Solution to 12.13.4.3

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are the possible values of X? Also find the Probability distribution of X.

Solution:

It is given that the coin is tossed 6 times.

Let H be a random variable which denotes the number of heads,

$$H = \{0, 1, 2, 3, 4, 5, 6\} \tag{1}$$

Let T be a random variable which denotes the number of tails,

$$T = 6 - H \tag{2}$$
$$= \{6, 5, 4, 3, 2, 1, 0\} \tag{3}$$

Let X be a random variable which denotes the absolute value of the difference between the number of heads and number of tails,

$$X = |H - T| \tag{4}$$
$$= |H - (6 - H)| \tag{5}$$
$$= |2H - 6| \tag{6}$$
$$= \{0, 2, 4, 6\} \tag{7}$$

Therefore, X can take values from the set {0,2,4,6}.

Now we will find the probability distribution of X,

$$\Pr(H(k)) = \frac{{}^6C_k}{2^6} \tag{8}$$

$$\Pr(X(0)) = \Pr(H(3)) \tag{9}$$
$$\Pr(X(6)) = \Pr(H(6)) + \Pr(H(0)) \tag{10}$$
$$\Pr(X(4)) = \Pr(H(4)) + \Pr(H(2)) \tag{11}$$
$$\vdots \tag{12}$$

so for k = 0,

$$H + T = 6 \tag{13}$$
$$H - T = k \tag{14}$$
$$\implies 2H = 6 + k \tag{15}$$
$$\implies H = 3 + \frac{k}{2} \tag{16}$$

Now we can directly write,

$$\Pr(X(k)) = \begin{cases} \frac{{}^6C_{3+\frac{k}{2}}}{2^6}, & k = 0 \\ 2\frac{{}^6C_k}{2^6}, & k = 2, 4, 6 \end{cases} \tag{17}$$