

Solution to 12.13.4.3

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are the possible values of X ? Also find the Probability distribution of X .

Solution:

It is given that the coin is tossed 6 times.

Let H be a random variable which denotes the number of heads,

$$H = \{0, 1, 2, 3, 4, 5, 6\} \quad (1)$$

Let T be a random variable which denotes the number of tails,

$$T = 6 - H \quad (2)$$

$$= \{6, 5, 4, 3, 2, 1, 0\} \quad (3)$$

Let X be a random variable which denotes the difference between the number of heads and number of tails,

$$X = H - T \quad (4)$$

$$= H - (6 - H) \quad (5)$$

$$= 2H - 6 \quad (6)$$

$$= \{-6, -4, -2, 0, 2, 4, 6\} \quad (7)$$

Therefore, X can take values from the set $\{-6, -4, -2, 0, 2, 4, 6\}$.

Now we will find the probability distribution of X ,

$$p_H(k) = \begin{cases} \frac{{}^6C_k}{2^6}, & 0 \leq k \leq 6 \\ 0, & k > 6 \end{cases} \quad (8)$$

$$p_T(k) = 1 - p_H(k) \quad (9)$$

so,

$$p_X(k) = P(X = k) \quad (10)$$

$$= P(H - T = k) \quad (11)$$

But we already know,

$$H + T = 6 \quad (12)$$

$$H - T = k \quad (13)$$

$$\implies 2H = 6 + k \quad (14)$$

$$\implies H = 3 + \frac{k}{2} \quad (15)$$

So from (11),

$$p_X(k) = P(H - T = K) \quad (16)$$

$$= P(H = 3 + \frac{k}{2}) \quad (17)$$

$$= \frac{{}^6C_{3+\frac{k}{2}}}{2^6} \quad (\text{from(8)}) \quad (18)$$