B.Sc. Mathematics (Hons.)
Mid Term Examination

Gautam Buddha University

November 14, 2024

Course: B.Sc. Mathematics (Hons.)
Duration: 1.5 Hours
Total Marks: 25

Instructions:

- All questions are compulsory.
- Marks for each question are indicated against it.
- Use of calculator is not permitted.
- Neatness and clarity of presentation will be rewarded.

Section A: Objective Type Questions (5 Marks)

Answer the following multiple-choice questions (1 mark each):

1. If $f(x) = x^2 + 3x + 2$, what is $f'(x)$?
   - (a) $2x + 3$
   - (b) $2x + 3 + 2$
   - (c) $3x + 2$
   - (d) $2x + 3 + 3$
2. Which of the following is a solution to the differential equation $y'' + 2y' + y = 0$?
   - (a) $y = e^{-x}$
   - (b) $y = e^{2x}$
   - (c) $y = \sin(x)$
   - (d) $y = x^2$
3. The matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ has:
   - (a) Determinant = -2

- (b) Determinant = 0
- (c) Determinant = 2
- (d) Determinant = 1

4. The sum of the infinite geometric series $1 + \frac{1}{2} + \frac{1}{4} + \dots$ is:
   - (a) 2
   - (b) 1
   - (c) $\frac{3}{2}$
   - (d) 4
5. Which of the following is the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$?
   - (a) $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$
   - (b) $\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$
   - (c) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
   - (d) $\begin{pmatrix} 2 & -1 \\ -3 & 1 \end{pmatrix}$

Section B: Short Answer Questions (10 Marks)

Answer the following questions. Each question carries 2 marks.

1. Find the integral of $f(x) = 3x^2 + 5x - 7$ with respect to x.
2. Solve the system of equations:
$$2x + 3y = 5 \quad \text{and} \quad 4x - y = 7$$
3. Find the eigenvalues of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.
4. Differentiate the function $f(x) = \sin(x) \cdot e^x$.
5. Find the limit of $\lim_{x \to 0} \frac{\sin(x)}{x}$.

Section C: Long Answer Questions (10 Marks)

Answer the following questions. Each question carries 5 marks.

1. Prove that the product of two continuous functions is continuous.
2. Find the solution of the differential equation:
$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1$$
using the method of separation of variables.