INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER I 2020 − 21
MTL100 CALCULUS
QUIZ 1

DATE: 06/12/2020 DURATION: 2:30 PM - 3:30 PM

Total Marks: 20

(Each question carries 5 marks)

Question 1: Compute the $\limsup_{n\to\infty} a_n$ and $\liminf_{n\to\infty} a_n$ for the sequence $(a_n)_{n\geq 1}$, where

$$a_n = \frac{(-1)^n(n+5)}{n} \quad \text{for } n \in \mathbb{N}.$$

Question 2: Let $(x_n)_{n\geq 1}$ be a sequence defined by

$$x_1 = 1 \text{ and } x_{n+1} = 3 + \frac{1}{x_n} \quad \text{for } n \in \mathbb{N}.$$

Show that the sequence $(x_n)_{n\geq 1}$ is Cauchy, and find the limit of the sequence.

Question 3: Let $x_n = (2^n + 3^n)^{\frac{1}{n}}$ for $n \in \mathbb{N}$. Discuss whether the sequence $(x_n)_{n\geq 1}$ is convergent or not. If the sequence $(x_n)_{n\geq 1}$ is convergent then find its limit.

Question 4: Let $a_n = \sqrt{n^4+1} - \sqrt{n^4-1}$ for $n \in \mathbb{N}$. Discuss whether the series $\sum_{n=1}^{\infty} a_n$ is convergent or not.

—ALL THE BEST—