INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
SEMESTER I 2020 − 21
MTL 100 (CALCULUS)
Quiz-2 Examination

DATE: 24/01/2021 Total Marks: 20 Time: 2.30 – 3:15 pm

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

Question 1: Let the function $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ be given by

$$f(x,y) = \frac{x \sin(x^2+2y^2)}{x^2+y^2}. \qquad [5]$$

Is it possible to define $f(0,0)$ so that f is continuous at $(0,0)$? Justify your answer.

Question 2: Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \frac{xy(x^2+y^4)}{x^4+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Discuss the differentiability of f at $(0,0)$. [5]

Question 3: Find the second order Taylor polynomial $P_2(x,y)$ for

$$f(x,y) = 2e^{x+y} - \sin(xy),$$

at $(0,0)$. Estimate the error approximation of $|f(x,y) - P_2(x,y)|$ if $|x| < 0.1$, $|y| < 0.1$. [5]

Question 4: Using Lagrange Multiplier, find the points on the surface given by

$$z^2 = xy + 4$$

closest to the origin. [5]

—ALL THE BEST—