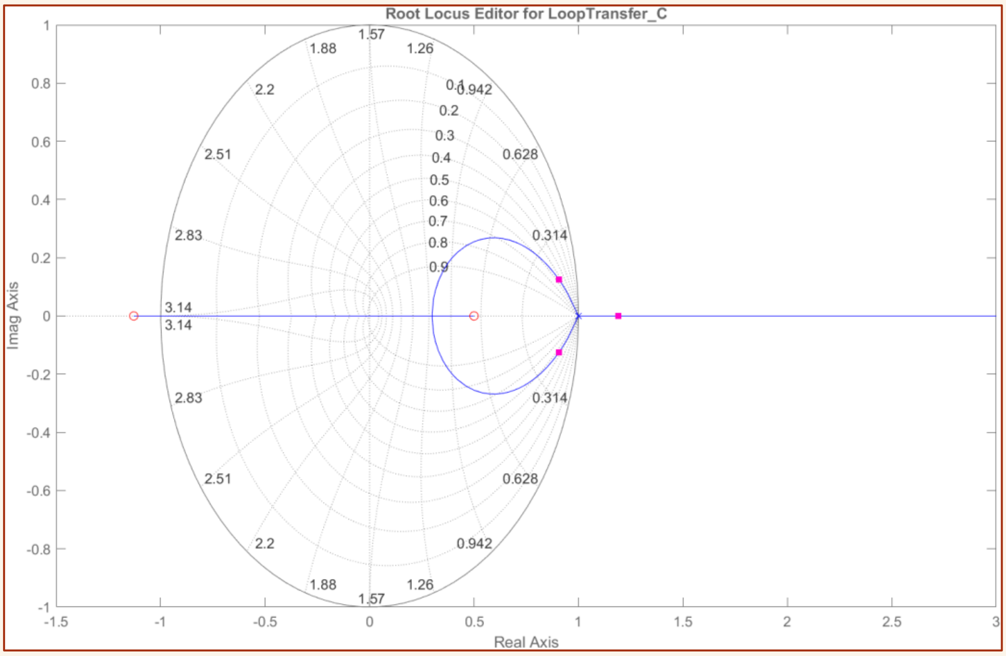
a = -1.2, b = 0.5, **+ve K**



No sustained oscillations are possible if one zero is taken outside the unit circle.

a = -1.2, b = 0.5, **-ve K**

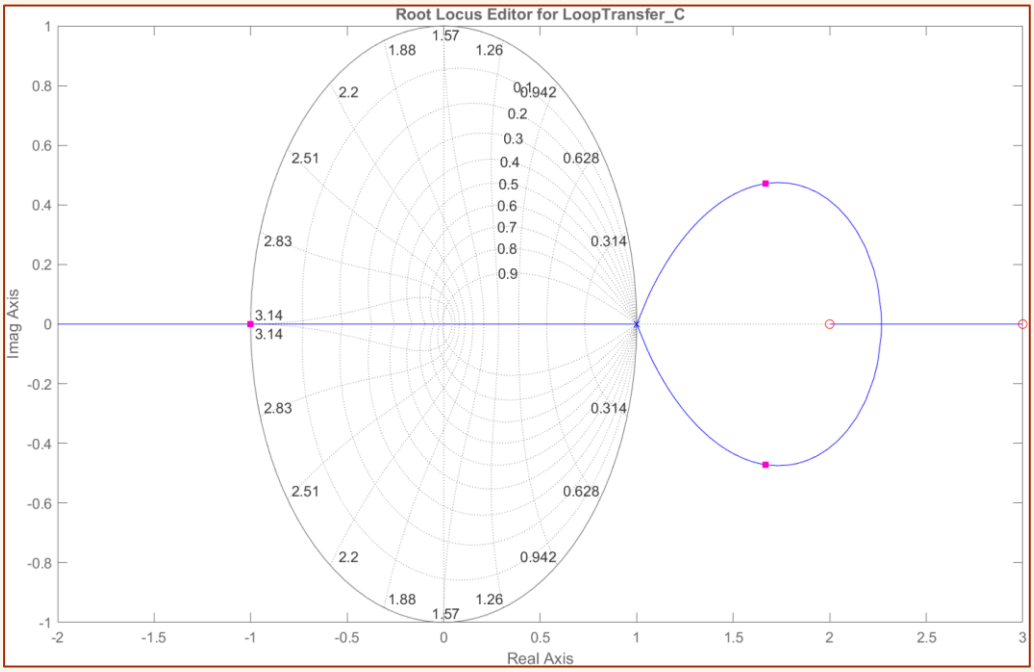
With negative K we can achieve sustained oscillations (that too at z = -1) but we will unavoidably get an unstable pole.



A similar situation is observed when we place **a** on the right half side of the real line, so we are not showing it here.

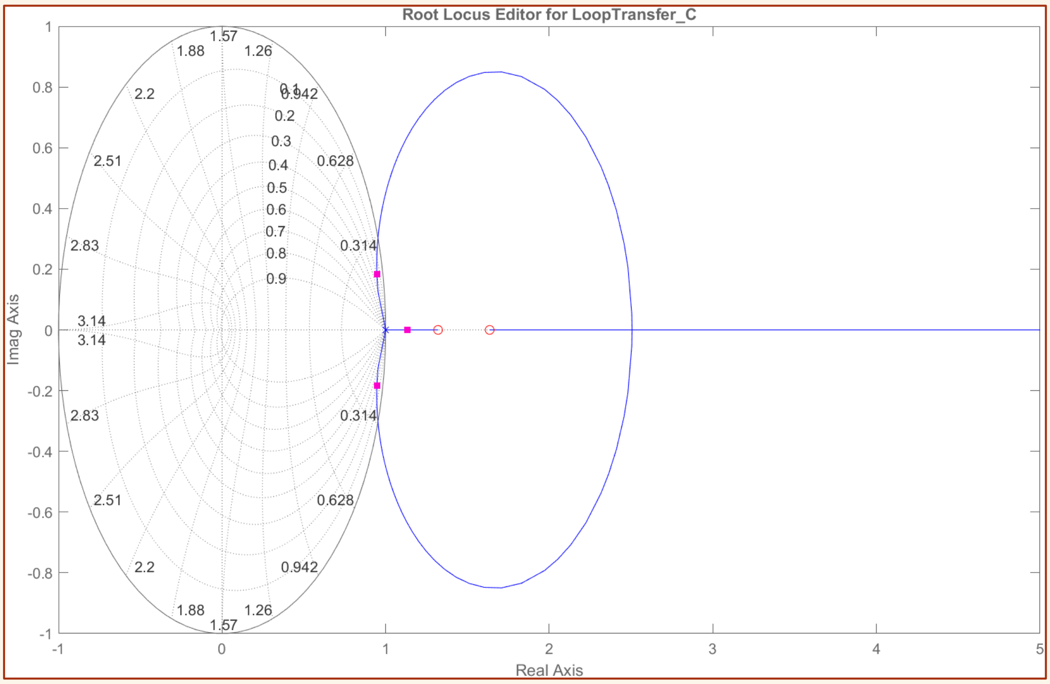
Both zeros outside the unit circle

a = 2, b = 3, **+ve K**



We can only obtain sustained oscillations at $z = -1$, so this configuration is not very useful.

a = 1.32, b = 1.635, **-ve K**



Negative gain can help achieve sustained oscillations but again, we have an unstable pole.

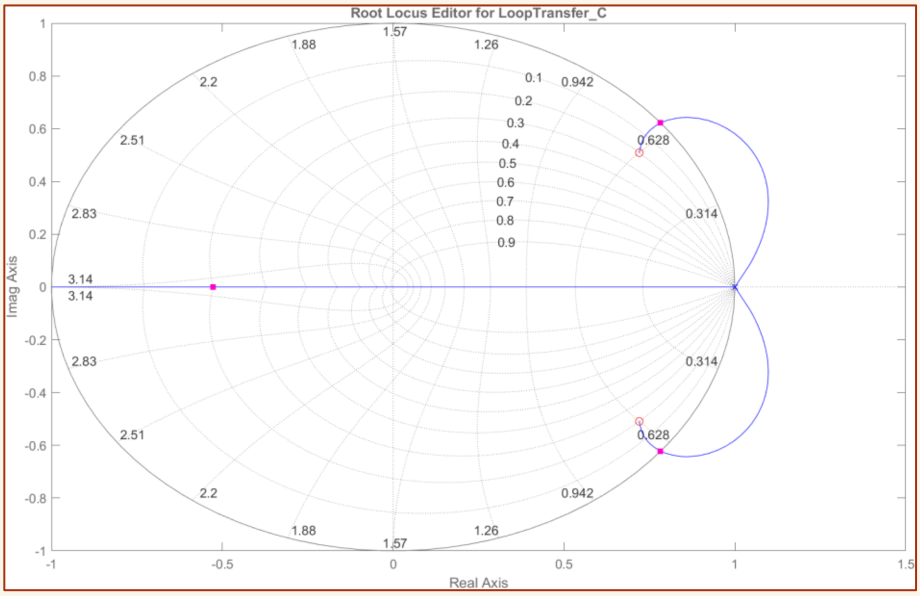
A very similar scenario occurs if we place both zeros on the negative half of the real axis, or one on each half so we are not discussing it.

Complex zeros

We will place complex pairs at various locations on the z-plane, and see how we can change closed loop sustained oscillation frequencies.

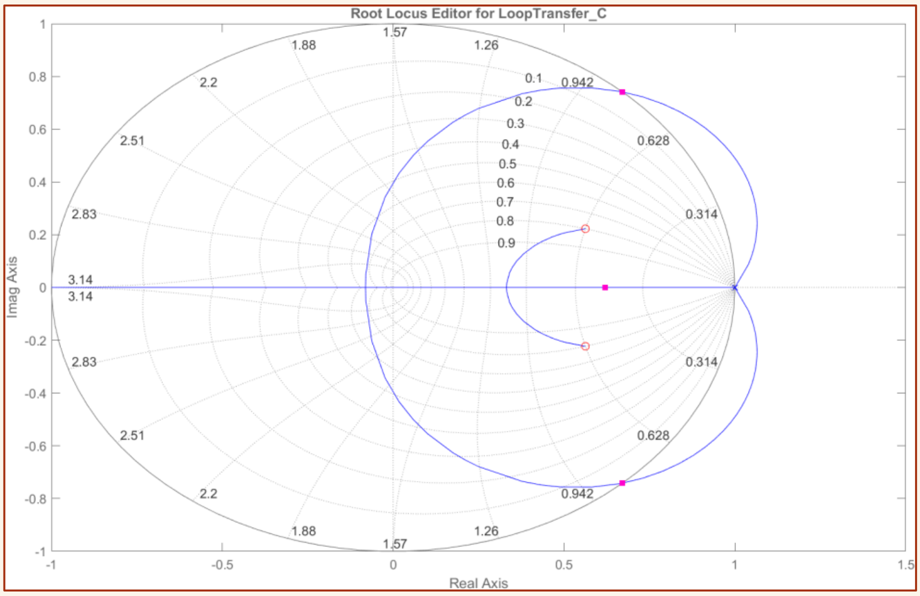
Inside the unit circle

Damping = 0.2, Frequency = 0.628 rad/s



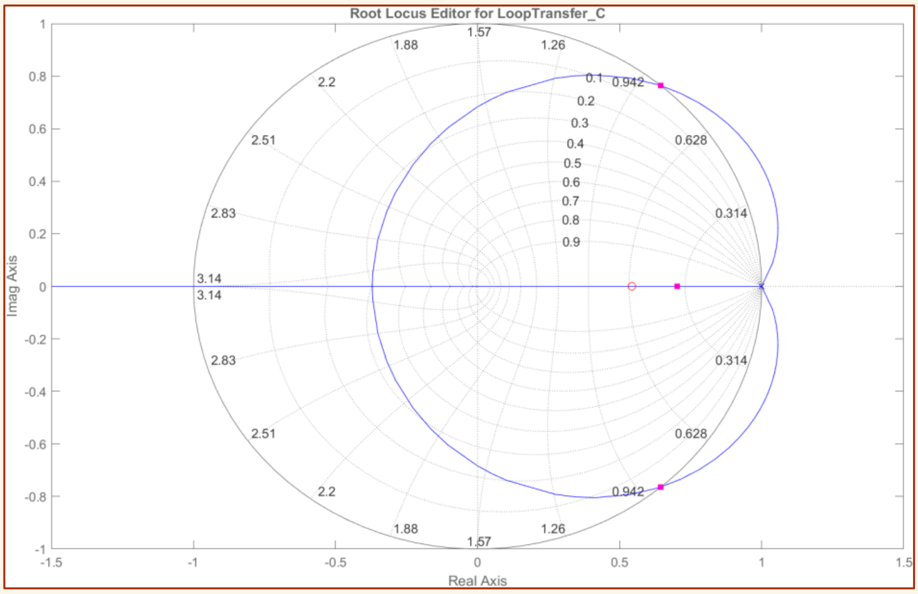
In this case we can achieve oscillations at 0.671 rad/s, with a gain of **1.9344**.

Damping = 0.8, Frequency = 0.628 rad/s



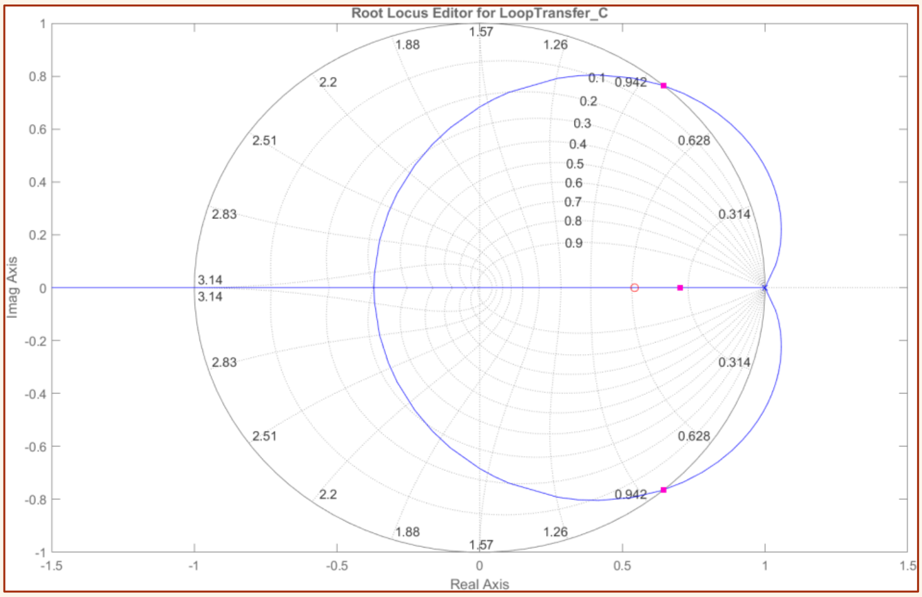
We have greatly increased the damping, and the root loci have changed accordingly. We are still able to achieve stable and sustained oscillations. The frequency is 0.836 rad/s, with a gain of **1.401.**

Damping = 1, Frequency = 0.628 rad/s



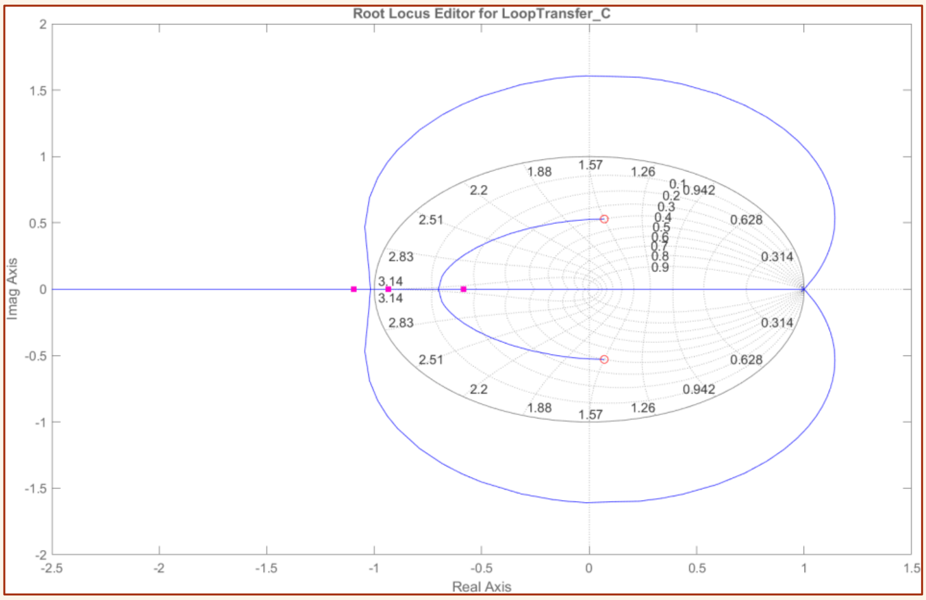
We have made the damping ratio equal to 1 now. We obtain oscillations at 0.906 rad/s at a gain of **1.0708**.

Damping = 0.37, Frequency = 1.57 rad/s



We obtain oscillations of 2.69 rad/s using a gain of **5.5254.**

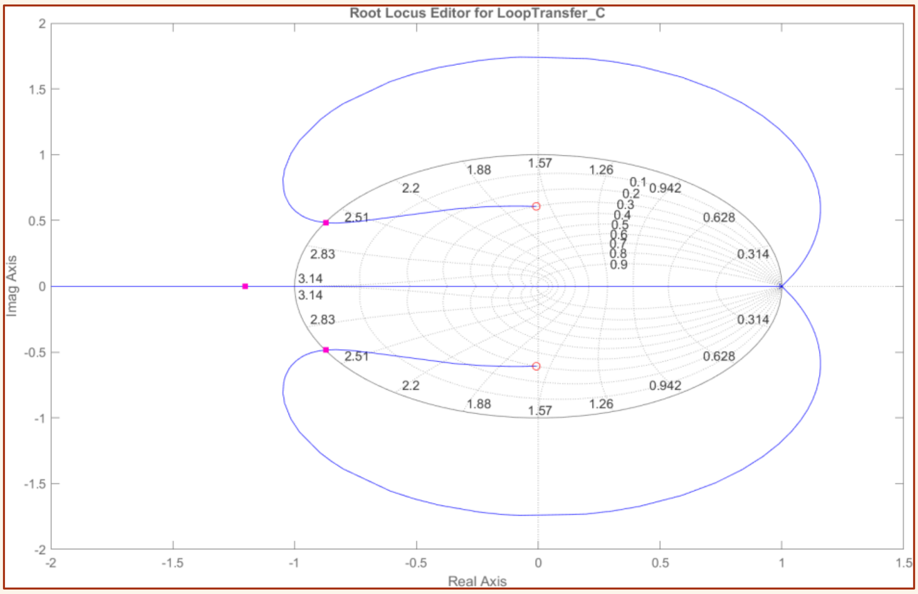
Damping = 0.4, Frequency = 1.57 rad/s



As we slightly increase damping, we become unable to achieve stable, sustained oscillations.

Beyond a certain range of frequency and damping ratio, we become unable to obtain stable, sustained oscillations.

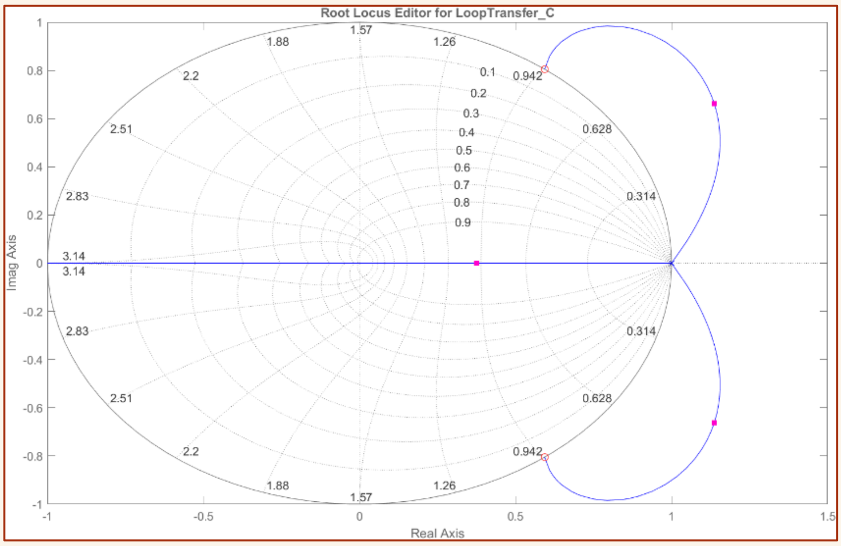
Damping = 0.3, Frequency = 1.66



For example, in this case, even though the loci cross the unit circle, there is an unstable pole, meaning that this configuration is not suitable.

On the unit circle

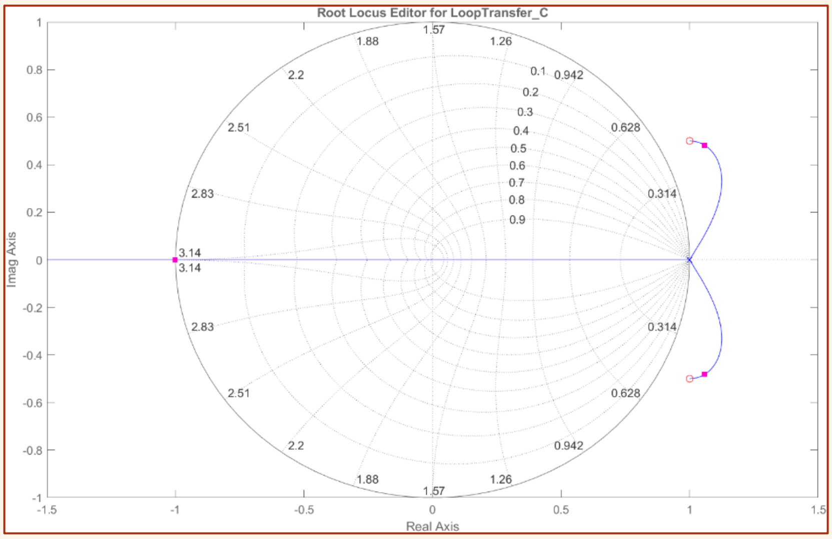
It is clearly not useful to place complex zeros on the unit circle, since we will require infinite gain to achieve sustained oscillations.



We will not discuss this configuration further since there is not much difference in the root loci shapes as compared to the next case (which is, of course, because the root loci are the same for analog and digital transfer functions, only our interpretation is changing).

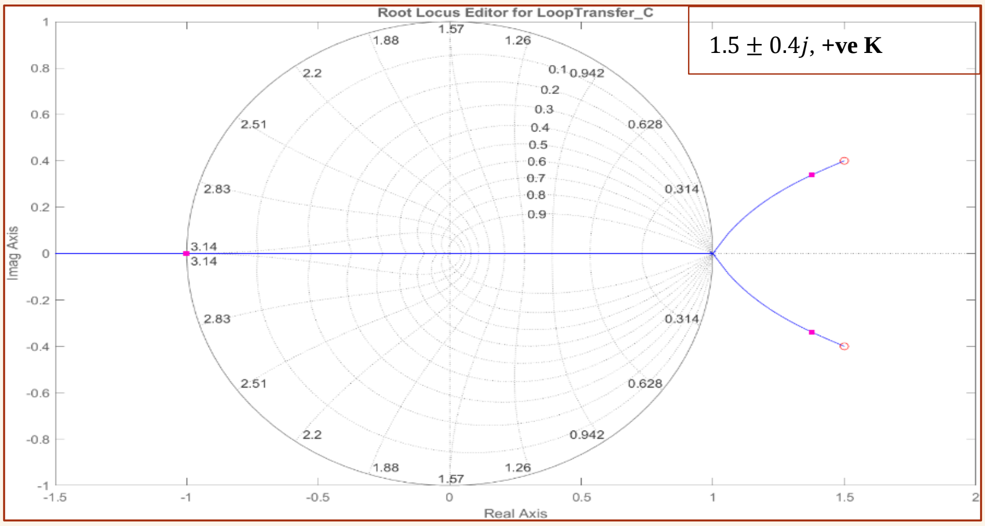
Outside the unit circle

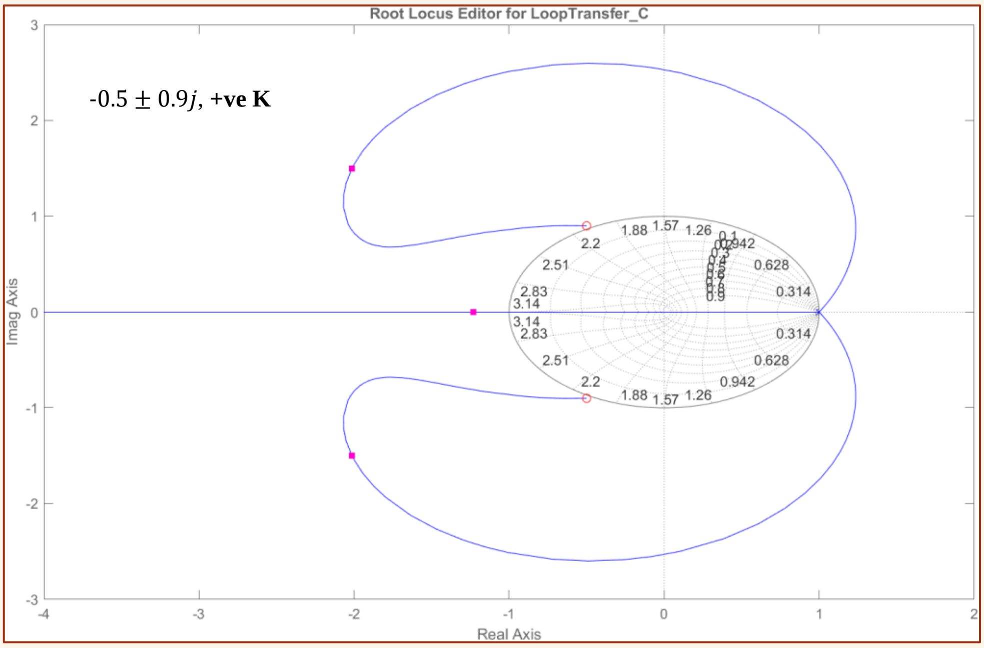
$1 \pm 0.5j$, **+ve K**

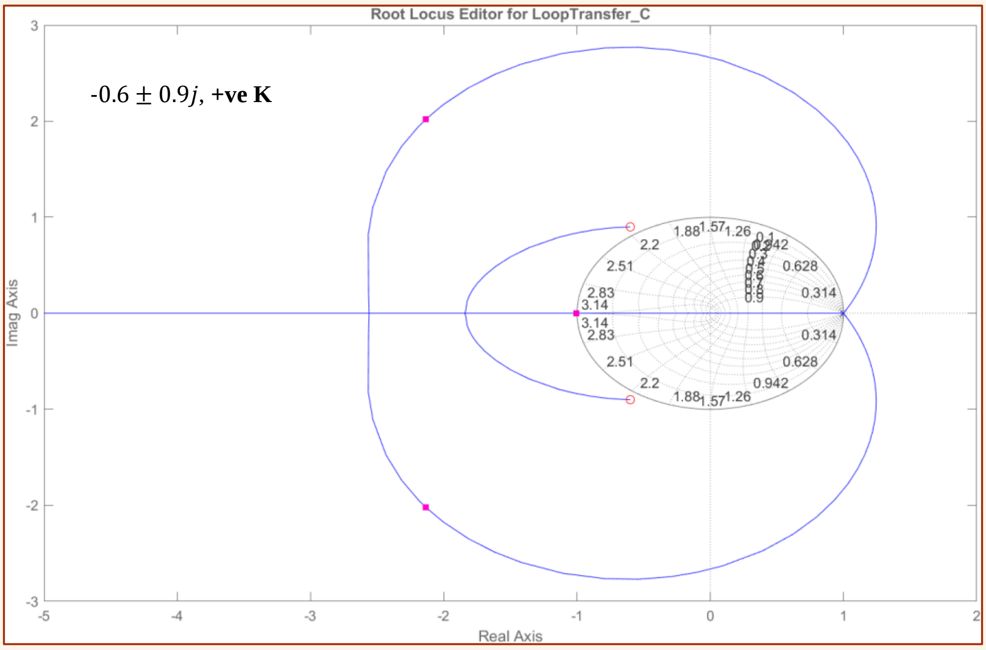


The loci are simple to understand – there are 3 poles, and 2 zeros. One pole therefore will approach the zero at $-\infty$, while the other two poles will approach the complex zeros. The only sustained oscillations possible are those at 3.14 rad/s (and a gain of +1.833).

There is not much difference wrt sustained oscillation frequencies as we change the location of the zeros outside the unit circle. We will nevertheless add some plots to help visualize the root loci at various zero locations.

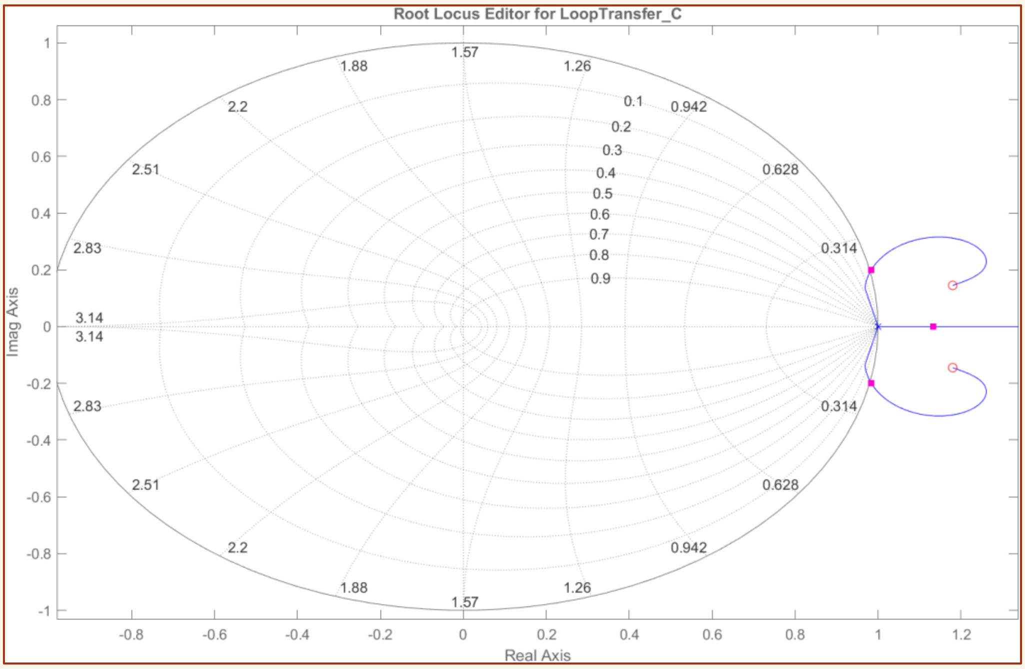

Root Locus Editor for LoopTransfer_C
$1.5 \pm 0.4j$, +ve K
Imag Axis
Real Axis


Root Locus Editor for LoopTransfer_C
$-0.5 \pm 0.9j$, +ve K
Imag Axis
Real Axis


Root Locus Editor for LoopTransfer_C
$-0.6 \pm 0.9j$, +ve K
Imag Axis
Real Axis

Clearly, in none of these cases do we obtain sustained oscillations at frequencies other than 3.14 rad/s. Also, they are all unstable systems.

$1.18 \pm 0.145j$, **-ve K**



With a negative gain of -0.10053, we can obtain sustained oscillations at $0.2\ rad/s$, however we now have an unstable pole.

As was the case with positive K, with negative K too we have unstable poles at the gains needed for sustained oscillations.

So, placing the zeros outside the unit circle is not useful.

Conclusions

In this experiment we studied the effect of changing OL zeros on the frequencies of sustained oscillations in closed loop. We can summarize our findings as follows –

- In general, placing zeros outside the unit circle is not useful, since it will add an unstable pole, and in many cases the root loci will never cross the unit circle.
- **For real zeros**, we obtain a multitude of sustained oscillation frequencies for certain configurations, i.e., as long as the break-in point remains inside the unit circle.
    - In other configurations, the break-in point is moved beyond the unit circle, so the root loci never cross it, hence sustained oscillations cannot be obtained without making the system stable.
- **For *complex zeros***, we showed that sustained oscillations could be achieved for frequencies below 1.4 rad/s (approx.) and over the complete range of damping ratios ($\neq 1$).
    - As we increased frequency, the range of damping ratios for which we could obtain sustained oscillations becomes more restricted.
    - Around approx. 2.05 rad/s, we become unable to obtain *stable* sustained oscillations regardless of the damping ratio.