EE208 Experiment 3

Controller Design on MATLAB Platform by Analog Frequency Response

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Objective

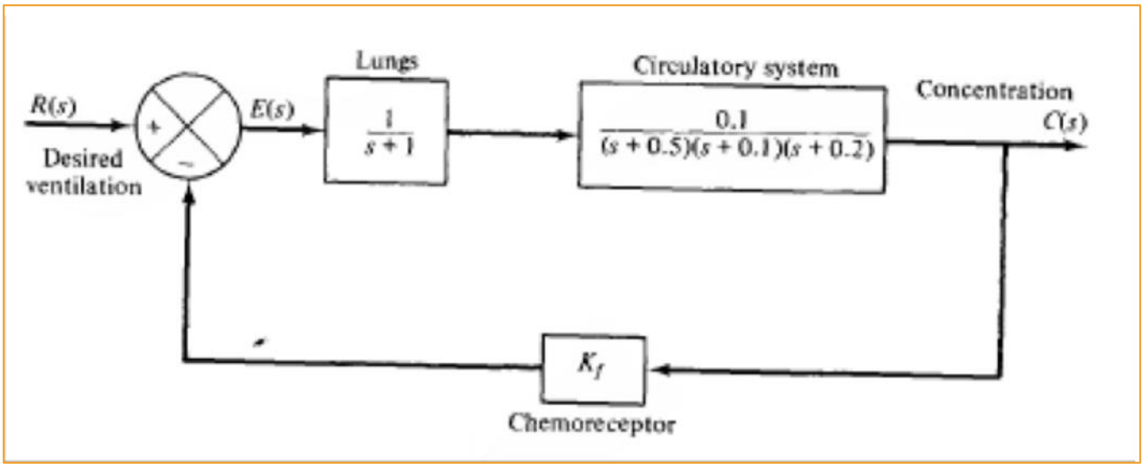
The project requires design of a cascade ventilator transfer function for a given analog respiratory system transfer function, according to desired specifications provided.

Project

This project deals with Analog Frequency Domain Design in MATLAB.

Given

The Block Diagram of the Human Respiratory System is given below.



Create a Ventilator transfer function, to be placed in cascade with lungs, to meet the requirements in the following sections.

Constraints

1. Kf may vary between 0.01 and 1, with 0.1 as the nominal value.
2. Time constant for the lungs may vary from 1 to 10.

Task

1. Design Ventilator TF to **maintain $\boldsymbol{Phase\ Margin \geq 45°}$ for the system**.
2. Also find Gain Margin and Phase Margins of the uncompensated system, for the typical value of chemoreceptor gain provided.

MATLAB Functions Used

```
bode, margin, tf, phlead, zeros, SamplingGrid, ndgrid, ControlSystemDesigner
```

MATLAB start-up code

```
tau = [1 2.5 5 7.5 10];
kf = [0.01 0.05 0.1 0.5 1];
B = tf(zeros(1,1,length(kf), length(tau)));
s = tf('s');
for i = 1:length(kf)
    for j = 1:length(tau)
        B(:,:,i,j) = 0.1/((tau(j)*s+1)*(s+0.5)*(s+0.2)*(s+0.1) + 0.1*kf(i));
    end
end
[kfgrid,taugrid] = ndgrid(kf,tau);
B.SamplingGrid = struct('kf',kfgrid,'tau',taugrid);
```

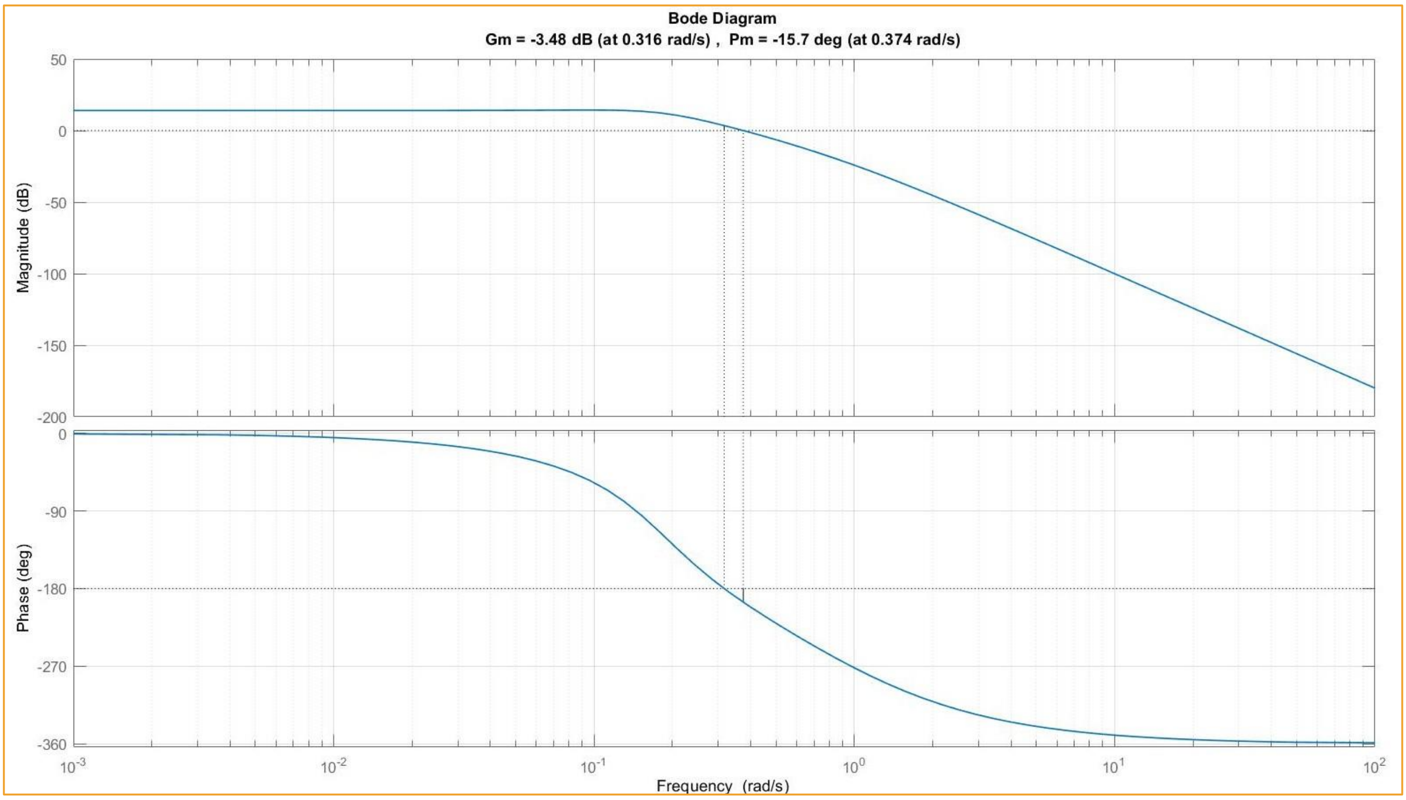
This code creates the CLTF of the body for various values of Kf and Lung Time constant in the form of a 1x1x5x5 Model Array.

Gain and Phase Margins of uncompensated system

The original system is $\frac{0.1}{s^4 + 1.8\,s^3 + 0.97\,s^2 + 0.18\,s + 0.02}$. We can view the Gain and Phase margins as follows –

```
margin(B(1,1,3,1)), grid on % Kf = 0.1, Tau = 1
```

- Gm = -3.48 dB at 0.316 rad/s
- Pm = -15.7 dB at 0.374 rad/s

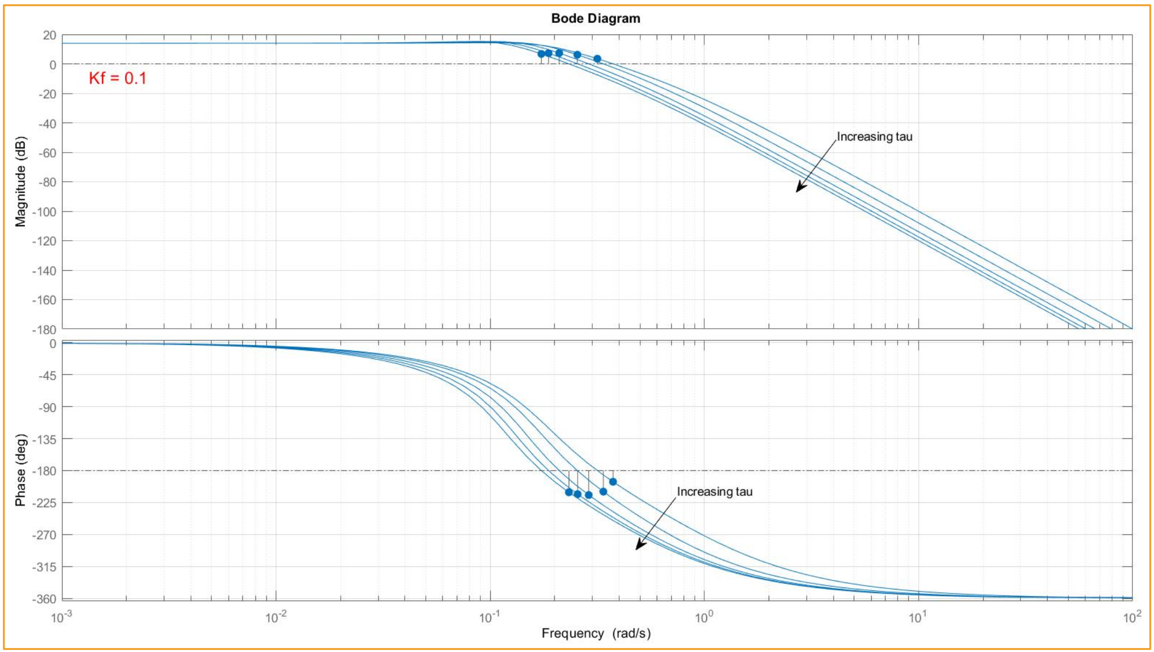


Variation of parameters

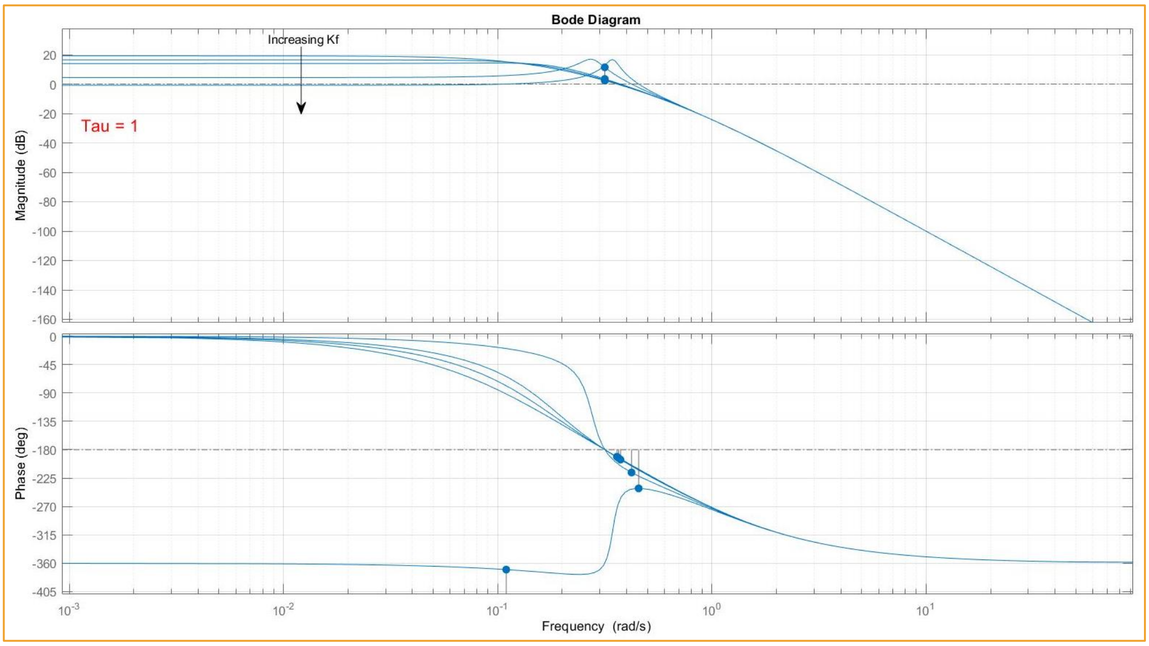
Let’s view some Bode plots to see the effect of varying Kf and tau.

```
bode(B), grid on
```

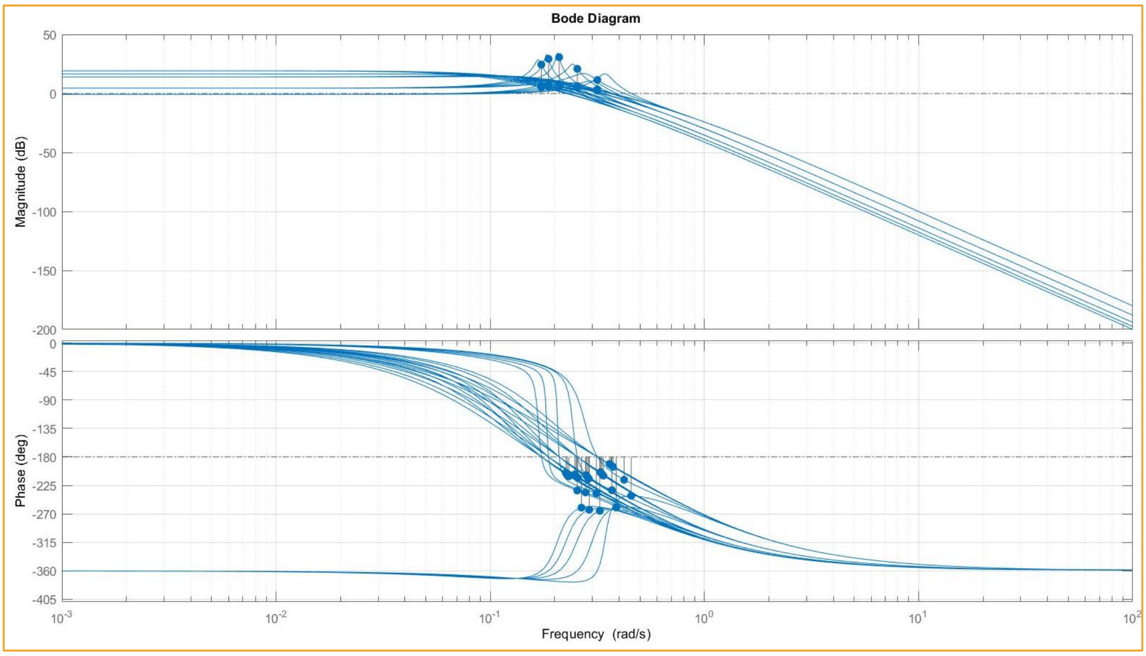
- **Kf = 0.1:**



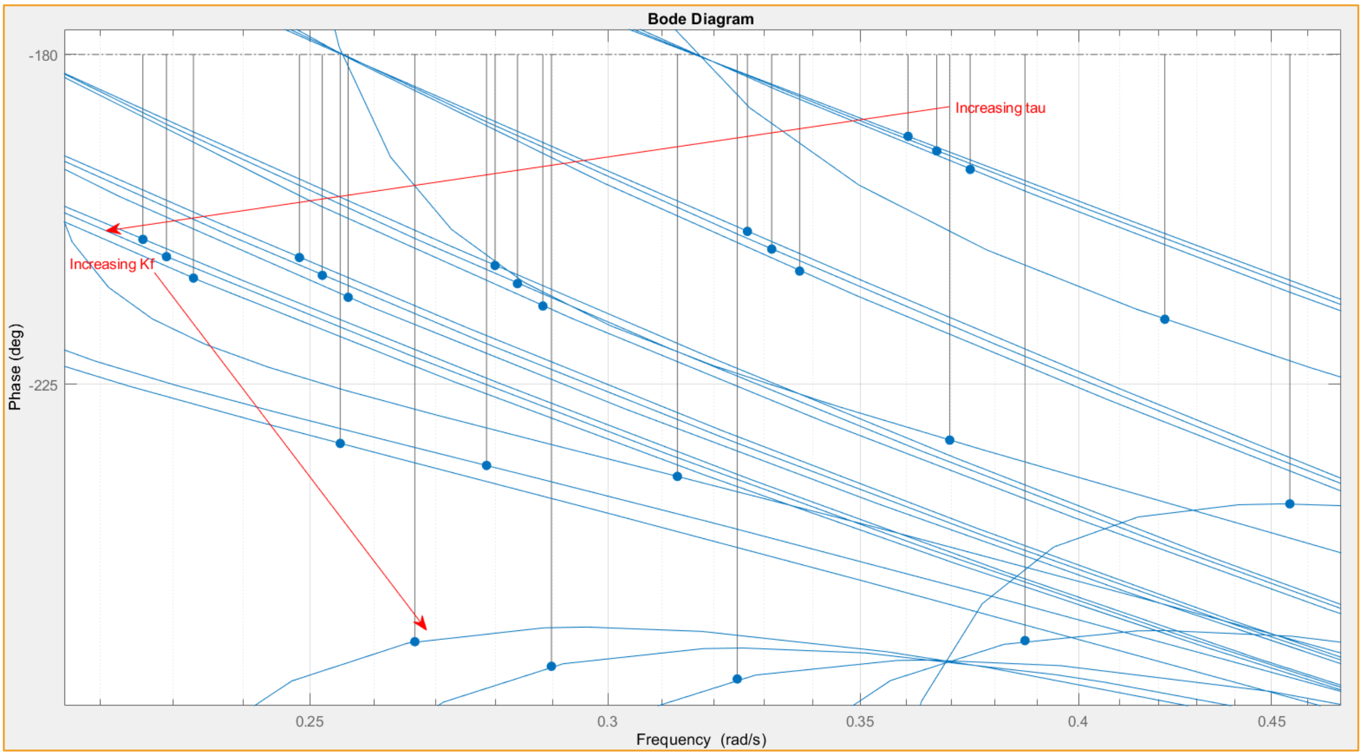
- **Tau = 1:**



We can also view the bode plots for the complete range of variations –



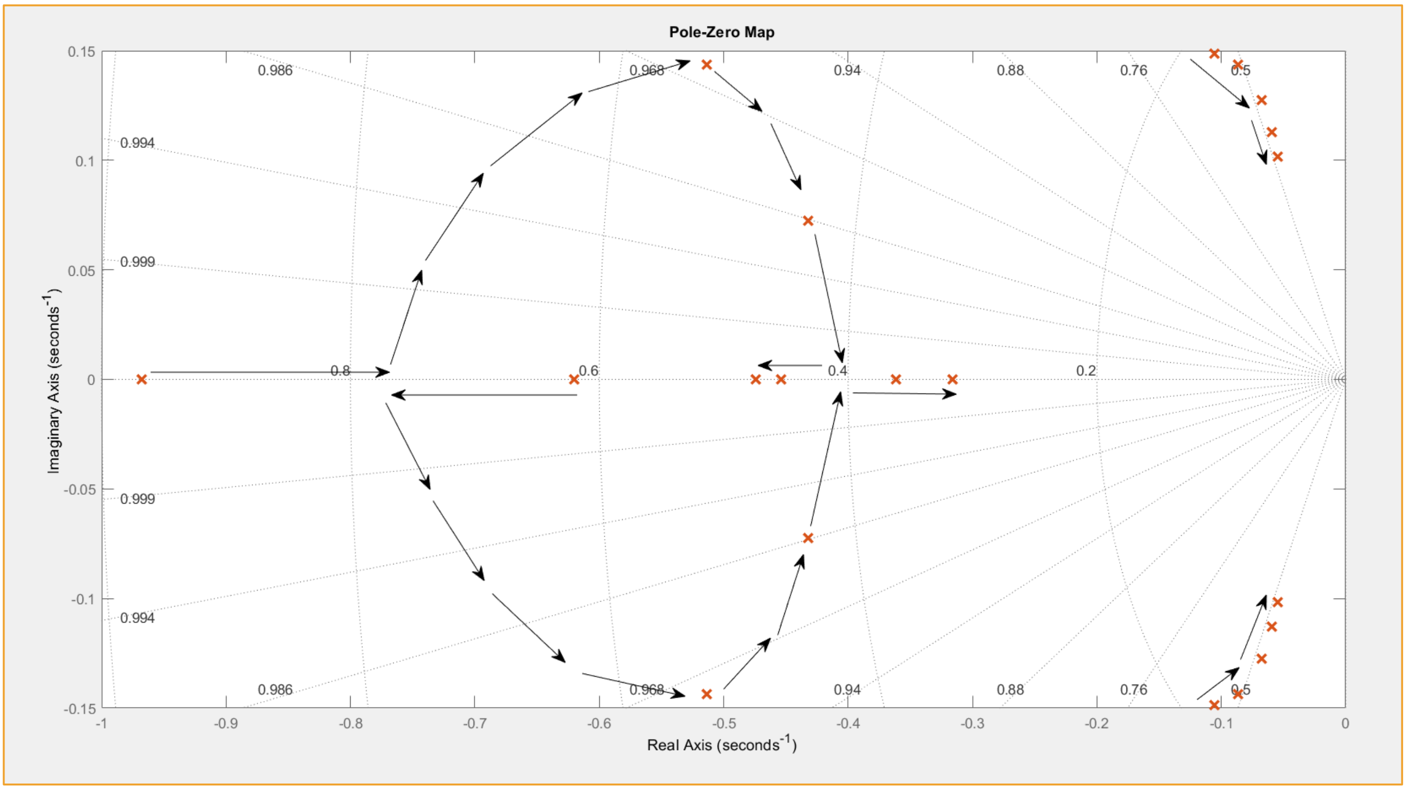
Let us zoom into the phase plot to see the phase margins.



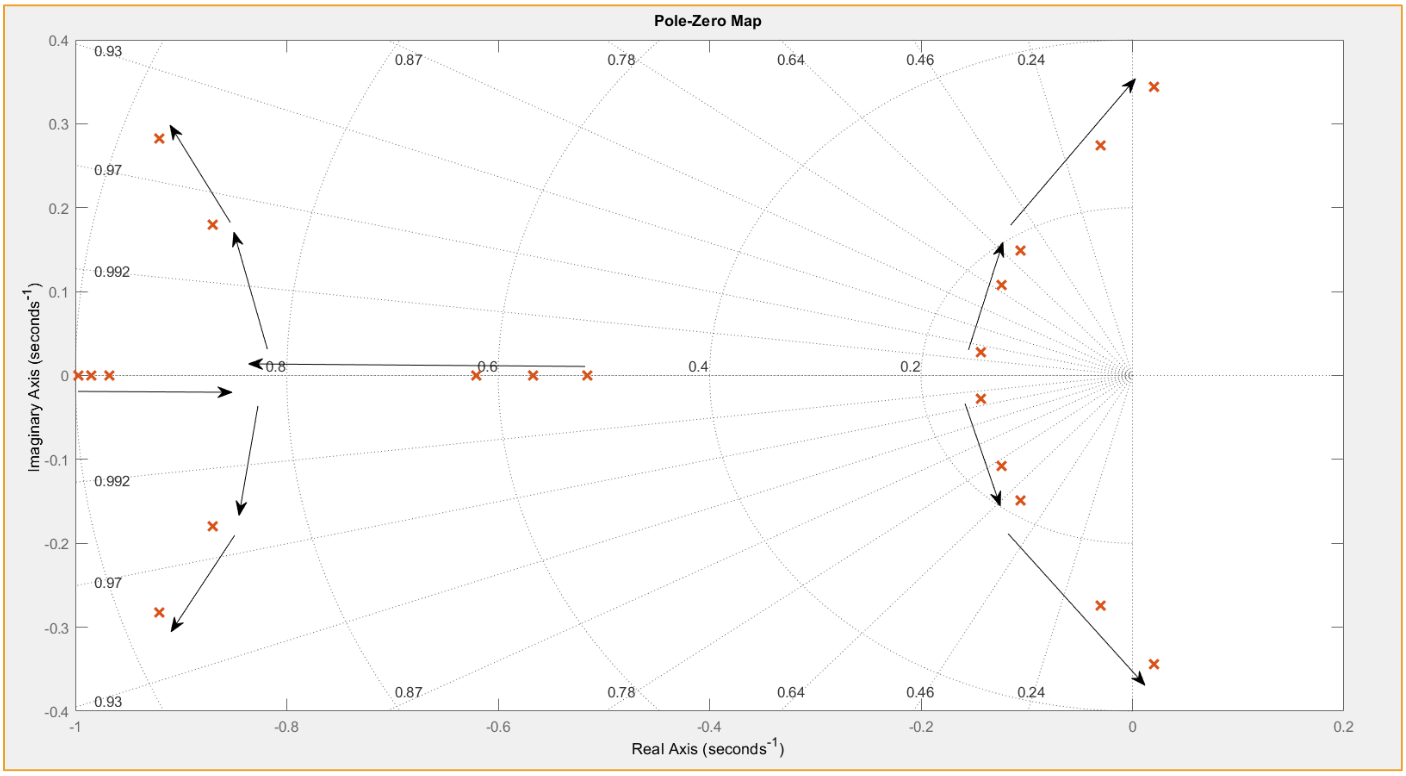
- The minimum phase margin is -85.2deg at 0.325 rad/s for kf=1, tau =5.
- The overall range of PMs is from -11.2deg to -85.2deg. The range of GCO frequencies is 0.226 rad/s to 0.455 rad/s.

We will also look at the pole-zero plots to better understand what is happening as the parameters vary.

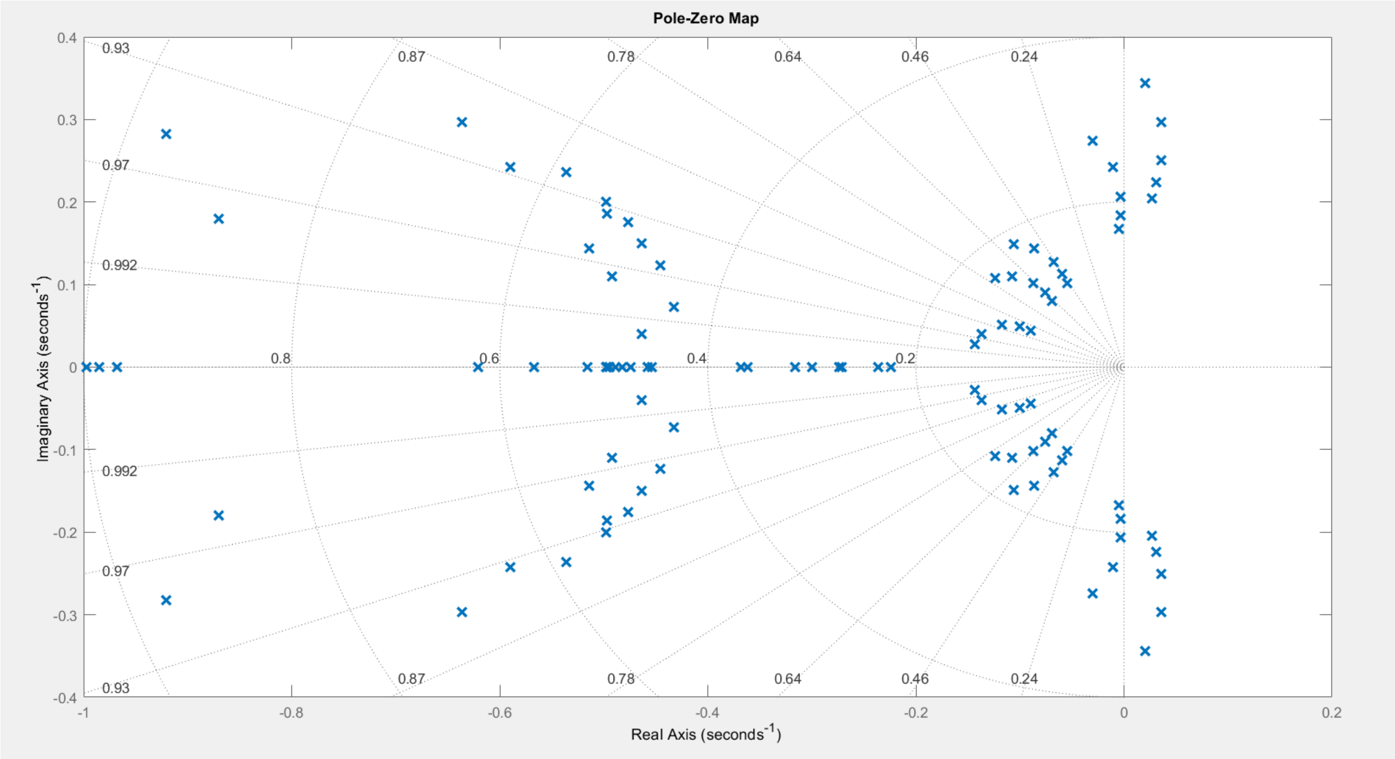
Kf=0.1, varying Tau (black lines show path of increasing Tau)



Tau=1, varying Kf (black lines show path of increasing Kf)



We can also look at the complete range of Tau and Kf:



There is a large variation in CL poles, giving rise to a great variation in PM and GCO frequencies.

In the next section, we will attempt to design the Transfer function for the Ventilator.

Design of the Ventilator TF

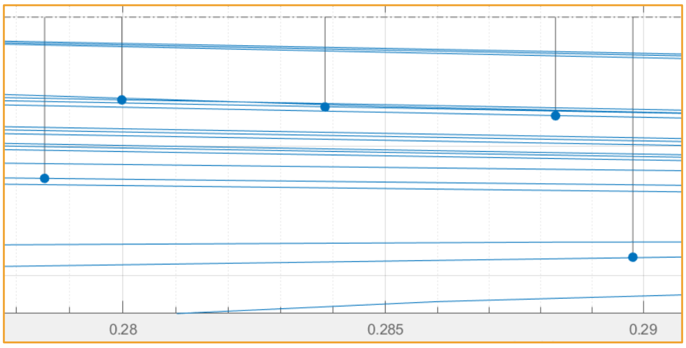
We cannot add the Ventilator *inside* the Body, i.e. we cannot break into the given loop. We must place a ventilator in the feedforward path before the lungs and add a second loop to achieve the desired specifications.

Our task is to maintain a PM of 45deg so the closed loop system is stable. We shall use a Lead Compensator for this purpose.

Let us look at the data we obtained in the previous section:

- Lower limit of PM: -85.2deg @ 0.325 rad/s
- Upper limit of PM: -11.2deg @ 0.36 rad/s
- Range of GCO Frequencies:
    - 0.226 rad/s, PM = -25.2
    - 0.455 rad/s, PM = -61.3

There is a wide variation in both PM and GCO frequencies. Furthermore, nearby frequencies also have a large variation in PM. For example –



Due to this, there will *unavoidably* be some extra phase margin for many configurations of Kf and Tau, since we must design keeping the worst case in mind.

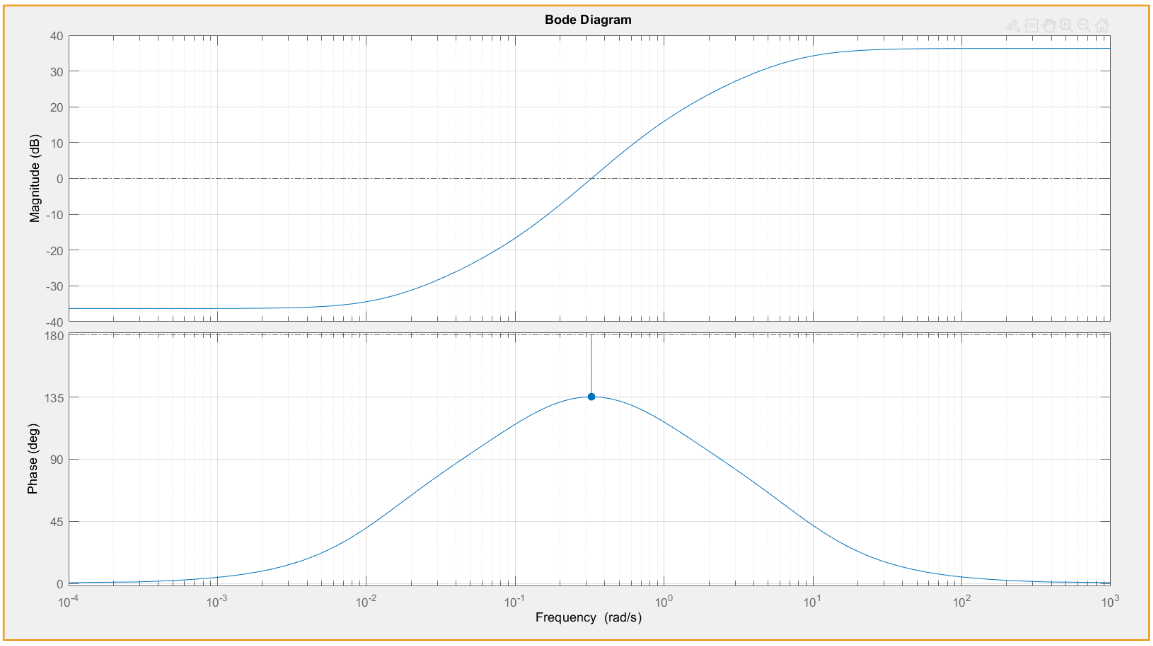
The worst case is -85.2deg at 0.325 rad/s for kf=1, tau =5. We shall try to design for this case.

We require a minimum PM of 45deg after compensation. Thus we need to add 45+85.2 degrees of Phase at 0.325 rad/s. We shall also add an extra 5 degrees of phase as a safety margin.

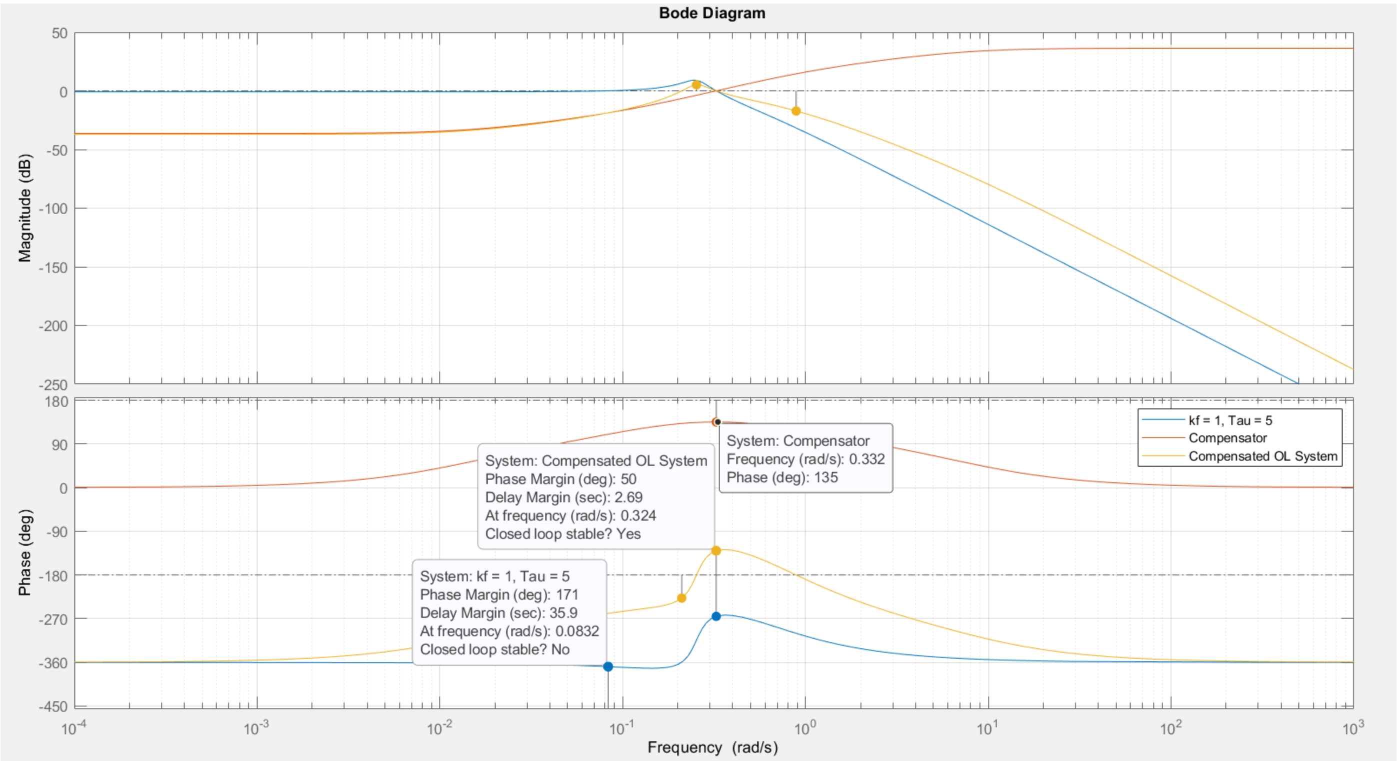
```
v1 = tf(phlead(0.325, 85.2))*tf(phlead(0.325, 50));
```

We are using two Lead Compensators, since we require 135.2 degrees of phase addition. With the help of the PHLEAD function, we can quickly create a lead compensator that provides unity gain at the centre frequency.

The Bode plot of this compensator is shown below.

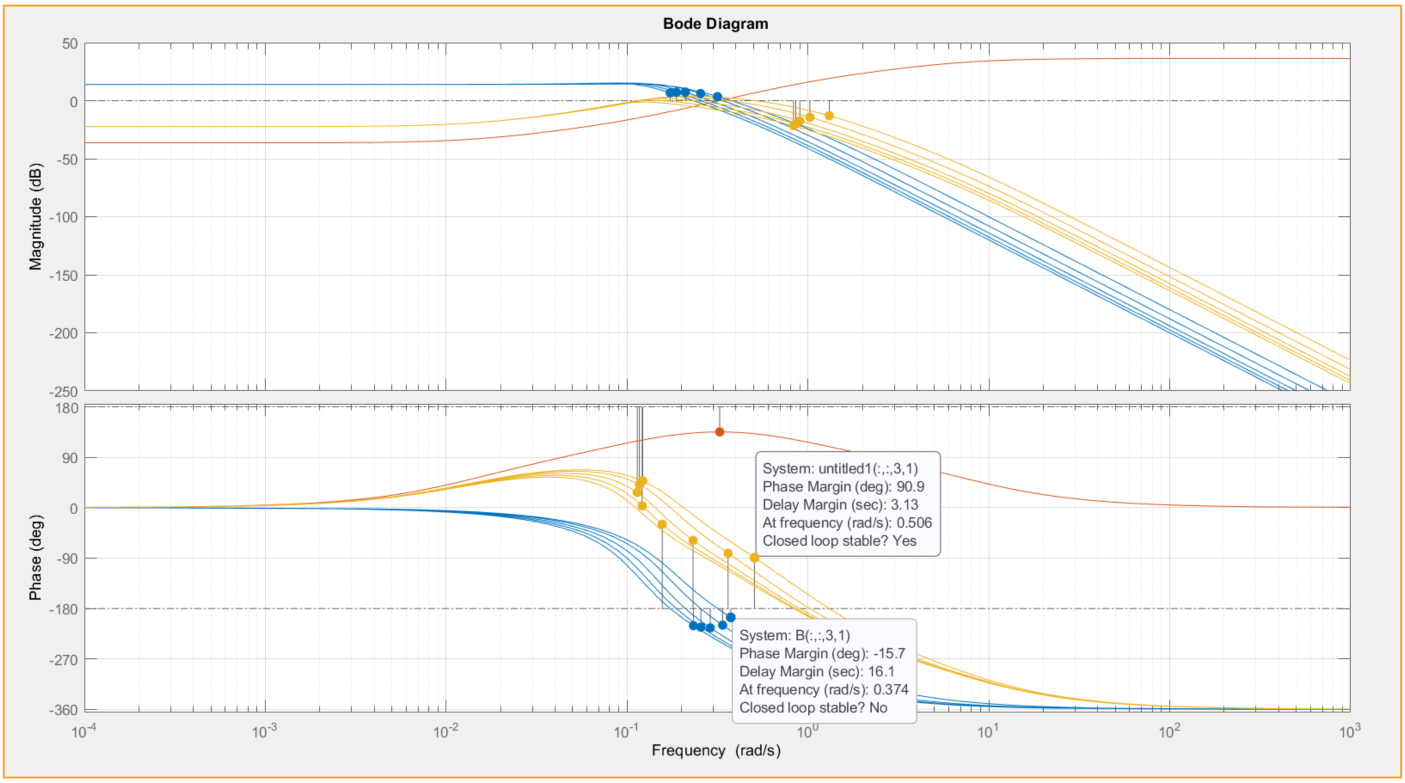


Let us see the effect on the worst case TF –



We see that the compensator works perfectly in this case, increasing the PM to 50 degrees without changing the GCO frequency. We use 50deg instead of 45deg as a safety margin, since the GCO is changed for other configurations of Kf and Tau.

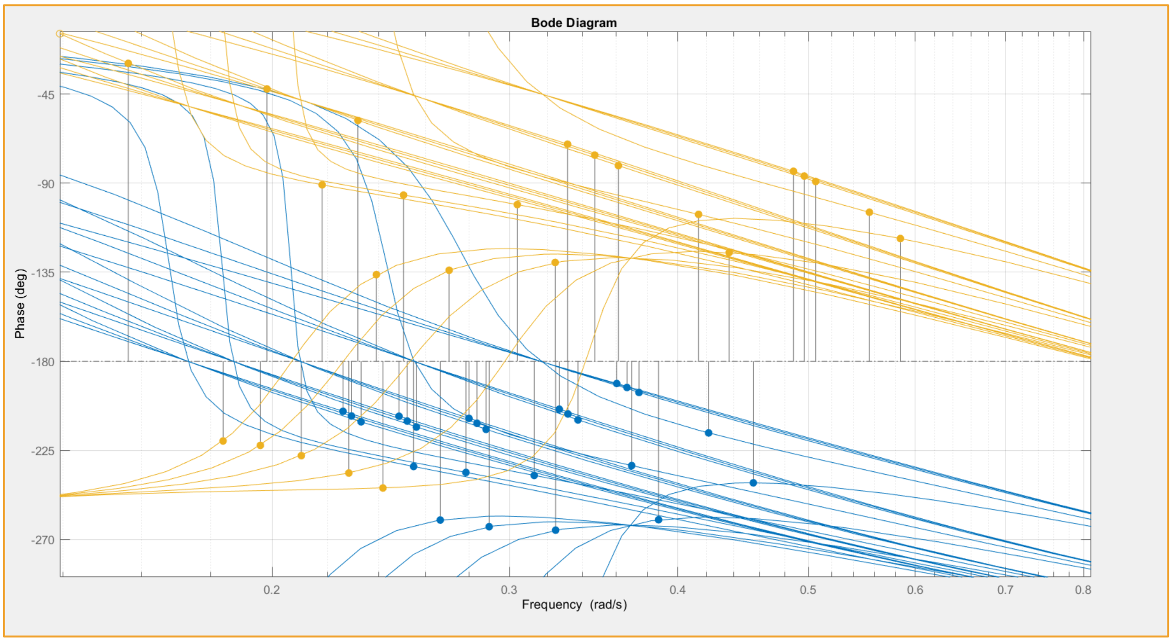
Let’s look at the effect on Kf = 0.1.



Here too the PM is greater than 45, but the GCO frequency has changed greatly and the PM reaches 90 degrees. This is unavoidable due to the gain response of the Lead Compensator.

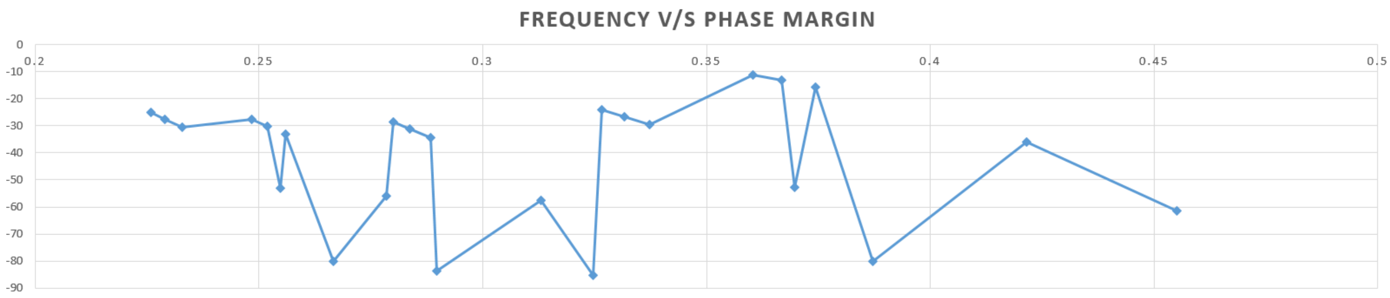
Overall, the Compensator does induce a gain margin of >= 45deg for all the cases, but inevitably adds some shift in the GCO frequency.

The compensator TF is $\frac{65.55\ s^2 + 8.647\ s + 0.105}{s^2 + 8.647\ s + 6.924}$, it ensures the appropriate PM for all parameter variations, thus ensuring closed loop stability.



Analysis

- The Lead Compensator works perfectly for the worst case, **hence ensuring a phase margin of at least 45deg across the range of parameter variations**.
- However, excessive phase is added for some configurations.
- This is a consequence of the great variation in phase margins due to parameter variation, which placed close together in frequency.
- We tried to resolve this by using a lag compensator at 0.226 rad/s, but the frequency spread due to parameter variation is very small – 0.226 rad/s to 0.455 rad/s. As a result, the lag compensator had very little effect and was thus discarded.
- It is not possible to achieve exactly 45deg phase margin across all parameter variations because the phase margin as a function of frequency changes very abruptly. In order to ensure that the system *always* has a PM of at least 45 degrees, we must focus on the configuration requiring greatest addition to the phase, which will then cause excess phase in other cases.



Conclusions

1. In this experiment, we examined a model of the human respiratory system. We observed it frequency response and found the Gain and Phase margins.
2. We were tasked to design a ventilator transfer function to maintain a phase margin of 45 degrees in light of variations in the system TF.
3. The "worst" case scenario was found and compensated appropriately with 2 lead compensators in series.
4. Thus the Design Requirement has been fulfilled.
5. The Compensator TF is $\frac{65.55\, s^2 + 8.647\, s + 0.105}{s^2 + 8.647\, s + 6.924}$.

References

- *Automatic Control Systems, 9th Ed.* – 8.2, 8.3, 8.4, 8.11, Chapter 9
- *Feedback Control of Dynamic Systems, 8th Ed.* – Chapter 6
- Relevant MATLAB Documentation
- Daniel J. Auger (2022). Phase Lead Compensator (https://www.mathworks.com/matlabcentral/fileexchange/40480-phase-lead-compensator), MATLAB Central File Exchange. Retrieved January 30, 2022.