Minimum Spanning Tree

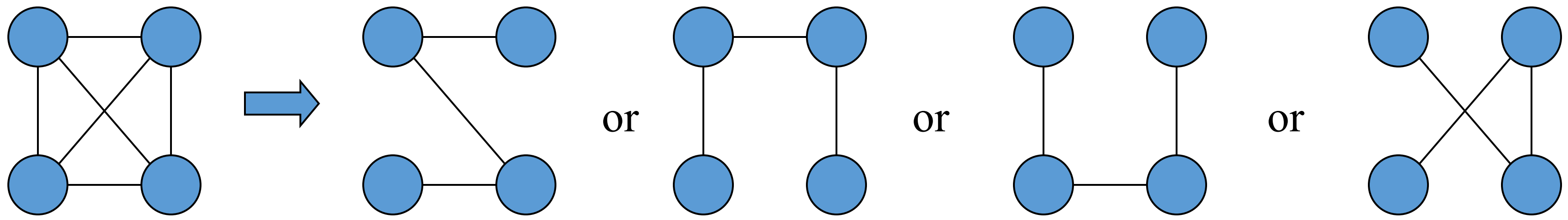
Spanning Trees

A spanning tree of a graph is just a subgraph that contains all the vertices and is a tree (no cycle).

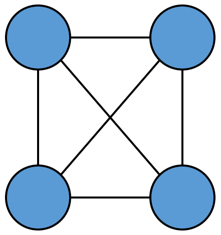
A graph may have many spanning trees.

Some Spanning Trees from Graph A

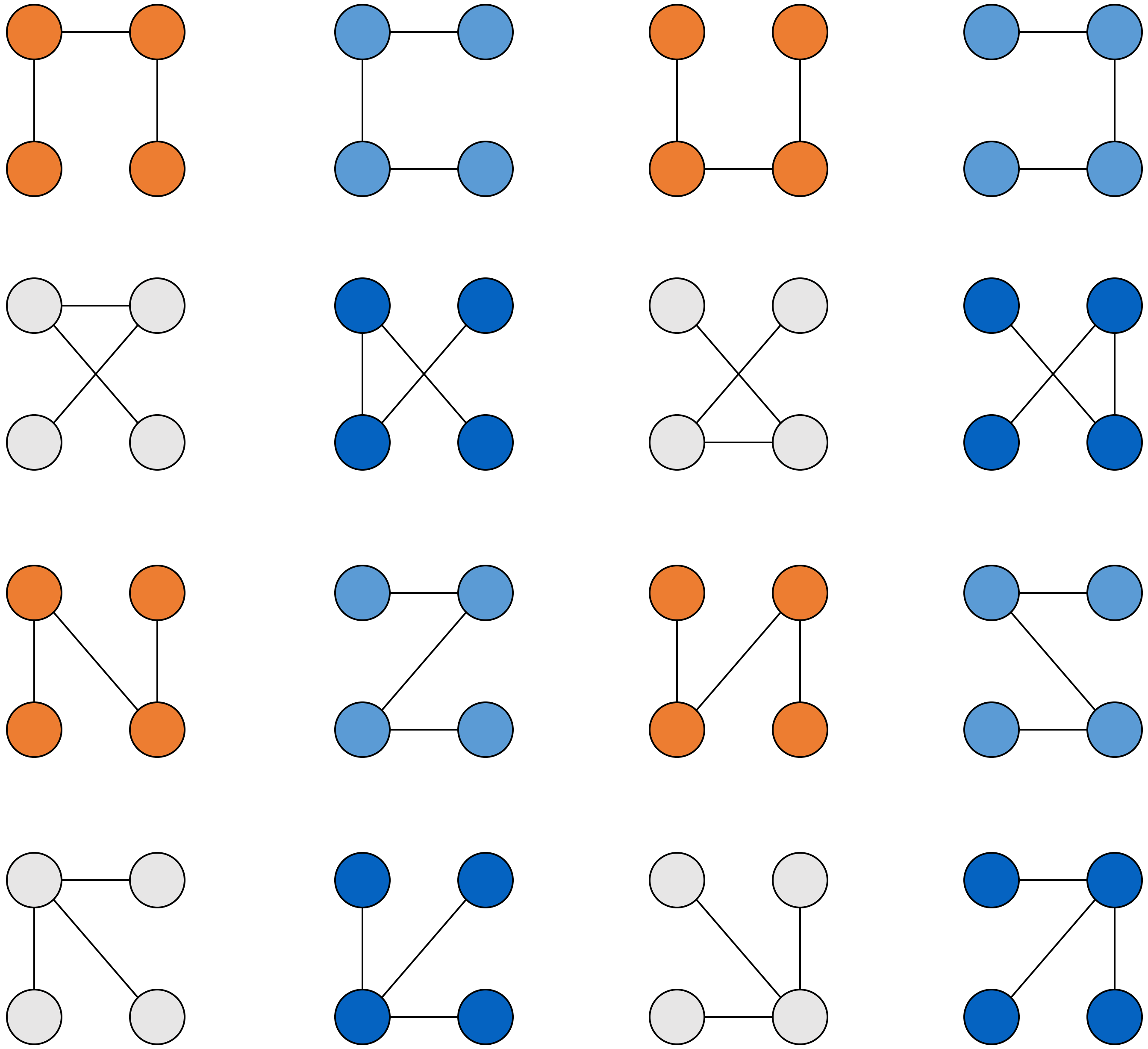
Graph A



Complete Graph



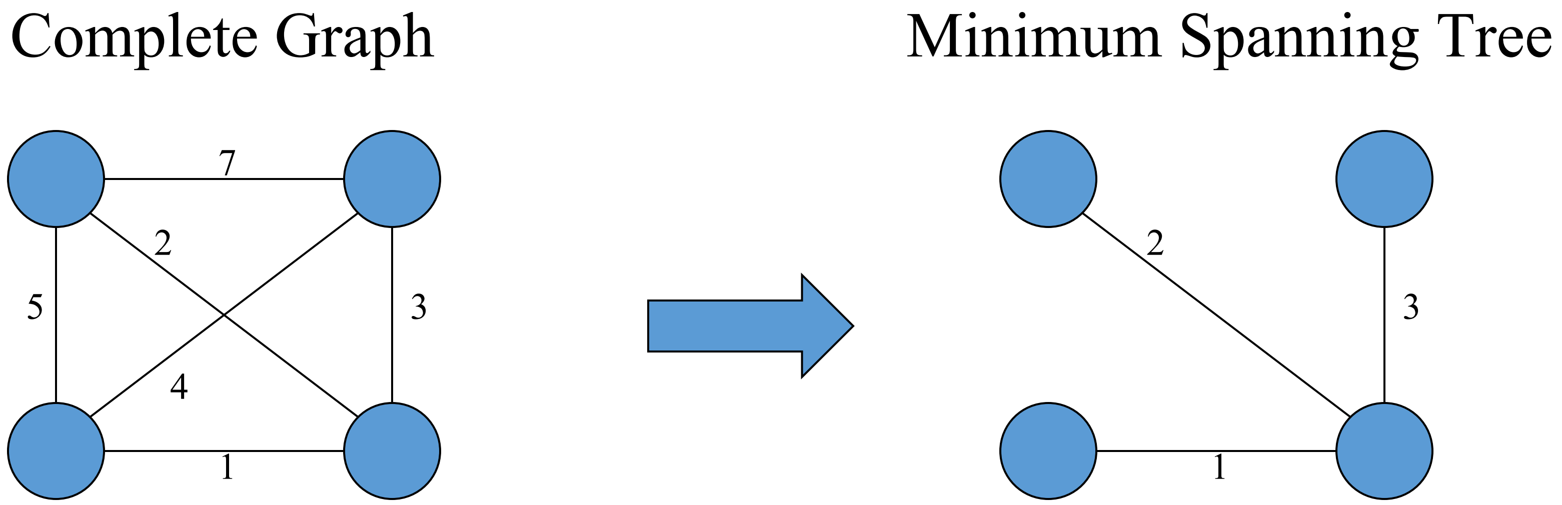
All 16 of its Spanning Trees



Minimum Spanning Trees

Minimum Spanning Trees

The Minimum Spanning Tree (MST) for a given graph is the Spanning Tree of minimum cost for that graph.



A spanning tree of G is a free tree (i.e., a tree with no root) with **| V | - 1** edges that connects all the vertices of the graph.

Minimum Spanning Tree(MST)

- A minimum spanning tree connects all nodes in a given graph.
- A MST must be a connected and undirected graph.
- A MST can have weighted edges.
- Multiple MSTs can exist within a given undirected graph.

More about Multiple MSTs

- Multiple MSTs can be generated depending on which algorithm is used.

- If you wish to have an MST start at a specific node However, if there are weighted edges and all weighted edges are unique, only **one MST** will exist.

Algorithms for Obtaining the Minimum Spanning Tree

- Kruskal's Algorithm

- Prim's Algorithm

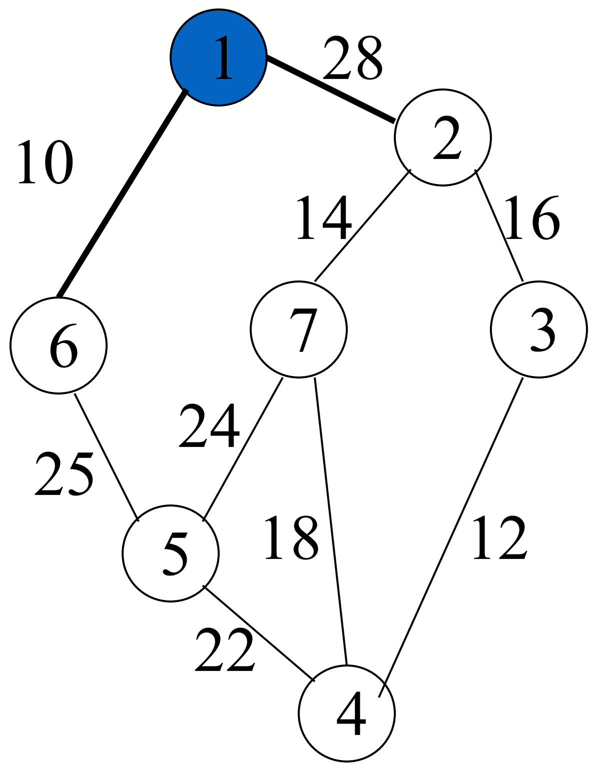
Prim's Algorithm

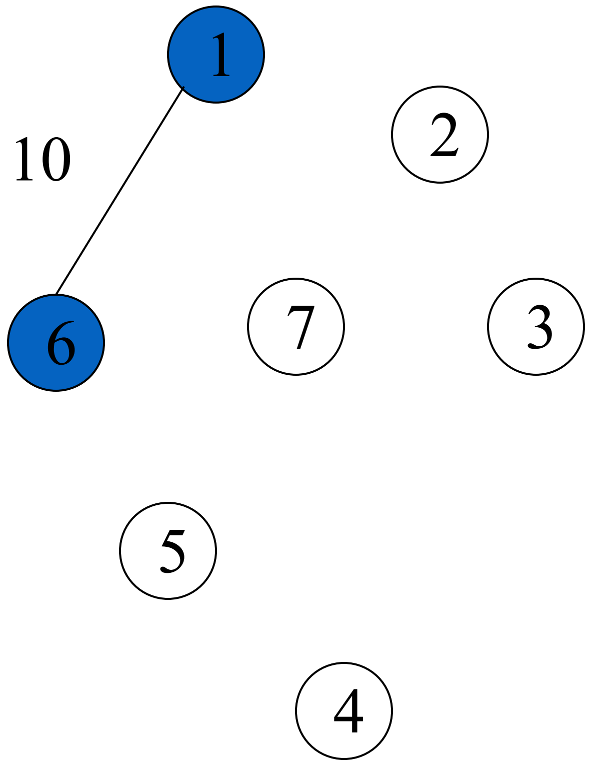
Prim’s Algorithm

- Prim’s algorithm finds a minimum cost spanning tree by selecting edges from the graph one-by-one as follows:
- It starts with a tree, T, consisting of the starting vertex, x.
- Then, it adds the shortest edge emanating from x that connects T to the rest of the graph.
- It then moves to the added vertex and repeats the process.

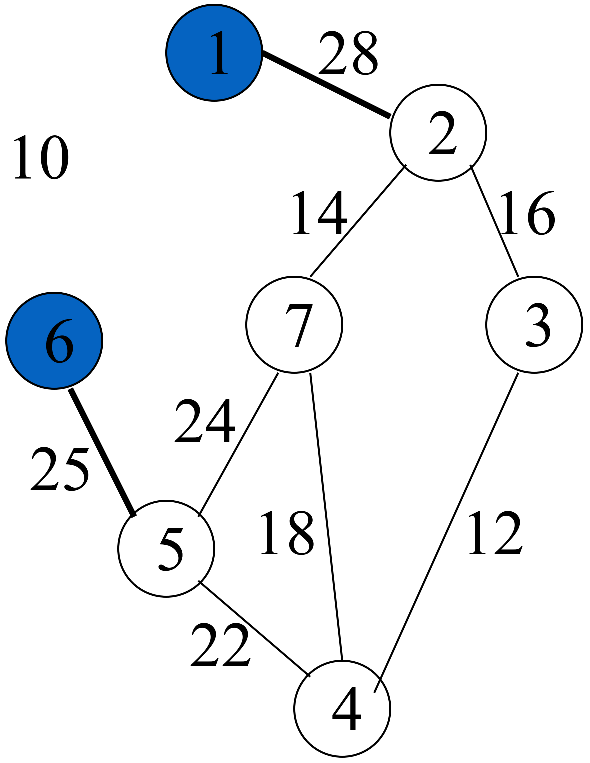
```
Consider a graph G=(V, E);
Let T be a tree consisting of only the starting vertex x;
while (T has fewer than |V| vertices)
{
        find a smallest edge connecting T to G-T;
        add it to T;
}
```

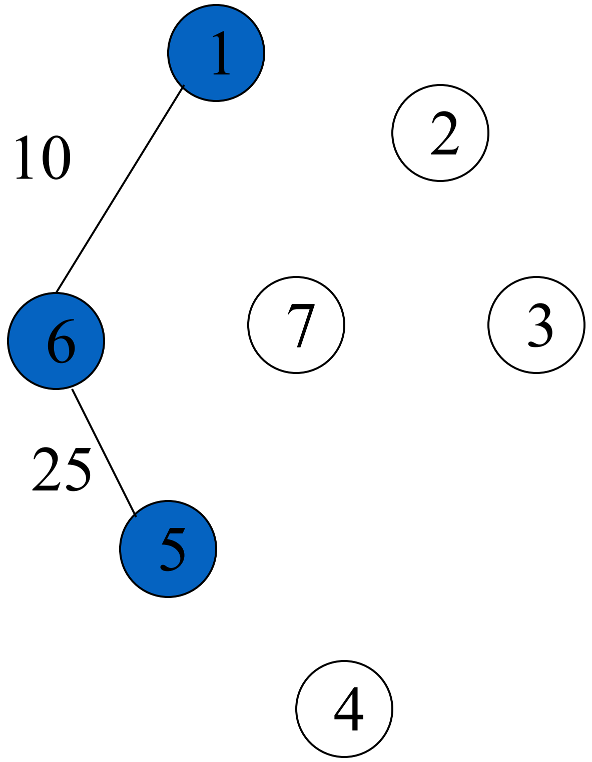
Prim’s algorithm





Prim’s algorithm





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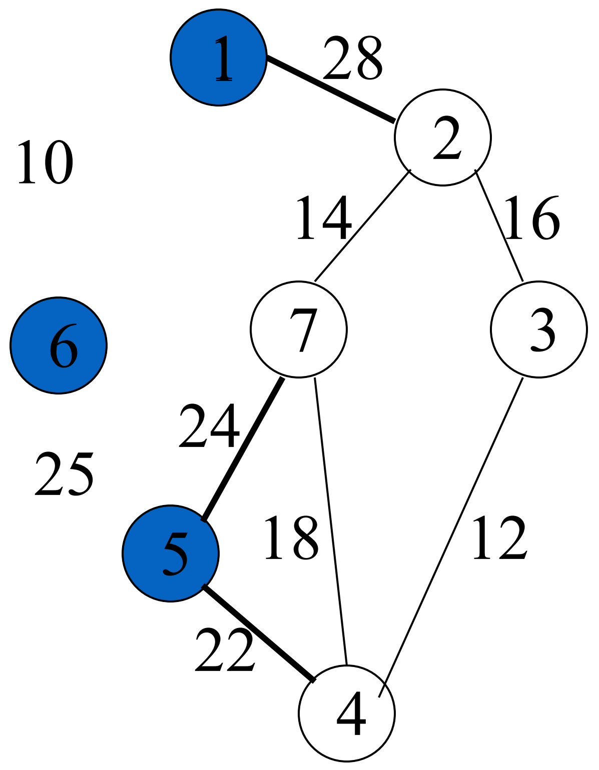

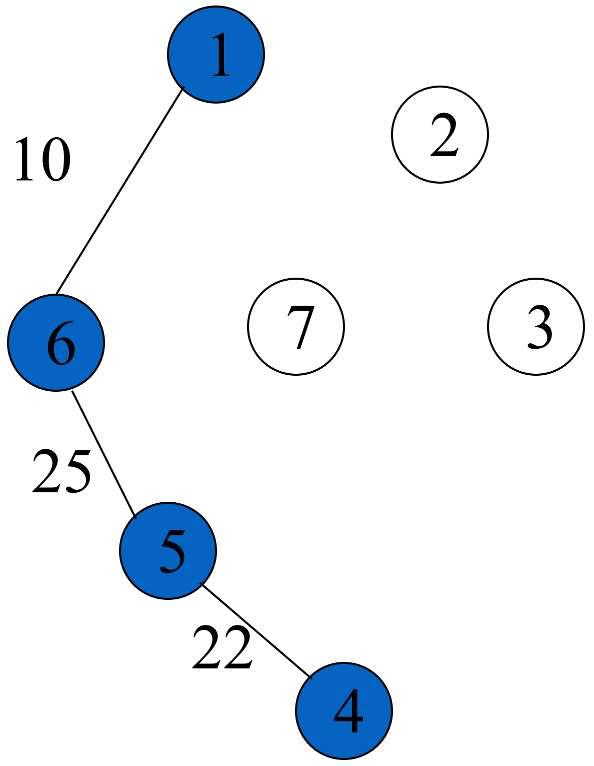

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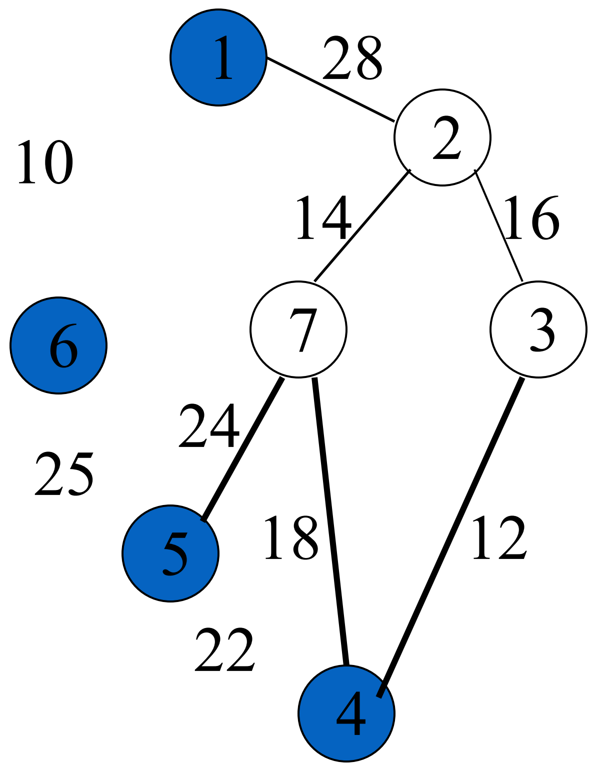
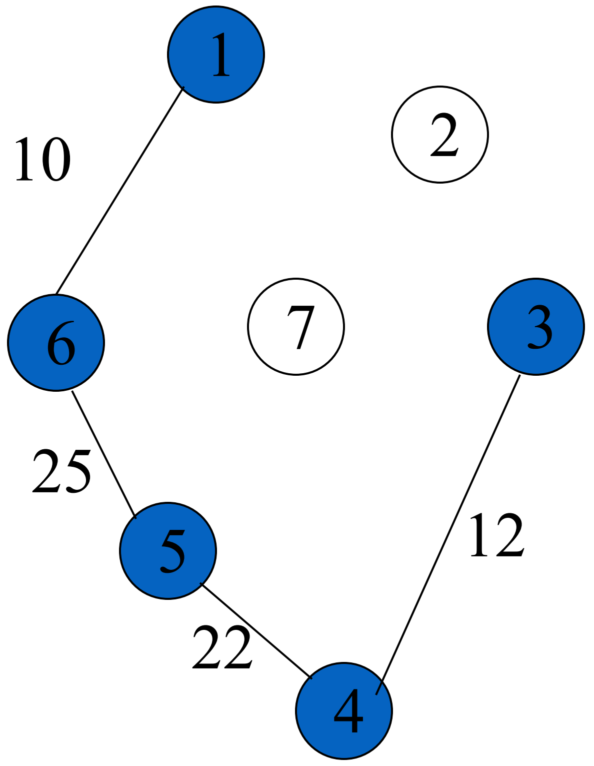
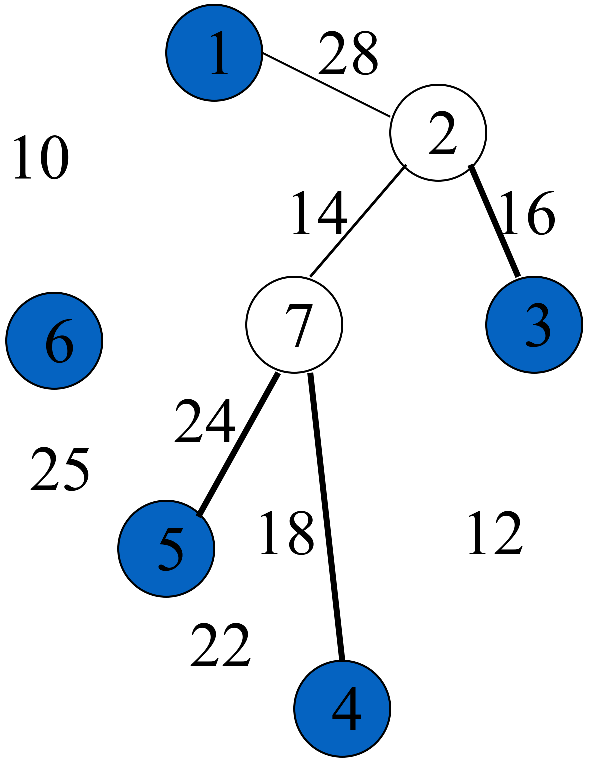
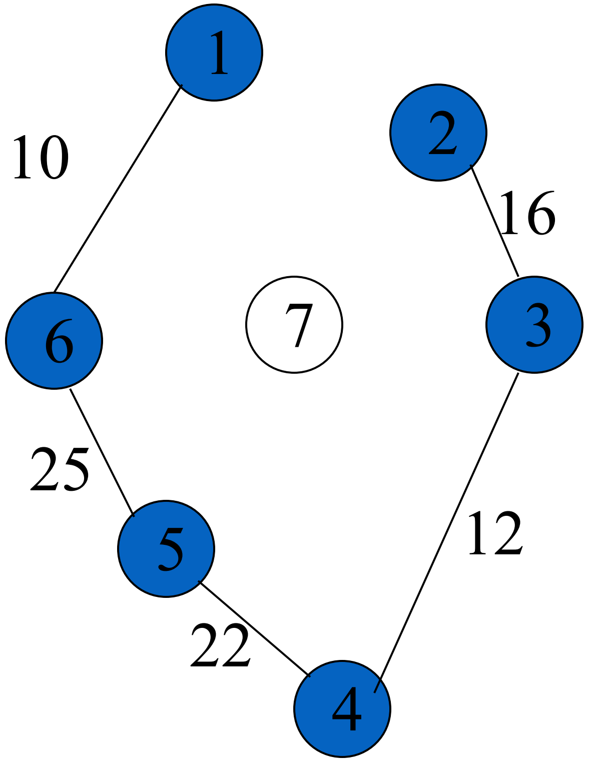


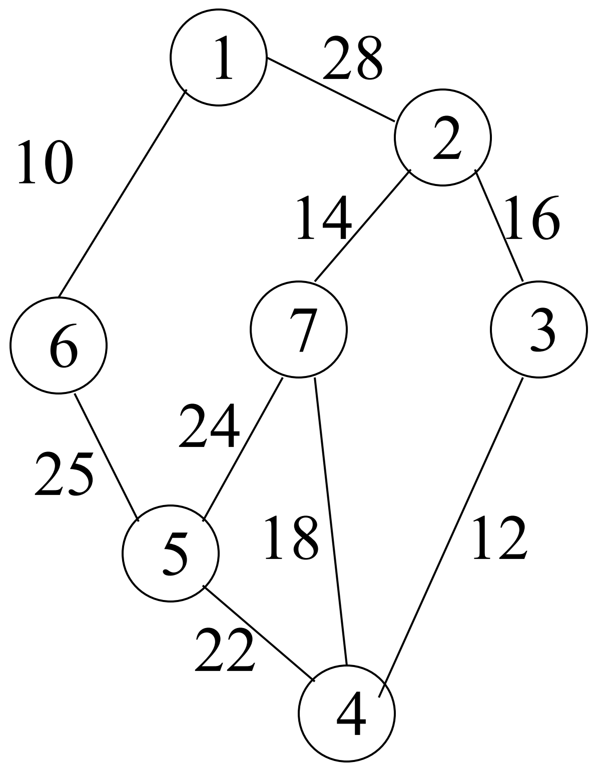


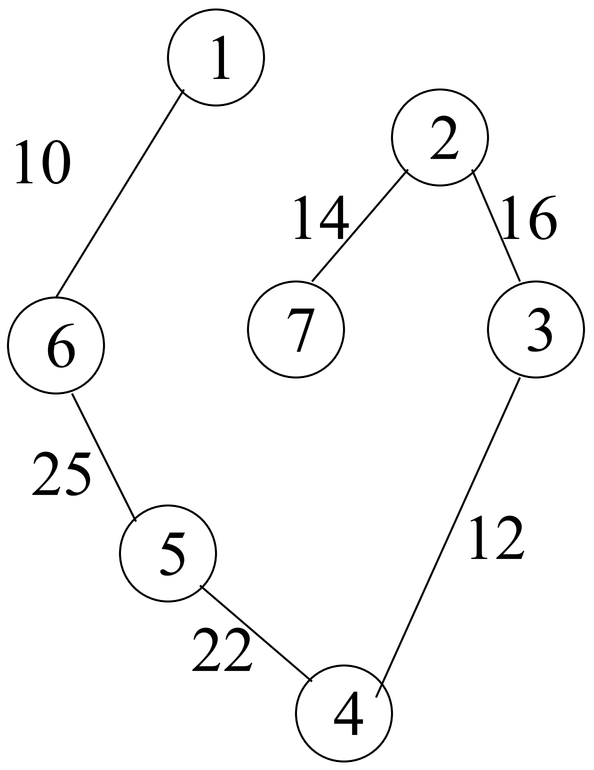
Prim’s algorithm





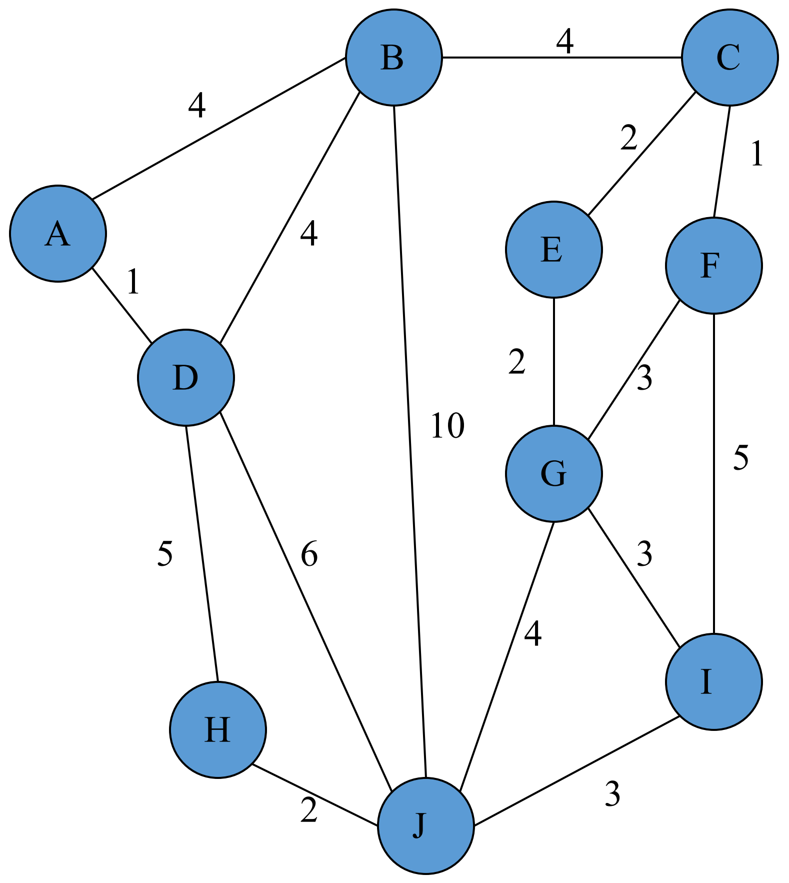
Prim’s algorithm



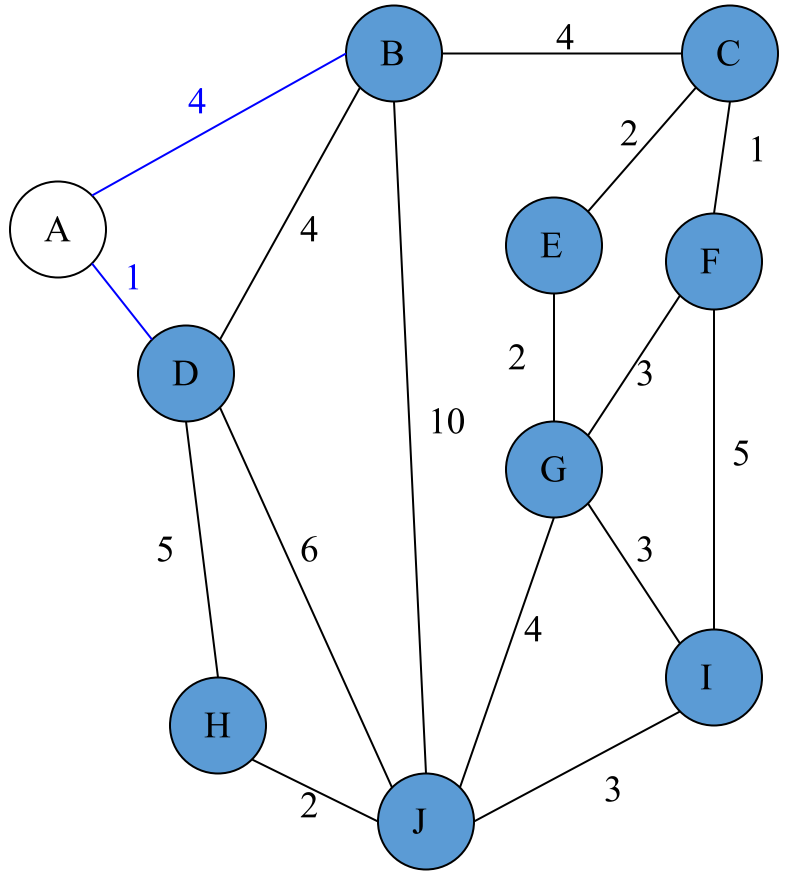


Cost = 99

Complete Graph



Old Graph



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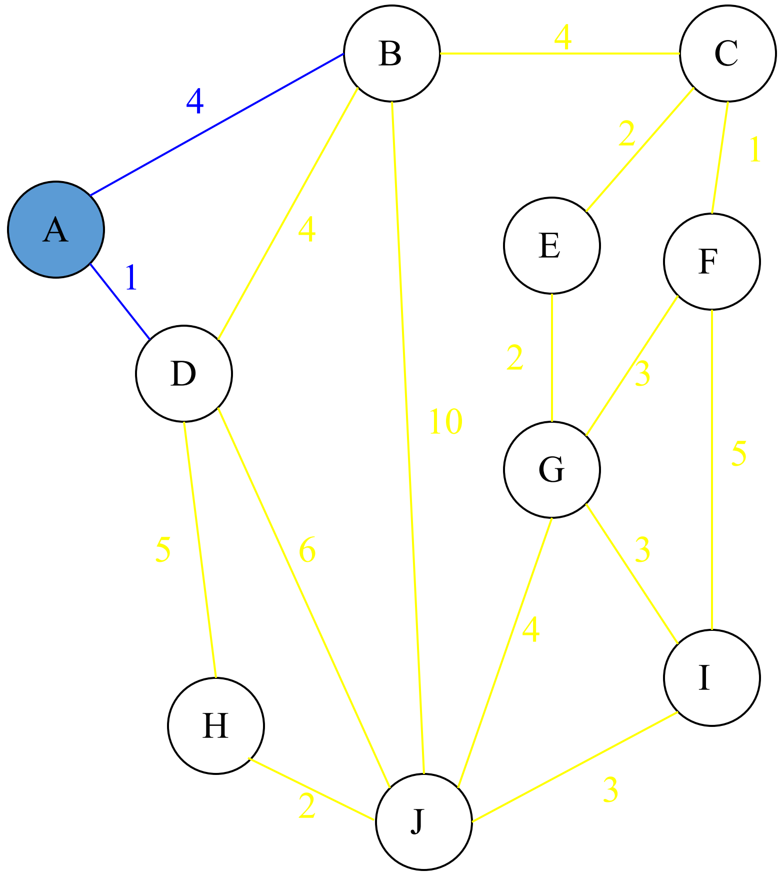

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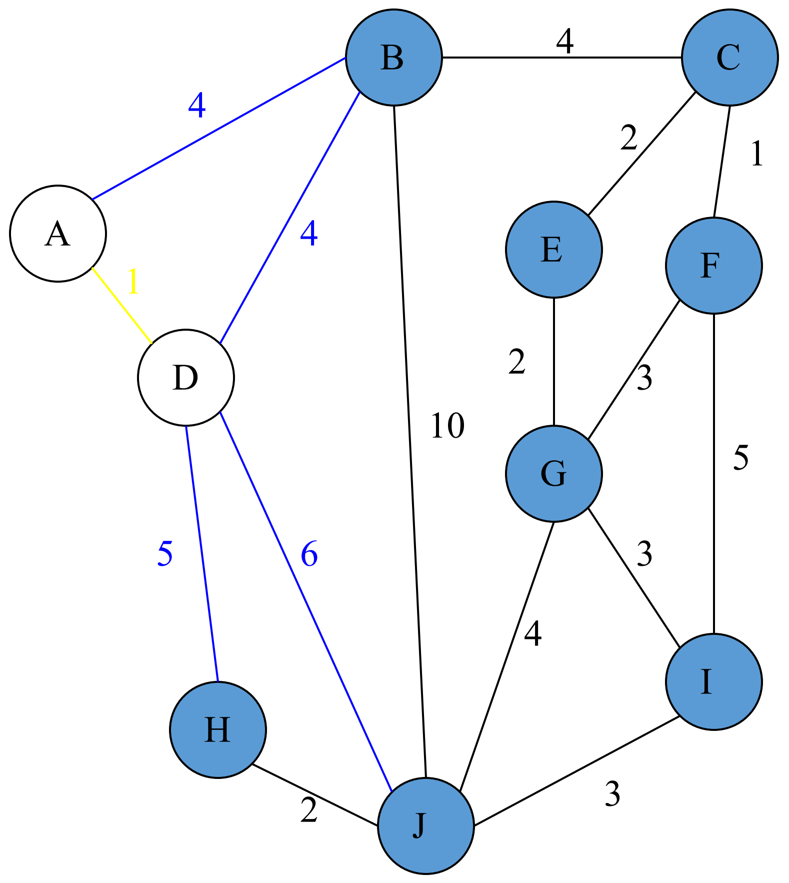

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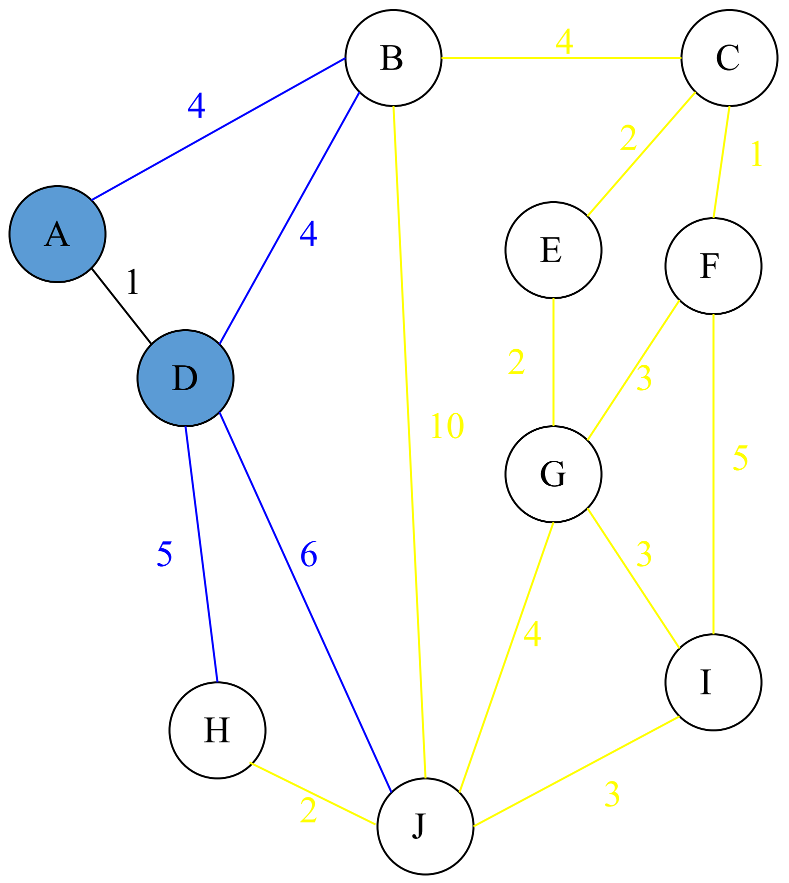

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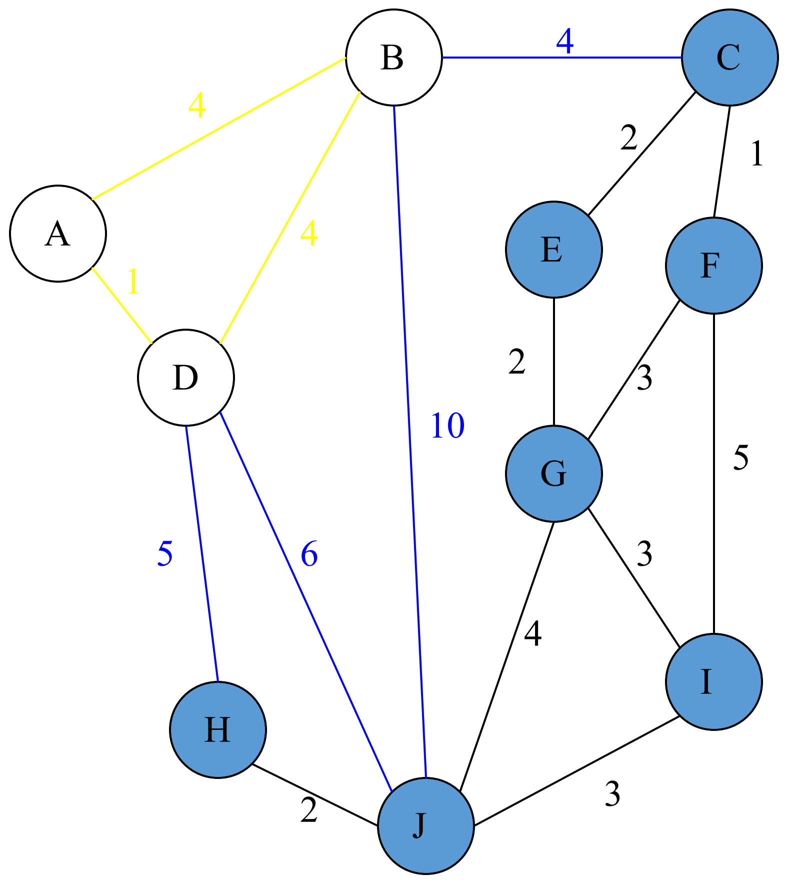

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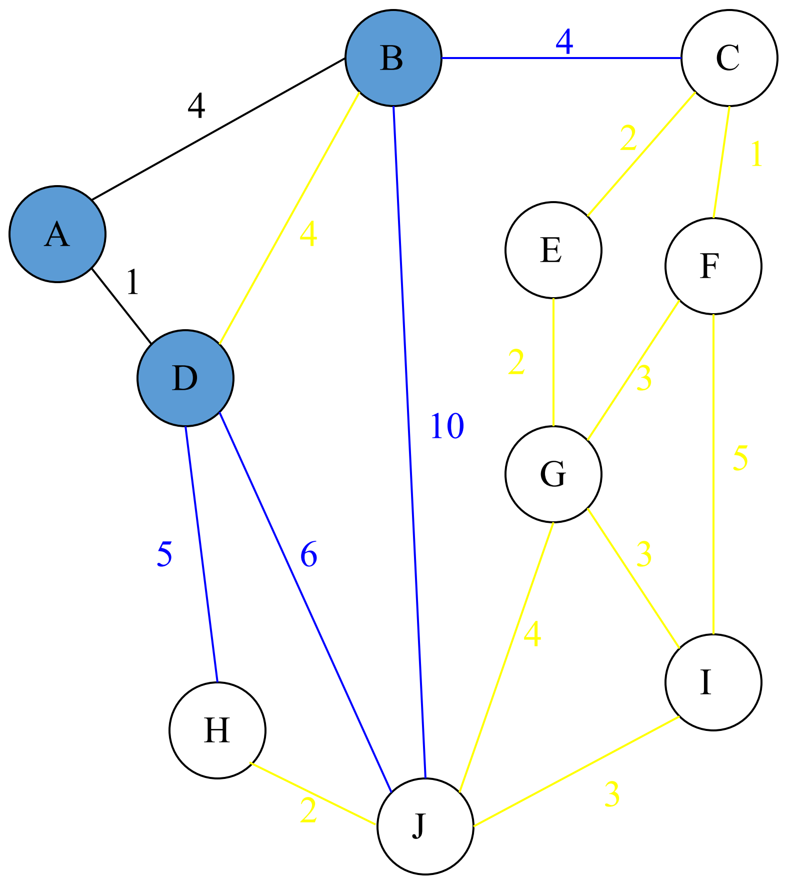

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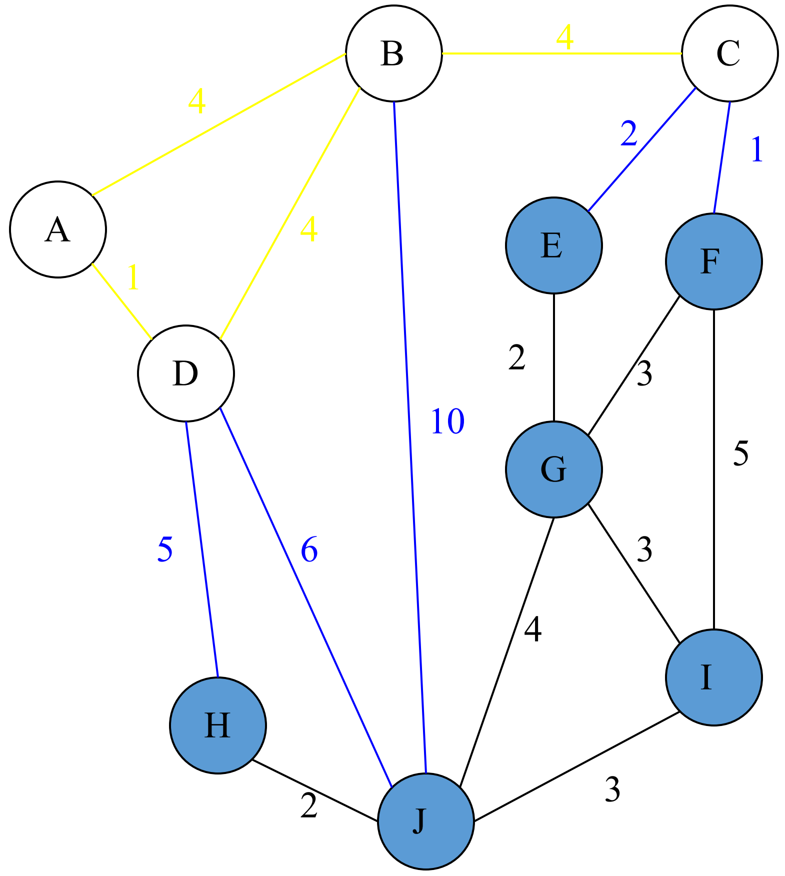

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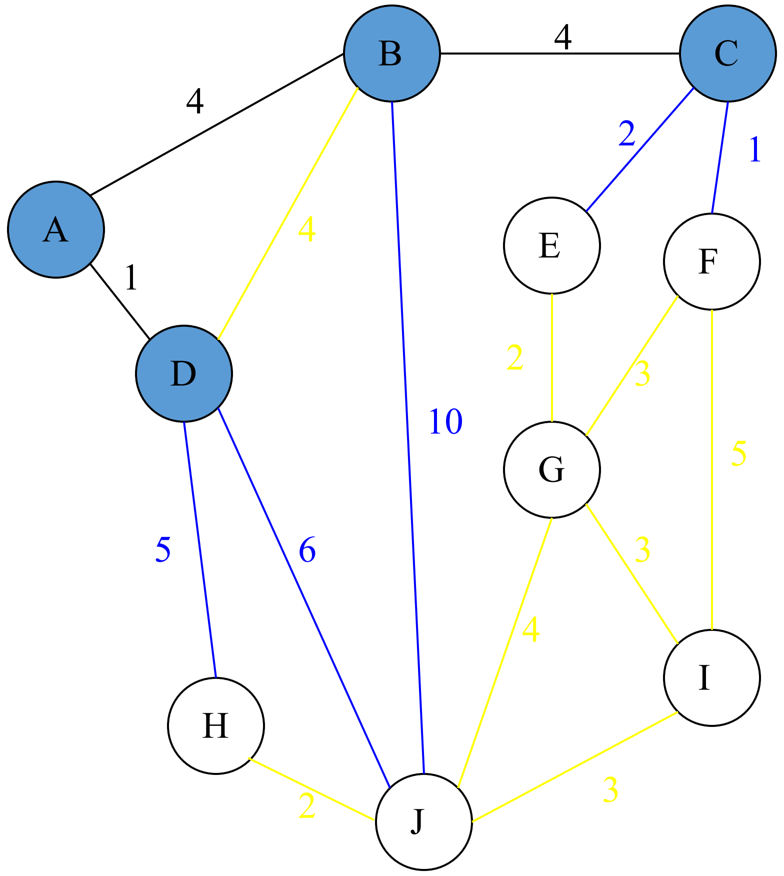

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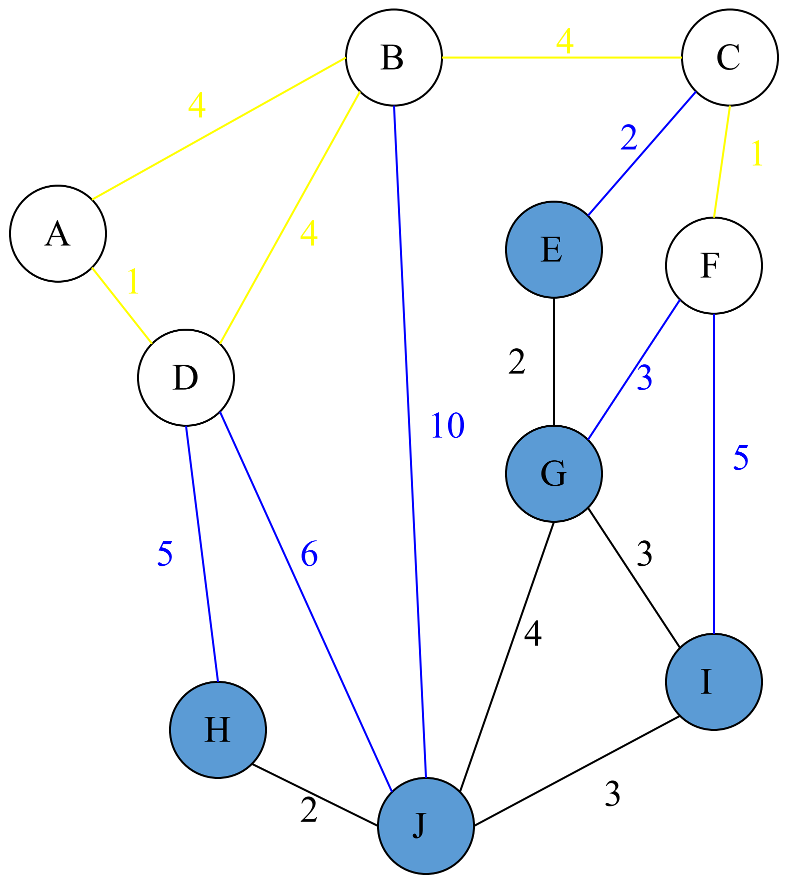

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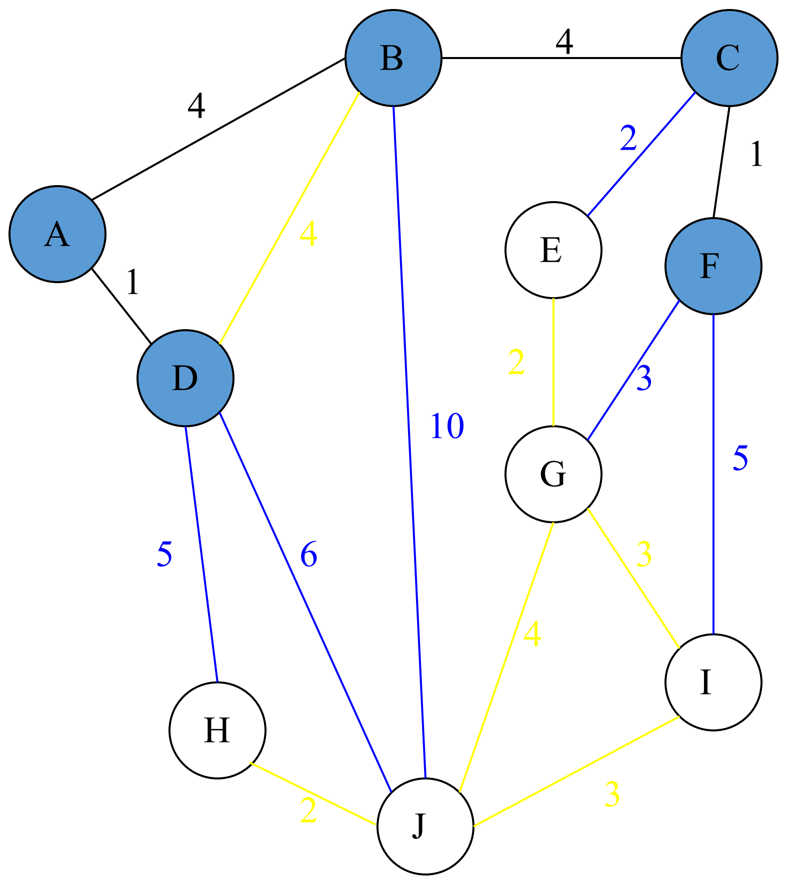

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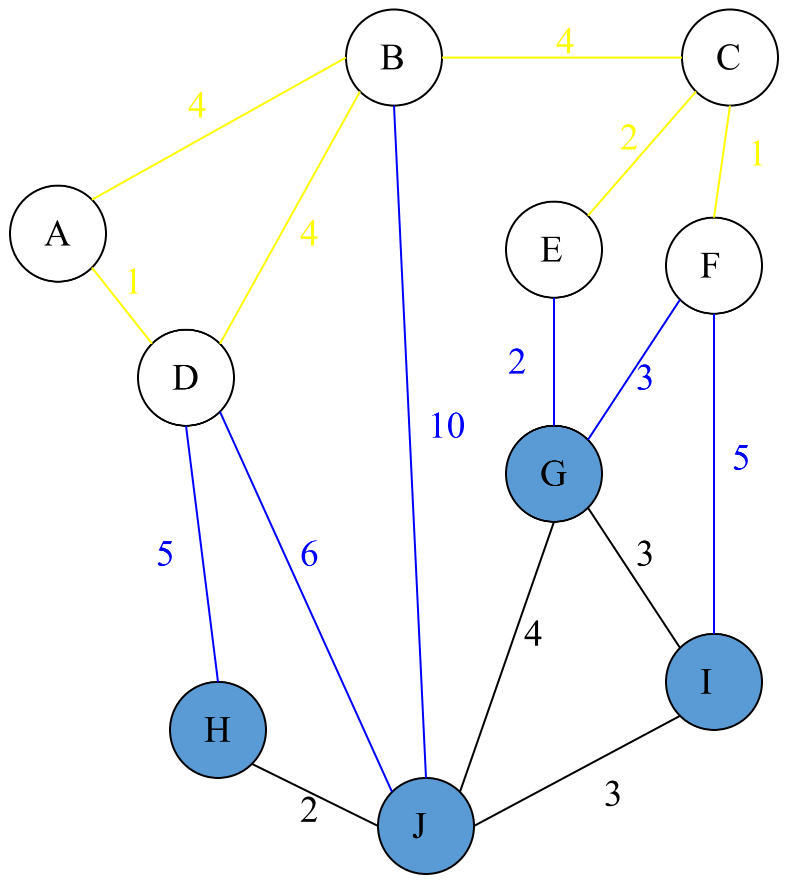

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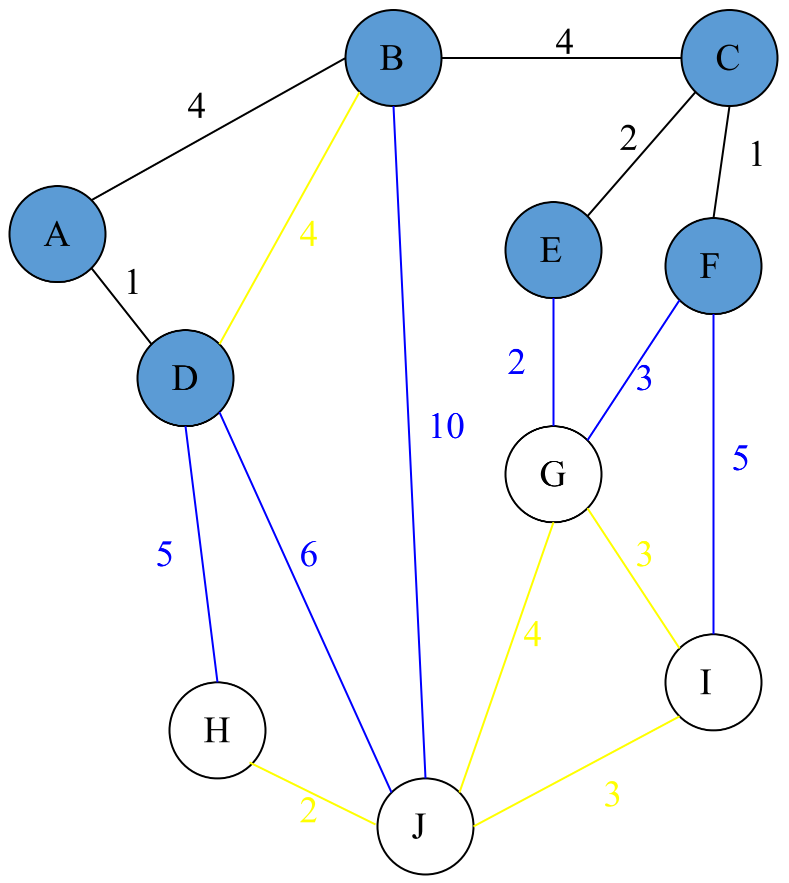

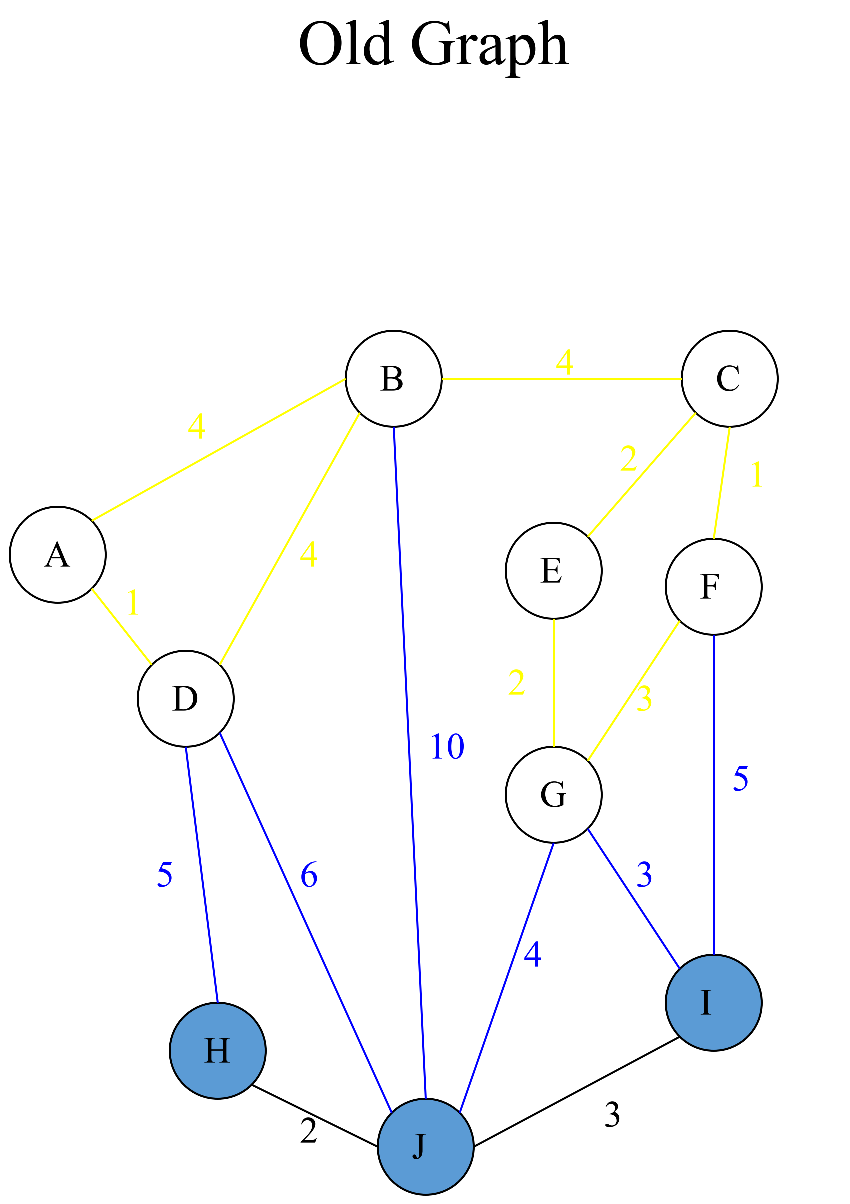
Old Graph
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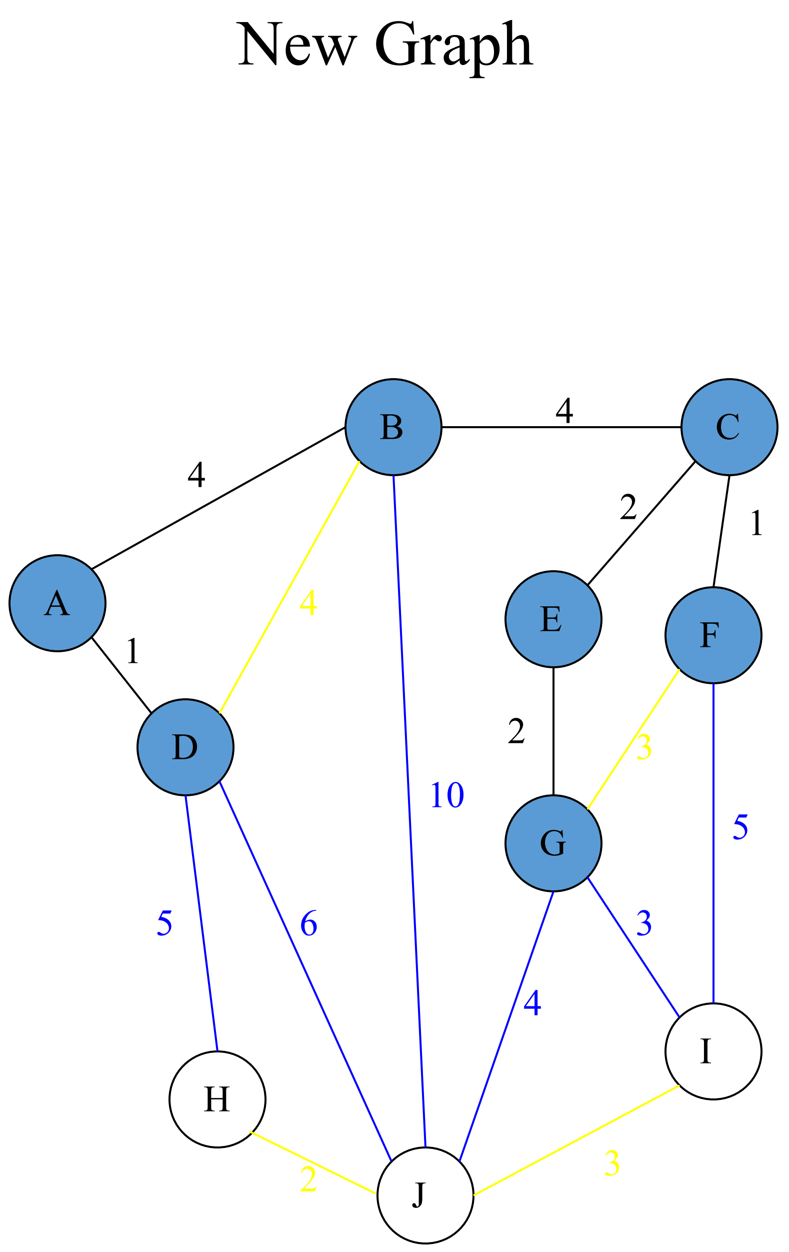
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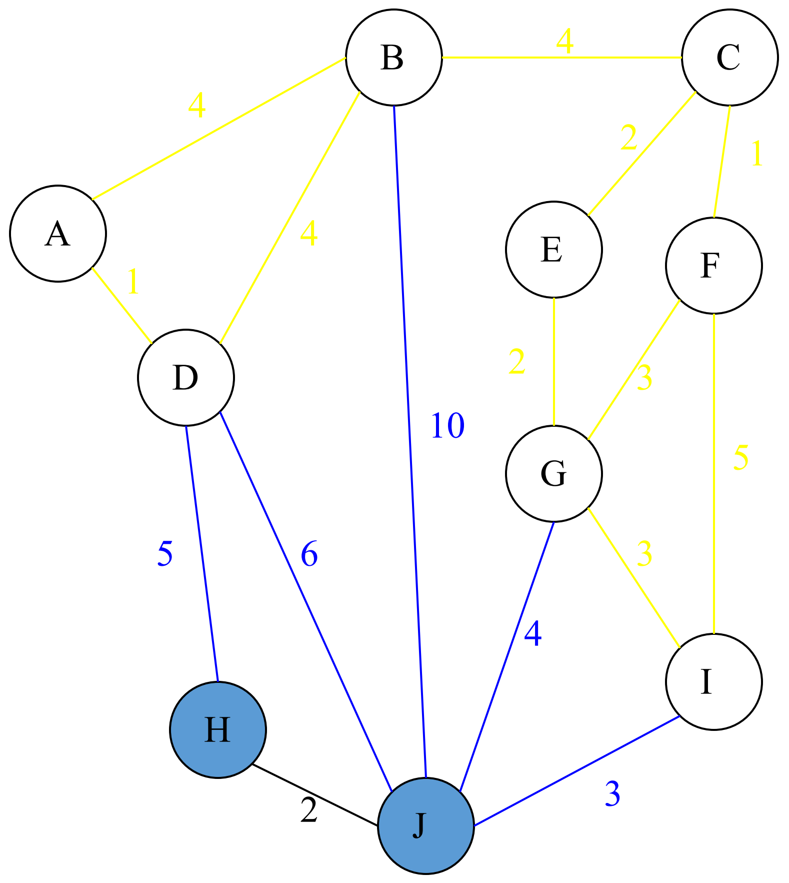

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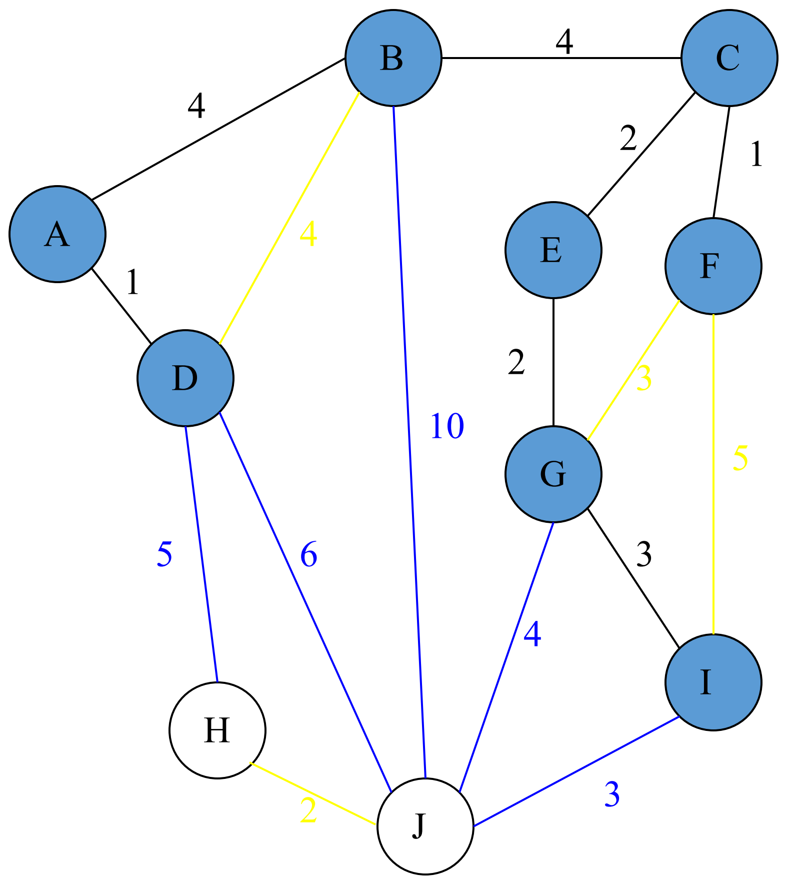

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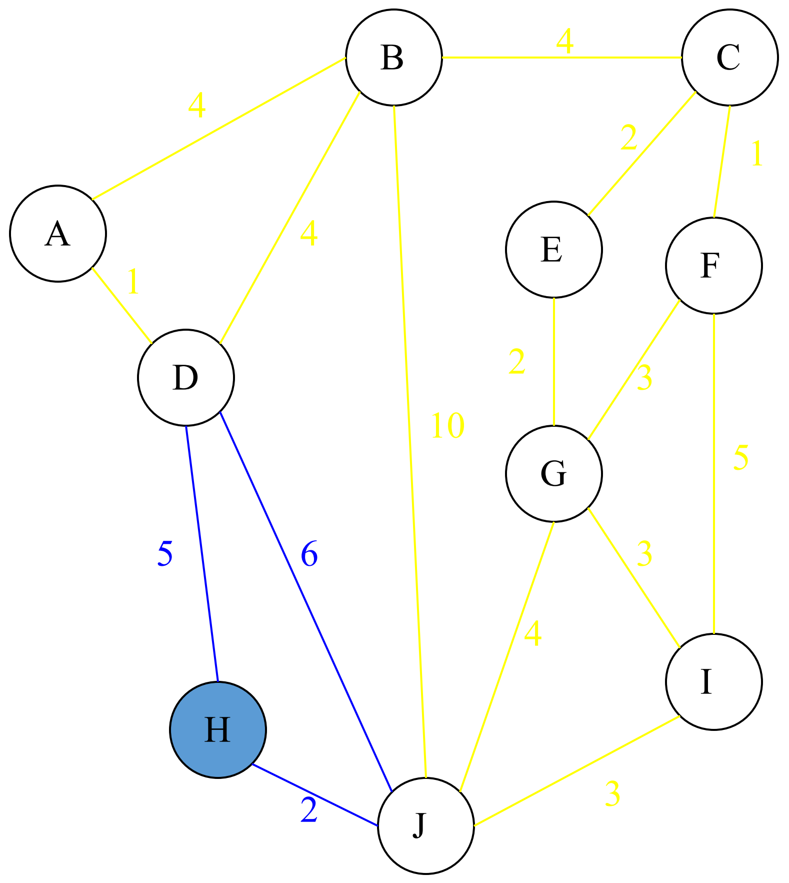

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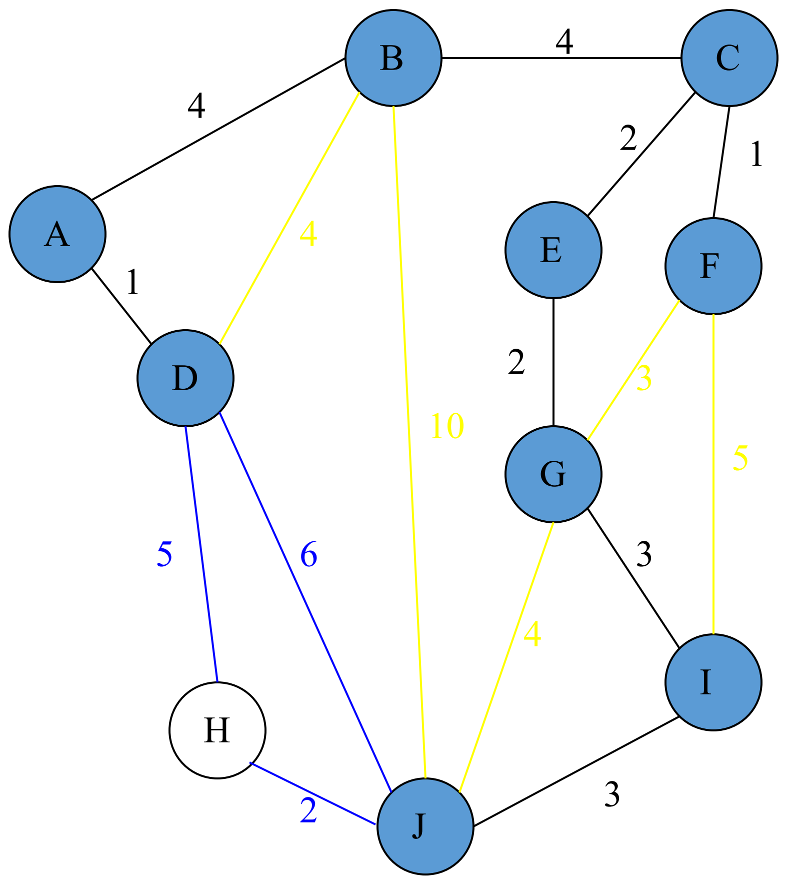

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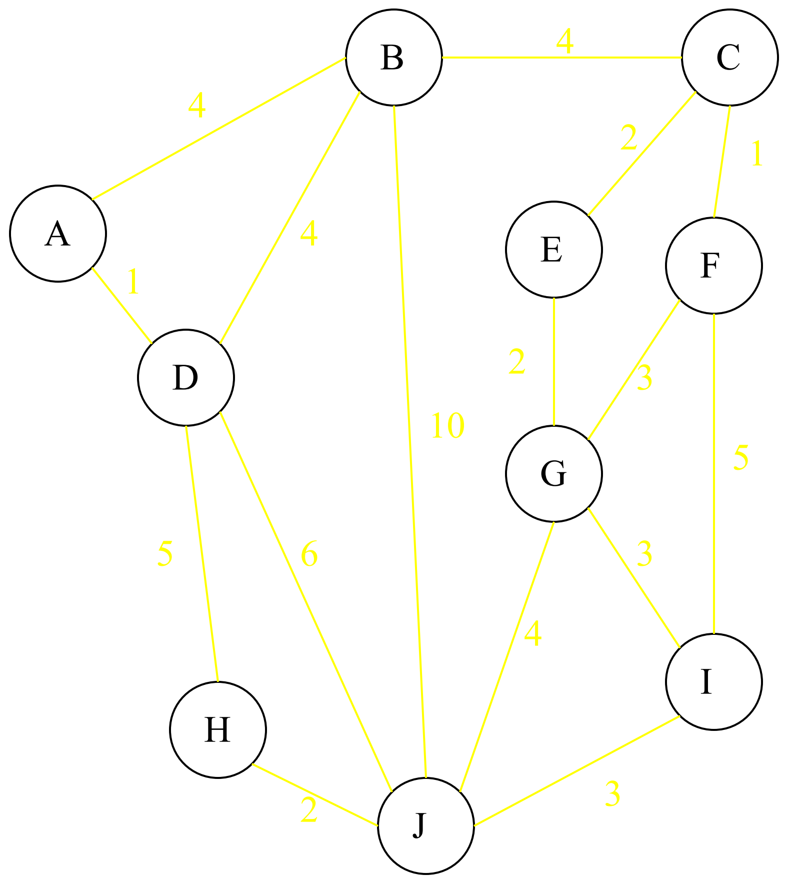

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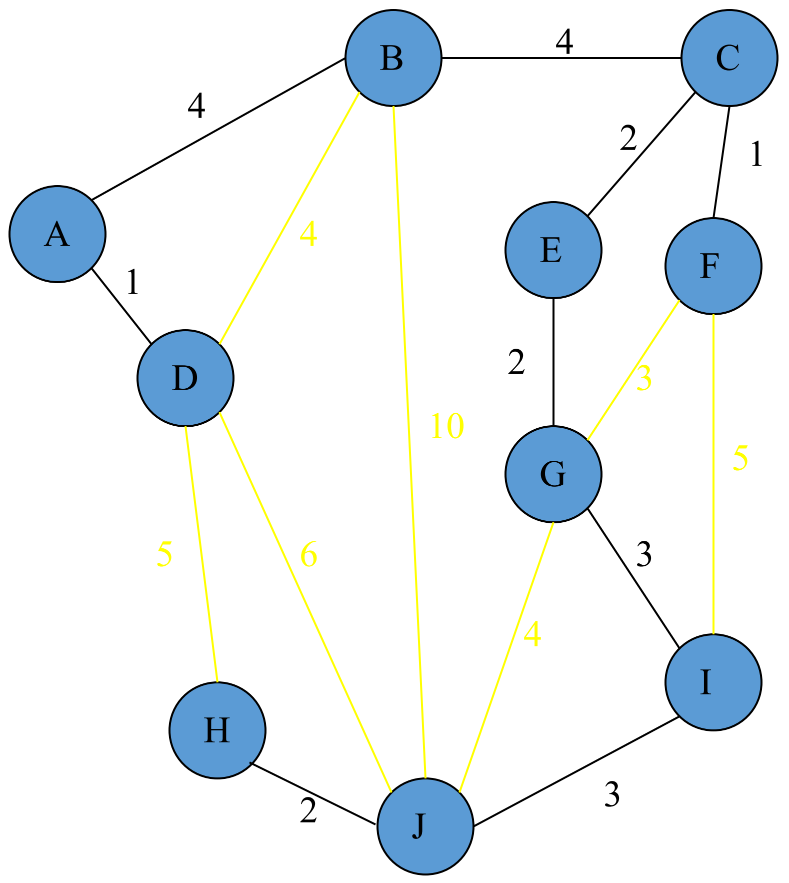
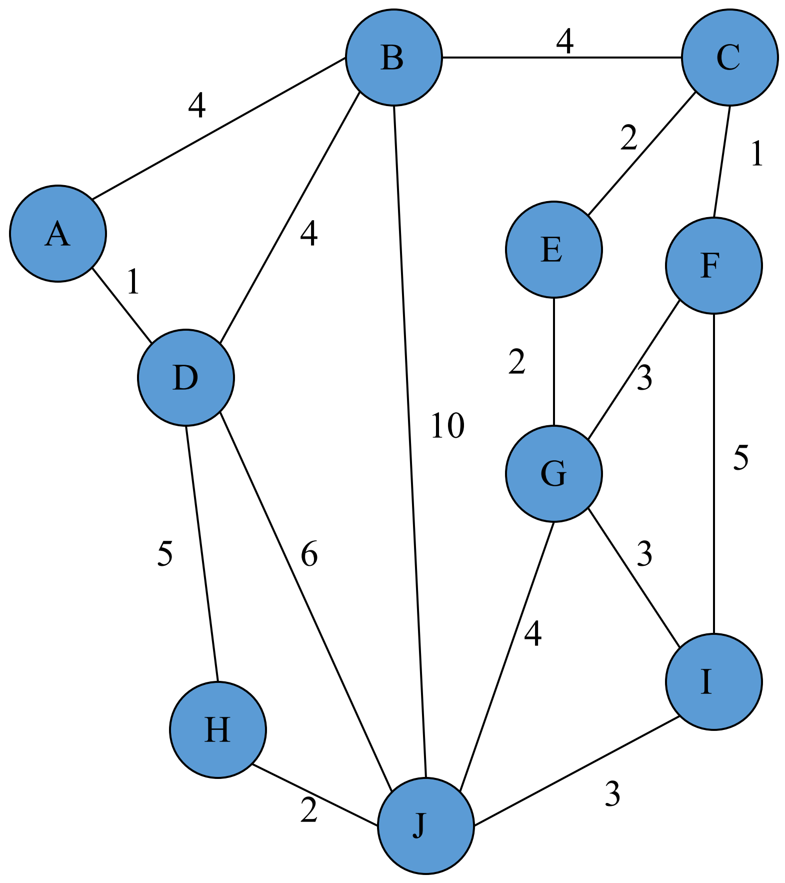
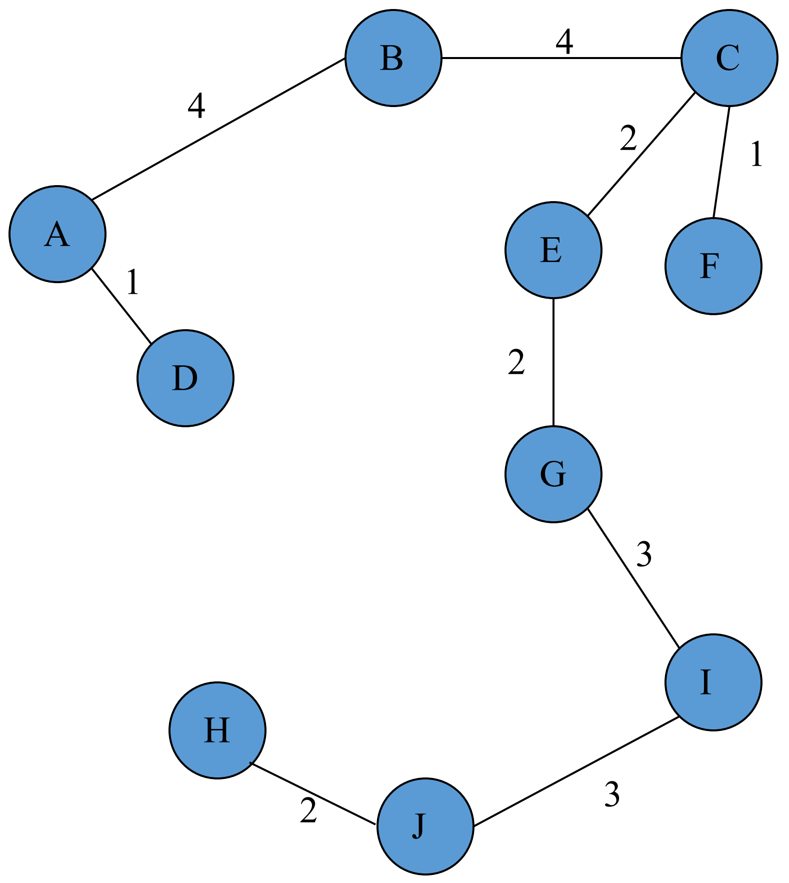


Complete Graph



Minimum Spanning Tree



Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
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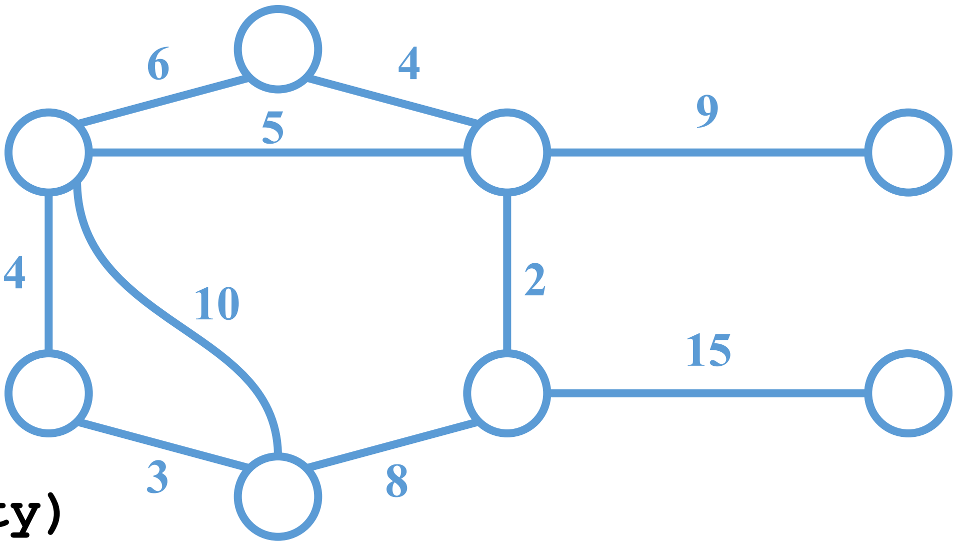

Run on example graph

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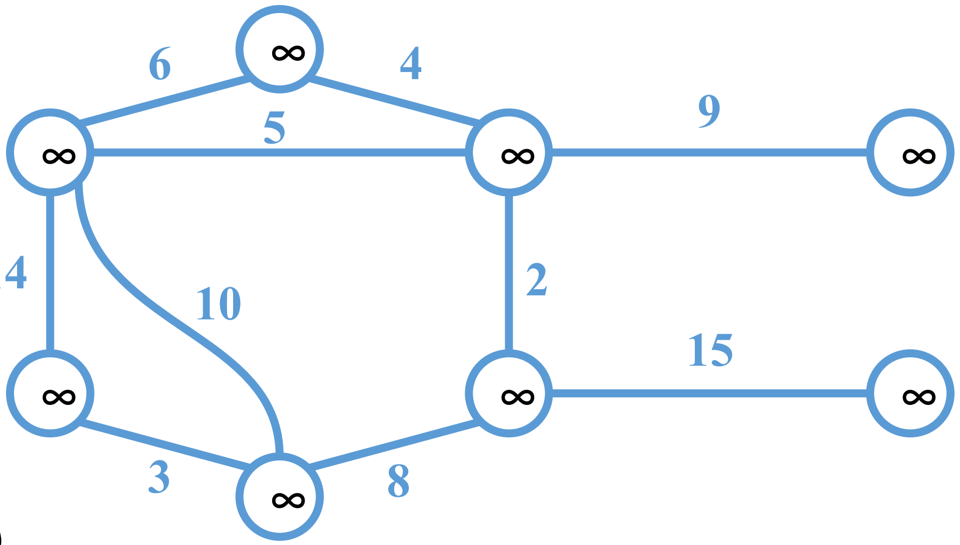

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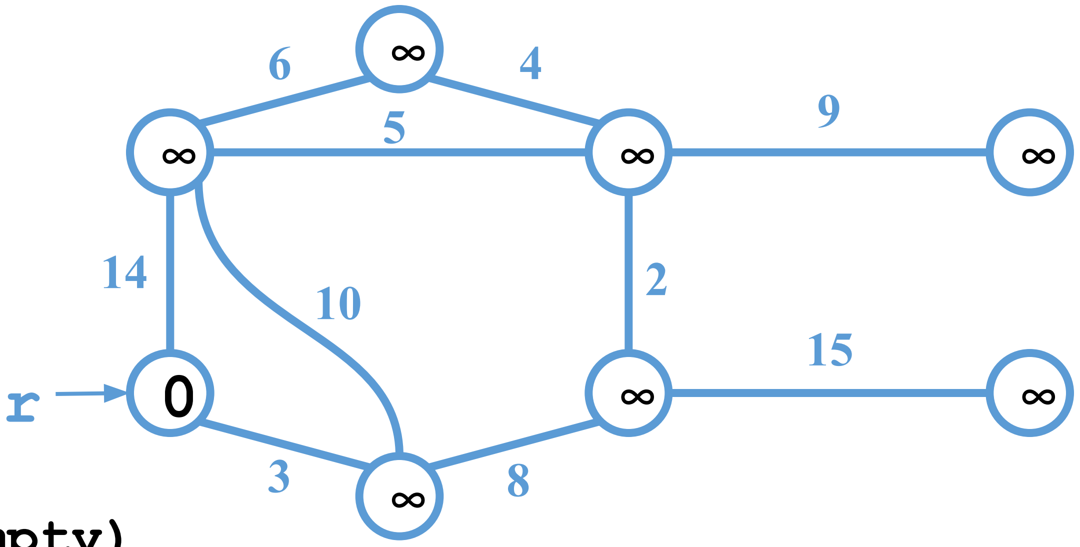

Pick a start vertex r

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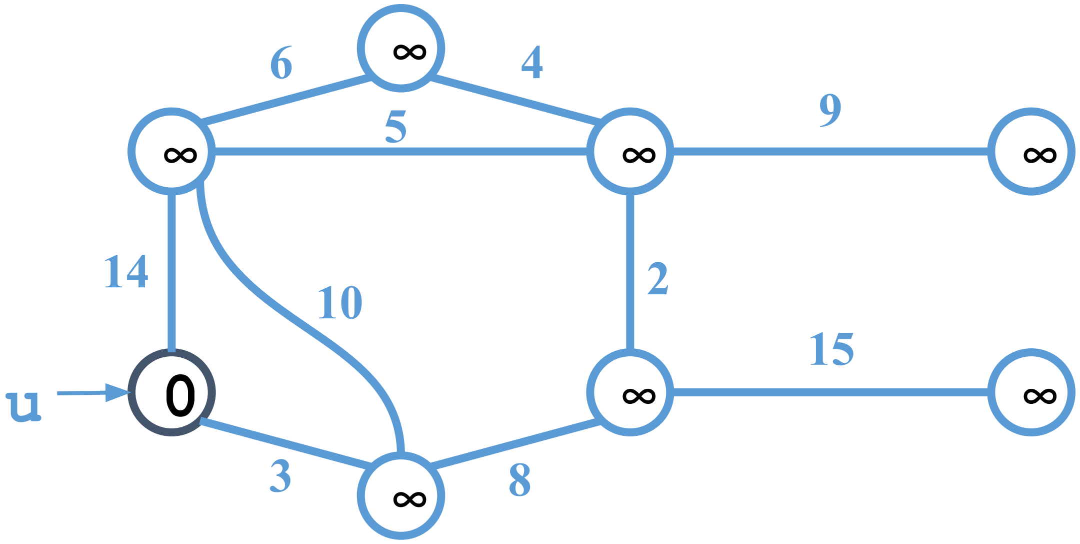

Black vertices have been removed from Q

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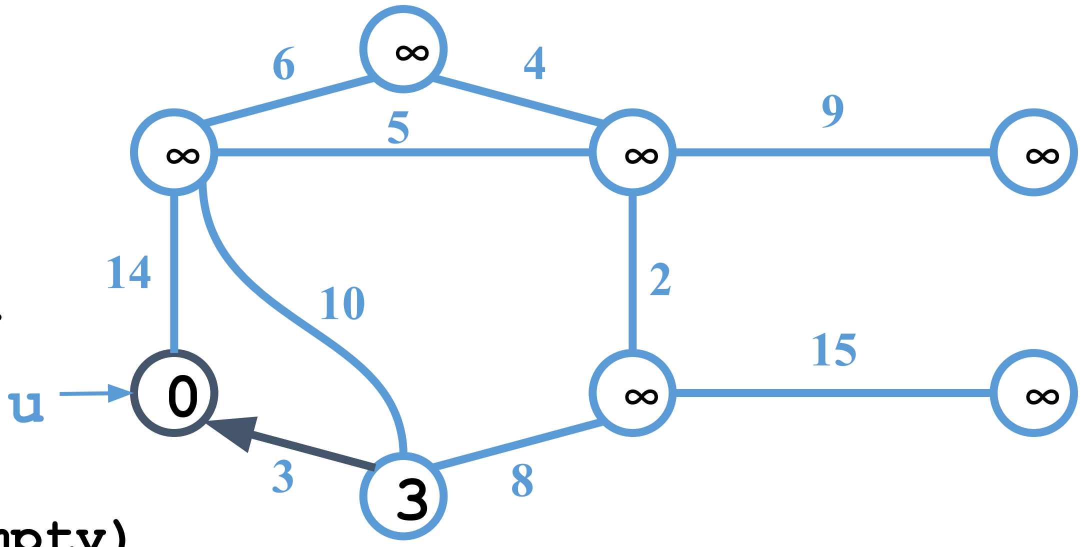

Black arrows indicate parent pointers

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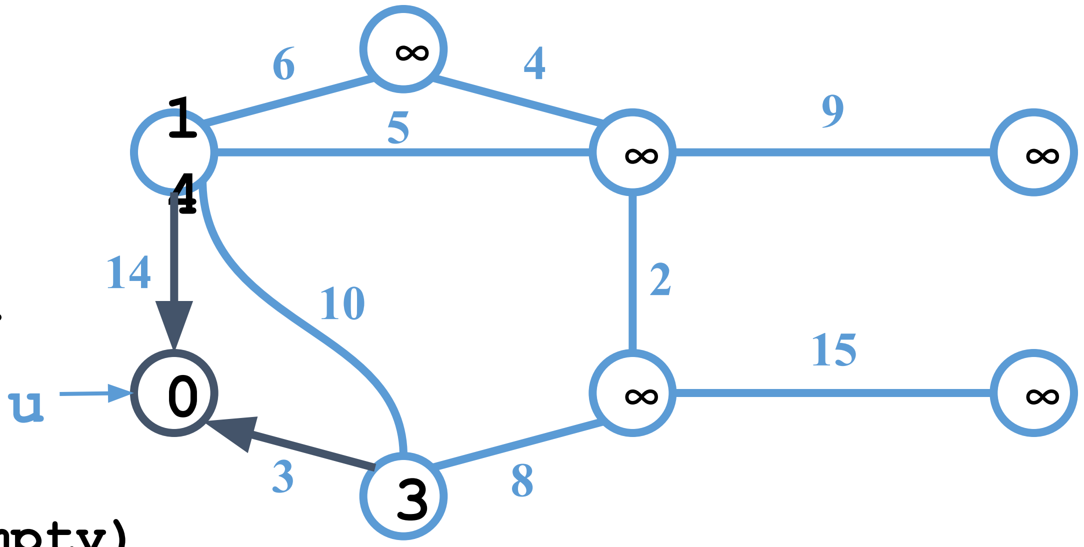

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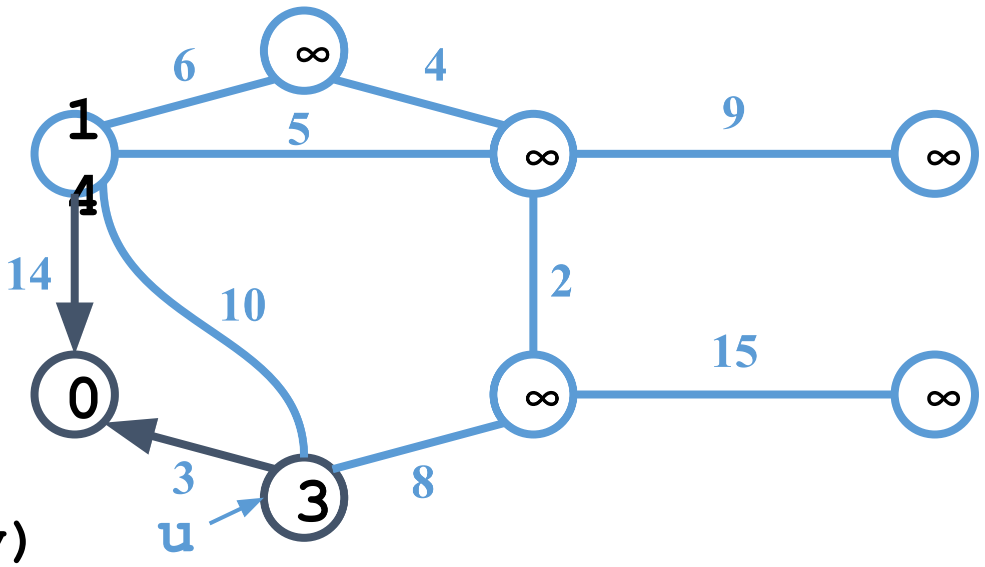

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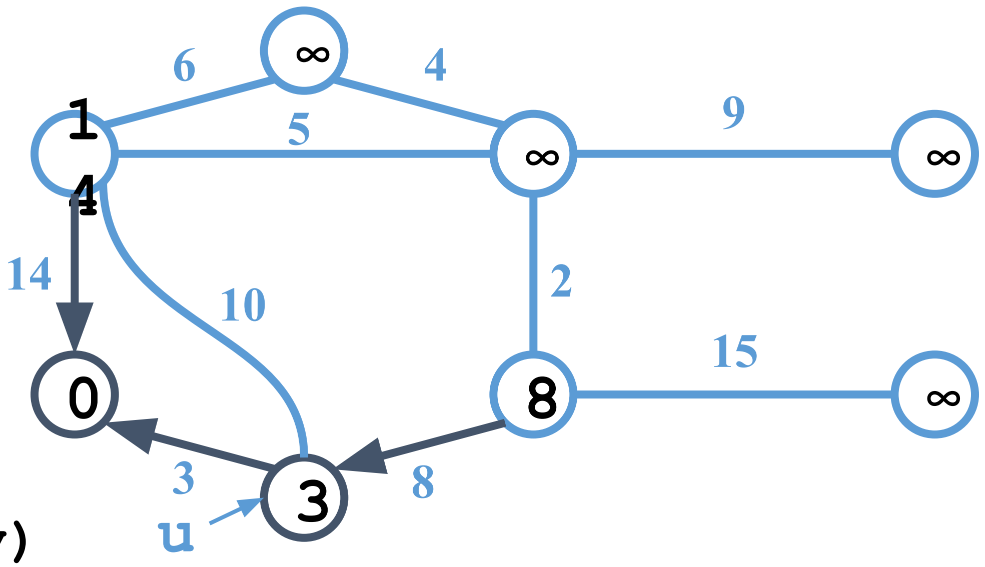

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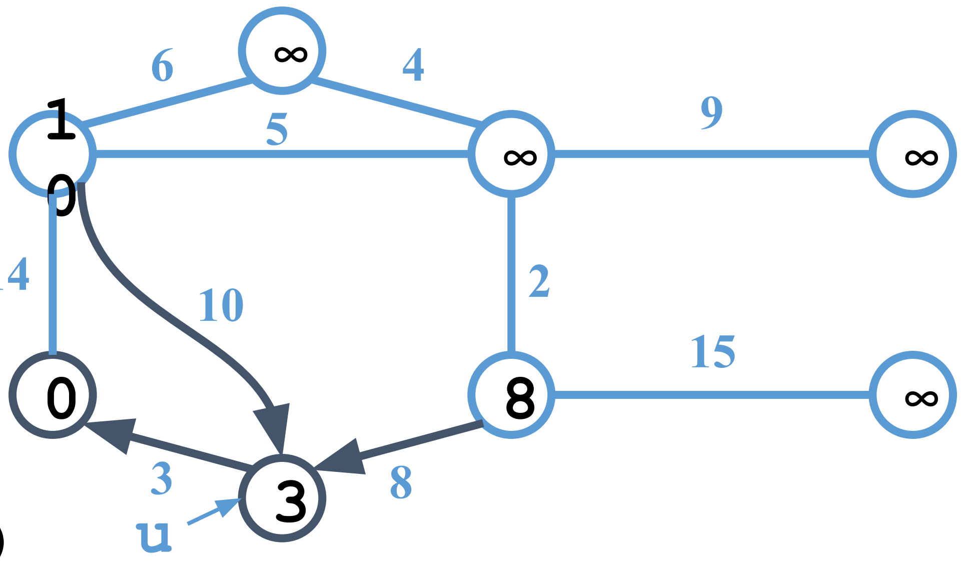

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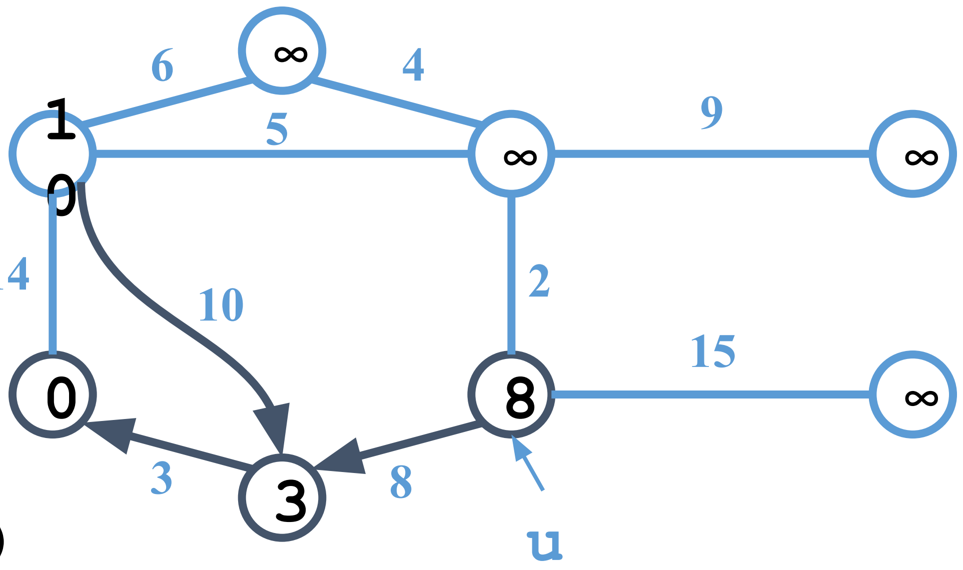

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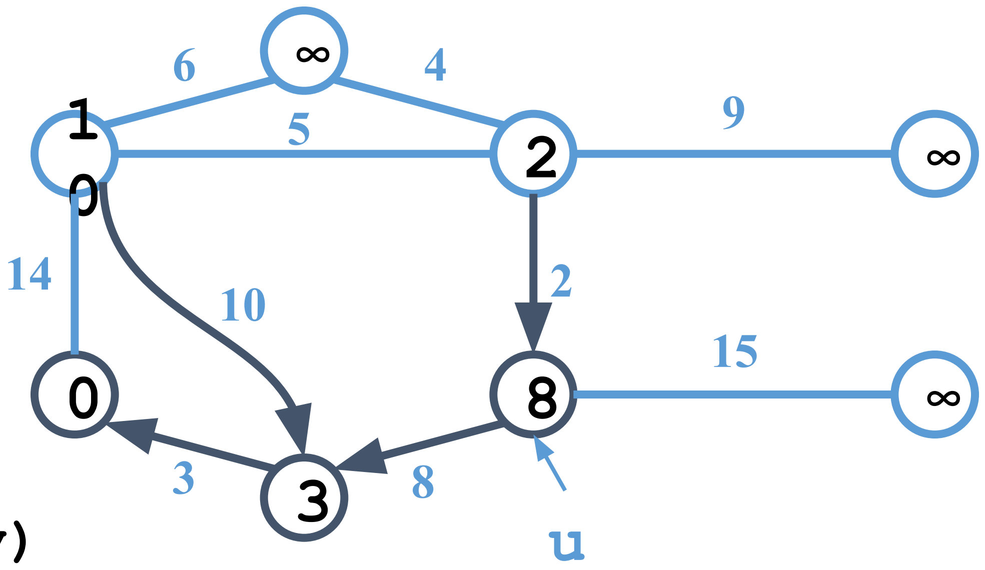

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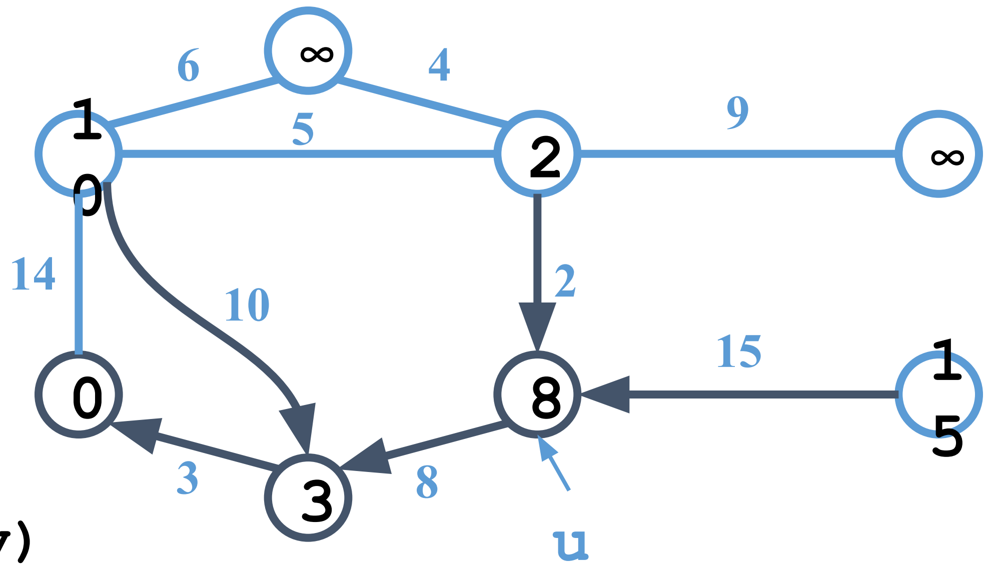

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                p[v] = u;
                key[v] = w(u,v);
```
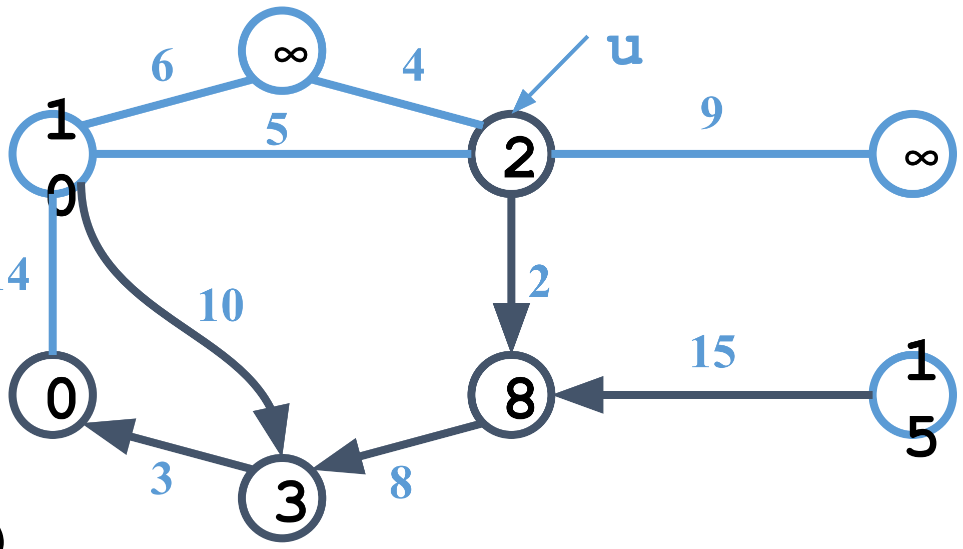
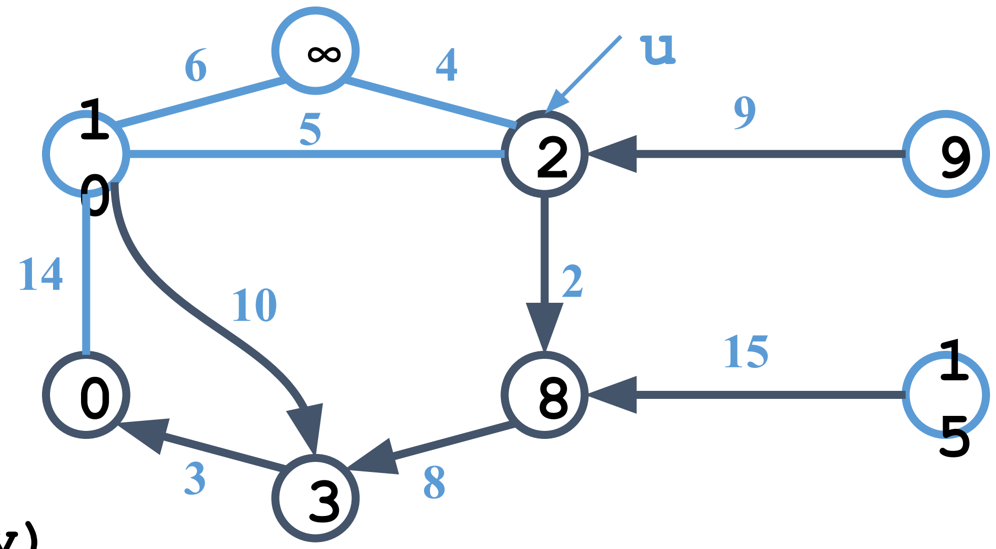



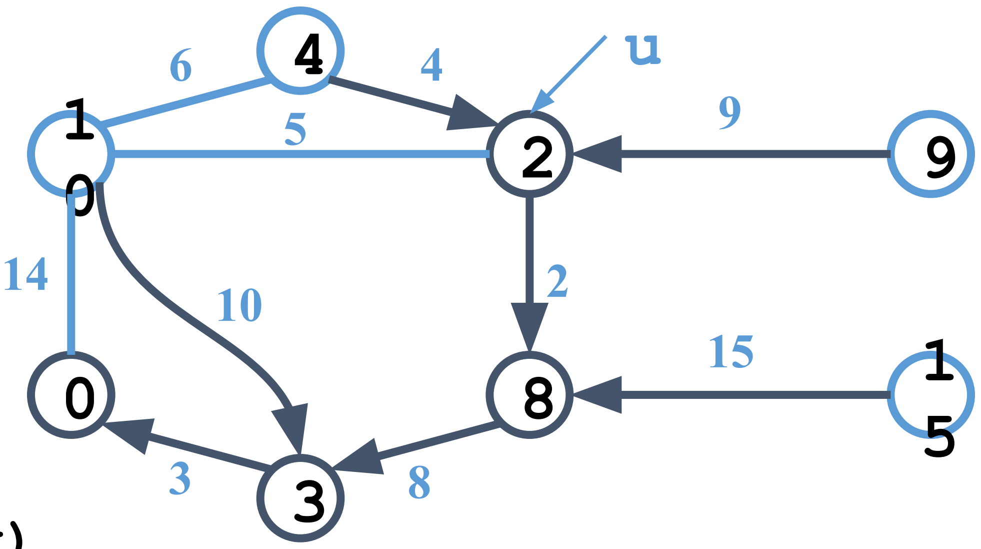
Prim's Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



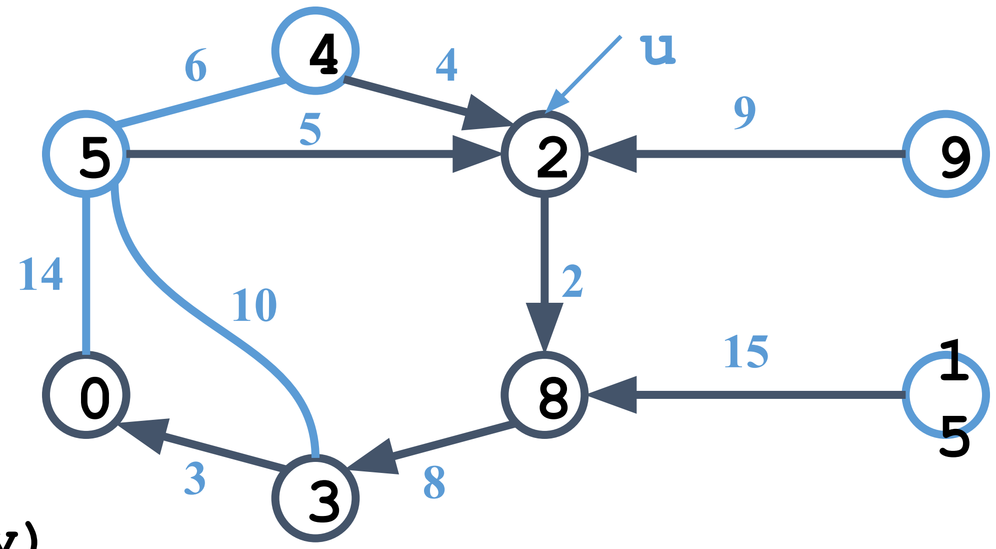
Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



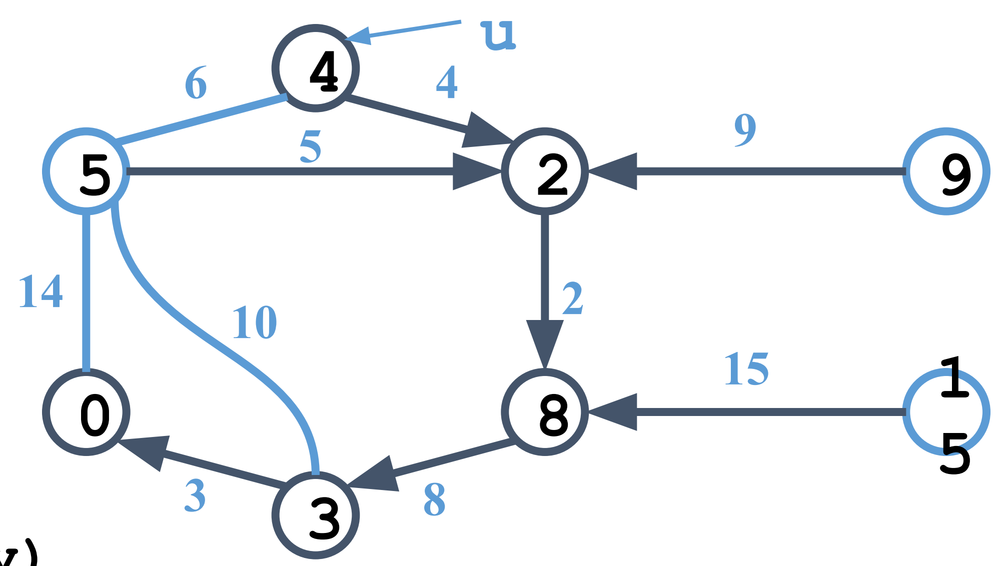
Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



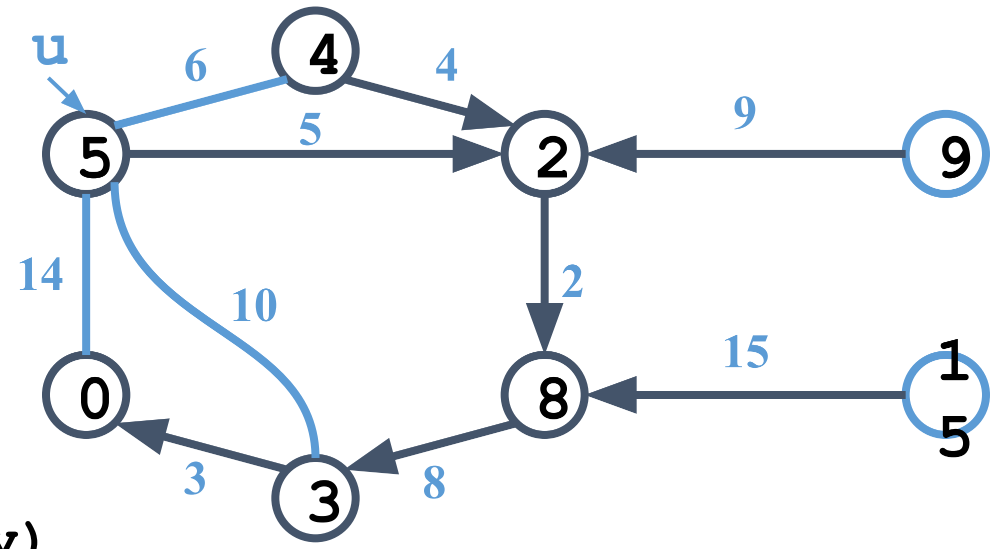
Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



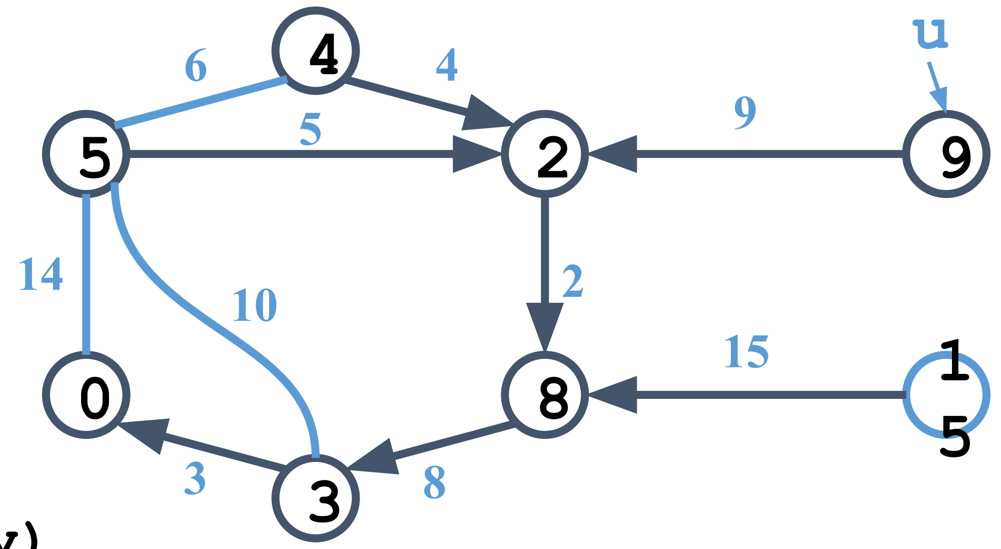
Prim's Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



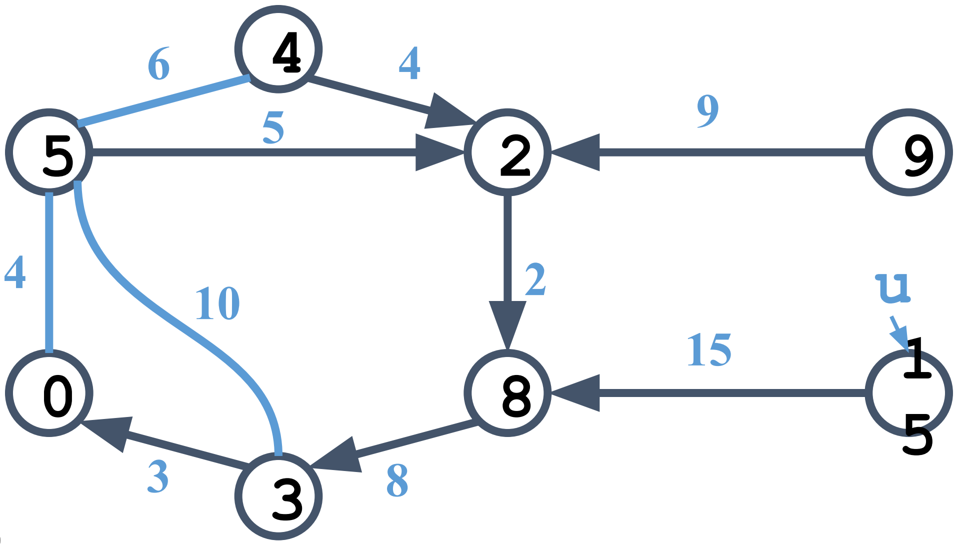
Prim's Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



Prim's Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```



Review: Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);
        for each v ∈ Adj[u]
            if (v ∈ Q and w(u,v) < key[v])
                p[v] = u;
                key[v] = w(u,v);
```

Runtime complexity: Prim’s Algorithm

```
MST-Prim(G, w, r)
    Q = V[G];
    for each u ∈ Q
        key[u] = ∞;                          →   O(V)
    key[r] = 0;
    p[r] = NULL;
    while (Q not empty)
        u = ExtractMin(Q);                   →   O(lg V)
        for each v ∈ Adj[u]                  →   O(E)
          if (v ∈ Q and w(u,v) < key[v])
             p[v] = u;
             DecreaseKey(v, w(u,v));         →   O(lg V)
```

O(V) +O(Vlog V +Elog V)= O(Elog V)

Analysis of Prim's Algorithm

Running Time :

$O(E \log V + V \log V) = O(E\log V)$

Where E = edges, V = nodes

If a heap is not used, the run time will be $O(V^2)$ instead of $O(E\log V)$

Kruskal's Algorithm

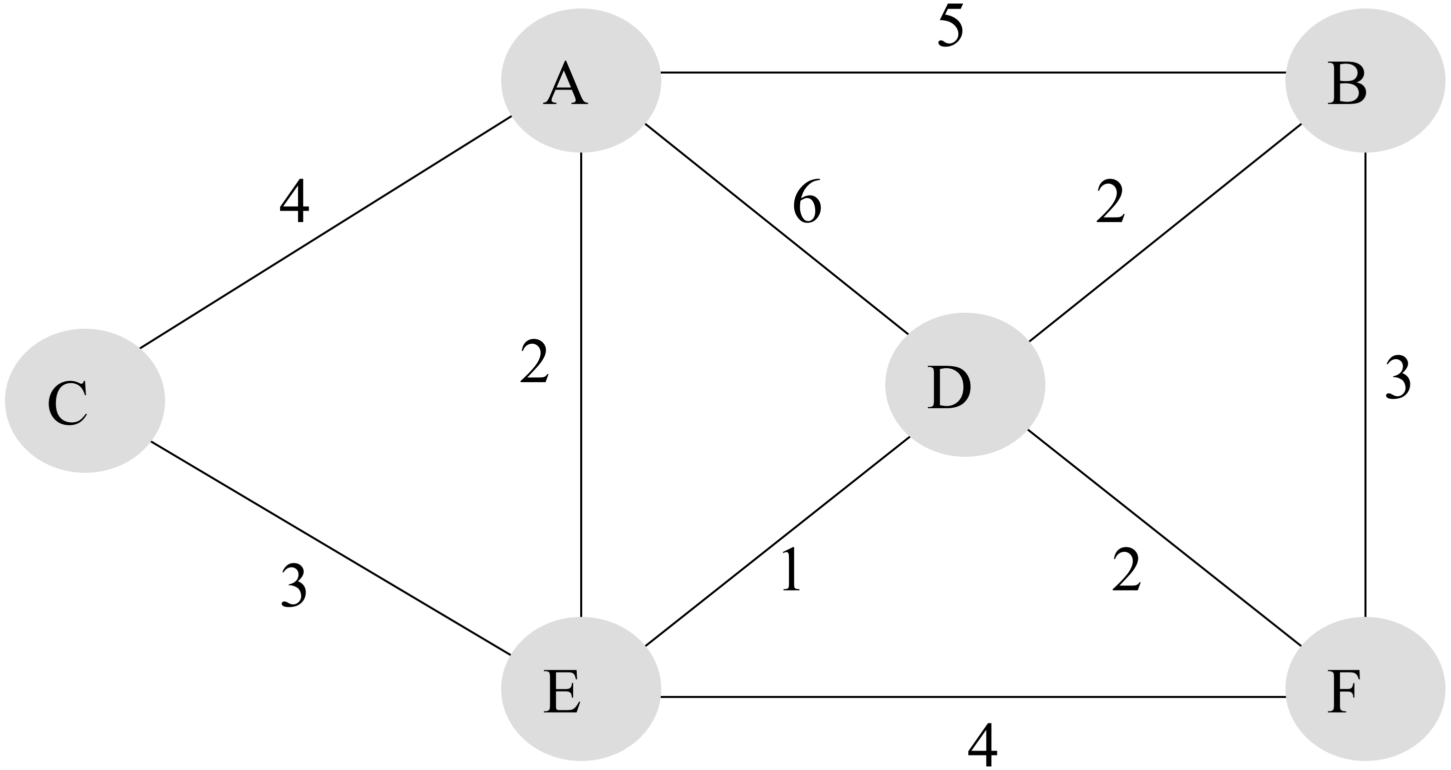
Kruskal's Algorithm

1. Each vertex is in its own set.

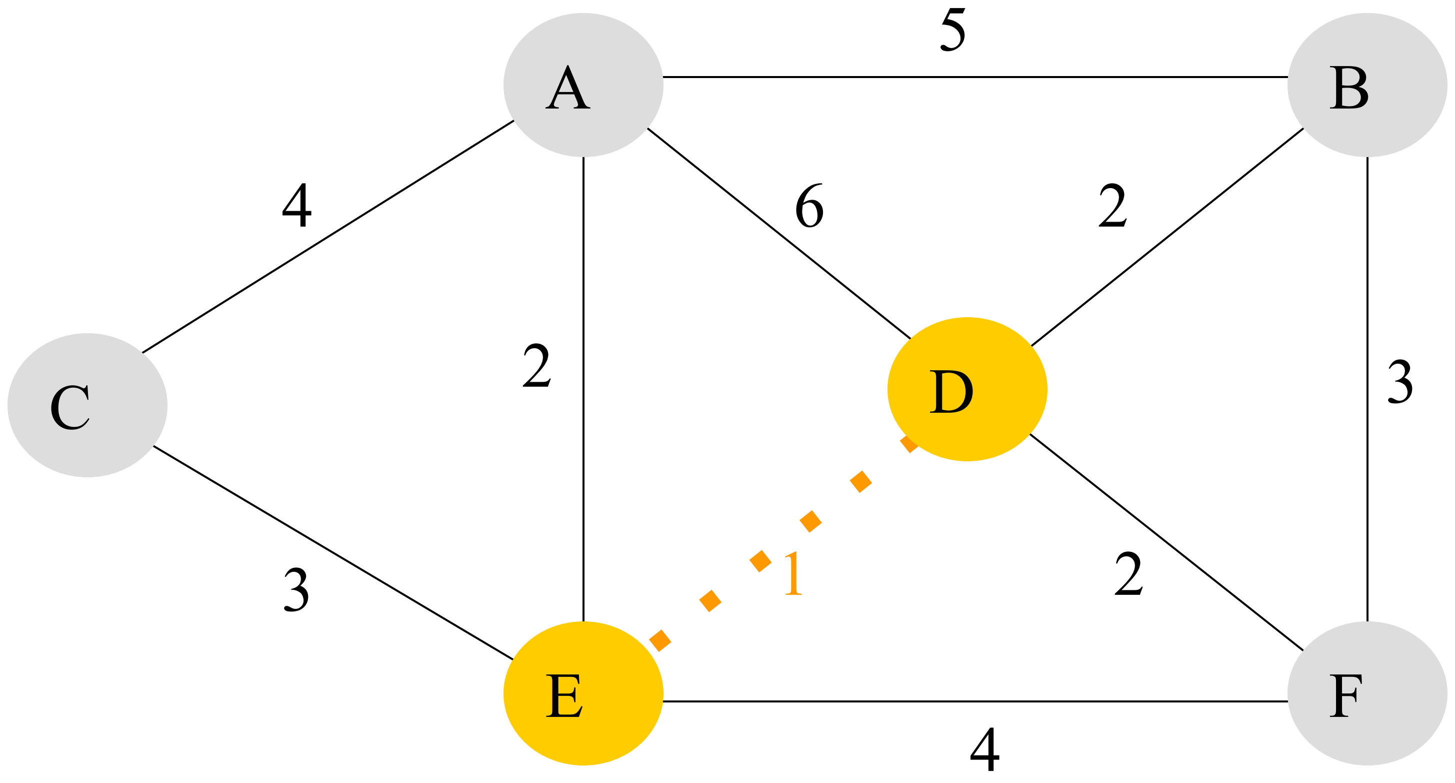
2. Take the edge *e* with the smallest weight

    - if *e* connects two vertices in different sets, then *e* is added to the MST and the two sets, which are connected by *e*, are merged into a single set.
    - if *e* connects two vertices, which are already in the same set, ignore it (cycle)

3. Continue until *V-1* edges were selected.



Kruskal's Algorithm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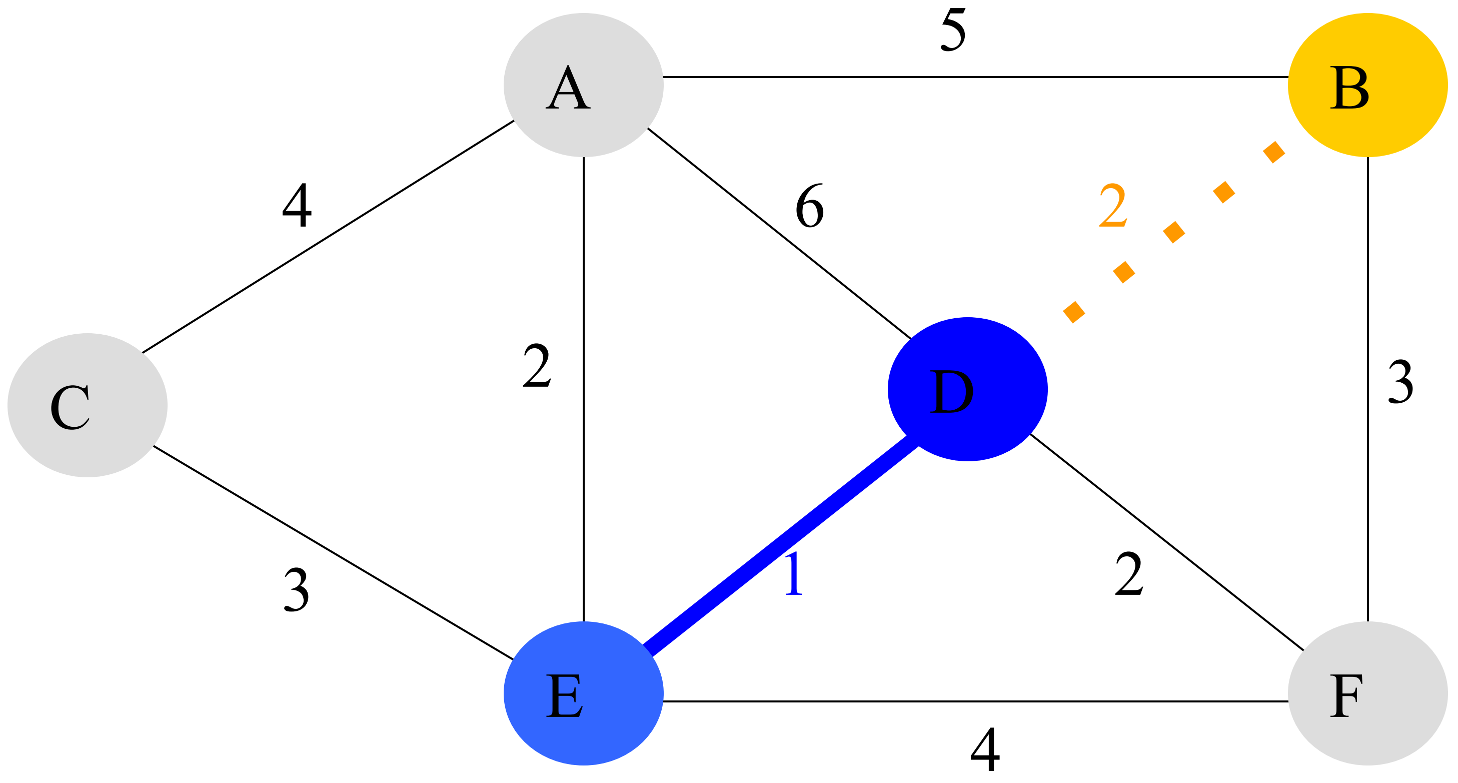
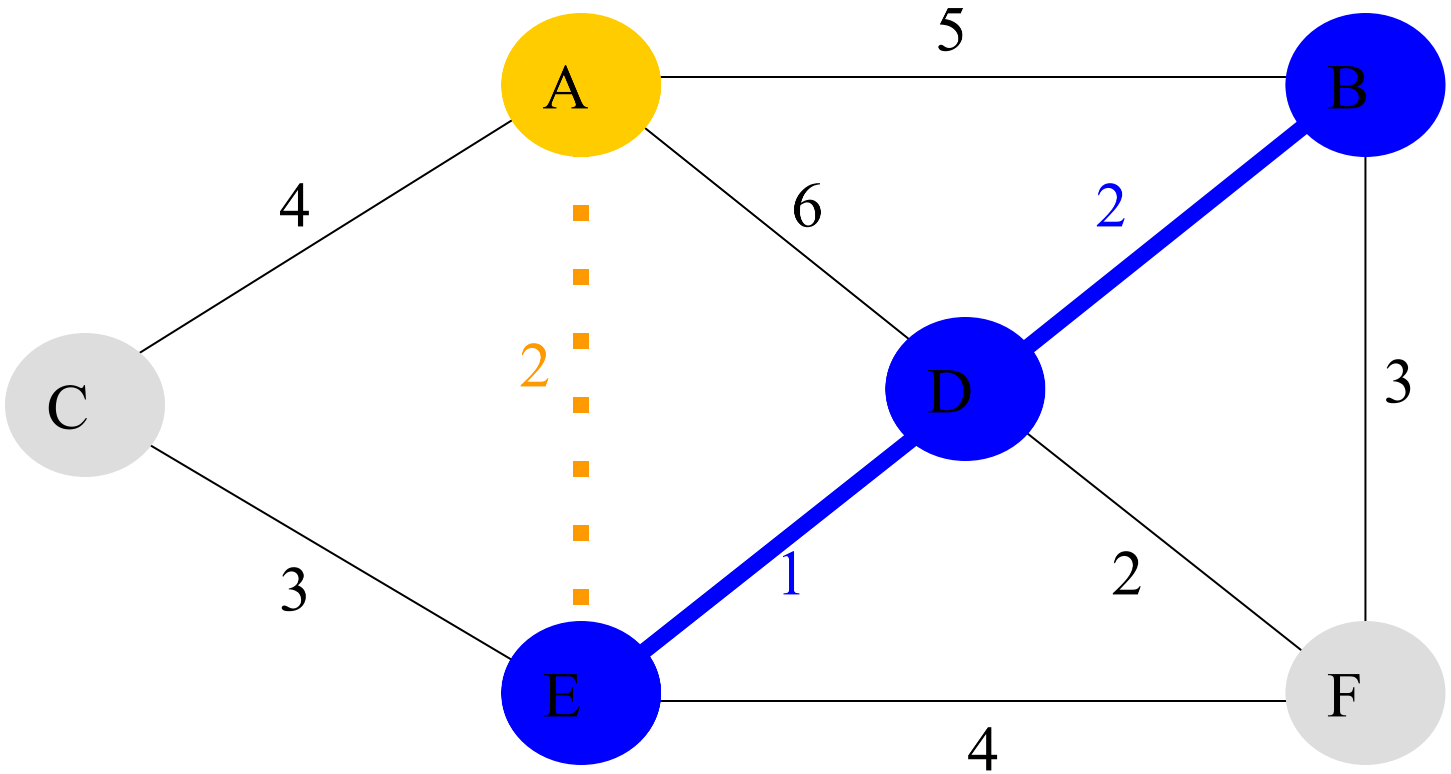
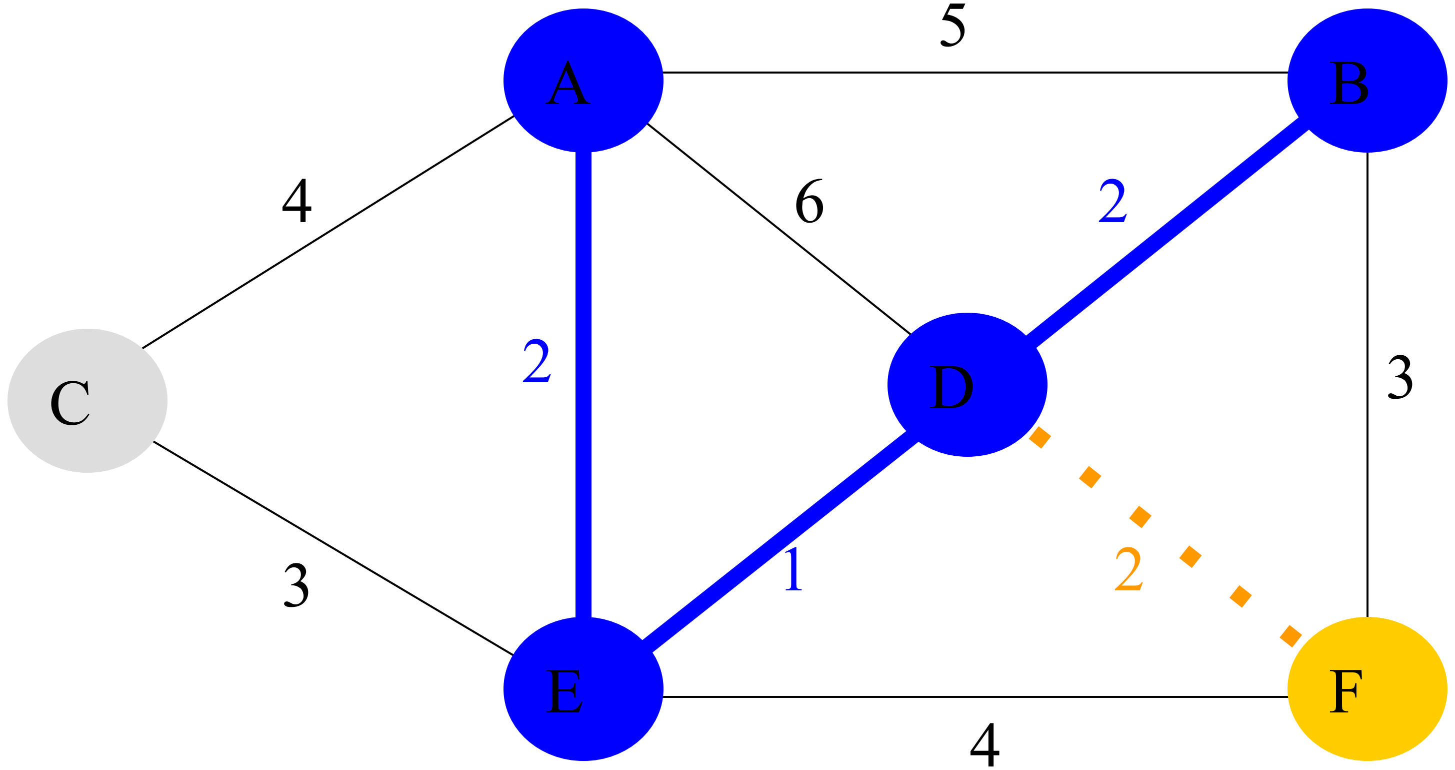
4



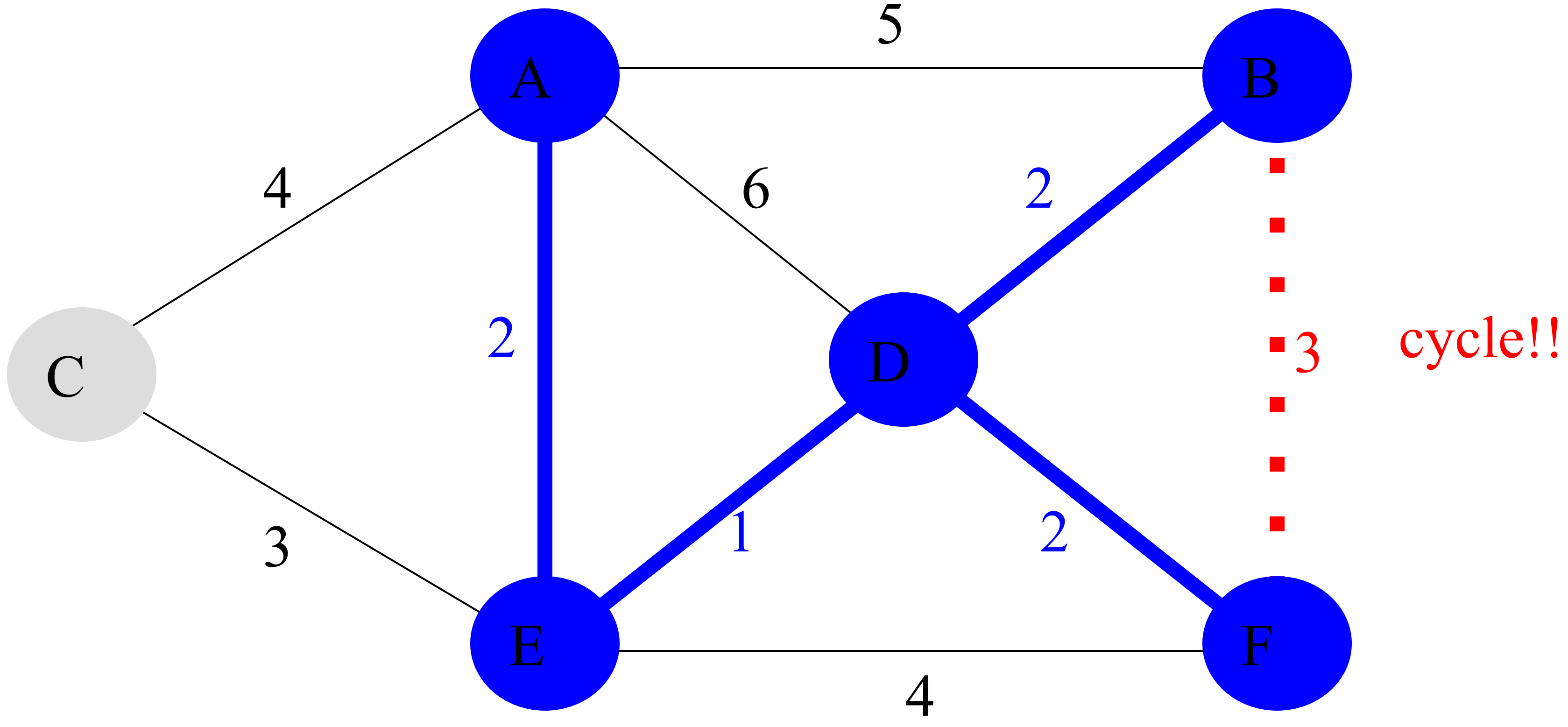
Kruskal's Algorithm



Kruskal's Algorithm



Kruskal's Algorithm


A
B
C
D
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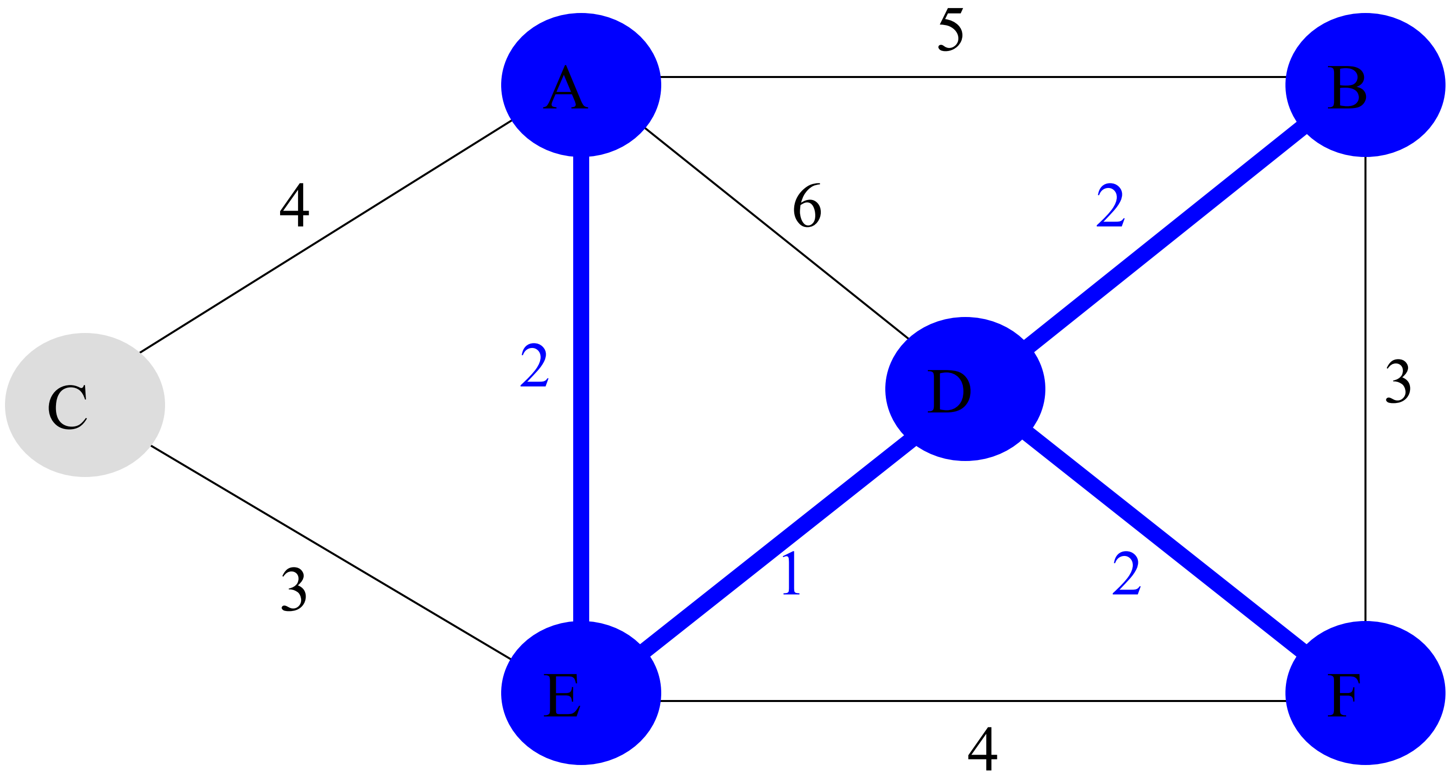
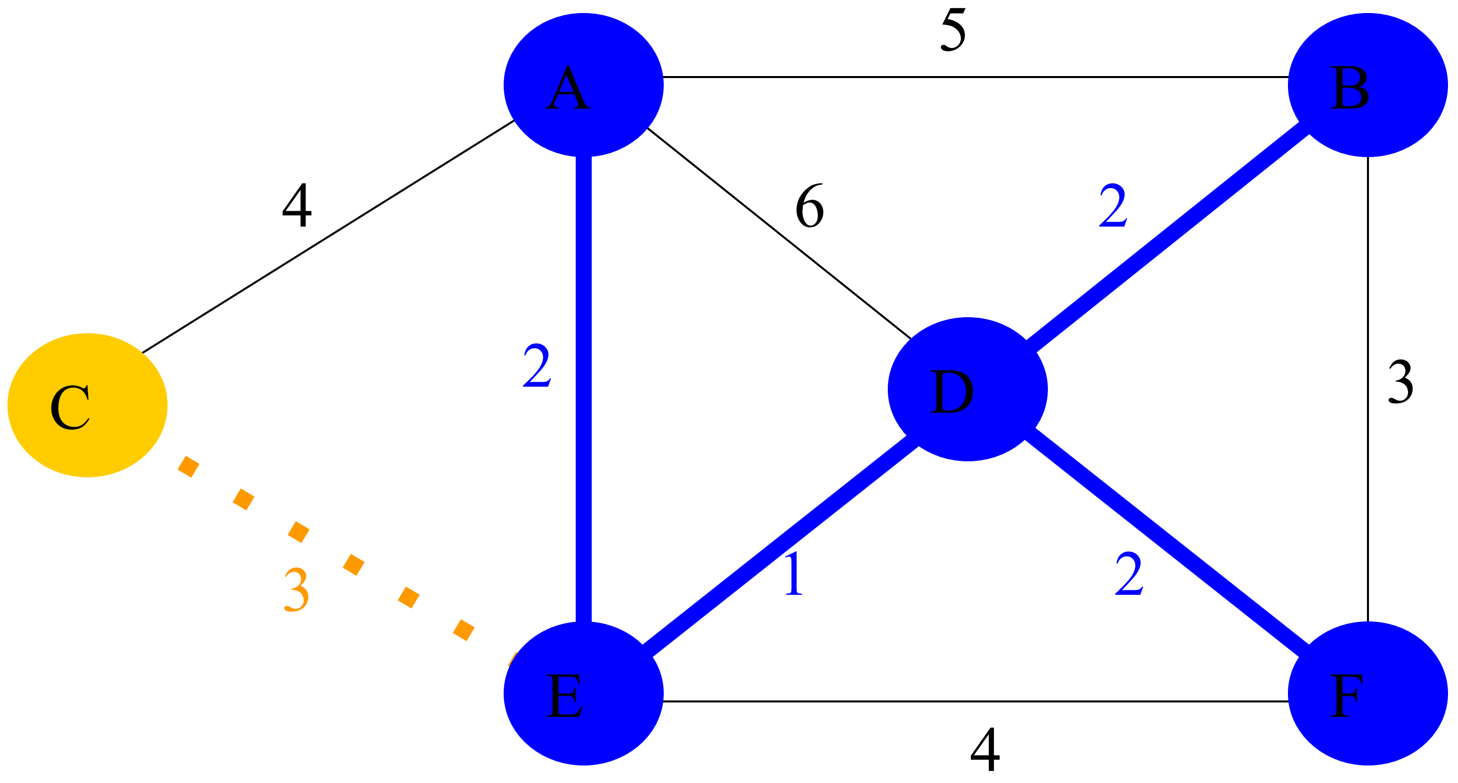
3
1
2
4
3
cycle!!

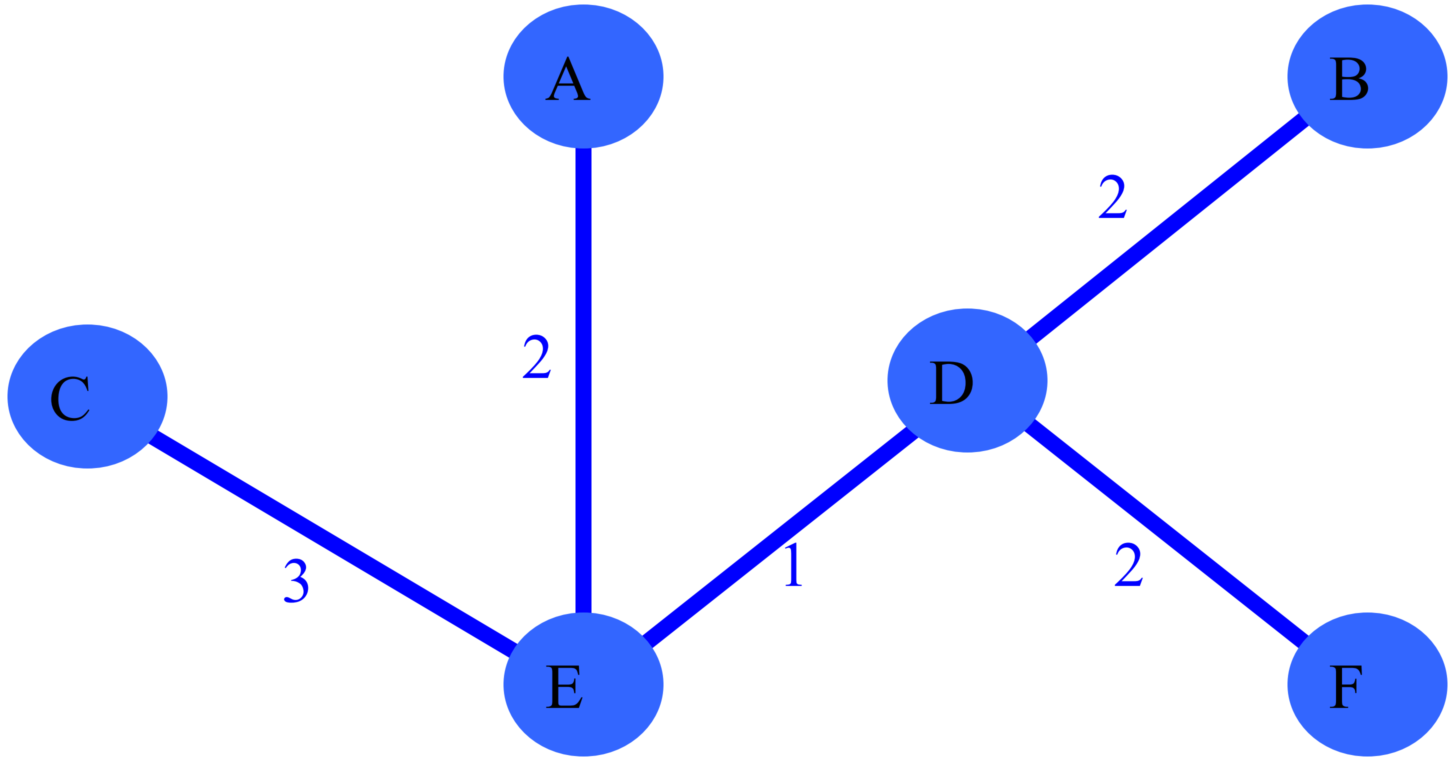


Kruskal's Algorithm

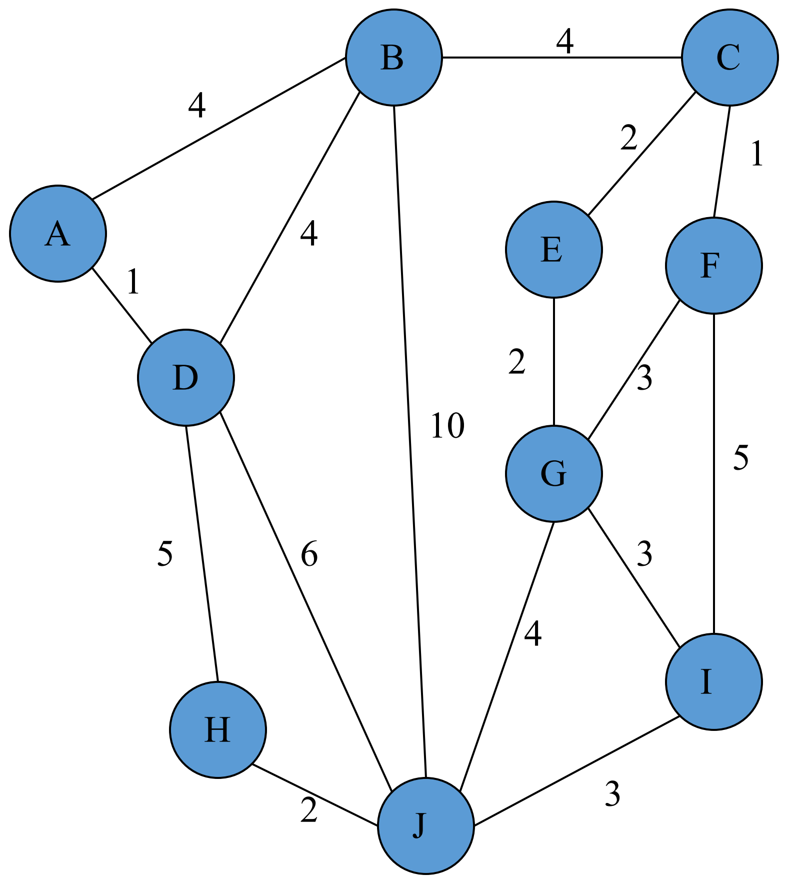


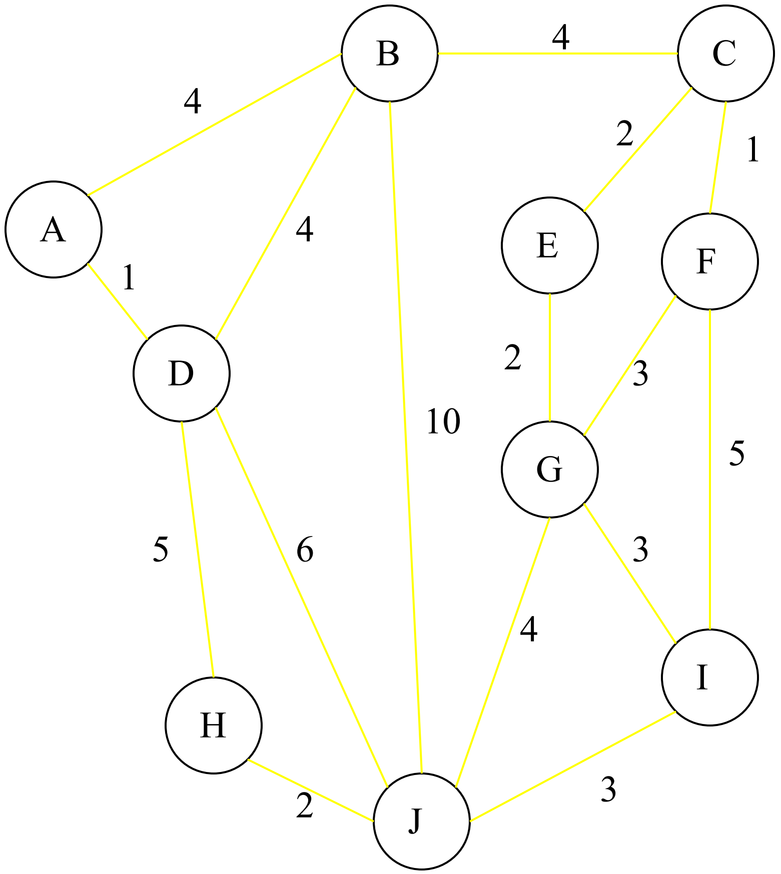
Kruskal's Algorithm

minimum- spanning tree



Complete Graph




A
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4
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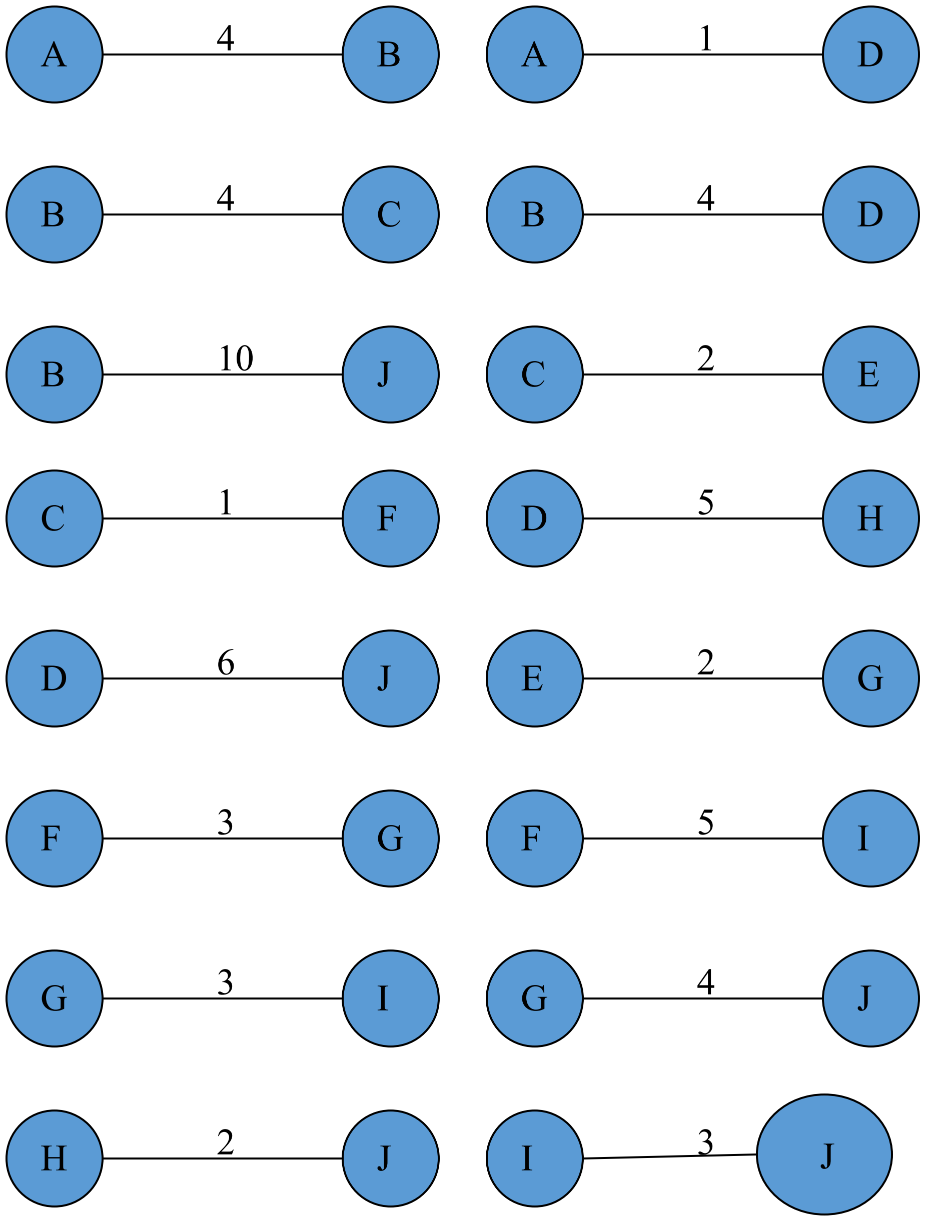
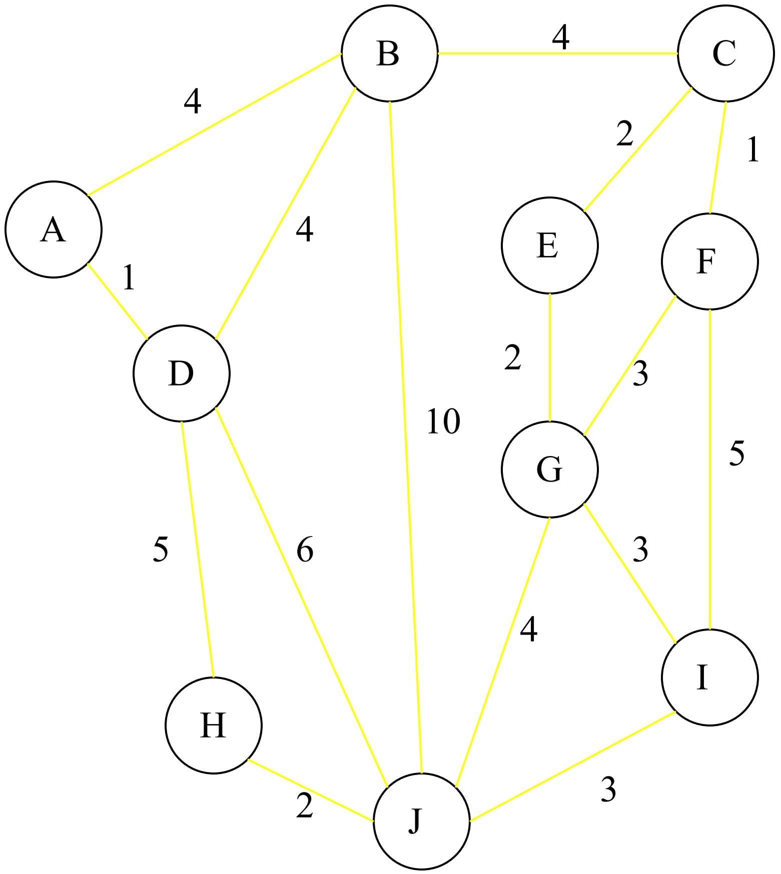
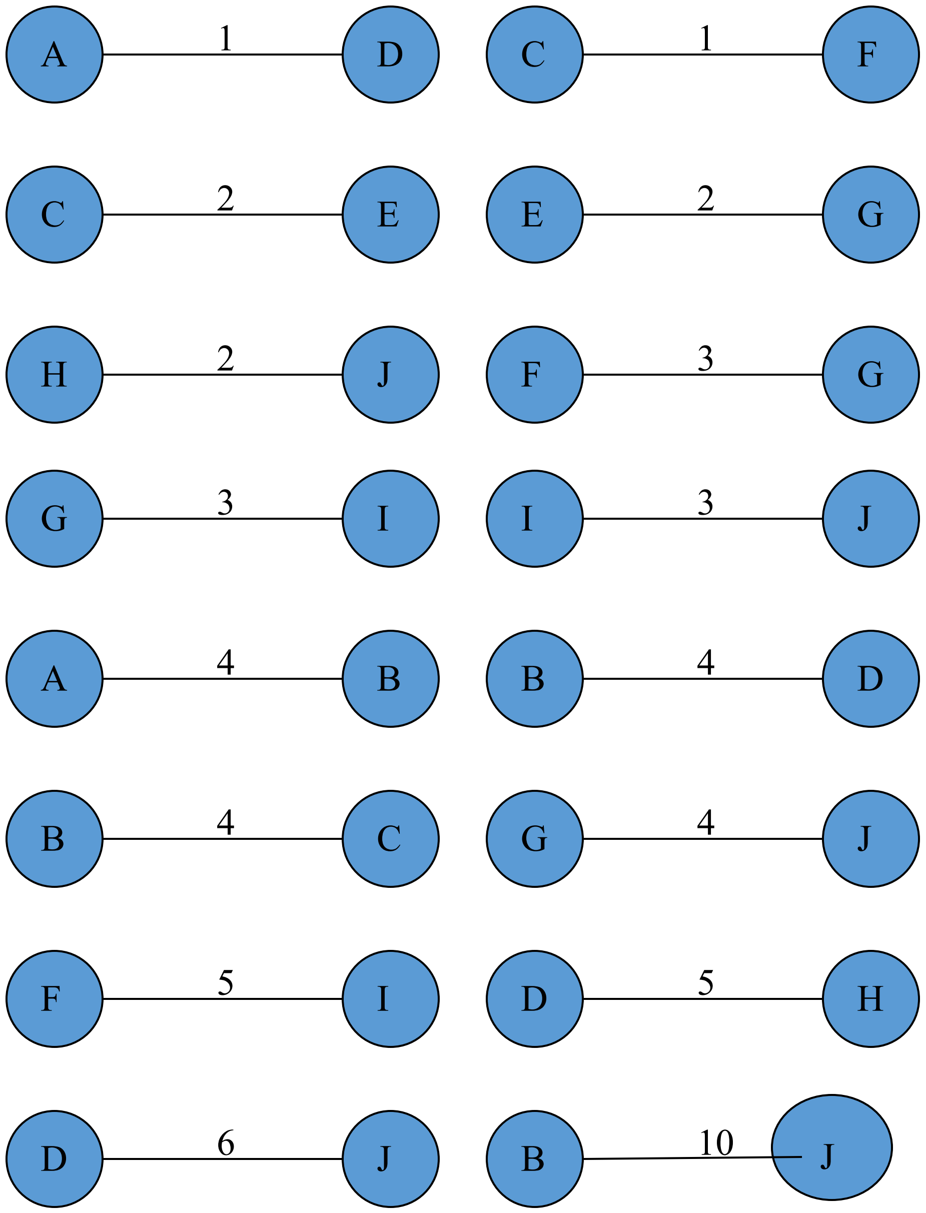
3


A 4 B
A 1 D
B 4 C
B 4 D
B 10 J
C 2 E
C 1 F
D 5 H
D 6 J
E 2 G
F 3 G
F 5 I
G 3 I
G 4 J
H 2 J
I 3 J

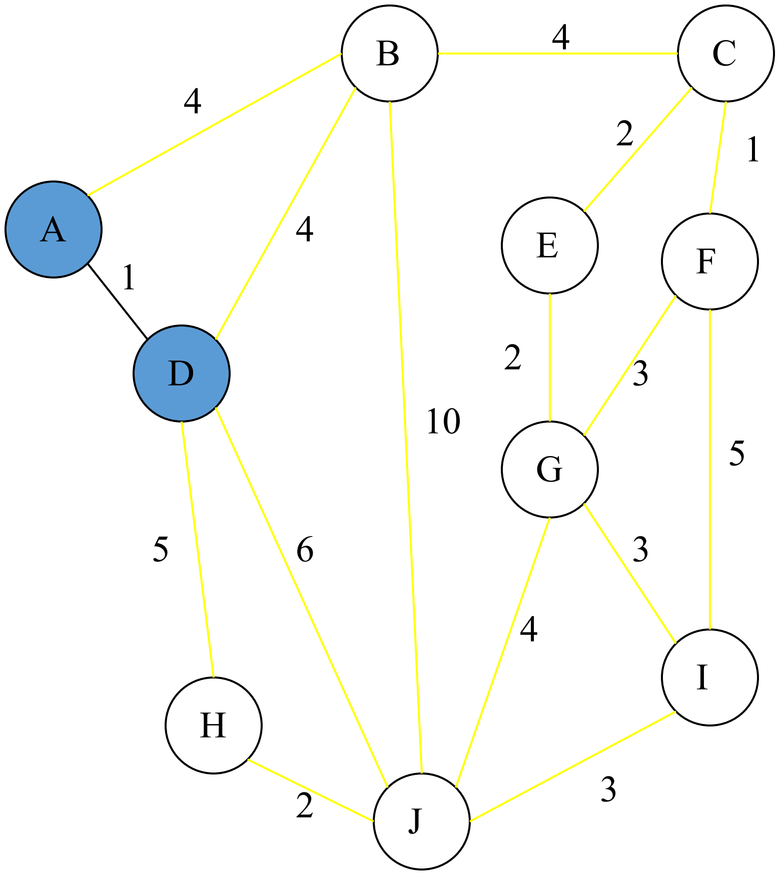
Sort Edges

(in reality they are placed in a priority queue - not sorted - but sorting them makes the algorithm easier to visualize)





Add Edge



A
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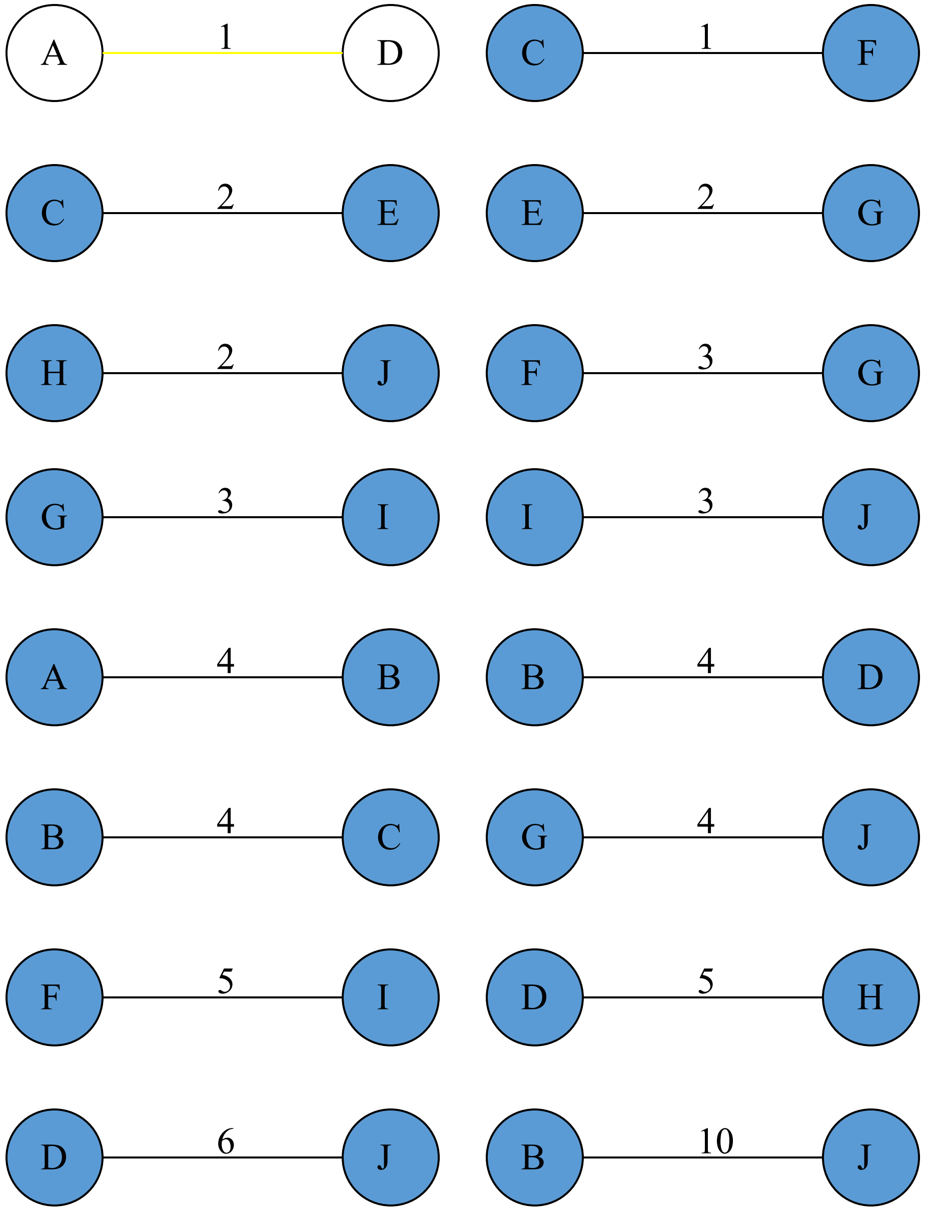
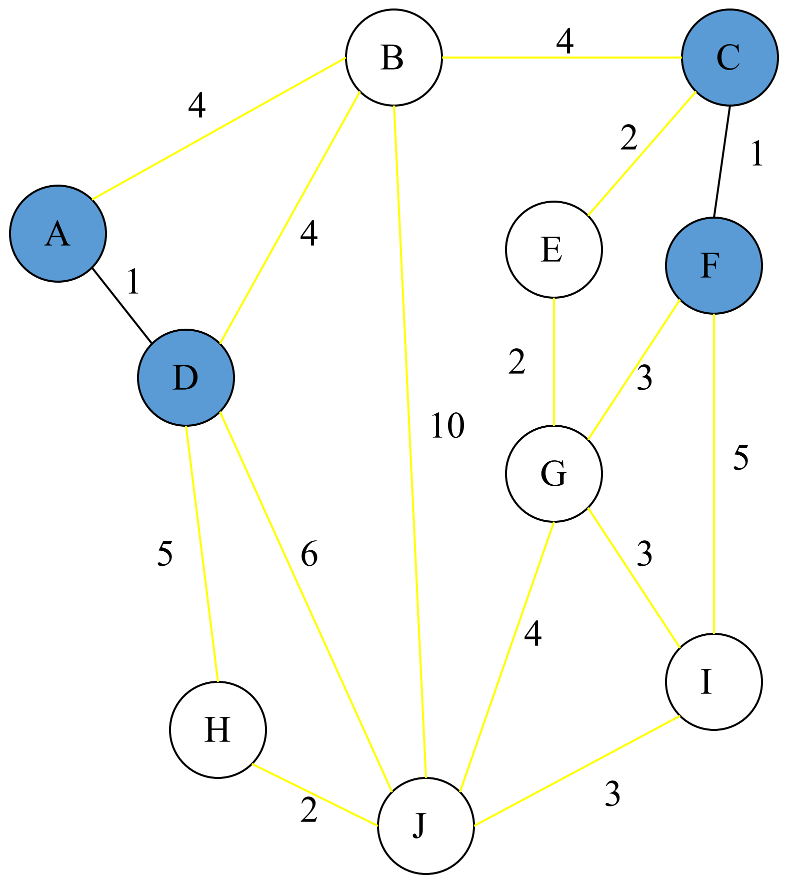


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



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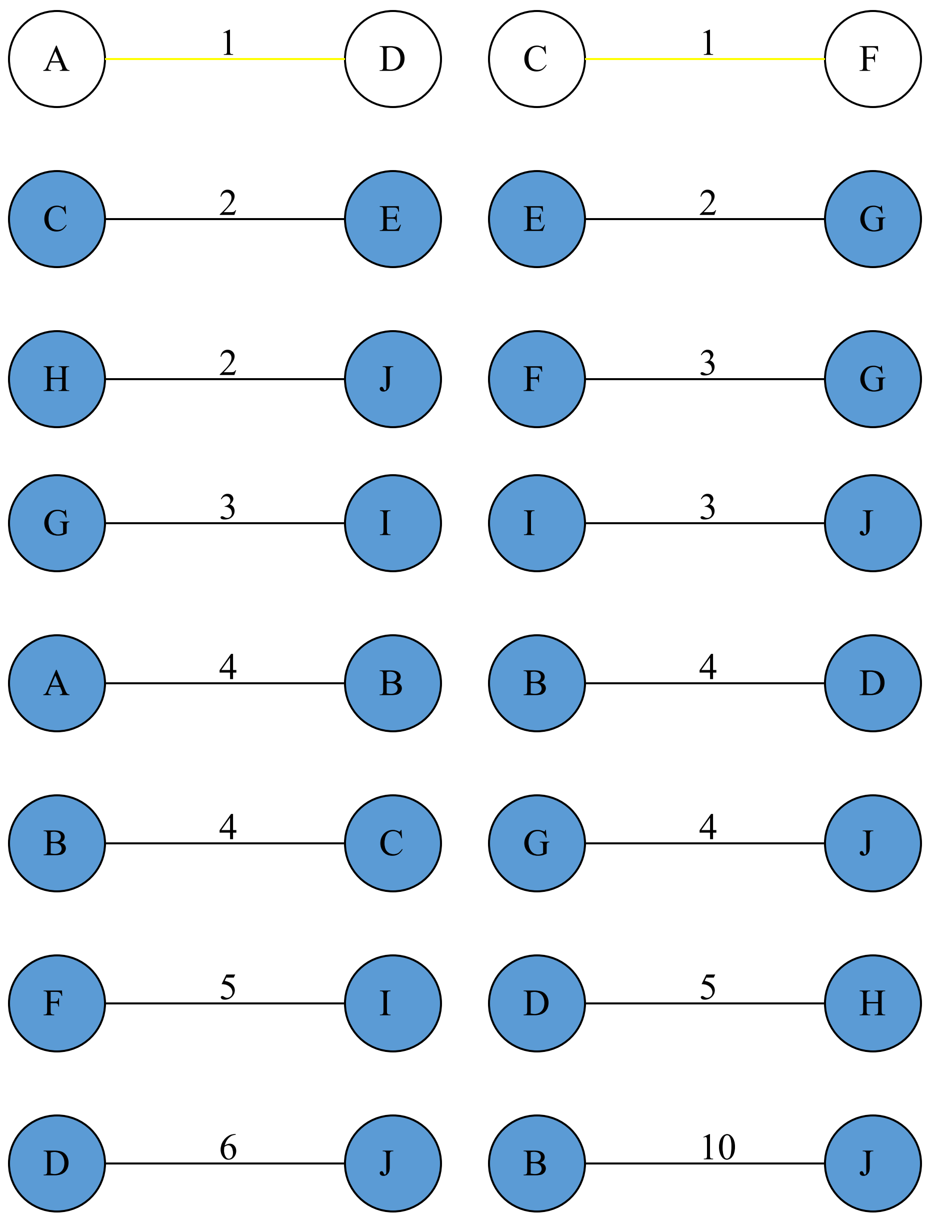
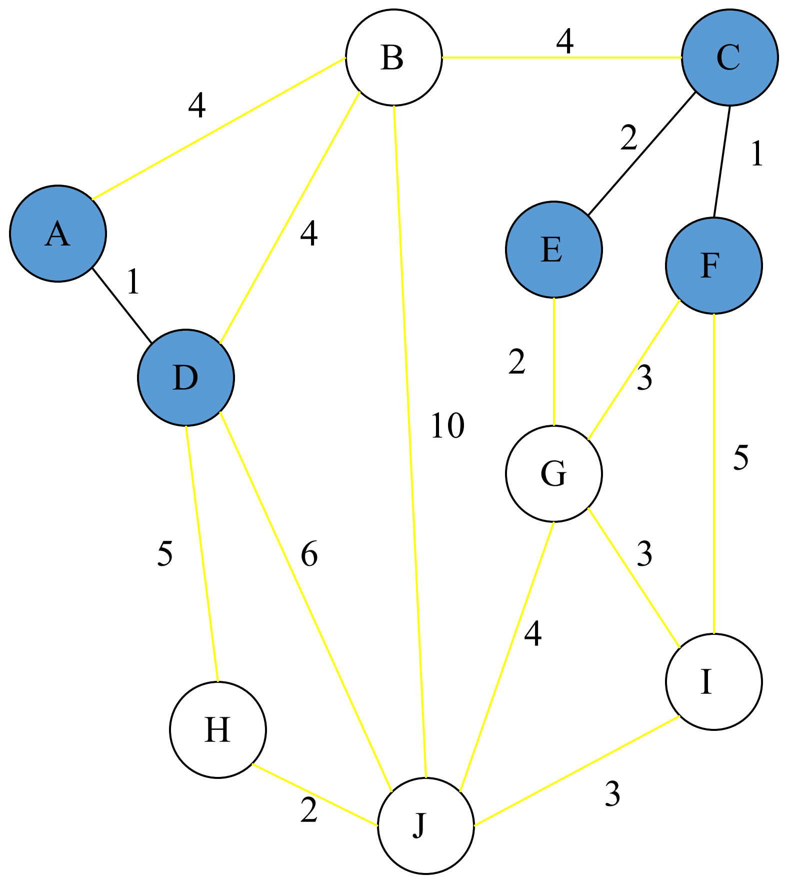


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



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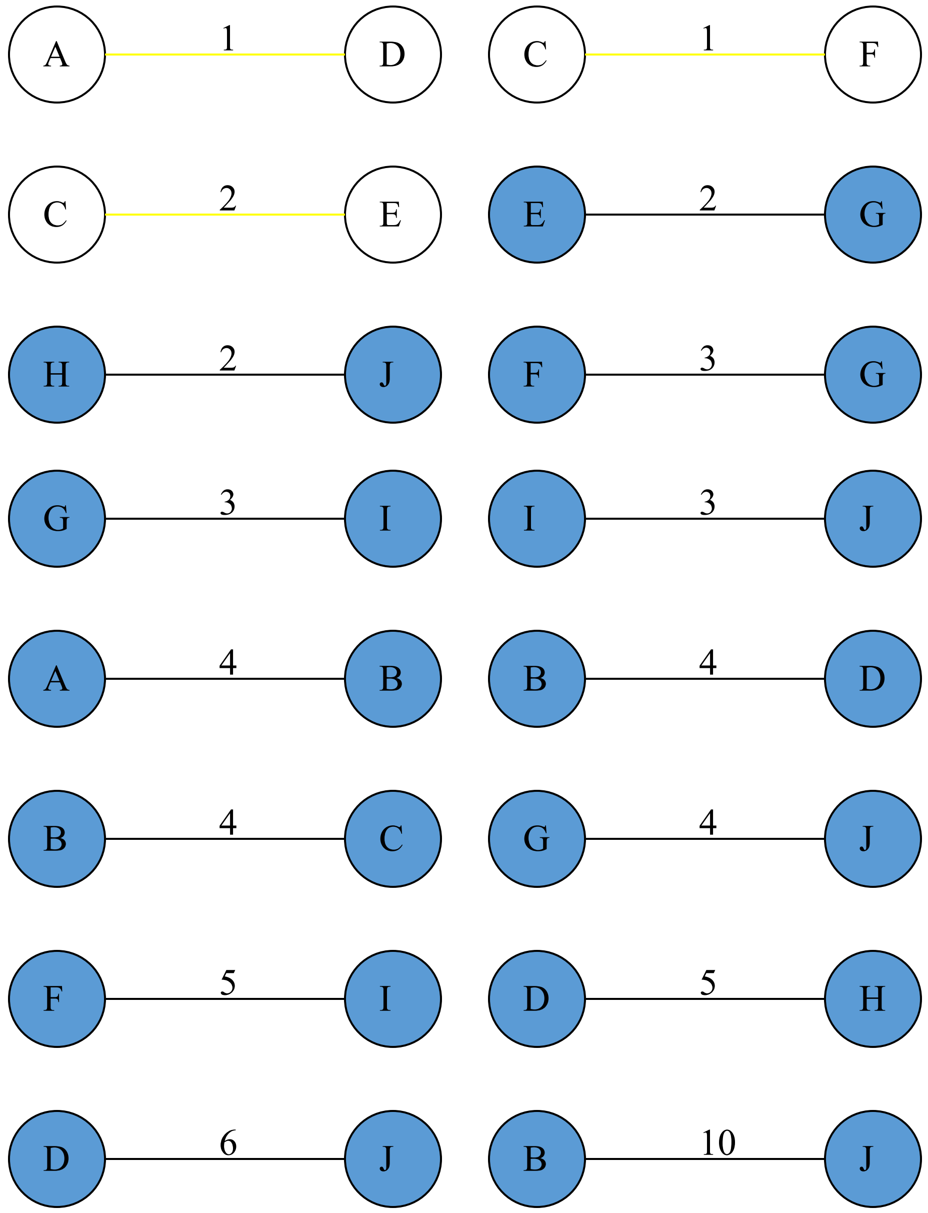

A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
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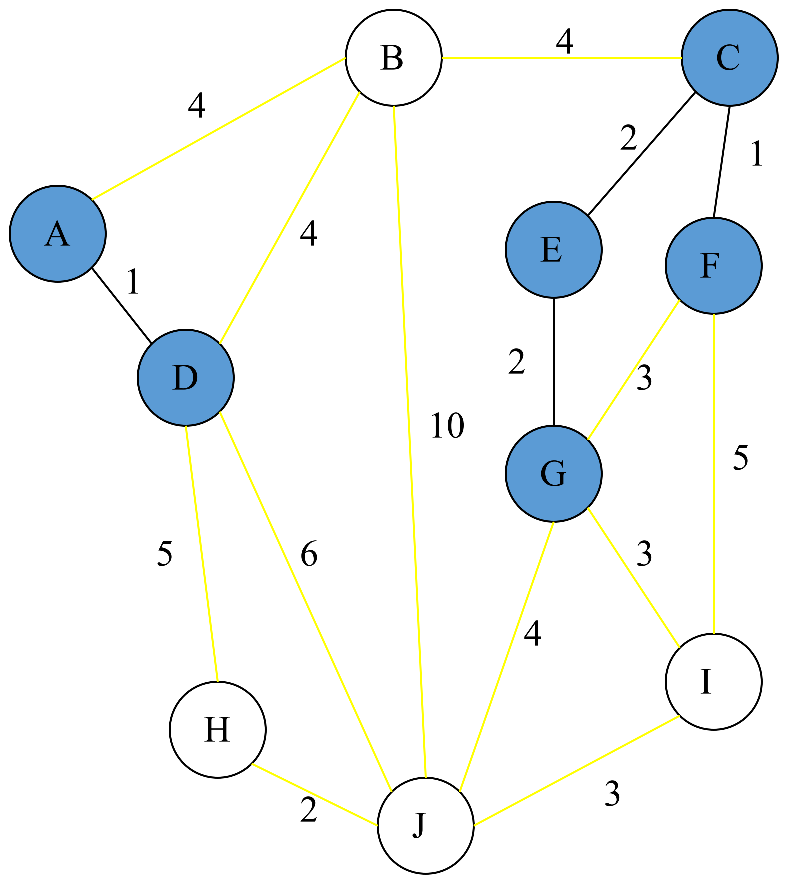
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



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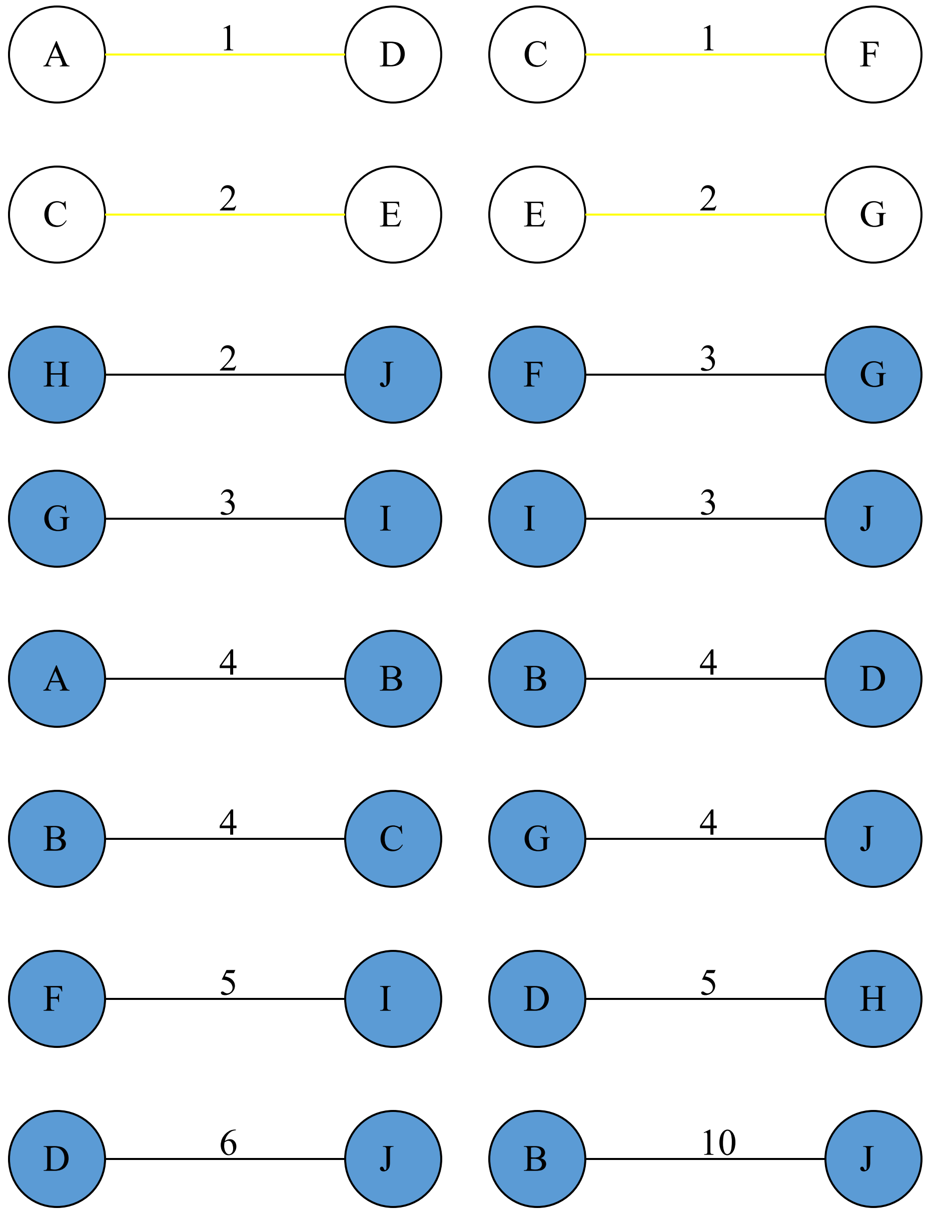
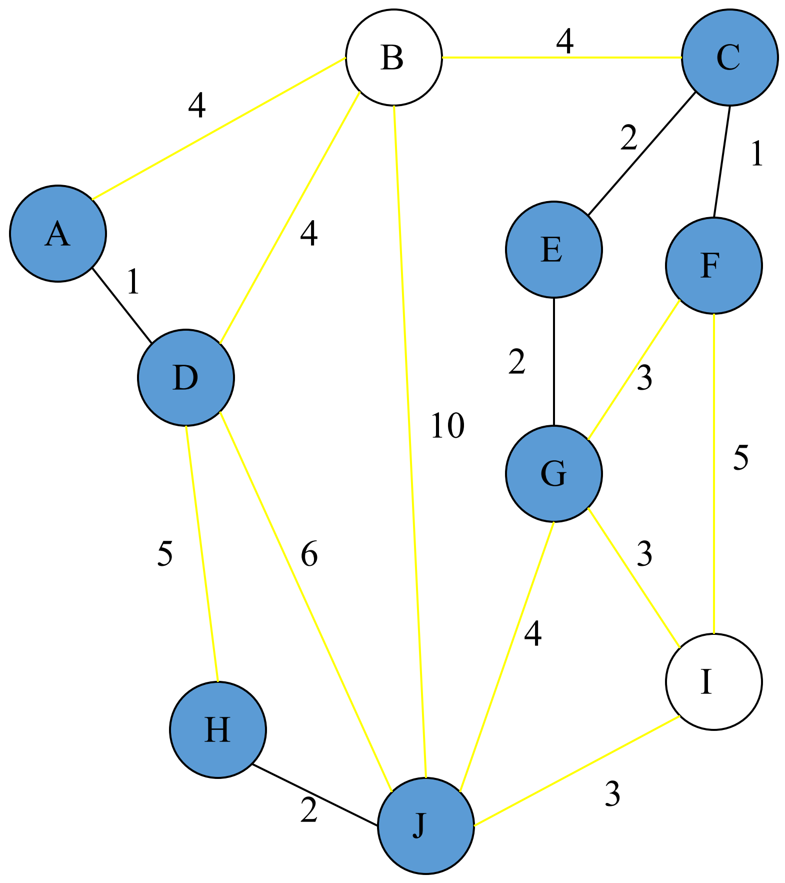


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



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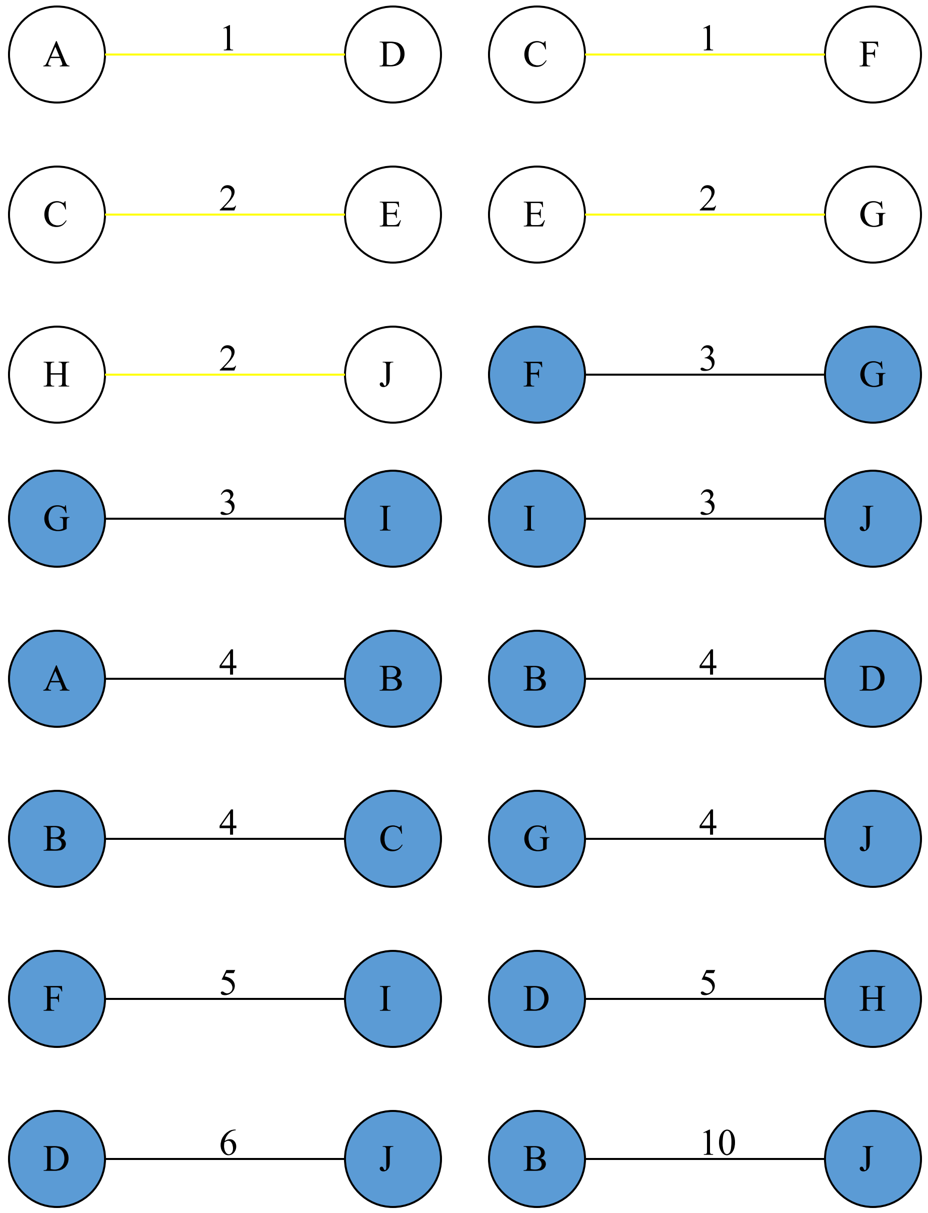

A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
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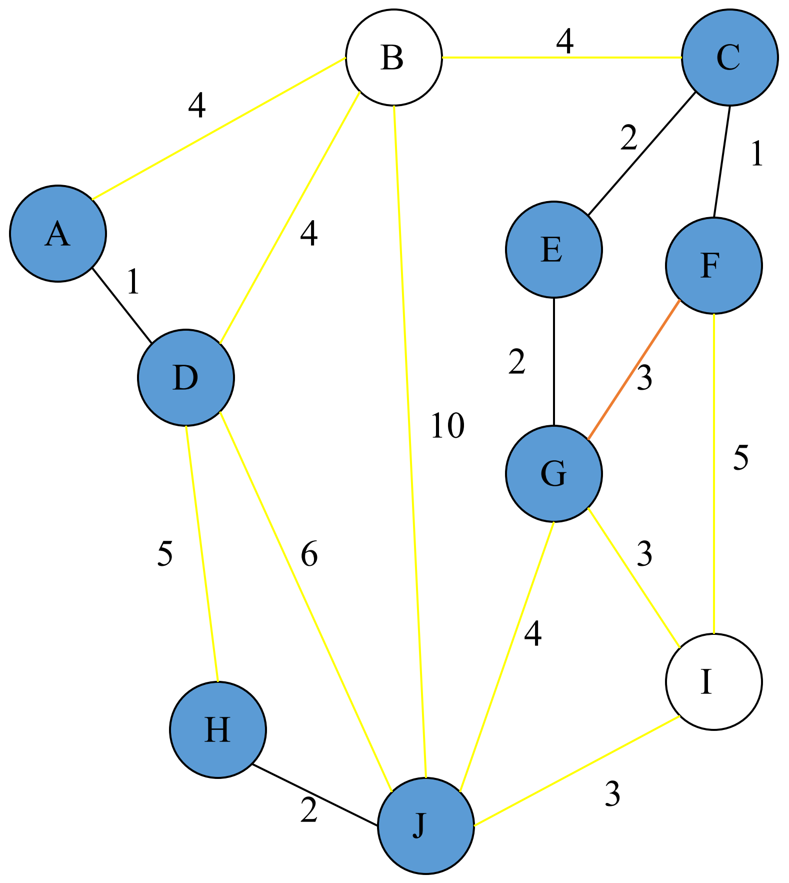
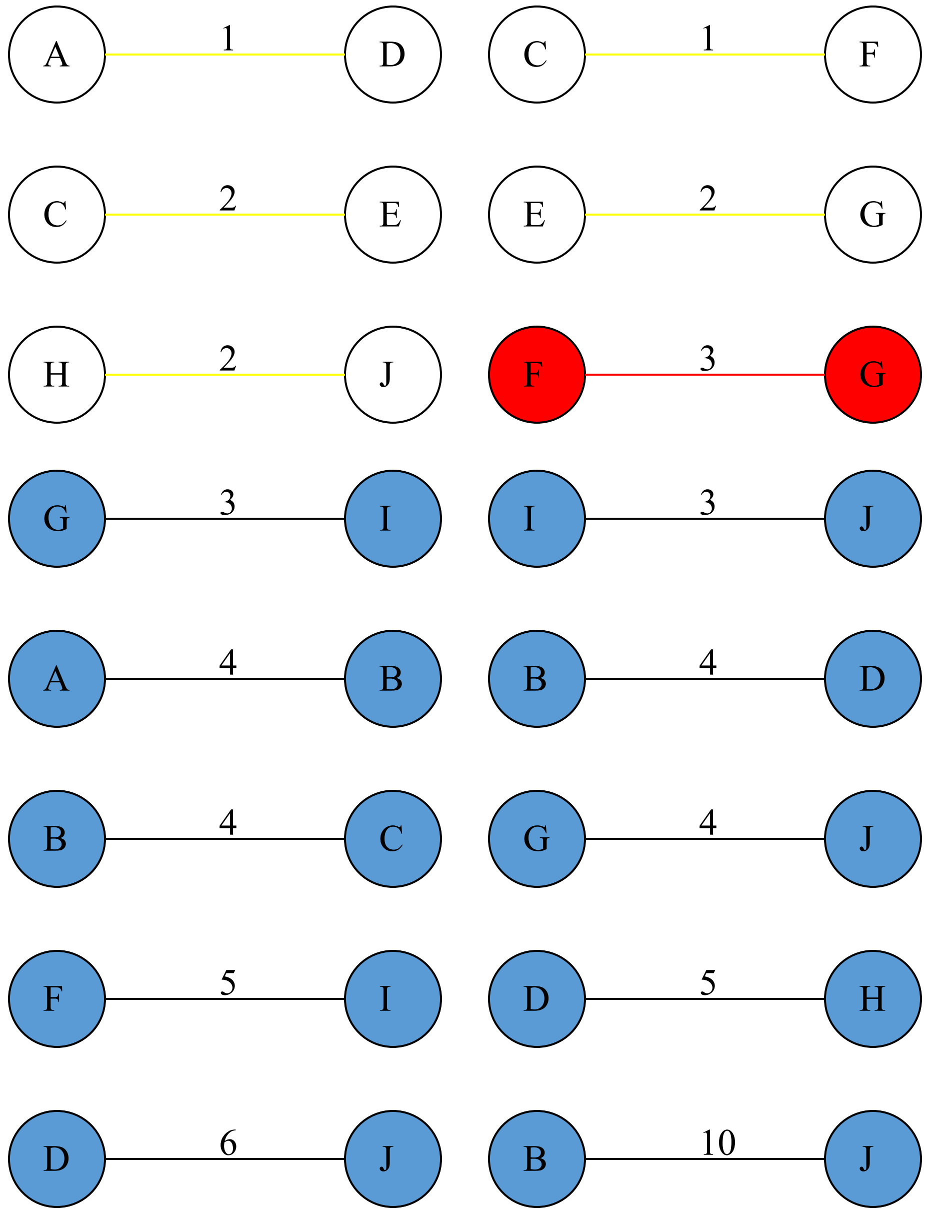
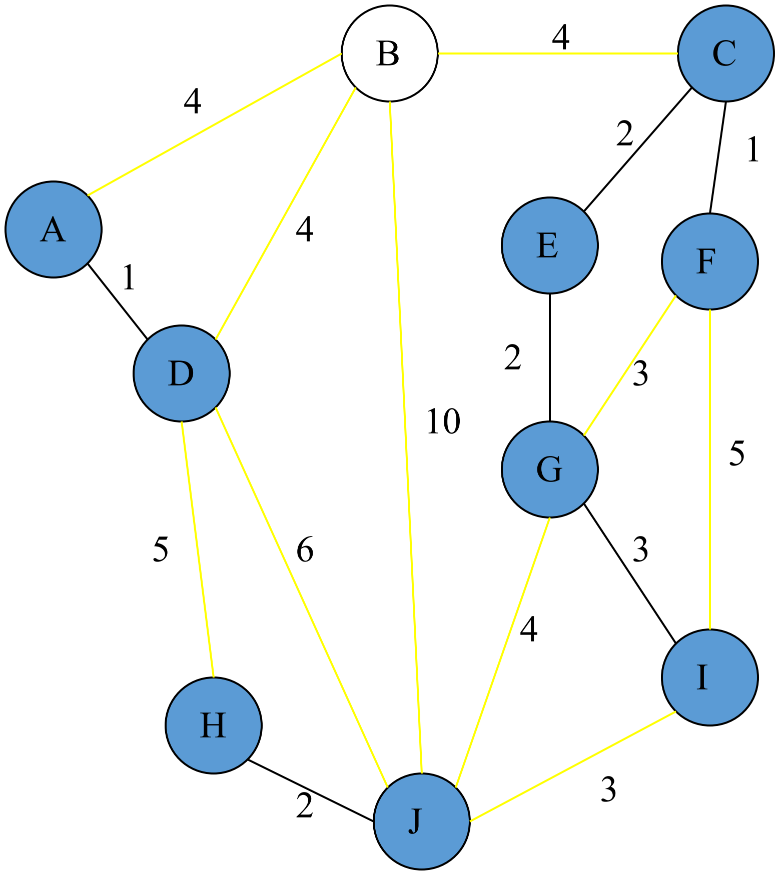
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Cycle
Don’t Add Edge





Add Edge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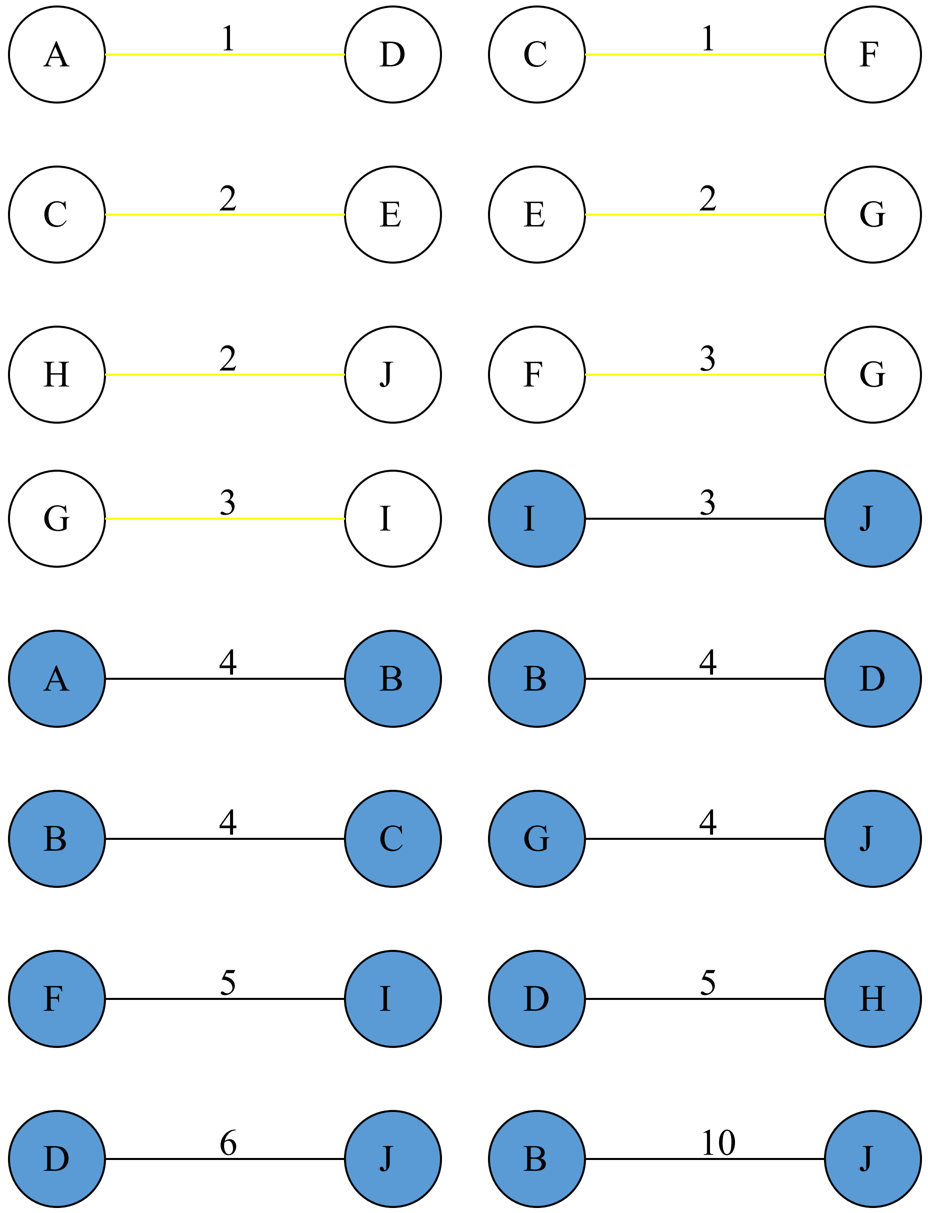
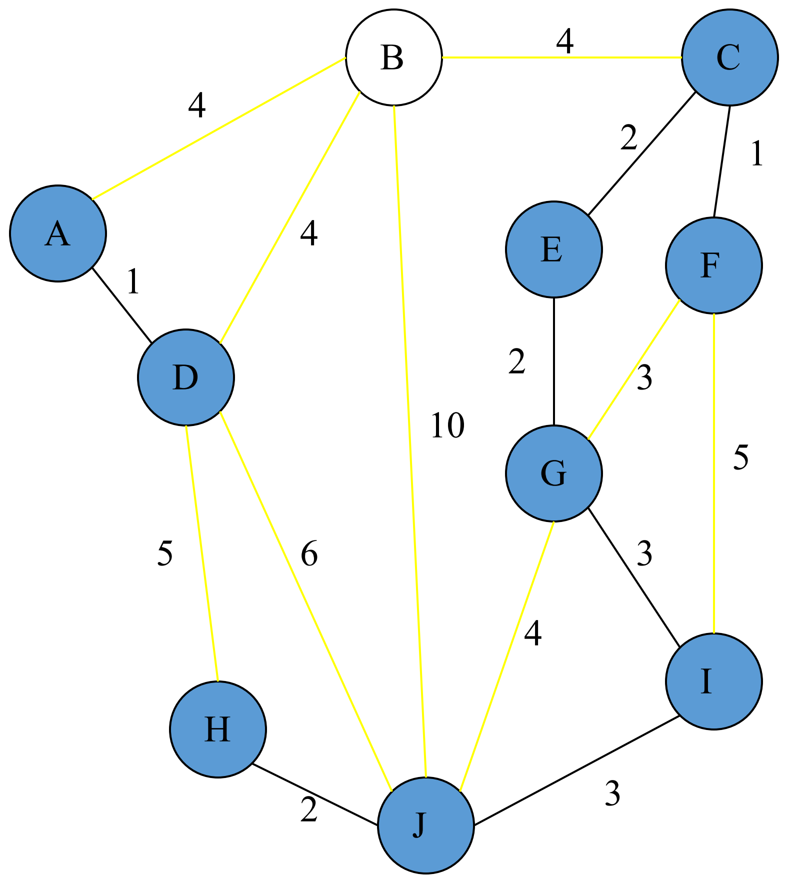

A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge



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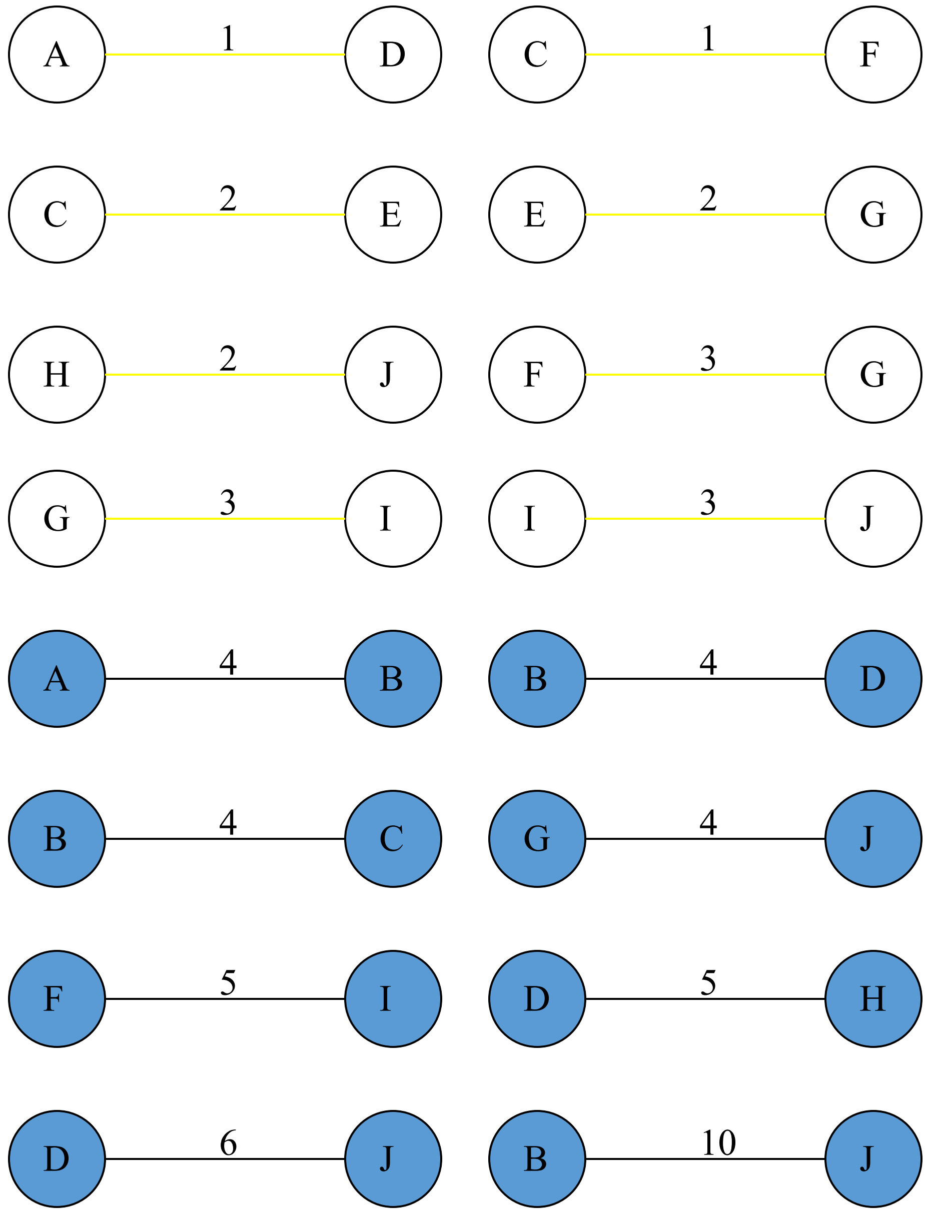
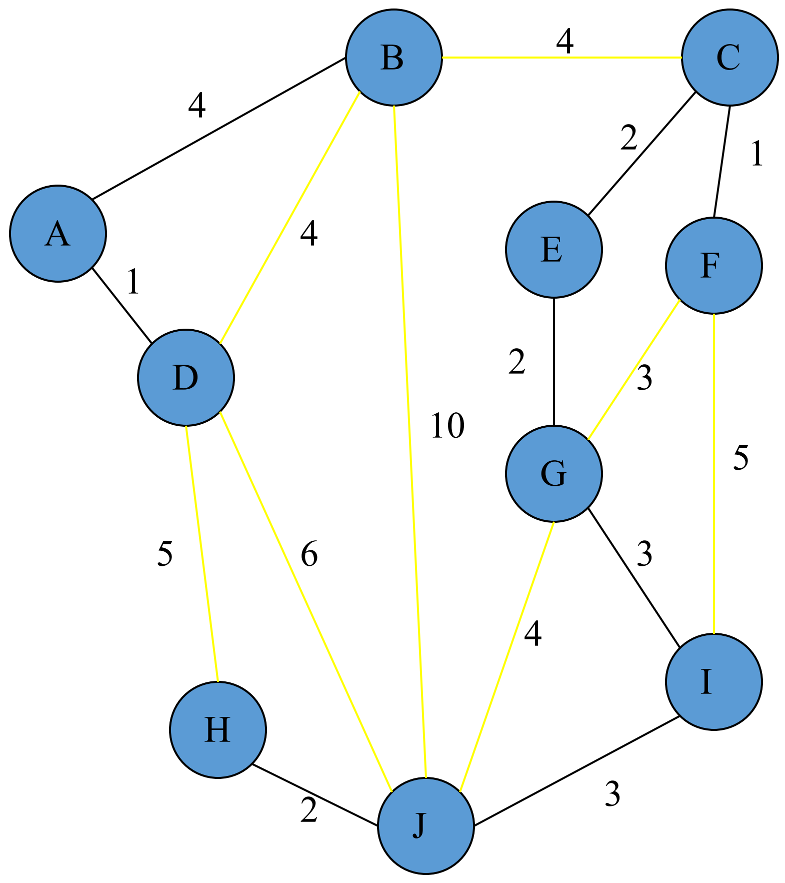


A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Add Edge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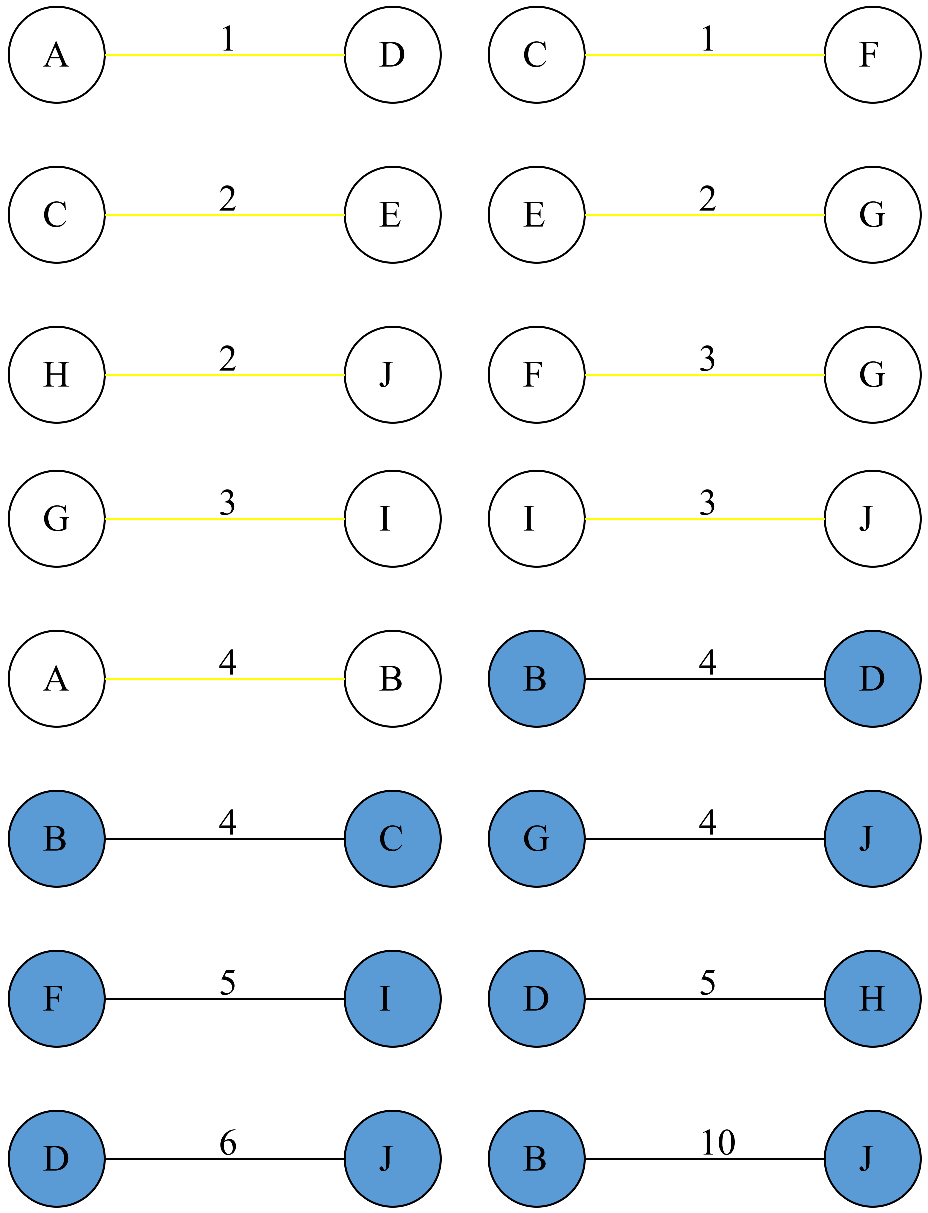
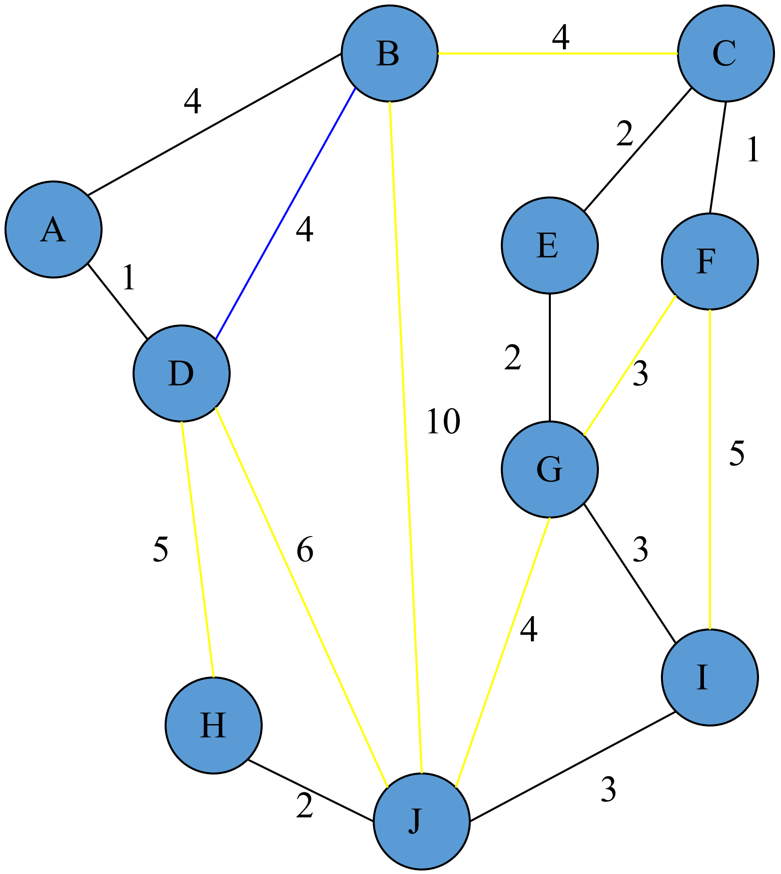
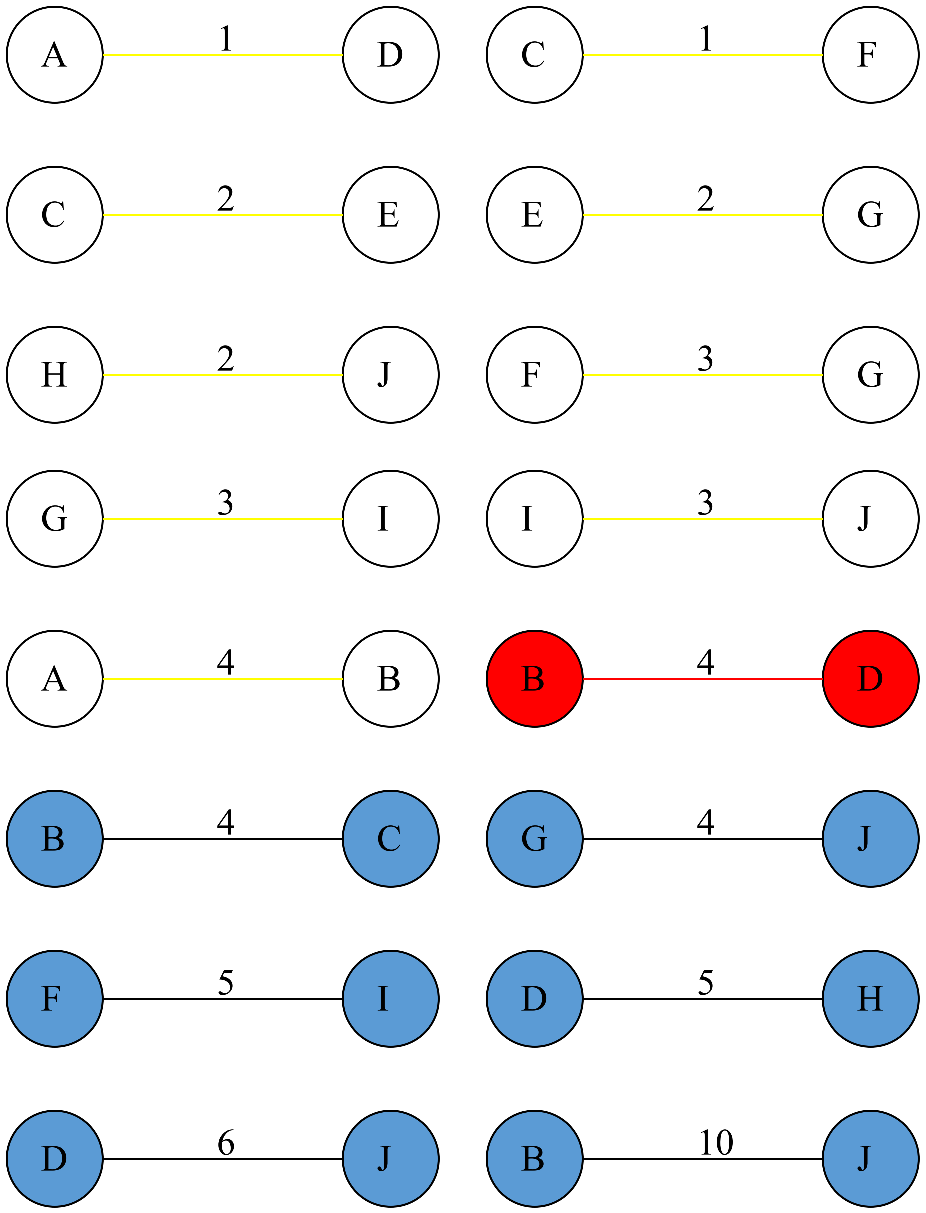

A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

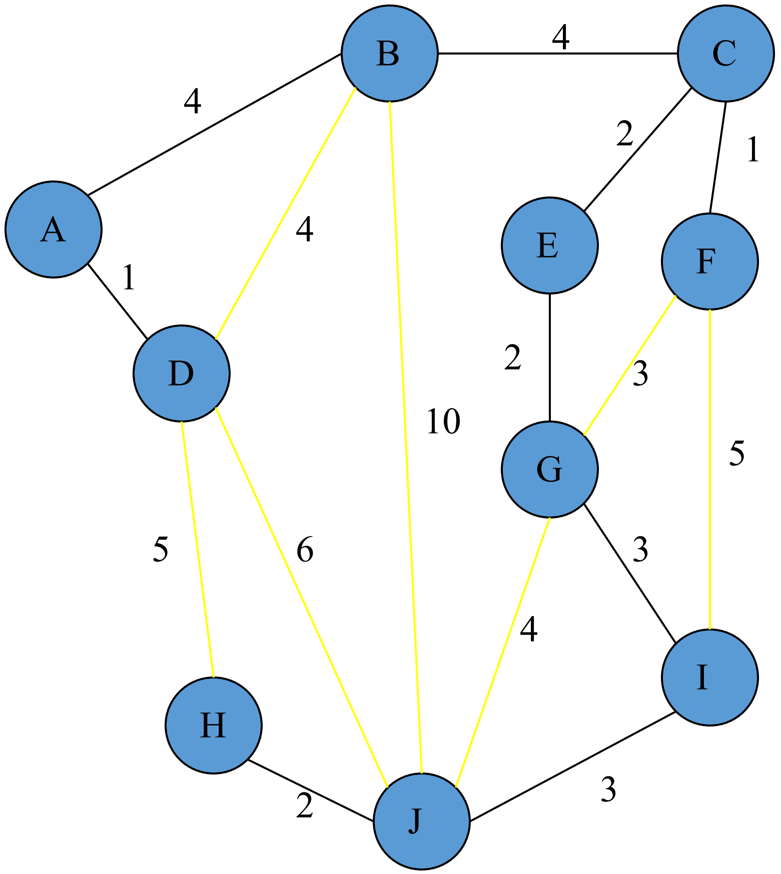
Cycle

Don't Add Edge





Add Edge


A
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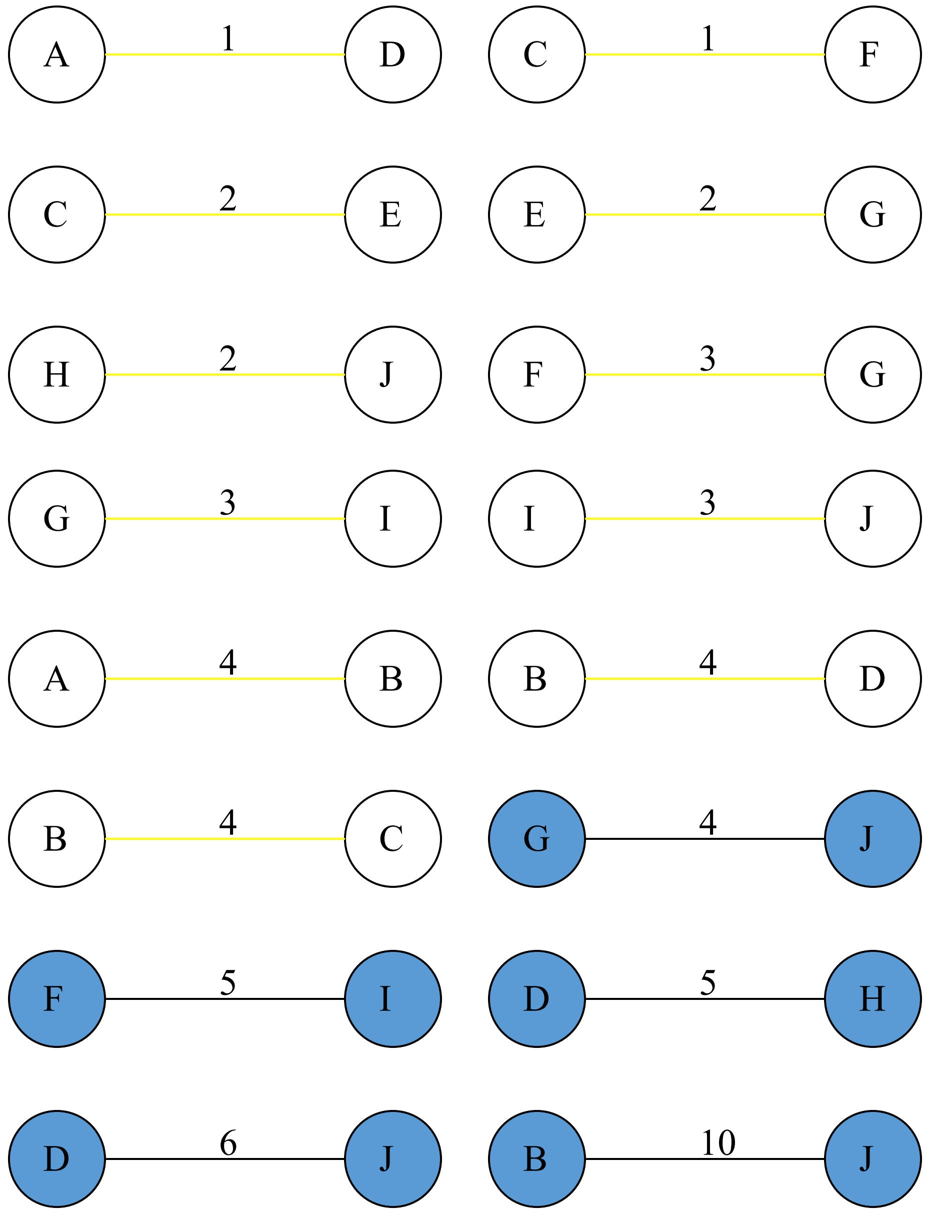
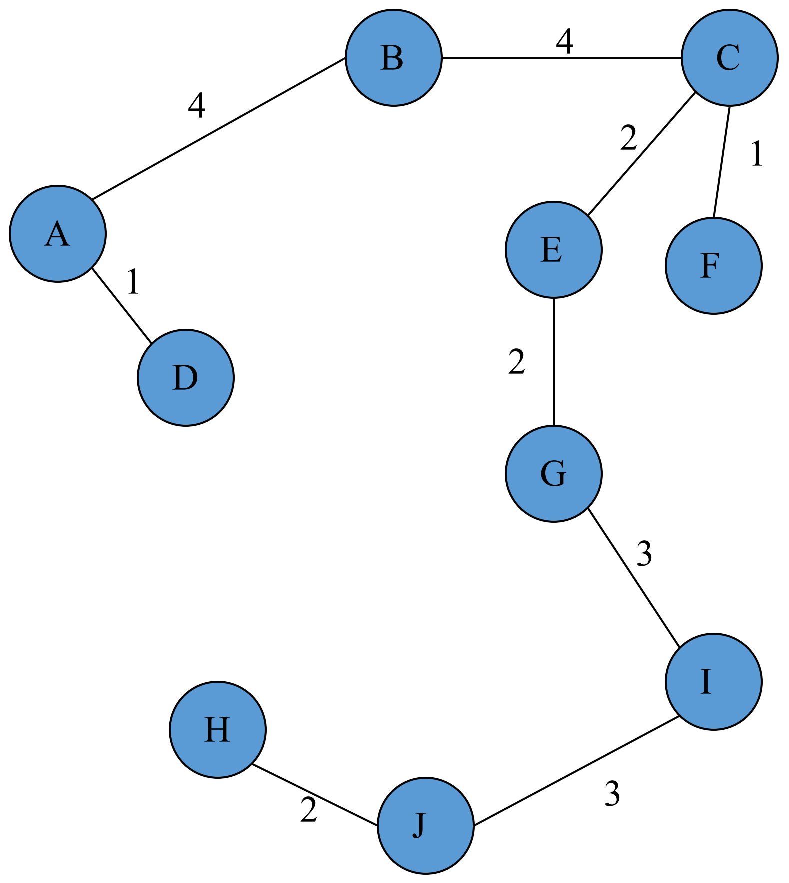
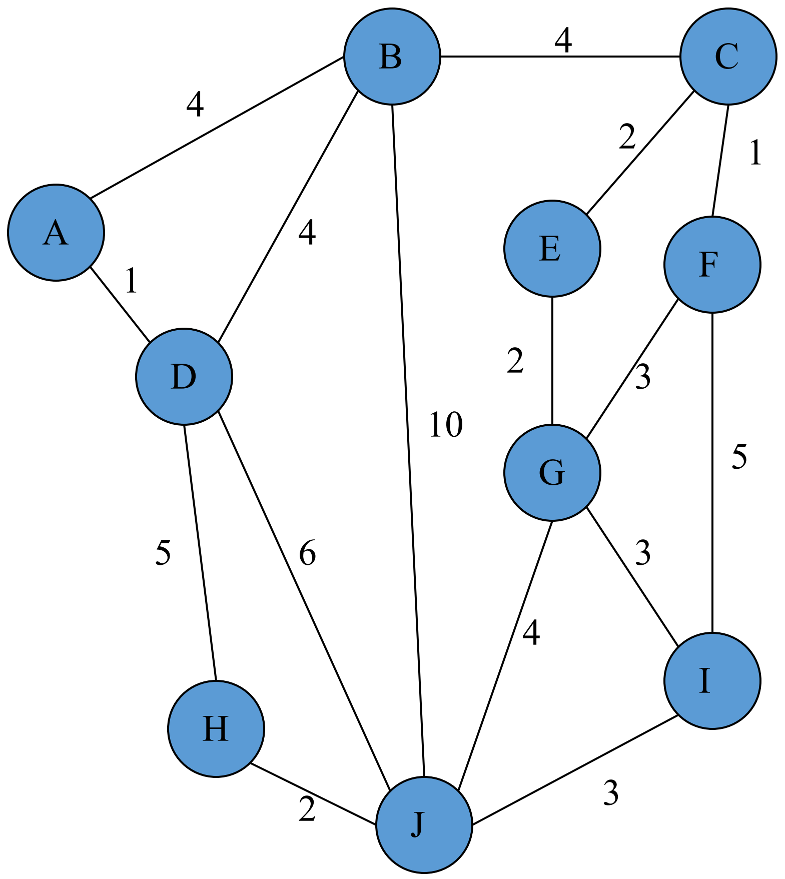

A 1 D
C 1 F
C 2 E
E 2 G
H 2 J
F 3 G
G 3 I
I 3 J
A 4 B
B 4 D
B 4 C
G 4 J
F 5 I
D 5 H
D 6 J
B 10 J

Minimum Spanning Tree



Complete Graph

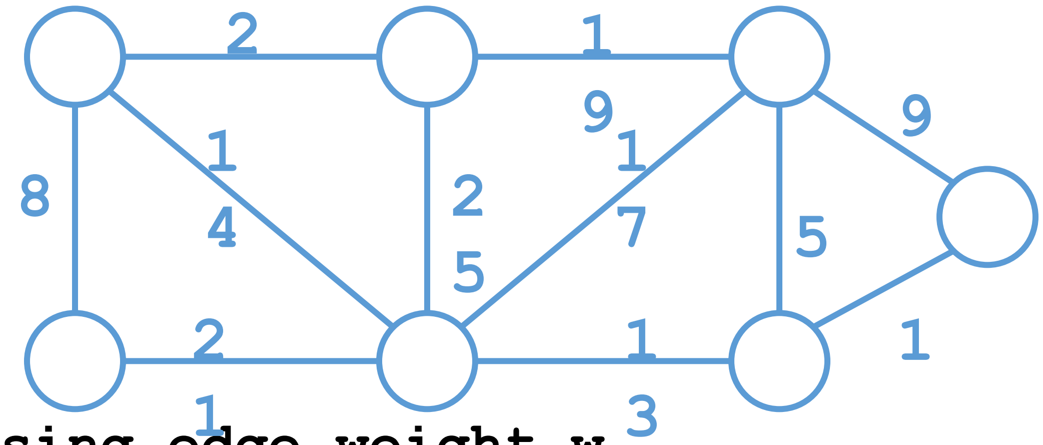


Kruskal’s Algorithm

```
Kruskal()
{
   T = ∅;
   for each v ∈ V
      MakeSet(v);
   sort E by increasing edge weight w
   for each (u,v) ∈ E (in sorted order)
      if FindSet(u) ≠ FindSet(v)
         T = T U {{u,v}};
         Union(FindSet(u), FindSet(v));
}
```

Kruskal's Algorithm

Run the algorithm:



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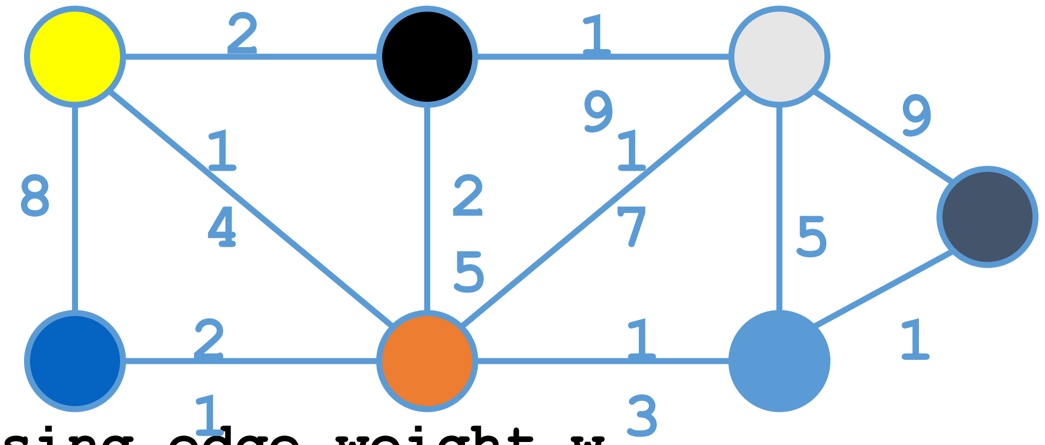
        if FindSet(u) ≠ FindSet(v)
            T = T ∪ {{u,v}};
            Union(FindSet(u), FindSet(v));
}
```

Kruskal's Algorithm

Run the algorithm:



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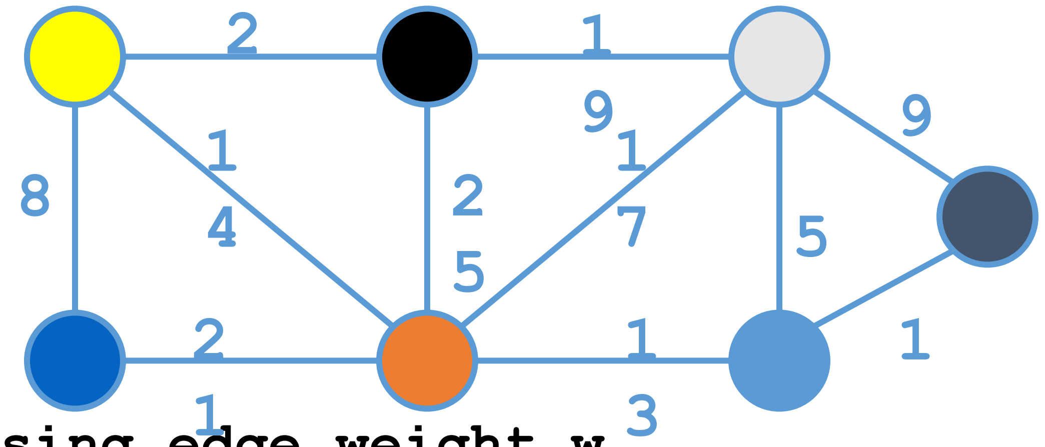

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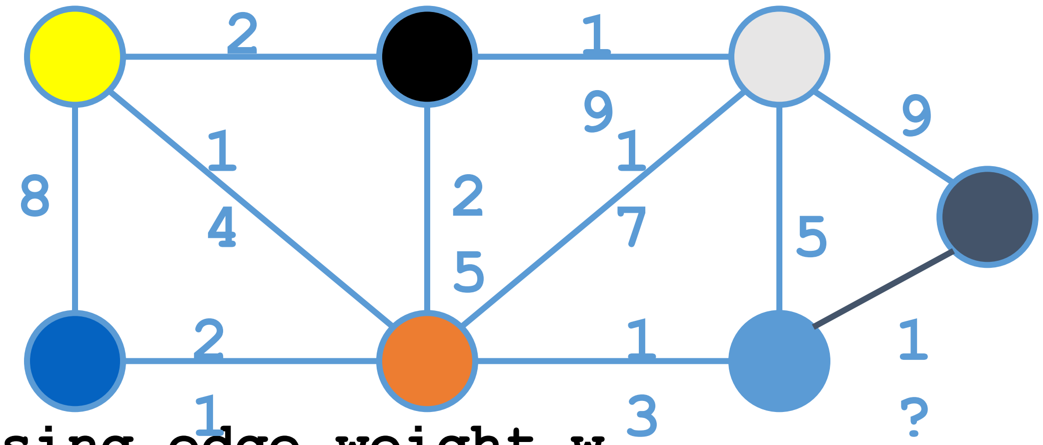

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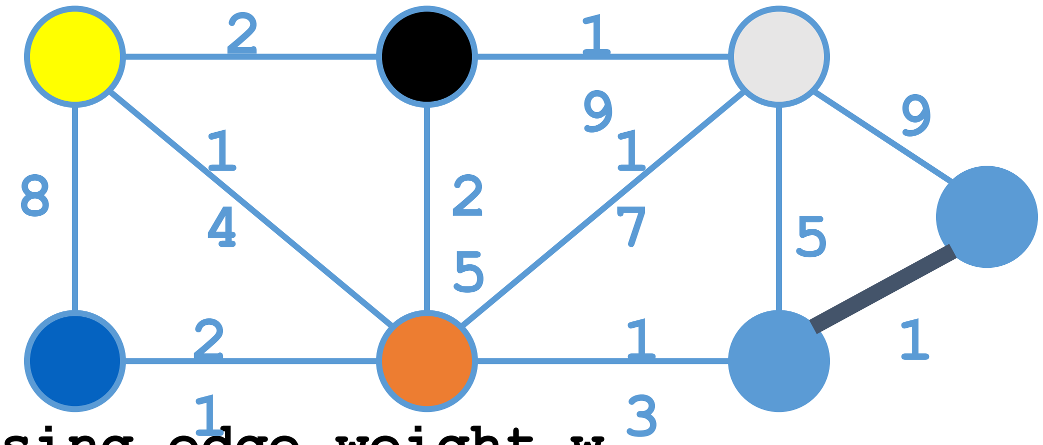

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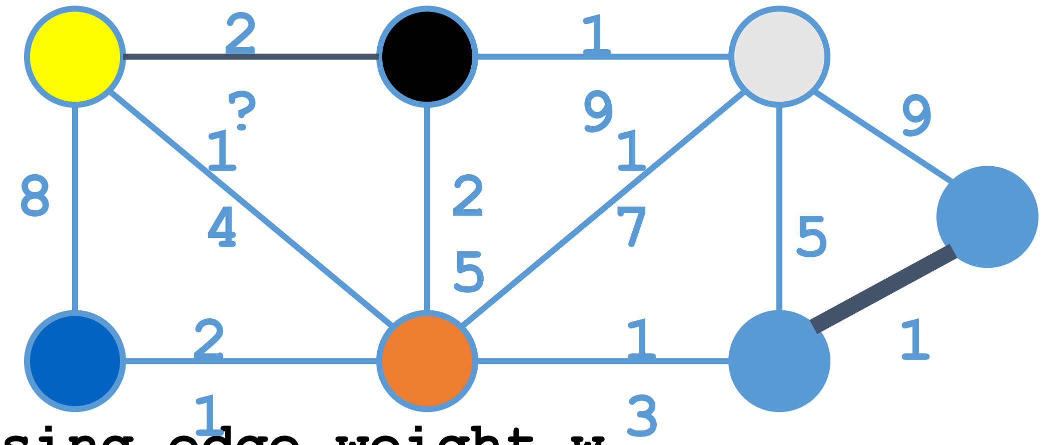
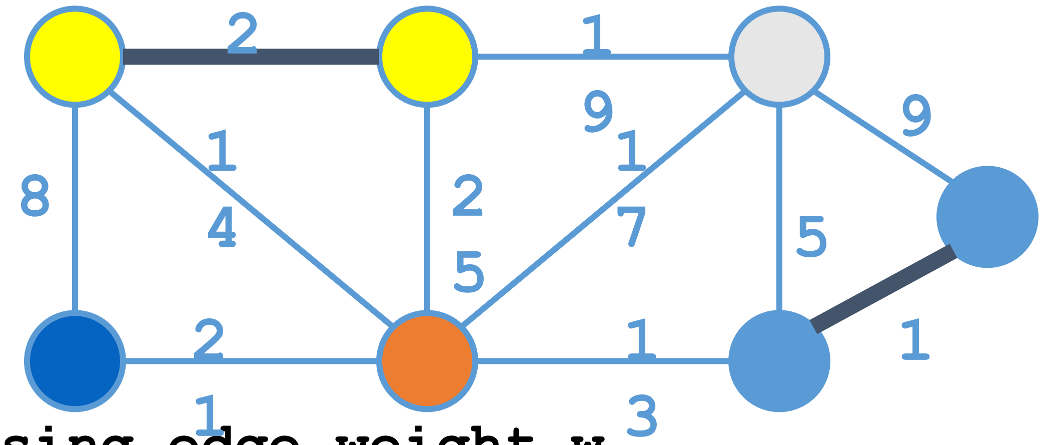


Kruskal’s Algorithm

Run the algorithm:



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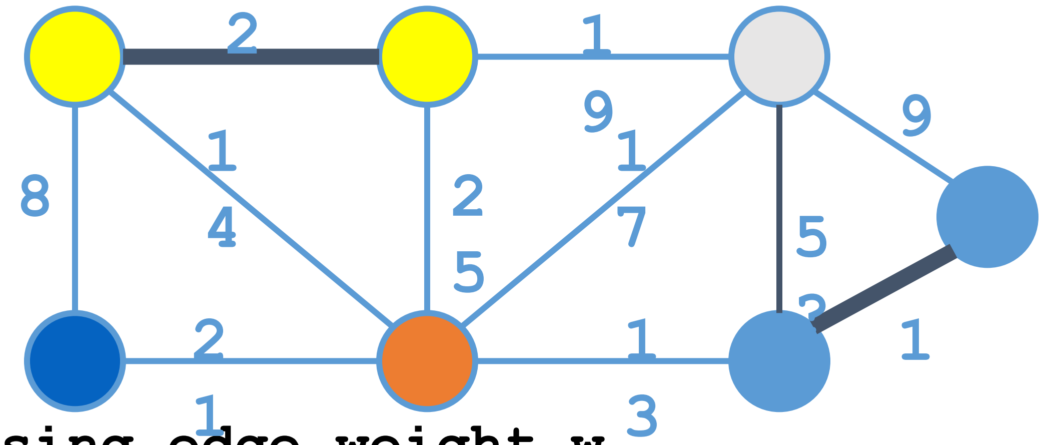

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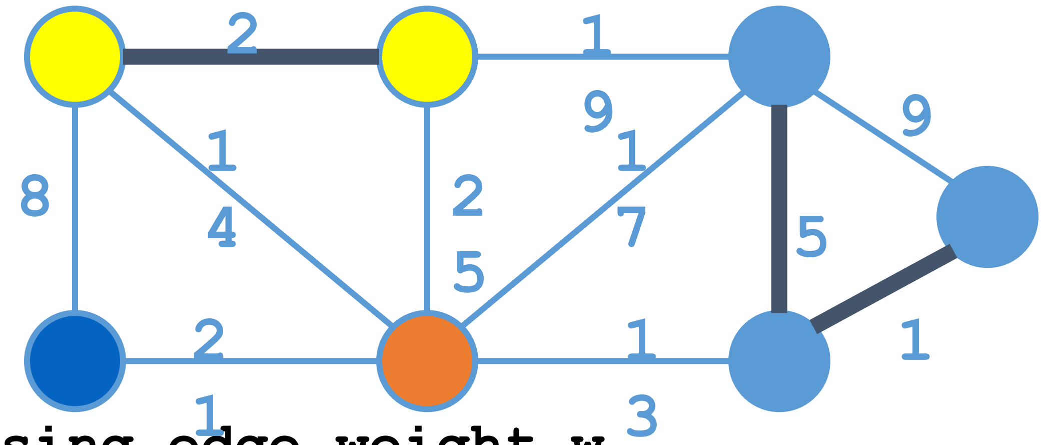

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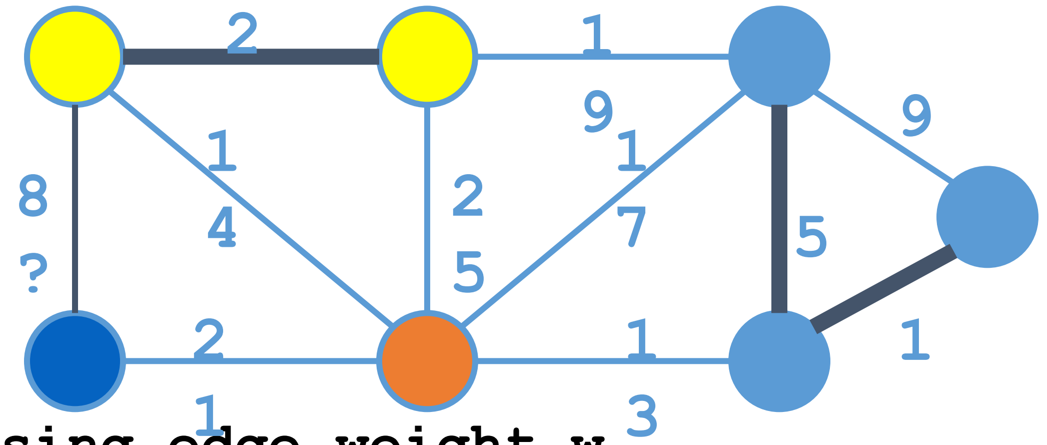

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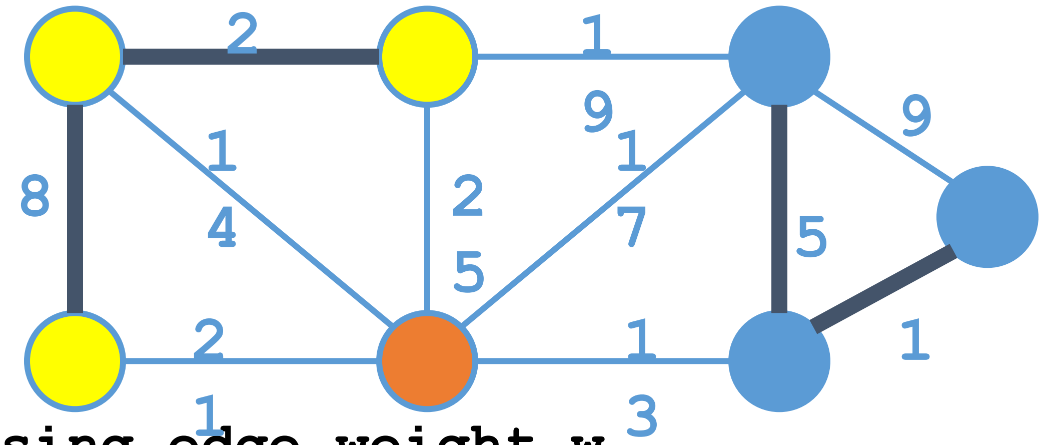

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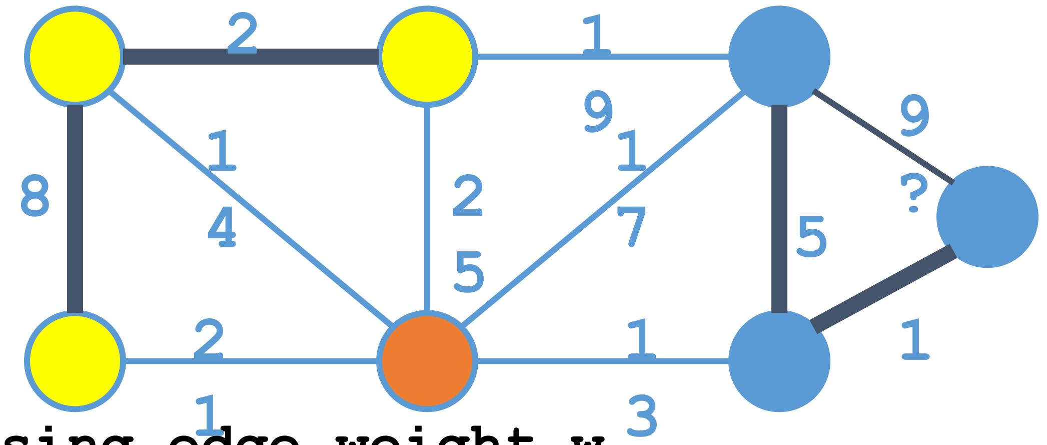

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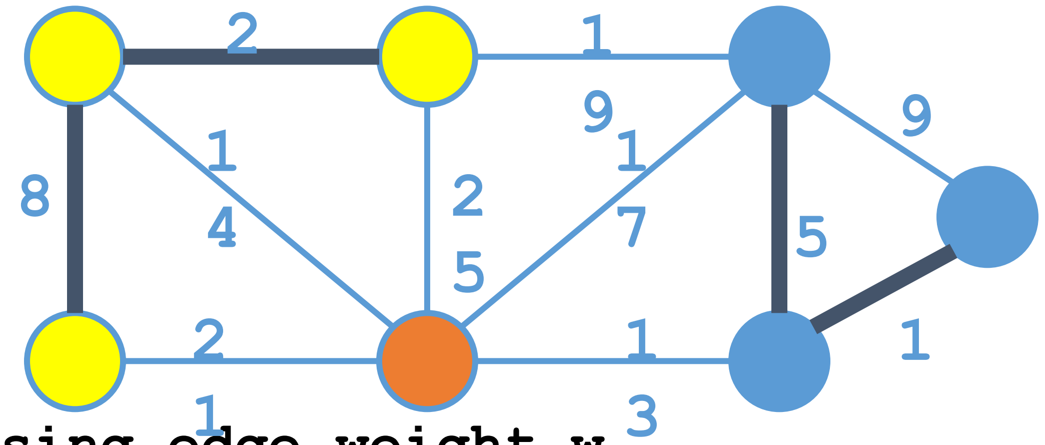

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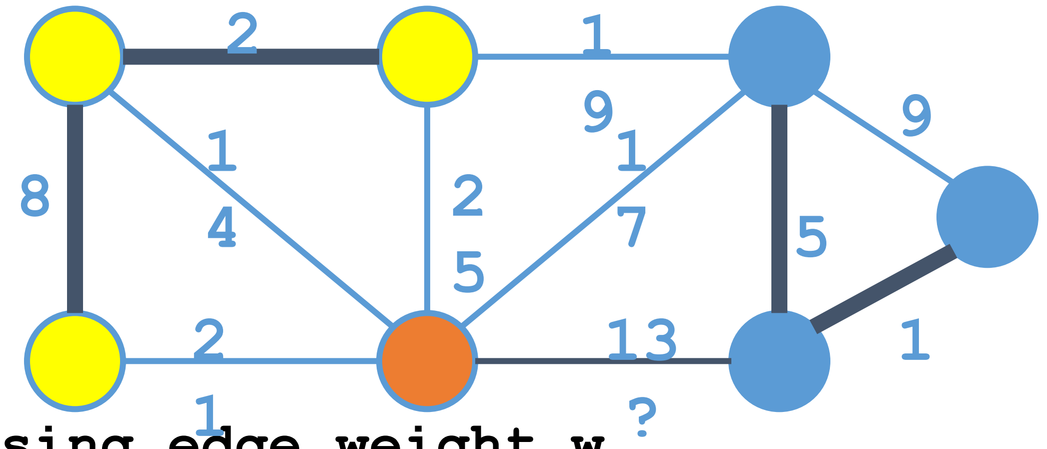

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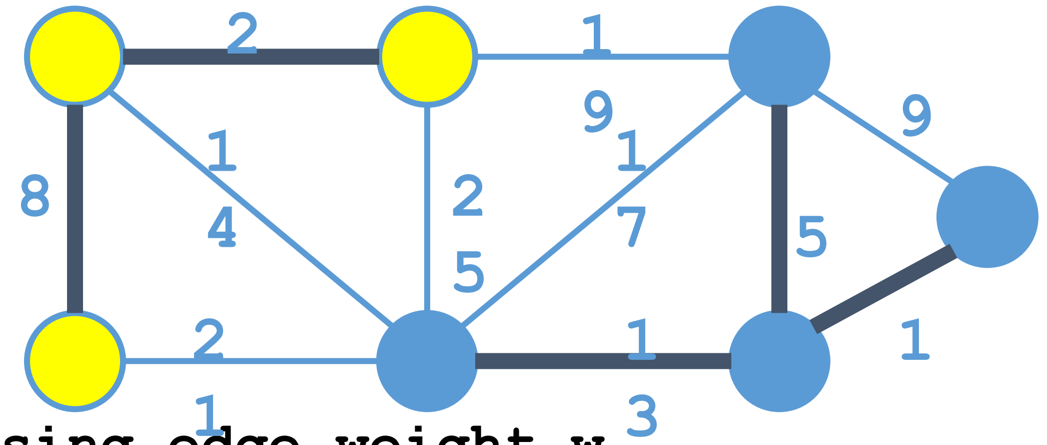

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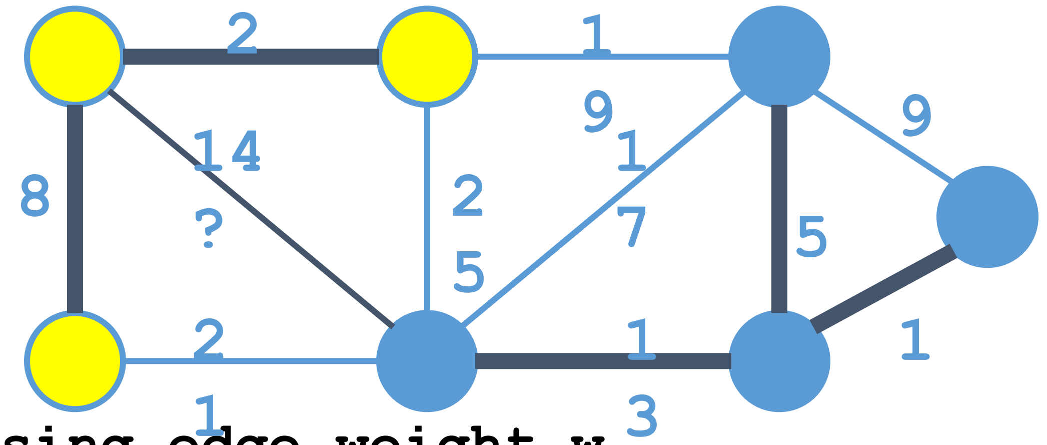

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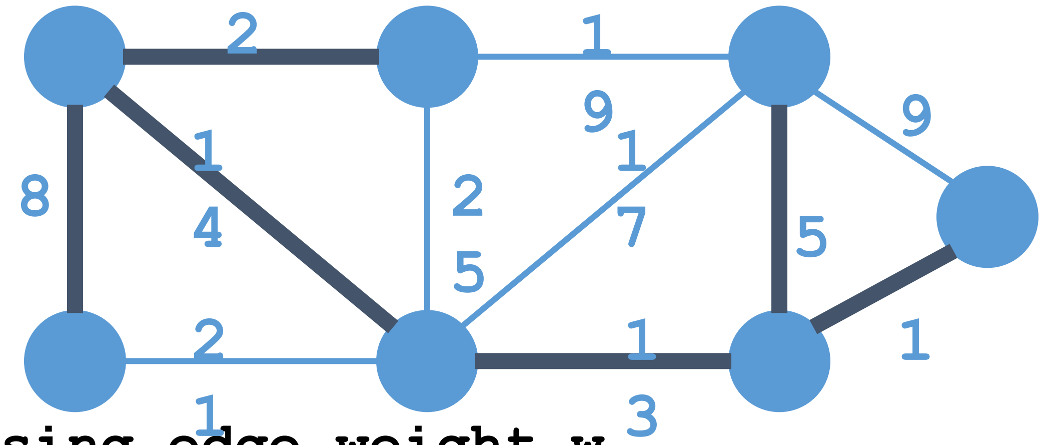

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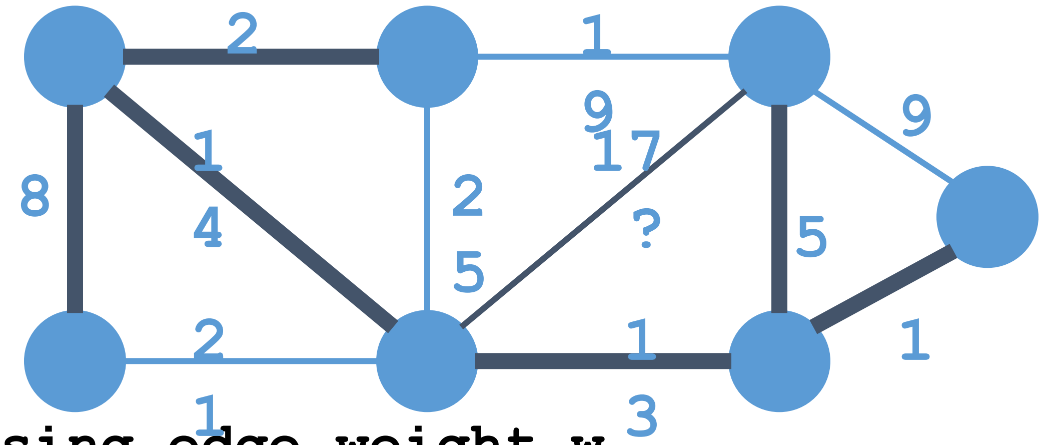

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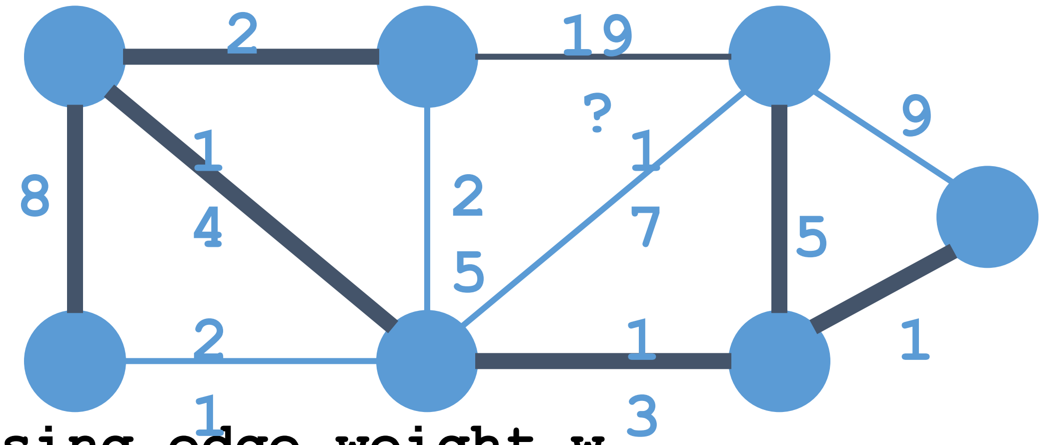

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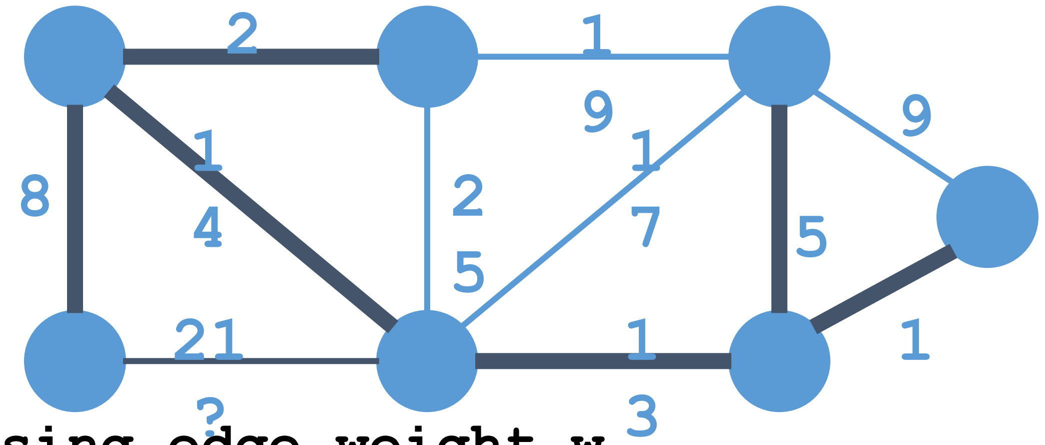

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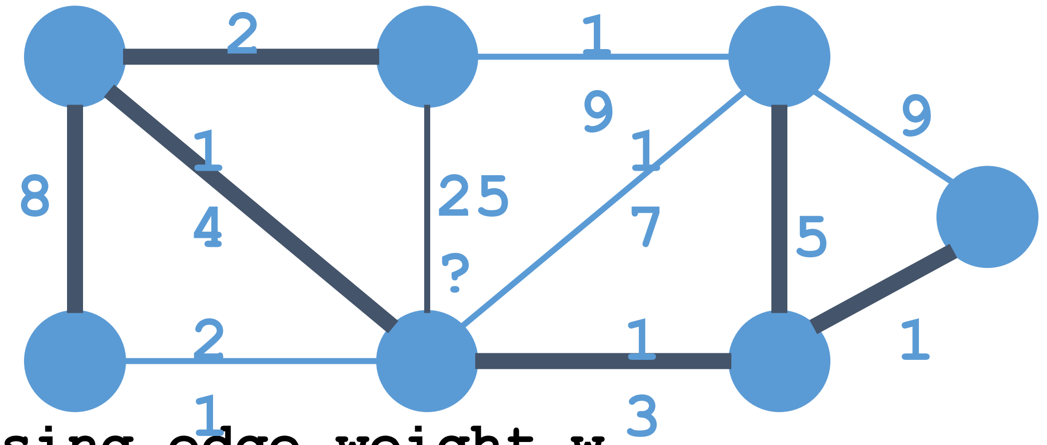

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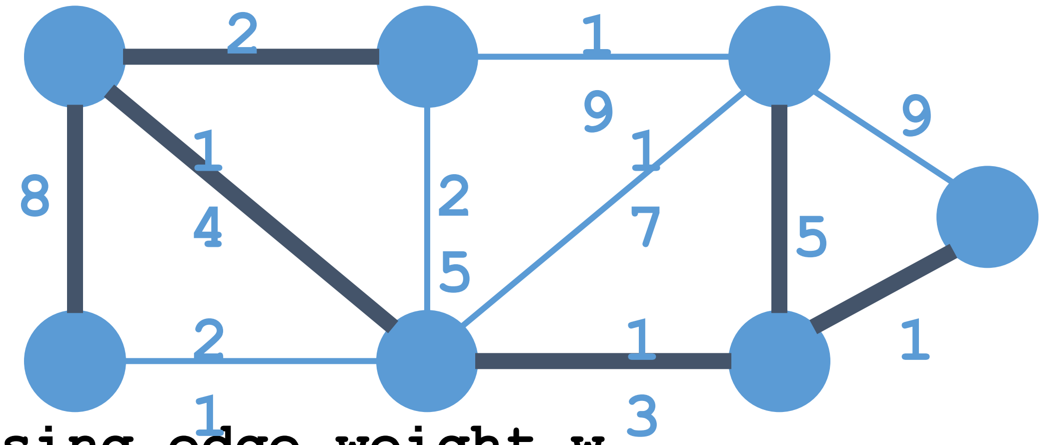

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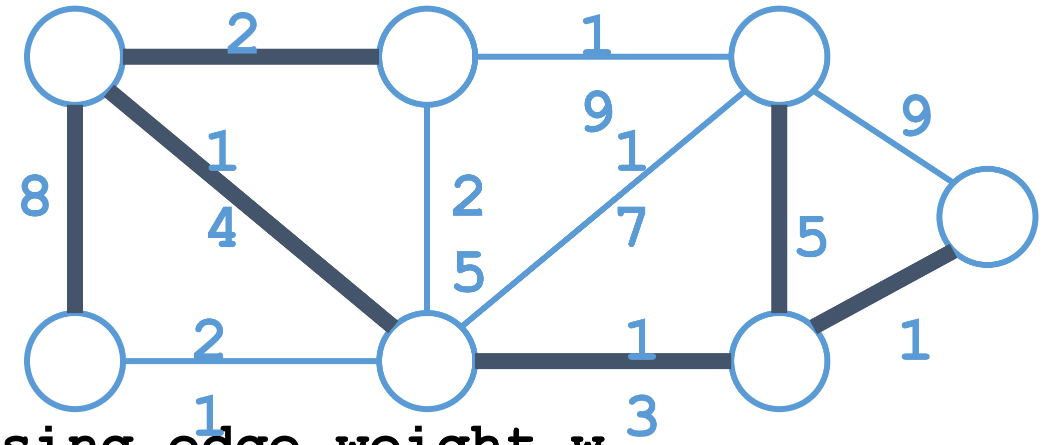

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{
    T = ∅;
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        MakeSet(v);
    sort E by increasing edge weight w
    for each (u,v) ∈ E (in sorted order)
        if FindSet(u) ≠ FindSet(v)
            T = T U {{u,v}};
            Union(FindSet(u), FindSet(v));
}
```

Kruskal’s Algorithm

```
Kruskal()
{
   T = $\emptyset$;
   for each v $\in$ V
      MakeSet(v);
   sort E by increasing edge weight w
   for each (u,v) $\in$ E (in sorted order)
      if FindSet(u) $\neq$ FindSet(v)
         T = T $\cup$ {{u,v}};
         Union(FindSet(u), FindSet(v));
}
```

Kruskal’s Algorithm

O(V) MakeSet() calls
O(E) FindSet() calls
O(V) Union() calls

```
Kruskal()
{
   T = ∅;
   for each v ∈ V
      MakeSet(v);
   sort E by increasing edge weight w
   for each (u,v) ∈ E (in sorted order)
      if FindSet(u) ≠ FindSet(v)
         T = T U {{u,v}};
         Union(FindSet(u), FindSet(v));
}
```

Kruskal’s Algorithm: Running Time

- To summarize:
  - Sort edges: O(E lg E)
  - O(V) MakeSet()’s
  - O(E) FindSet()’s
  - O(V) Union()’s

Analysis of Kruskal's Algorithm

Running Time = O(Elog V)   (E = edges, V = nodes)

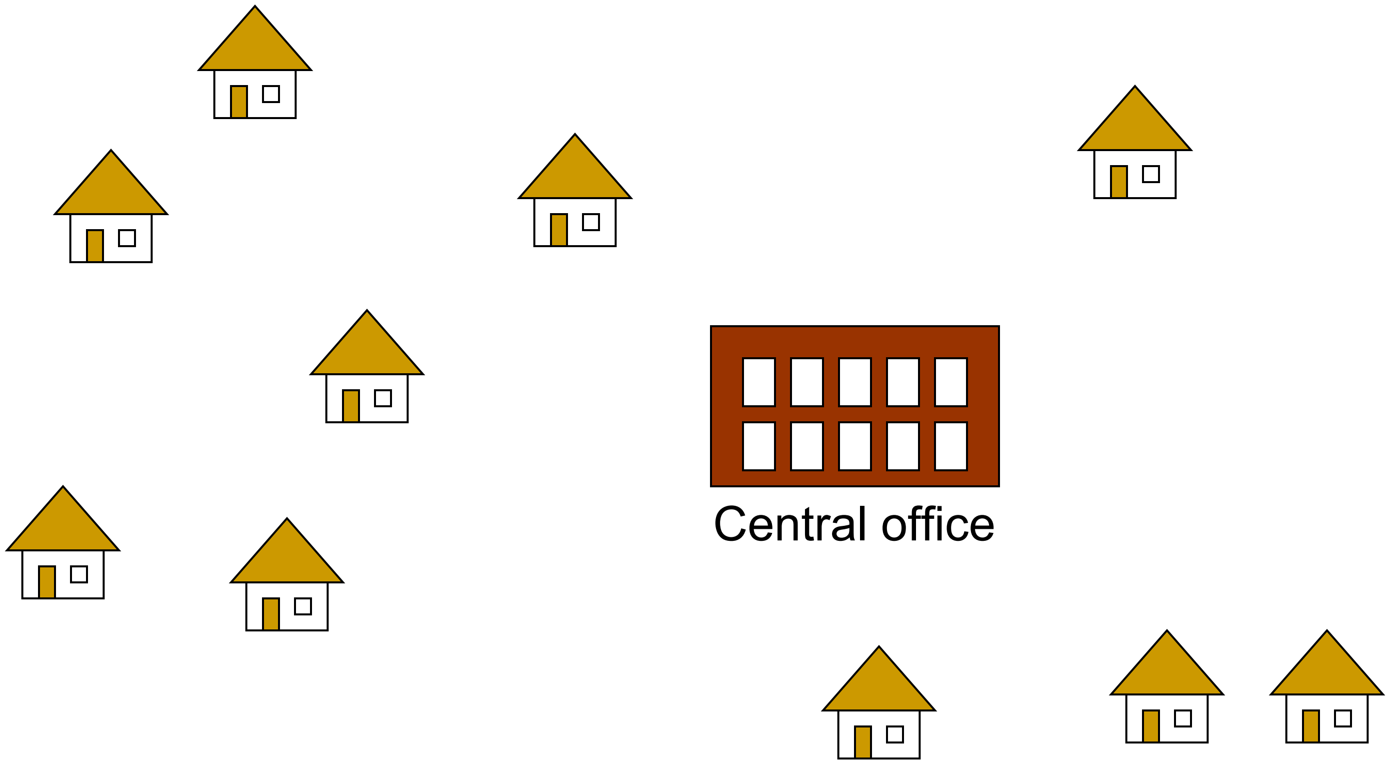
By implementing queue *Q* as a heap, *Q* could be initialized in $O(E)$ time and a vertex could be extracted in each iteration in $O(\log V)$ time

Testing if an edge creates a cycle can be slow unless a complicated data structure called a “union-find” structure is used.

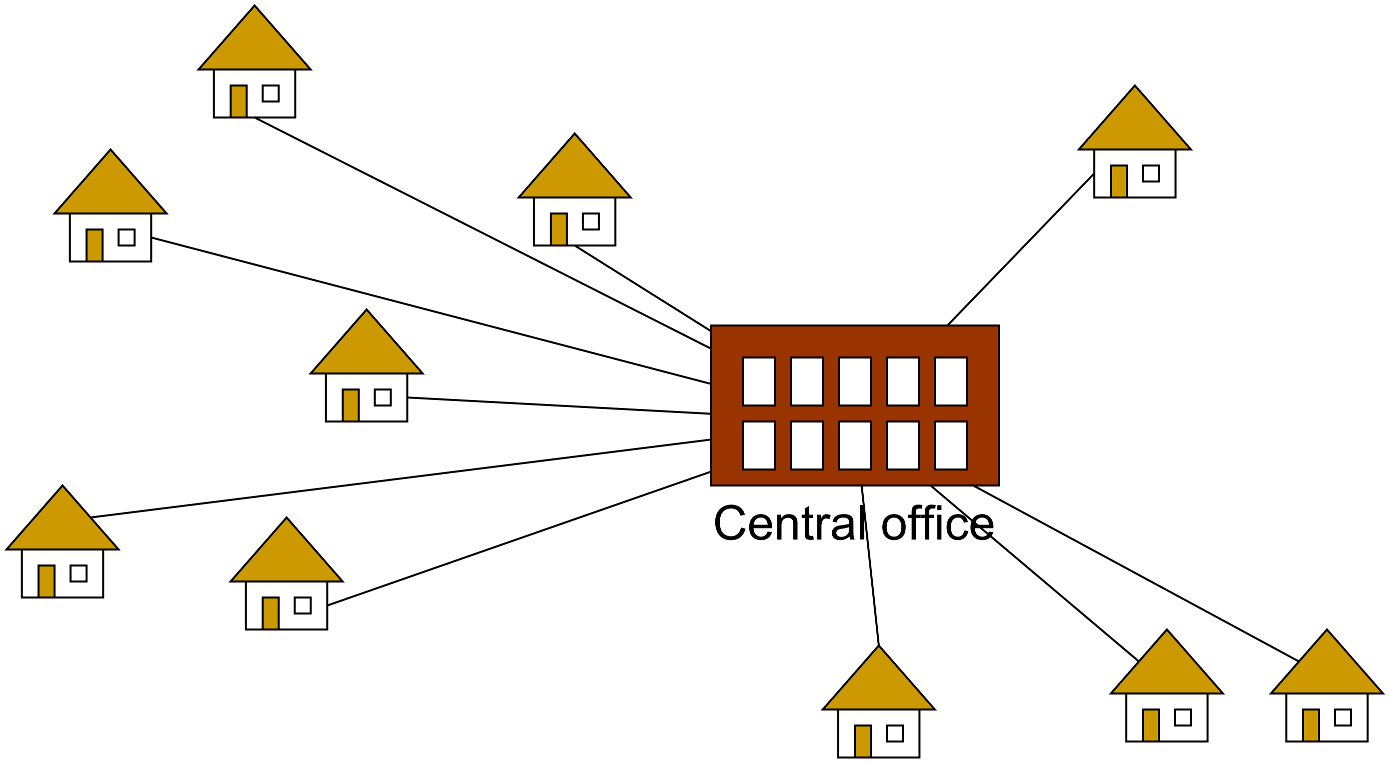
It usually only has to check a small fraction of the edges, but in some cases (like if there was a vertex connected to the graph by only one edge and it was the longest edge) it would have to check all the edges.

This algorithm works best, of course, if the number of edges is kept to a minimum.

Problem: Laying Telephone Wire

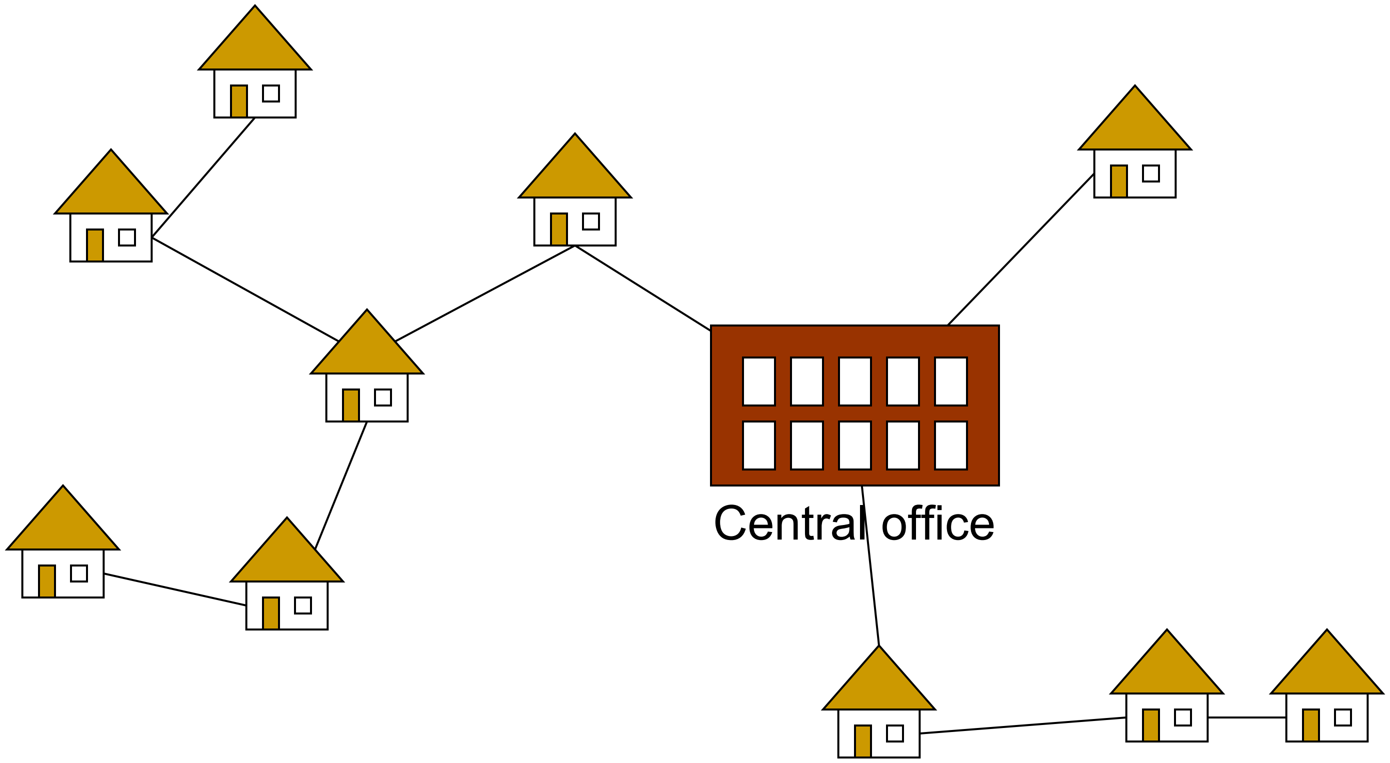


Wiring: Random Approach



Expensive!

Wiring: Better Approach



Minimize the total length of wire connecting the customers

Real Life Application of a MST

A cable TV company is laying cable in a new neighborhood. If it is constrained to bury the cable only along certain paths, then there would be a graph representing which points are connected by those paths. Some of those paths might be more expensive, because they are longer, or require the cable to be buried deeper; these paths would be represented by edges with larger weights. A *minimum spanning tree* would be the network with the lowest total cost.

Real Life Application of a MST

- One practical application of a MST would be in the design of a network. For instance, a group of individuals, who are separated by varying distances, wish to be connected together in a telephone network. Because the cost between two terminal
- is different, if we want to reduce our expenses, Prim's Algorithm is a way to solve it .
- Connect all computers in a computer science building using least amount of cable.
- A less obvious application is that the minimum spanning tree can be used to approximately solve the traveling salesman problem. A convenient formal way of defining this problem is to find the shortest path that visits each point at least once.
- Another useful application of MST would be finding airline routes. The vertices of the graph would represent cities, and the edges would represent routes between the cities.
- Obviously, the further one has to travel, the more it will cost, so MST can be applied to optimize airline routes by finding the least costly paths with no cycles.