Solution to 12.13.4.3

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are the possible values of X? Also find the Probability distribution of X.

Solution:

It is given that the coin is tossed 6 times.

Let H be a random variable which denotes the number of heads,

$$H = \{0, 1, 2, 3, 4, 5, 6\} \tag{1}$$

Let T be a random variable which denotes the number of tails,

$$T = 6 - H \tag{2}$$
$$= \{6, 5, 4, 3, 2, 1, 0\} \tag{3}$$

Let X be a random variable which denotes the absolute value of the difference between the number of heads and number of tails,

$$X = |H - T| \tag{4}$$
$$= |H - (6 - H)| \tag{5}$$
$$= |2H - 6| \tag{6}$$
$$= \{0, 2, 4, 6\} \tag{7}$$

Therefore, X can take values from the set {0,2,4,6}. Now we will find the probability distribution of X,

$$p_H(k) = \begin{cases} \frac{^6C_k}{2^6}, & 0 \le k \le 6 \\ 0, & k > 6 \end{cases} \tag{8}$$

$$p_T(k) = 1 - p_H(k) \tag{9}$$

so,

$$p_X(k) = P(X = k) \tag{10}$$
$$= P(H - T = k) \tag{11}$$

for k = 0, we know,

$$H + T = 6 \tag{12}$$
$$H - T = k \tag{13}$$
$$\implies 2H = 6 + k \tag{14}$$
$$\implies H = 3 + \frac{k}{2} \tag{15}$$

So from (11),

$$p_X(k) = P(H - T = K) \tag{16}$$
$$= P(H = 3 + \frac{k}{2}) \tag{17}$$
$$= P(H = 3 + \frac{k}{2}) \tag{18}$$
$$= \frac{^6C_{3+\frac{k}{2}}}{2^6} \tag{19}$$

for k = 2,4,6, we know,

$$H + T = 6 \tag{20}$$
$$H - T = \pm k \tag{21}$$
$$\implies 2H = 6 \pm k \tag{22}$$
$$\implies H = 3 \pm \frac{k}{2} \tag{23}$$

So from (11),

$$p_X(k) = P(H - T = K) \tag{24}$$
$$= P(H = 3 \pm \frac{k}{2}) \tag{25}$$
$$= P(H = 3 + \frac{k}{2}) + P(H = 3 - \frac{k}{2}) \tag{26}$$
$$= \frac{^6C_{3+\frac{k}{2}}}{2^6} + \frac{^6C_{3-\frac{k}{2}}}{2^6} \tag{27}$$
$$= 2(\frac{^6C_{3+\frac{k}{2}}}{2^6}) \tag{28}$$

Therefore,

$$p_X(k) = \begin{cases} \frac{^6C_{3+\frac{k}{2}}}{2^6}, & k = 0 \\ 2(\frac{^6C_{3+\frac{k}{2}}}{2^6}), & k = 2, 4, 6 \end{cases} \tag{29}$$