

Assignment

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Question: Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

- 1) $\frac{1}{2}$
- 2) $\frac{1}{3}$
- 3) $\frac{2}{3}$
- 4) $\frac{4}{7}$

Solution:

Let X_0, X_1, X_2 be the random variables which denotes the three children, where X_0 is the eldest child and X_2 is the youngest child.

RV	Value	Description
X_i	0	child is boy
	1	child is girl

TABLE 4
RV DESCRIPTION TABLE

so the required probability is,

$$\Pr(X_0 = 1 | X_0 + X_1 + X_2 \geq 1) = \frac{\Pr(X_0 = 1, X_0 + X_1 + X_2 \geq 1)}{\Pr(X_0 + X_1 + X_2 \geq 1)} \quad (1)$$

$$= \frac{\Pr(X_0 = 1)}{\sum_{k=1}^3 {}^3C_k \times \frac{1}{2}^k \times \frac{1}{2}^{3-k}} \quad (2)$$

$$= \frac{\frac{1}{2}}{\frac{3}{8} + \frac{3}{8} + \frac{1}{8}} \quad (3)$$

$$= \frac{4}{7} \quad (4)$$

Therefore, the probability that the eldest child is a girl given that the family has atleast one girl is $\frac{4}{7}$