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Combinatorial (1)

1.1 Permutations

1.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

1.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

1.1.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

1.2 Partitions and subsets

1.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

1.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$.

1.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$. 037a49, 6 lines

```
ll multinomial(VI& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  REP(i,1,SZ(v)) REP(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

1.3 General purpose numbers

1.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^\infty f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

1.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

1.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

1.3.4 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

1.3.5 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

1.3.6 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.
$S(n,k) = S(n-1,k-1) + kS(n-1,k)$
$S(n,1) = S(n,n) = 1$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

1.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.

- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

1.4 DP Optimizations

Quadrangle Inequality. f satisfies it if $\forall a \le b \le c \le d, f(a,d) - f(a,c) \ge f(b,d) - f(b,c)$.

1D-1D.h

Description: Applicable if $dp_i = min_{j>i}(dp_j + cost(i,j))$ s.t. $opt_i \le opt_j$ when $i \le j$ (which holds if quadrangle)
Time: $\mathcal{O}(n \log n)$

d76777, 27 lines

```
#define until first
#define opt second
ll dp[100000];
ll cost(int i, int j) {
  return dp[j] /* + cost to jump from i to j*/;
}
void solve(int n) {
  dp[n] = 0;
  vector<PII> v;
  v.EB(n - 1, n);
  for (int i = n - 1, ipos = 0; i >= 1; i--) {
    while (ipos+1 < SZ(v) && i <= v[ipos+1].until) ipos
        ++;
    dp[i] = cost(i, v[ipos].opt);
    while (v.back().until < i
      && cost(v.back().until, i)
         <= cost(v.back().until, v.back().opt))
      { v.pop_back(); }
    int l = 1, r = min(i - 1, v.back().until);
    while (l <= r) {
      int mid = (l + r)/2;
      if (cost(mid, i) <= cost(mid, v.back().opt)) {
        l = mid + 1;
      } else { r = mid - 1; }
    }
    if (l - 1 >= 1) v.EB(l - 1, i);
  }
}
```

Divide-and-Conquer.h

Description: Works when $dp_{k,i} = min_{j<i}(dp_{k-1,j} + cost(j,i))$ and $opt_k(i) \le opt_k(i+1)$. (This holds when quadrangle)
Usage: `find dp[1], then: for(i = 2 to n) solve(i, 1, n, 1, n)`
Time: $\mathcal{O}(kn \log n)$

1db0cf, 15 lines

```
ll dp[100][100]; // set correctly
ll cost(int i, int j); // cost to go from i to j, 1-
    indexed.
void solve(int i, int l, int r, int optl, int optr) {
  const ll inf = 1e18; // set correctly
  if (l > r || optl > optr) return;
  int mid = (l + r)/2; pair<ll, int> best = {inf, -1};
  for (int j = optl; j <= min(mid, optr); j++) {
    pair<ll,int> cand(dp[i - 1][j] + cost(j, mid), j);
    if (best.second == -1) best = cand;
    else best = min(best, cand);
  }
  dp[i][mid] = best.first;
  solve(i, l, mid - 1, optl, best.second);
  solve(i, mid + 1, r, best.second, optr);
}
```

Dynamic-CHT.h

Description: Add lines $y = ax + b$ and query for min at given x
Time: $\mathcal{O}(logn)$ per update/query.

f03adc, 55 lines

```
const dbl INF = 1e16;
struct HullDynamic {
#define CLREF const Line&
  struct Line {
    int a; ll b, val=0; dbl xLeft = -INF; bool type=0;
    Line(int a = 0, ll b = 0): a(a), b(b) {}
    ll eval(int x) const{ return a * 1ll * x + b; }
    bool operator< (CLREF l2) const {
      return l2.type ? (xLeft > l2.val) : (a < l2.a);
    }
  };
  using Iter = set<Line>::iterator;
  bool parallel(CLREF l1, CLREF l2) { return l1.a == l2
      .a; }
  dbl meetX(CLREF l1, CLREF l2) {
    return parallel(l1, l2) ? INF :
      (l2.b-l1.b) / (dbl(l1.a-l2.a));
  }
  set<Line> hull;
  bool hasPrev(Iter it) { return it != hull.begin(); }
  bool hasNext(Iter it) {
    return it != hull.end() && next(it) != hull.end();
  }
  bool bad(CLREF l1, CLREF l2, CLREF l3){
    return meetX(l1,l3) <= meetX(l1,l2);
  }
  bool bad(Iter it) {
    return hasPrev(it) && hasNext(it)
        && (bad(*next(it), *it, *prev(it)));
  }
  Iter upd_left_border(Iter it) {
    if(!hasNext(it)) return it;
    dbl val = meetX(*it, *next(it));
    Line buf(*it); it = hull.erase(it);
    buf.xLeft = val; return hull.insert(it, buf);
  }
  void insert_line(int a, ll b) {
    Line l3 = Line(a, b); auto it = hull.lower_bound(l3
        );
    if (it != hull.end() && parallel(*it , l3)) {
      if (it->b <= b) return;
      it = hull.erase(it);
    }
    it = hull.insert(it, l3);
    if (bad(it)) { hull.erase(it); return; }
    while (hasPrev(it) && bad(prev(it))) hull.erase(
        prev(it));
    while (hasNext(it) && bad(next(it))) hull.erase(
        next(it));
    it = upd_left_border(it);
    if (hasPrev(it)) upd_left_border(prev(it));
    if (hasNext(it)) upd_left_border(next(it));
  }
  ll eval(int x) {
    Line q; q.val = x; q.type = 1;
    auto best = hull.lower_bound(q);
    return (best == hull.end()) ? INF : best->eval(x);
  }
};
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \ge f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

Numerical (2)

2.1 Polynomials and recurrences

Polynomial.h

2ad8d0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = SZ(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    REP(i,1,SZ(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=SZ(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0
        +b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 7118fe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double
    xmax) {
  if (SZ(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(ALL(dr));
  REP(i,0,SZ(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      REP(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \ldots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

a4f803, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  REP(k,0,n-1) REP(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  REP(k,0,n) REP(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

2.2 Matrices

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

669167, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = SZ(a); ll ans = 1;
  REP(i,0,n) {
    REP(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) REP(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$

d6dca7, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = SZ(A), m = SZ(x), rank = 0, br, bc;
  if (n) assert(SZ(A[0]) == m);
  VI col(m); iota(ALL(col), 0);

  REP(i,0,n) {
    double v, bv = 0;
    REP(r,i,n) REP(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      REP(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    REP(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    REP(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      REP(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    REP(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 5cad07, 7 lines

```
REP(j,0,n) if (j != i) // instead of REP(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
REP(i,0,rank) {
  REP(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2 m\right)$

c9c00b, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, VI& b, bs& x, int m) {
  int n = SZ(A), rank = 0, br;
  assert(m <= SZ(x));
  VI col(m); iota(ALL(col), 0);
  REP(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      REP(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    REP(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    REP(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    REP(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

43464f, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = SZ(A); VI col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  REP(i,0,n) tmp[i][i] = 1, col[i] = i;

  REP(i,0,n) {
    int r = i, c = i;
    REP(j,i,n) REP(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    REP(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
          ]);
    swap(col[i], col[c]);
    double v = A[i][i];
    REP(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      REP(k,i+1,n) A[j][k] -= f*A[i][k];
      REP(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    REP(j,i+1,n) A[i][j] /= v;
    REP(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) REP(j,0,i) {
    double v = A[j][i];
    REP(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  REP(i,0,n) REP(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

a74eda, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>&
    super,
    const vector<T>& sub, vector<T> b) {
  int n = SZ(b); VI tr(n);
  REP(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[
        i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i
          ];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

2.3 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

a333b0, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = SZ(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if
      double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    REP(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i
        /2];
  }
  VI rev(n);
  REP(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  REP(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) REP(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-
          rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(SZ(a) + SZ(b) - 1);
  int L = 32 - __builtin_clz(SZ(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(ALL(a), begin(in));
  REP(i,0,SZ(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  REP(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  REP(i,0,SZ(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" 860295, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  vl res(SZ(a) + SZ(b) - 1);
  int B=32-__builtin_clz(SZ(res)), n=1<<B, cut=int(sqrt
      (M));
  vector<C> L(n), R(n), outs(n), outl(n);
  REP(i,0,SZ(a)) L[i] = C((int)a[i] / cut, (int)a[i] %
      cut);
  REP(i,0,SZ(b)) R[i] = C((int)b[i] / cut, (int)b[i] %
      cut);
  fft(L), fft(R);
  REP(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) /
        1i;
  }
  fft(outl), fft(outs);
  REP(i,0,SZ(res)) {
```

```
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])
        +.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5)
        ;
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
}
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" 14d0bb, 33 lines

```
const ll mod = (119 << 23) + 1, root = 62; // =
    998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26,
    479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = SZ(a), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    REP(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  VI rev(n);
  REP(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  REP(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) REP(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i
          + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = SZ(a) + SZ(b) - 1, B = 32 - __builtin_clz(s),
      n = 1 << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  REP(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod
      * inv % mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$

b1dc68, 16 lines

```
void FST(VI& a, bool inv) {
  for (int n = SZ(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) REP(j,i,i+
        step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? PII(v - u, u) : PII(v, u + v); // AND
        inv ? PII(v, u - v) : PII(u + v, u); // OR
        PII(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= SZ(a); // XOR only
}
VI conv(VI a, VI b) {
  FST(a, 0); FST(b, 0);
  REP(i,0,SZ(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

WalshHadamard.h

Description: $C_k = \sum_{i \otimes j = k} A_i B_j$

Usage: Apply the transform, point multiply and invert

Time: $\mathcal{O}(N \log N)$

905e71, 11 lines

```
void WalshHadamard(poly &P, bool invert) {
  for (int len = 1; 2 * len <= SZ(P); len <<= 1) {
    for (int i = 0; i < SZ(P); i += 2 * len) {
      REP(j, 0, len) {
        auto u = P[i + j], v = P[i + len + j];
        P[i + j] = u + v, P[i + len + j] = u - v; //
            XOR
      }
    }
  }
  if (invert) for (auto &x : P) x /= SZ(P);
}
```

OnlineFFT.h

Description: Given $B_1, \ldots B_m$, compute $A_i = \sum_{j=1}^{i-1} A_j * B_{i-j}$

Usage: 1-indexed, pad B[i] = 0 for i > m

Time: $\mathcal{O}\left(N \log^2 N\right)$

211338, 18 lines

```
void online(const Poly &B, CD a1, int n, Poly &A) {
  const int m = SZ(B) - 1;
  A.assign(n + 1, 0); A[1] = a1;
  auto bst = B.begin(), ast = A.begin();
  REP(i, 1, n) {
    A[i + 1] += A[i] * B[1];
    if (i + 2 <= n) A[i + 2] += A[i] * B[2];
    for (int pw = 2; i % pw == 0 && pw + 1 <= m; pw <<=
        1) {
      Poly blockA(ast + i - pw, ast + i);
      Poly blockB(bst + pw + 1, bst + min(pw * 2, m) +
          1);
      Poly prod = conv(blockA, blockB);
      REP(j, 0, SZ(prod)) {
        if (i + 1 + j <= n)
          A[i + 1 + j] += prod[j];
      }
    }
  }
}
```

Number theory (3)

3.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

84f78c, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
REP(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] %
    mod;
```

ModPow.h

fe17f7, 13 lines

```
int power(long long x, unsigned int y, int p){
  int res = 1;
  x = x % p;
  if (x == 0)
    return 0;
  while (y > 0){
    if (y & 1)
      res = (res * x) % p;
    y = y >> 1;
    x = (x * x) % p;
  }
  return res;
}
```

NCR.h

Description: Calculates ncr for large N and prime Mod

8f060e, 12 lines

```
#define MAX_N_FACT (int)3e5 + 5
vector<long long> factorial(MAX_N_FACT, 1),
    inverse_factorial(MAX_N_FACT, 1), inv(MAX_N_FACT,
    1);
void prec_factorials() {
    for (int i = 2; i < MAX_N_FACT; i++) {
        factorial[i] = factorial[i - 1] * i % mod;
        inv[i] = (mod - (mod / i) * inv[mod % i] % mod)
            ;
        inverse_factorial[i] = (inv[i] *
            inverse_factorial[i - 1]) % mod;
    }
}
long long ncr(int n, int k) {
```

```
    return factorial[n] * inverse_factorial[k] % mod *
        inverse_factorial[n - k] % mod;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.
Time: $\mathcal{O}\left(\sqrt{m}\right)$

0ff368, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    REP(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
`modsum(to, c, k, m)` = $\sum_{i=0}^{\text{to}-1} (ki + c)\%m$. `divsum` is similar but for floored division.
Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m,
      k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k,
      m);
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}\left(\log p\right)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no
      solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8
      == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

3.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 $\approx$ 1.5s

bdcb93, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
VI eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  VI pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)
      *1.1));
  vector<PII> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] =
        1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L]
          = 1;
    REP(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504,
      1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" 65f857, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd
        = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), ALL(r));
  return l;
}
```

3.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

3.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$ then $\phi(n) = (p_1-1)p_1^{k_1-1}...(p_r-1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1-1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}$ $\forall a$.

e4742a, 7 lines

```
const int LIM = 5000000;
int phi[LIM];
void calculatePhi() {
  REP(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] /
        i;
}
```

3.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1
    e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y
      = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q :
        inf),
      a = (ll)floor(y), b = min(a, lim),
      NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that
          gives us a
      // better approximation; if b = a/2, we *may*
          have one.
      // Return {P, Q} here for a more canonical
          approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (
          d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search
      (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)
          ) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

3.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ b = k \cdot (2mn), \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

3.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful.
For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit).
There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

3.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

3.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Mobius.h

Description: Dirichlet - $H(n) = \sum_{xy=n} a_x b_y, 1 \leq n \leq N$

306cf1, 21 lines

```
VI mobius(int N) {
  VI mu(N + 1, 1);
  vector<bool> ispr(N + 1, 1);
  for (int i = 2; i <= N; ++i) {
    if (!ispr[i]) continue;
```

```
    for (int j = i; j <= N; j += i) {
      ispr[j] = 0;
      mu[j] *= -1;
    }
    if (i * 1ll * i > N) continue;
    for (int j = i * i, ii = i * i; j <= N; j += ii)
      mu[j] = 0;
  }
  return mu;
}
VI DirichletConvolution(const VI &a, const VI &b, int N
    ) {
  VI h(N + 1, 0);
  REP(i, 1, N + 1)
    for (int j = i; j <= N; j += i) h[j] += a[i] * b[j
        / i];
  return h;
}
```

Data structures (4)

4.1 Set and Map like

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2
      into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(
      x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{
    1<<16});
```

Trie.h

Description: krishna
Time: $\mathcal{O}(\log N)$

1cba01, 26 lines

```
const int NX = int(1e6) + int(5e5);
int arr[NX][26];
int root;
int lastocc;
void Trie() {
    root = 0, lastocc = 0;
    memset(arr, 0, sizeof(int) * NX * 26);
}
void insert(const string &x) {
    int curptr = root;
    for (auto ch : x) {
        if (arr[curptr][ch - 'a'] == 0)
            arr[curptr][ch - 'a'] = ++lastocc;
        curptr = arr[curptr][ch - 'a'];
    }
}
int search(const string &x) {
    int curptr = root;
    for (auto ch : x) {
        if (arr[curptr][ch - 'a'] == 0)
            return 0;
        else
            curptr = arr[curptr][ch - 'a'];
    }
    return 1;
}
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

9556fc, 55 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rand()) {}
  void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  if (cnt(n->l) >= k) { // "n->val >= k" for
      lower_bound(k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and
        just "k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
  }
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;
  if (!r) return l;
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index
    k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k;
      }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
```

```
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}(\log(N))$

f79afc, 18 lines

```
struct RollbackUF {
  VI e; vector<PII> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return SZ(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);}
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;}};
```

4.2 Matrix

Matrix.h

Description: Basic operations on square matrices.
Usage: `Matrix<int, 3> A;`
`A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

d8e33f, 26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    REP(i,0,N) REP(j,0,N)
      REP(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    REP(i,0,N) REP(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    REP(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

4.3 Range DS

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

710629, 19 lines

```
template <typename T>
struct segTree {
  T unit;
  T (*f) (T obj1, T obj2);
  vector<T> s;
  int n;
  segTree(int n, T (*c)(T obj1, T obj2), T def) : s(2 *
       n, def), n(n), f(c), unit(def) {}
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  }
  T query(int b, int e) {  // query [b, e]
    e++;
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);}};
```

LazySegmentTree.h

Description: Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.
Usage: `Node* tr = new Node(v, 0, SZ(v));`
Time: $\mathcal{O}(\log N)$.

faa5ec, 58 lines

```
struct node {
    int sum, width; // width = # of leaves for node
    node operator+(const node &n) {
        return {max(sum, n.sum), width + n.width};}};
struct update {
    int value; bool type;// 0 = add, 1 = reset
    node operator()(const node &n) {
        if (type) return {n.width * value, n.width};
        else return {n.sum + value, n.width};}
    update operator+(const update &u) {
        if (u.type) return u;
        return {type, value + u.value};}};
template <typename T, typename U>
struct seg_tree_lazy {
    int S, H; T zero;
    vector<T> value; U noop;
    vector<bool> dirty; vector<U> prop;
    seg_tree_lazy<T, U>(int _S, T _zero = T(), U _noop
        = U()) {
        zero = _zero, noop = _noop;
        for (S = 1, H = 1; S < _S;) S *= 2, H++;
        value.resize(2 * S, zero), dirty.resize(2 * S,
            false);
        prop.resize(2 * S, noop);}
    void set_leaves(vector<T> &leaves) {
        copy(leaves.begin(), leaves.end(), value.begin
            () + S);
        for (int i = S - 1; i > 0; i--)
            value[i] = value[2 * i] + value[2 * i + 1];
                }
    void apply(int i, U &update) {
        value[i] = update(value[i]);
        if (i < S) {
            prop[i] = prop[i] + update;
            dirty[i] = true;}}
    void rebuild(int i) {
        for (int l = i / 2; l; l /= 2) {
            T combined = value[2 * l] + value[2 * l +
                1];
            value[l] = prop[l](combined);}}
    void propagate(int i) {
        for (int h = H; h > 0; h--) {
            int l = i >> h;
            if (dirty[l]) {
                apply(2 * l, prop[l]);
                apply(2 * l + 1, prop[l]);
                prop[l] = noop;
                dirty[l] = false;}}}
    void upd(int i, int j, U update) {
        i += S, j += S;
        propagate(i), propagate(j);
        for (int l = i, r = j; l <= r; l /= 2, r /= 2)
            {
            if ((l & 1) == 1) apply(l++, update);
            if ((r & 1) == 0) apply(r--, update);}
        rebuild(i), rebuild(j);}
    T query(int i, int j) {
```

```
        i += S, j += S;
        propagate(i), propagate(j);
        T res_left = zero, res_right = zero;
        for (; i <= j; i /= 2, j /= 2) {
            if ((i & 1) == 1) res_left = res_left +
                value[i++];
            if ((j & 1) == 0) res_right = value[j--] +
                res_right;}
        return res_left + res_right;}};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

b06af0, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < SZ(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos
      ] >= sum
    // Returns n if no sum is >= sum, or -1 if empty
        sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= SZ(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" d5c1b7, 22 lines

```
struct FT2 {
  vector<VI> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < SZ(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    for (VI& v : ys) sort(ALL(v)), ft.emplace_back(SZ(v
        ));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(ALL(ys[x]), y) - ys[x].
        begin()); }
  void update(int x, int y, ll dif) {
    for (; x < SZ(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: RMQ rmq(values);
rmq.query(inclusive, exclusive);
Time: $\mathcal{O}(|V| \log |V| + Q)$

9a1bbf, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= SZ(V); pw *= 2,
        ++k) {
      jmp.emplace_back(SZ(V) - pw * 2 + 1);
      REP(j,0,SZ(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j +
            pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).
Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

3015c5, 43 lines

```
void add(int ind, int end) { ... } // add a[ind] (end =
    0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

VI mo(vector<PII> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  VI s(SZ(Q)), res = s;
#define K(x) PII(x.first/blk, x.second ^ -(x.first/blk
    & 1))
  iota(ALL(s), 0);
  sort(ALL(s), [&](int s, int t){ return K(Q[s]) < K(Q[
      t]); });
  for (int qi : s) {
    PII q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();}
  return res;}
VI moTree(vector<array<int, 2>> Q, vector<VI>& ed, int
    root=0){
  int N = SZ(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  VI s(SZ(Q)), res = s, I(N), L(N), R(N), in(N), par(N)
      ;
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) ->
      void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;};
  dfs(root, -1, 0, dfs);
#define K(x) PII(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] /
    blk & 1))
  iota(ALL(s), 0);
  sort(ALL(s), [&](int s, int t){ return K(Q[s]) < K(Q[
      t]); });
  for (int qi : s) REP(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0;
    } \
                  else { add(c, end); in[c] = 1; } a =
                      c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();}
  return res;}
```

MoWithUpdates.h

Description: Supports point updates at position
Time: $\mathcal{O}\left(n^{5/3}\right)$ when block = $n^{2/3}$

303d07, 14 lines

```
struct Query { int l, r, id, t; }
struct Update { int pos, pre, now; };
void MoWithUpdates(vector<Query> qs, vector<Update> upd
    ) {
  int BLK; // set block size
  sort(qs.begin(), qs.end(), [&](Query a, Query b) {
    return {a.l/BLK, a.r/BLK, a.t} < {b.l/BLK, b.r/BLK,
        b.t};});
  for (auto q : qs) {
    while (t < q.t) ++t, apply(upd[t].pos, upd[t].now);
    while (t > q.t) apply(upd[t].pos, upd[t].pre), --t;
```

```
    while (l > q.l) add(--l);
    while (l < q.l) remove(l++);
    while (r < q.r) add(++r);
    while (r > q.r) remove(r--);
    ans[q.id] = get();}}
```

Strings (5)

5.1 String Matching

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

578c4b, 16 lines

```
VI pi(const string& s) {
  VI p(SZ(s));
  REP(i,1,SZ(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

VI match(const string& s, const string& pat) {
  VI p = pi(pat + '\0' + s), res;
  REP(i,SZ(p)-SZ(s),SZ(p))
    if (p[i] == SZ(pat)) res.push_back(i - 2 * SZ(pat))
        ;
  return res;
}
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

a706e2, 12 lines

```
VI Z(string S) {
  VI z(SZ(S));
  int l = -1, r = -1;
  REP(i,1,SZ(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < SZ(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

f7553c, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1,
        nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  VI backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = SZ(N); N.emplace_back(-1);
          }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    REP(i,0,SZ(pat)) insert(pat[i], i);
    N[0].back = SZ(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      REP(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].
              start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  VI find(string word) {
    int n = 0;
    VI res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<VI> findAll(vector<string>& pat, string word)
      {
    VI r = find(word);
    vector<VI> res(SZ(word));
    REP(i,0,SZ(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - SZ(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

Hashing.h

Description: Self-explanatory methods for string hashing.

cc1766, 30 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and
    more
// code, but works on evil test data (e.g. Thue-Morse,
    where
// ABBA... and BAAB... of length 2^10 hash the same mod
    2^64).
// "typedef ull H;" instead if you think test data is
    random,
// or work mod 10^9+7 if the Birthday paradox is not a
    problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
  (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) : B);
      return r; }
  OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
  H operator-(H o) { return *this + ~o.x; }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get();
      }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random
    also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(SZ(str)+1), pw(ha) {
    pw[0] = 1;
    REP(i,0,SZ(str))
```

```
    ha[i+1] = ha[i] * C + str[i],
    pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b) and 0
      indexed
    return ha[b] - ha[a] * pw[b - a];
  }
};
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

43e521, 8 lines

```
int minRotation(string s) {
  int a=0, N=SZ(s); s += s;
  REP(b,0,N) REP(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
        break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

5.2 Palindromes

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

1deebd, 13 lines

```
array<VI, 2> manacher(const string& s) {
  int n = SZ(s);
  array<VI,2> p = {VI(n+1), VI(n)};
  REP(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

5.3 Suffix DS

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

3e14f8, 23 lines

```
struct SuffixArray {
  VI sa, lcp;
  SuffixArray(string& s, int lim=256) { // or
      basic_string<int>
    int n = SZ(s) + 1, k = 0, a, b;
    VI x(ALL(s)+1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = y, iota(ALL(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2),
        lim = p) {
      p = j, iota(ALL(y), n - j);
      REP(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(ALL(ws), 0);
      REP(i,0,n) ws[x[i]]++;
      REP(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      REP(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1
            : p++;
    }
    REP(i,1,n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
  }
};
```

SuffixAutomaton.h

Description: Each path in the automaton is a substring (if it ends in a terminal node, it is a suffix) And no. of occurences = no. of ways to reach a terminal node. Or keep reverse edges of suffix links(all prefixes for that substring), then no. of ways to reach a root.
Time: $\mathcal{O}(len)$ map accesses, map can be at most of size alphabet, can also use unordered_map

37fe84, 34 lines

```
struct SuffixAutomaton {
  vector<map<char, int>> edges;
  VI link, length; // length[i]: longest string in i-th
      class
  int last;        // index of equivalence class of
      whole string
  SuffixAutomaton(string s) : edges{}, link{-1}, length
      {0}, last(0) {
    edges.emplace_back();
    REP(i, 0, SZ(s)) {
      edges.emplace_back();
      length.push_back(i + 1);
      link.push_back(0);
      int r = SZ(edges) - 1, p = last;
      while (p >= 0 && edges[p].find(s[i]) == edges[p].
          end()) {
        edges[p][s[i]] = r, p = link[p];
      }
      if (p != -1) {
        const int q = edges[p][s[i]];
        if (length[p] + 1 == length[q]) link[r] = q;
        else {
          edges.push_back(edges[q]);
          length.push_back(length[p] + 1);
          link.push_back(link[q]);
          const int qq = SZ(edges) - 1;
          link[q] = link[r] = qq;
          for (; p >= 0 && edges[p][s[i]] == q; p =
              link[p])
            edges[p][s[i]] = qq;
        }
      }
      last = r;
    }
    VI terminals;
    for (int p = last; p > 0; p = link[p])
      terminals.push_back(p);
  }
};
```

Graph (6)

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

11ca03, 17 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }
    };
struct Node { ll dist = inf; int prev = -1; };
void bellmanFord(vector<Node>& nodes, vector<Ed>& eds,
    int s) {
  nodes[s].dist = 0;
  sort(ALL(eds), [](Ed a, Ed b) { return a.s() < b.s();
      });
  int lim = SZ(nodes) / 2 + 2; // /3+100 with shuffled
      vertices
  REP(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);}}
  REP(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;}}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

0ff4bf, 10 lines

```
const ll inf = 1LL << 62;
void floydWarshALL(vector<vector<ll>>& m) {
  int n = SZ(m);
```

```
  REP(i,0,n) m[i][i] = min(m[i][i], 0LL);
  REP(k,0,n) REP(i,0,n) REP(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);}
  REP(k,0,n) if (m[k][k] < 0) REP(i,0,n) REP(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -
        inf;}
```

6.1 Network flow

Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

abfd54, 42 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you
        need flows
  };
  VI lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, SZ(adj[b]), c, c});
    adj[b].push_back({a, SZ(adj[a]) - 1, rcap, rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < SZ(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  }
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    REP(L,0,31) do { // 'int L=30' maybe faster for
        random data
      lvl = ptr = VI(SZ(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

MCMF-SPFA.h

Description: Multiedges and negative costs allowed.
Time: Approximately $\mathcal{O}\left(V^2E^2\right)$

19b593, 62 lines

```
template <typename FLOW, typename COST> struct MCMF {
  const COST INFC = 1e9, EPSC = 0;
  const FLOW INFF = 1e9, EPSF = 0;
  struct Edge {
    int from, to;
    FLOW flow, cap;
    COST cost;
  };
  int nodes, src, dest, m = 0;
  vector<vector<int>> adj;
  vector<Edge> edges;
  void add(int u, int v, FLOW cap, COST cost) {
    edges.EB(u, v, 0, cap, cost);
    adj[u].PB(m++);
    edges.EB(v, u, 0, 0, -cost);
    adj[v].PB(m++);
  }
  vector<COST> dis;
  vector<bool> inQ;
  VI par;
  pair<FLOW, COST> SPFA() {
    fill(ALL(dis), INFC);
    fill(ALL(inQ), false);
    queue<int> Q;
    Q.push(src), dis[src] = 0, inQ[src] = true;
    while (!Q.empty()) {
      int u = Q.front(); Q.pop();
      inQ[u] = false;
      for (int i : adj[u]) {
        auto &e = edges[i];
        if (e.cap - e.flow > EPSF
            && dis[e.to] - (dis[u] + e.cost) > EPSC) {
          dis[e.to] = dis[u] + e.cost;
          par[e.to] = i;
          if (!inQ[e.to]) { Q.push(e.to), inQ[e.to] =
              true; }
        }
      }
    }
    if (dis[dest] + EPSC >= INFC) return {0, 0};
    FLOW aug = INFF;
    for (int u = dest; u != src; u = edges[par[u]].from
        ) {
      aug = min(aug, edges[par[u]].cap - edges[par[u]].
          flow);
    }
    for (int u = dest; u != src; u = edges[par[u]].from
        ) {
      edges[par[u]].flow += aug;
      edges[par[u] ^ 1].flow -= aug;
    }
    return {aug, aug * dis[dest]};
  }
  MCMF(int n, int s, int t)
  : nodes(n), src(s), dest(t), adj(n), dis(n), inQ(n),
      par(n) {}
  pair<FLOW, COST> mincostmaxflow() {
    pair<FLOW, COST> ans(0, 0);
    while (true) {
      auto cur = SPFA();
      if (cur.first <= EPSF) break;
      ans.first += cur.first;
      ans.second += cur.second;
    }
    return ans;
  }
};
```

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}\left(E^2\right)$

92261c, 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<VI> ed, red;
  vector<VL> cap, flow, cost;
  VI seen;
  VL dist, pi;
  vector<PII> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost
        (cap),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(ALL(seen), 0);
    fill(ALL(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});
```

```
    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i
            ], i});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1)
            ;
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    REP(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x
          = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[
            x][p]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x
          = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    REP(i,0,N) REP(j,0,N) totcost += cost[i][j] * flow[
        i][j];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this before
      maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(ALL(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      REP(i,0,N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

1d69cc, 21 lines

```
pair<int, VI> globalMinCut(vector<VI> mat) {
  pair<int, VI> best = {INT_MAX, {}};
  int n = SZ(mat);
  vector<VI> co(n);
  REP(i,0,n) co[i] = {i};
  REP(ph,1,n) {
    VI w = mat[0];
    size_t s = 0, t = 0;
    REP(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
        queue
      w[t] = INT_MIN;
      s = t, t = max_element(ALL(w)) - w.begin();
      REP(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), ALL(co[t]));
    REP(i,0,n) mat[s][i] += mat[t][i];
    REP(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

6.2 Matching

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" 57dfe3, 20 lines

```
VI cover(vector<VI>& g, int n, int m) {
  VI match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false
      ;
  VI q, cover;
  REP(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1)
        {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  REP(i,0,n) if (!lfound[i]) cover.push_back(i);
  REP(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(SZ(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.
Time: $\mathcal{O}\left(N^2M\right)$

8d4fc6, 31 lines

```
pair<int, VI> hungarian(const vector<VI> &a) {
  if (a.empty()) return {0, {}};
  int n = SZ(a) + 1, m = SZ(a[0]) + 1;
  VI u(n), v(m), p(m), ans(n - 1);
  REP(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    VI dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      REP(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      REP(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  REP(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

6.3 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(graph, [&](VI& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.
Time: $\mathcal{O}\left(E+V\right)$

c6a3ff, 20 lines

```
VI val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  for (auto e : g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));
```

```
  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;}
  return val[j] = low;}
template<class G, class F> void scc(G& g, F f) {
  int n = SZ(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  REP(i,0,n) if (comp[i] < 0) dfs(i, g, f);}
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const VI& edgelist) {...});`
Time: $\mathcal{O}(E+V)$

69fe6f, 33 lines

```
VI num, st;
vector<vector<PII>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = SZ(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(VI(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(SZ(ed), 0);
  REP(i,0,SZ(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||!b)\&\&(!a||c)\&\&(d||!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ($\sim$x).
Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

def624, 56 lines

```
struct TwoSat {
  int N;
  vector<VI> gr;
  VI values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }

  void atMostOne(const VI& li) { // (optional)
    if (SZ(li) <= 1) return;
    int cur = ~li[0];
    REP(i,2,SZ(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  VI val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    REP(i,0,2*N) if (!comp[i]) dfs(i);
    REP(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

8e52d2, 15 lines

```
VI eulerWalk(vector<vector<PII>>& gr, int nedges, int
    src=0) {
  int n = SZ(gr);
  VI D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = SZ(gr[x
        ]);
    if (it == end){ ret.push_back(x); s.pop_back();
        continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || SZ(ret) != nedges+1)
      return {};
  return {ret.rbegin(), ret.rend()};
}
```

6.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

8618ee, 31 lines

```
VI edgeColoring(int N, vector<PII> eds) {
  VI cc(N + 1), ret(SZ(eds)), fan(N), free(N), loc;
  for (PII e : eds) ++cc[e.first], ++cc[e.second];
```

```
  int u, v, ncols = *max_element(ALL(cc)) + 1;
  vector<VI> adj(N, VI(ncols, -1));
  for (PII e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i =
        0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) !=
        -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at
        ][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  REP(i,0,SZ(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++
        ret[i];
  return ret;
}
```

6.5 Trees

BinaryLifting.h

Description: Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

c4e44c, 19 lines

```
vector<VI> treeJump(VI& P){
  int on = 1, d = 1;
  while(on < SZ(P)) on *= 2, d++;
  vector<VI> jmp(d, P);
  REP(i,1,d) REP(j,0,SZ(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;}
int jmp(vector<VI>& tbl, int nod, int steps){
  REP(i,0,SZ(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;}
int lca(vector<VI>& tbl, VI& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = SZ(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;}
  return tbl[0][a];}
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" cbd116, 16 lines

```
struct LCA {
  int T = 0;
  VI time, path, ret;
  RMQ<int> rmq;
  LCA(vector<VI>& C) : time(SZ(C)), rmq((dfs(C,0,-1),
      ret)) {}
  void dfs(vector<VI>& C, int v, int par) {
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);}}
  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];}
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(
      a,b)];}
};
```

CompressTree.h

Description: Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

0292cd, 13 lines

```
bool cmp(int u,int v){return arr[u]<arr[v];}
int create_tree(){//return root of tree
set<int> S;//get distinct nodes
REP(i,k)S.insert(Q[i]);k=0;for(auto it : S)Q[k++]=it;
sort(Q,Q+k,cmp);int kk = k;//distinct initial nodes
//add lca of adjacent pairs
for(int i=0;i<kk-1;i++){int x = lca(Q[i],Q[i+1]);
if(S.count(x))continue;Q[k++]=x;S.insert(x);
}sort(Q,Q+k,cmp);stack<int> s;s.push(Q[0]);
for(int i=1;i<k;i++){
while(!anc(s.top(),Q[i]))s.pop();
tree[s.top()].PB(Q[i]);tree[Q[i]].PB(s.top());
s.push(Q[i]);}return Q[0];}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.
Time: $\mathcal{O}\left((\log N)^2\right)$

c4840c, 21 lines

```
VI sz, sc, hd, en, ex, par, dep;
seg_tree_lazy<node, update> st(1, {0, 0}, {0, 0}); //
    for alter accordingly
int timer = -1;
void hld(int u, int p, int ch, int d) {
    hd[u] = ch; en[u] = ++timer; par[u] = p; dep[u] = d
        ;
    if (sc[u] != -1) hld(sc[u], u, ch, d + 1);
    for (auto e : g[u]) {
        int v = U[e] ^ V[e] ^ u;
        if (v == p || v == sc[u]) continue;
        hld(v, u, v, d + 1);}
    ex[u] = timer;}
int path(int x, int y) {
    int ma = (int)-1e9;
    while (hd[x] != hd[y]) {
        if (dep[hd[x]] < dep[hd[y]]) swap(x, y);
        ma = max(st.query(en[hd[x]], en[x]).sum, ma);
            // for hd[x] -> x
        x = par[hd[x]];
    }
    if (dep[x] < dep[y]) swap(x, y);
    ma = max(ma, st.query(en[y], en[x]).sum); // for y
        -> x
    return ma;}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

5909e2, 72 lines

```
struct Node { // Splay tree. Root's pp contains tree's
    parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted
        )
    }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;}
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ?
        y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;}
    y->c[i ^ 1] = b ? this : x;
```

```
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);}
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);}}
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);}};
struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}
  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];}
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();}}
  bool connected(int u, int v) { // are u, v in the
      same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();}
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();}}
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;}
    return u;}};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 719f12, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r);
    }

pair<ll, VI> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new
      Node{e});
  ll res = 0;
  VI seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  REP(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    REP(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (
      optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  REP(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

CentriodDecomposition.h

Description: Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

f8ca72, 14 lines

```
VI U, V, W, isDel;
int dp[n][log2(n) + 1];
// (avoid deleted edges) in all 3 DFS
void decompose(int root, int p) {
    dfs_sz(root, -1); // calc sizes of subtrees
    int c = get_centroid(root, -1, sz[root]);  // if sz
        [v] * 2 > sz[root] return get_centroid(v) else
        return u
    if (p == -1) p = root;
    // Add edge btwn p and c here
    dfs(c);  // to compute functions
    for (auto e : g[root]) {
        if (isDel[e]) continue;
        isDel[e] = 1;
        int v = U[e] ^ V[e] ^ u;
        decompose(v, root);}}
```

6.6 Math

Number of Spanning Trees Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

Erdős–Gallai theorem A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Mirsky's Theorem Max length chain is equal to min partitioning into antichains. Max chain is height of poset.

Dilworth's Theorem Min partition into chains is equal to max length antichain. From poset create bipartite graph. Any edge from v_i - v_j implies LV_i - RV_j. Let A be the set of vertices such that neither LV_i nor RV_i are in vertex cover. A is an antichain of size n-max matching. To get min partition into chains, take a vertex from left side, keep taking vertices till a matching exist. Consider this as a chain. Its size is n - max matching.

Geometry (7)

7.1 Geometric primitives

Point.h

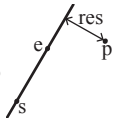
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

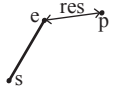


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



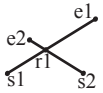
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



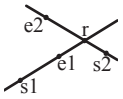
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (SZ(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 36c2d7, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {ALL(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

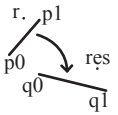
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; REP(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

0f0602, 35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y
      , t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x
      >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare
      distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
     between
// them, i.e., the angle that covers the defined line
     segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector
     b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle
     a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b
       < a)};
}
```

7.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P
    >* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 -
             p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2
      ) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2,
    double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr *
      dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" eea4d5, 10 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  if (a == b) return {};
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2()
      ;
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2()
      ;
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

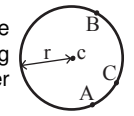
"../../content/geometry/Point.h" f5c096, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.
        dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt
        (det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2
        ;
  };
  auto sum = 0.0;
  REP(i,0,SZ(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % SZ(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c
      )/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" 69dd52, 13 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(ALL(ps), mt19937(time(0)));
```

```
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  REP(i,0,SZ(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    REP(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      REP(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
}}}return {o, r};}
```

7.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2261c4, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = SZ(p);
  REP(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !
        strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q
        ) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" e287fe, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  REP(i,0,SZ(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

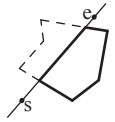
Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

"Point.h" 7d84e0, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = SZ(v) - 1; i < SZ(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h



Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f50354, 18 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  if (SZ(poly) <= 2) return {};
  vector<P> res;
  REP(i,0,SZ(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back()
        ;
    if (zero(s.cross(e, cur))) {
      res.push_back(cur);
      continue;
    }
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" c5c490, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (SZ(pts) <= 1) return pts;
  sort(ALL(pts));
  vector<P> h(SZ(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(ALL(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <=
          0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] ==
      h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

"Point.h" 261063, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = SZ(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  REP(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j
          ]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i
          ]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" efb6da, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true
    ) {
  int a = 1, b = SZ(l) - 1, r = !strict;
  if (SZ(l) < 3) return r && onSegment(l[0], l.back(),
      p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b],
      p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h" 331463, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly
    [(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n)
    < 0
template <class P> int extrVertex(vector<P>& poly, P
    dir) {
  int n = SZ(poly), lo = 0, hi = n;
```

```
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi :
        lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  REP(i,0,2) {
    int lo = endB, hi = endA, n = SZ(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + SZ(poly) + 1) % SZ(poly)
        ) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

7.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac393c, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(SZ(v) > 1);
  set<P> S;
  sort(ALL(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(
        p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

7.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(
      p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z
      (z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z
      ); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z
      ); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.
        x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval
      [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval
      [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z);
      }
  P unit() const { return *this/(T)dist(); } //makes
      dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around
      axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.
        unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 0754b0, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(SZ(A) >= 4);
  vector<vector<PR>> E(SZ(A), vector<PR>(SZ(A), {-1, -1
      }));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  REP(i,0,4) REP(j,i+1,4) REP(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  REP(i,4,SZ(A)) {
    REP(j,0,SZ(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = SZ(FS);
    REP(j,0,nw) {
```

```
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b,
    i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b)
        ;
  return FS;
};
```

Mathematics (8)

8.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

8.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

8.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \\ \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2} \end{aligned}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

8.4 Geometry

8.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

8.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

8.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2}(ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x) g(x) dx = [F(x) g(x)]_a^b - \int_a^b F(x) g'(x) dx$$

8.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

8.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

8.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

8.8.1 Discrete distributions

Binomial distribution

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

8.8.2 Continuous distributions

Uniform distribution

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Miscellaneous (9)

9.1 RNG, Intervals, T.S

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \ge \cdots \ge f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

a9cf52, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  REP(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

354a6a, 19 lines

```
template<class T>
VI cover(pair<T, T> G, vector<pair<T, T>> I) {
  VI S(SZ(I)), R;
  iota(ALL(S), 0);
  sort(ALL(S), [&](int a, int b) { return I[a] < I[b];
      });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < SZ(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, SZ(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T
    q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

564cdd, 23 lines

```
set<PII>::iterator addInterval(set<PII>& is, int L, int
    R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<PII>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

RNGs.h

5 lines

```
SEED = chrono::steady_clock::now().time_since_epoch().
    count(); // or use 'high_resolution_clock'
random_device rd; auto SEED = rd();
mt19937 rng(SEED);
uniform_int_distribution<> dis(MIN, MAX); // usage: dis
    (rng)
// others: uniform_real_distribution,
```

DebuggingTricks.cpp

Description: Debug
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

26e792, 4 lines

```
1. signal(SIGSEGV, [](int) { _Exit(0); });
converts segfaults into Wrong Answers. Similarly one
    can catch SIGABRT (assertion failures) and SIGFPE (
    zero divisions). _GLIBCXX_DEBUG failures generate.
    SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
2. feenableexcept(29);
kills the program on NaNs(1), 0-divs (4), infinities
    (8) and denormals (16).
```

9.2 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

9.2.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `REP(b,0,K) REP(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

9.2.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).
- `#pragma GCC optimize("unroll-loops")`
- `target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx, avx")`

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod b$ in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

FastInput.h

Description: Usage requires your program to pipe in input from file.
Usage: `./a.out < input.txt`
Time: About 5x as fast as cin/scanf.

7b3c70, 16 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}
int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

Contest (10)

template.cpp

37 lines

```
// #pragma GCC optimize("O3,unroll-loops")
// #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <bits/stdc++.h>
using namespace std;
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <class T>
using o_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// order_of_key (val): no. of values less than val
// find_by_order (k): kth largest element.(0-based)
#define int long long
#define FOR(i, a, b) for (int i = (a); i < (b); ++i)
#define REP(i, a, b) for (int i = (a); i < (b); ++i)
#define ALL(x) begin(x), end(x)
#define SZ(x) ((int)(x).size())
#define SET(a, v) memset((a), (v), sizeof(a))
```

```cpp
#define PB push_back
#define EB emplace_back
#define MP make_pair
#define F first
#define S second
using LL = long long;
using dbl = double;
using II = pair<int, int>;
using VI = vector<int>;
using VII = vector<II>;
using VVI = vector<VI>;
#define endl "\n"
const long long mod = 1e9 + 7;
signed main() {
  // freopen("sample.in", "r", stdin);
  // freopen("sample.out", "w", stdout);
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit); // RTE if out of bound
  return 0;
}
```