EE208 LAB 12

Transfer between digital states.

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Objective

In this experiment we consider a given three-variable digital system with parameters susceptible to arbitrariness of values (or for that matter, adjustable by the operator), and check out the scope and limits of possible deadbeat type performance.

Project statement

The following three variable system has arbitrary values or settings possible for parameters a and b:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & -a/b \end{bmatrix} \cdot \begin{bmatrix} x_1(k) \\ x_2(k) \\ x3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot u(k)$$

1. Given $\boldsymbol{x}(0) = [1\ 1\ 1]^T$, find out if $\boldsymbol{x}(3) = \mathbf{0}$ can be achieved.
2. Given $\boldsymbol{x}(0) = 0$, find out if $\boldsymbol{x}(3) = [1\ 1\ 1]^T$ can be achieved.

Give a solution (if any) in terms of the parameters and a suitable input.

It is not necessary that the performance will be always achievable, but in either case, the solution must be accompanied by sound set of reasons, and substantiated by appropriate studies on MATLAB.

Original System

We define the system in MATLAB as

```
syms a b;
A = [0 1 0;
     0 0 1;
     a b -a/b];
B = [0;1;0];
C = eye(3);
D = 0;
```

Using eig(A), the system eigenvalues are $\pm\sqrt{b}, \frac{-a}{b}$.

For deadbeat control, we require that the A matrix have all its eigenvalues equal to 0.

Putting $a = 0$ sets one pole to zero, but $b \neq 0$, so we can't directly set the other two poles equal to zero. The input $u(k)$ in open-loop will also have no effect on the system poles, so it is not useful here.

It isn't possible to achieve deadbeat dynamics by only OL control – hence we'll try to use feedback control and pole placement for this task.

State Feedback Control

We'll use a controller of the form $u = K(\boldsymbol{r} - \boldsymbol{x})$.

We need to find K such that $det(A - BK) = s^3$. Equivalently all the eigenvalues of $A - BK$ should be 0.

Now, $A - BK$ is:

$$\boxed{\begin{pmatrix} 0 & 1 & 0 \\ -k_1 & -k_2 & 1 - k_3 \\ a & b & -\frac{a}{b} \end{pmatrix}}$$

Having eigenvalues –

$$\begin{pmatrix} -\frac{k_2}{2} - \frac{\sqrt{{k_2}^2 + 4\,b - 4\,k_1 - 4\,b\,k_3}}{2} \\ \frac{\sqrt{{k_2}^2 + 4\,b - 4\,k_1 - 4\,b\,k_3}}{2} - \frac{k_2}{2} \\ -\frac{a}{b} \end{pmatrix}$$

The third eigenvalue is unaffected. Thus, we must deal with it directly – setting $\boldsymbol{a = 0}$.

We also wish to find $k_1,\ k_2,\ k_3$ such that the first two rows also equal 0.

We must set $k_2 = 0$.

If we can set $b - k_1 - bk_3$ equal to 0, our task is done. Setting $k_1 = b(1 - k_3)$ is sufficient.

If we set $b = 1$ and $k_3 = 1$ we obtain the CL system matrix as –

```
Ak = A-B*k
Ak = 3×3
     0     1     0
     0     0     0
     0     1     0
```

Which has the eigenvalues [0, 0, 0].

Now, by the Cayley-Hamilton theorem –

$$A_{\mathrm{k}}^3 = 0$$

This means that at $t = 3$, and beyond, the matrix is zero, so the state is completely determined by the input/reference. Thus, the system will reach the desired state in at most three time steps.

Deadbeat Control

Given $\boldsymbol{x}(0) = 0$, desired $\boldsymbol{x}(3) = [1\ 1\ 1]^T$

1. We have $\boldsymbol{x}(0) = [0\ 0\ 0]^T$. We set $\boldsymbol{r} = [1\ 1\ 1]^T$.
2. Using the state equations, we can obtain the next values –
3. $\boldsymbol{x}(1) = A_k\boldsymbol{x}(0) + BK\boldsymbol{r}(0)$
4. Thus, $\boldsymbol{x}(1) = [0\ 1\ 0]^T$.
5. Similarly, we obtain $\boldsymbol{x}(2) = [1\ 1\ 1]^T$ and $\boldsymbol{x}(3) = [1\ 1\ 1]^T$.

Thus, we can achieve the desired state at $t = 3$ with the following settings –

1. a = 0
2. b = 1 (otherwise the state x_3 does not settle down to 1)
3. State Feedback with K = [0 0 1]
4. Reference input of $[1\ 1\ 1]^T$

Given $\boldsymbol{x}(0) = [1\ 1\ 1]^T$, desired $\boldsymbol{x}(3) = \boldsymbol{0}$

In this case we can freely choose b and x_3, it will have no effect on the deadbeat nature.

Now, let us see the evolution of states –

1. $\boldsymbol{x}(0) = [1\ 1\ 1]^T$
2. $\boldsymbol{x}(1) = \begin{pmatrix} 1 \\ b\ (k_3 - 1) - k_3 + 1 \\ b \end{pmatrix}$
3. $\boldsymbol{x}(2) = \begin{pmatrix} b\ (k_3 - 1) - k_3 + 1 \\ 0 \\ b\ (b\ (k_3 - 1) - k_3 + 1) \end{pmatrix}$
4. $\boldsymbol{x}(3) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Thus this is also acheivable with –

1. a = 0
2. b = free variable
3. State Feedback with K = $\begin{pmatrix} -b\ (k_3 - 1) & 0 & k_3 \end{pmatrix}$
4. Reference input of $[0\ 0\ 0]^T$

As before, $A_k^3 = 0$ causes the system to settle down at the third time step.

Conclusions

In this experiment we studied a given digital system and achieved deadbeat control – settling down of states within a given number of time steps. We used state-feedback control to find solutions for the two problems given in the project statement.

MATLAB Code

https://drive.google.com/file/d/1L7qRPaKbP54uGJiQ4c8YI8PE7OG0M51Y/view?usp=sharing