

# Special Assignment

## Epidemic Modelling

Question 2(with all of its parts) is answered in this document.  
No Prolog implementation is done for this question.

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### Notes:

- Calculation and derivations are not shown, only the important equations and calculated data is presented to keep it concise.
- This pdf contains total 7 pages.

**a).** Choosing a model :

Perspective chosen – **Human Perspective (HP)**.

The modelling of the epidemic would yield similar results regardless of the perspective chosen if the scope of the model actually captured reality with all uncertain variables.

Hence, we chose a perspective through which we can have more insights and data.

Arguments in support :

- Human world is better known to us than pathogen world. Hence the number of uncertain variables will be less.
- Mechanism of the pathogen is unknown(as in any burgeoning epidemic) to us. Hence prediction with the perspective of the virus becomes difficult.
- The medium of spread of the pathogen is Humans. Expansion of the pathogen solely depends upon Human mobility and contact duration(according to the question). Hence choosing **HP** would be more beneficial to fit in the factors into the model.
- Of both, the **PP** model the **HP** model, the latter would be easily applicable to other Human related areas as the variable mapping would not change much(however the **PP** would also be applicable but the mapping would be tough).

**b). A mathematical model:**

We try to formulate the task into an equation which accounts for different variables.

Since each zone has different values for different factors, we create an equation with these factors and apply to each zone separately.

The spread of the virus can be seen as a reaction between two reactants, A and B (Humans and Pathogens). The rate of reaction will give the rate of cumulative population infection at time  $t$ .

So  $C(t)$  is the number of cumulative infectious people and  $C'(t) = dC(t)/dt$  gives the rate of growth of infection.

The rate  $C'(t)$  depends on the number of infectious people present at time  $t$  i.e.,  $C(t)$  and the growth factor  $r$  :

$$C'(t) = dC(t)/dt = r * C(t)^k \quad (1)$$

Where the growth parameter is  $k \in [0,1]$ .  $k = 0$  means the rate is linear and equals  $r$ , and  $k = 1$  means the rate is exponential. Considering the given task we chose  $k$  to be 0.95 as the infection is epidemic and  $k$  will be uniform for all zones (as given in the task).

The growth factor  $r$  depends upon the average inter-person (radial) space  $R$ , average duration of time between when two person contact  $\langle t_c \rangle$  and the probability  $P$  of transmission of infection, all according to the given zone.

$$r = \langle t_c \rangle * P / R \quad (2)$$

Upon solving eq.(1) we get,

$$C(t) = \left( (1/m) * r * t + A \right)^m \quad (3)$$

where  $m = 1 / (1 - k)$ , and  $A = C(0)^{1/m}$  when  $t = 0$  was put in solved equation to get the integration constant.

$C(0)$  is the infectious population at start time, which is

$$C(0) = N / q \quad (4)$$

$N$  is the total population of the zone and  $q$  defines **no. of carriers per  $q$  person** (in the task  $q$  is represented by  $r$ , here  $r$  is used for growth rate).

**(i).**

Time required for **x%** population to be infected(in a particular zone):

We can get the time **t** (in days) from eq. (3).

Choosing variable values,

**R = 2.5 to 1.5 meters** (Zone MSZ(2.5m-2m) and MICZ(2m-1.5m) as the variable values will be nearly similar for these two zones). Average is **2 meters**.

**n**(total population = 20 million) **or N = 7%(MSZ) + 10%(MICZ)** of 20 million = **3.4 million**.

**<t<sub>c</sub>>**(average contact duration b/w two people) = 5 seconds.

**P**(the probability of transmission of infection) = **0.7**(given in task).

We get from eq.(2),

$$r = \langle t_c \rangle * P / R$$

$$r = 5 * 0.7 / 2$$

$$r = 1.75.$$

**k = 0.95** (epidemic hence near to exponential growth)

$$m = 1 / (1 - k) = 1 / (1 - 0.95) = 20.$$

From eq.(4)

$$A = C(0)^{1/m} = (\text{Initial infectious population})^{1/m} = N / q.$$

Let **q = 1 million** (i.e, 1 carrier in a million people),

$$A = (3.4 / 1)^{1/20} = (3.4 / 1)^{0.05} = 1.06.$$

Now, for **x = 25**, we have to find the time required for **25%** population to get infected.

**C(t) = 25%** of **3.4 million = 850k**.

Put all calculated values of **A, r, C(t)** and **m** in eq.(3),

$$t = [ C(t)^{1/m} - A ] * m / r$$

$$= [ (850,000)^{0.05} - 1.06 ] * 20 / 1.75$$

$$= \mathbf{10.50 \text{ days.}}$$

Hence, time required for zones MSZ and MICZ to get 25% of their total population to get infected under given circumstances is **10.5 days**.

**(ii).**

MOP conditions on answer of **(i)**.

**a).** MOP is S, i.e, everyone stays at home.

Effect on variable **r** ,

$\langle t_c \rangle$  average contact time increases, say 100 seconds,

**P** remains same as average time increased had the effect = 0.7,

**R** should decrease as everyone is at home say 1 meter.

$$r = 70.$$

Effect on growth parameter **k**,

Since groups do not interact with each other **k** decreases to 0.4.

$$m = 1.6$$

Hence,

$$t = [ C(t)^{1/m} - A ] * m / r$$

$$t = 116.09 \text{ days.}$$

Hence staying at home increases infection spread time.

**b).** MOP is D, i.e, normal movement, which is the one we calculated in **(i)** as **10.5 days**.

c). MOP is SN, i.e, only sick stay at home.

Effect on **r**,

only **P** will decrease as less infectious persons are contacting, say 0.5.

Also **k** will decrease, say 0.6.

So,  $r = 5 * 0.5 / 2 = 1.25.$

$$m = 1 / (1 - 0.6) = 2.4.$$

$$t = [ C(t)^{1/m} - A ] * m / r$$

$$t = [ (850,000)^{0.41} - 1.06 ] * 2.4 / 1.25$$

$$t = 516.00 \text{ days.}$$

Hence applying this MOP would be the best to longer the time of spread of virus.

**(iii).**

The most important change as we could see in **(ii)** was to control the mobility by having only sick/infectious people stay at home.

It reduces the carrier contact power(**k**), infection probability (**P**) which account for most of the effect in our equation.

However, the major and most sensitive contributor is **k**. Even a slight change in it will affect the outcome by great extent.