Assignment

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Question: A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

(a) atleast once
(b) exactly once
(c) atleast twice?

Solution: Let us define:

Parameter	Value	Description
n	50	number of lotteries
p	0.01	probability of winning a prize
q	0.99	probability of not winning
$\mu = np$	0.5	mean of the distribution
$\sigma^2 = npq$	0.495	variance of the distribution
Y	0,1,2,3,...,50	Number of successes

(a) **using Gaussian**
the Gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (x \in Y) \tag{1}$$

the Q-function from the gaussian distribution:

$$Q(x) = \int_x^\infty p_Y(t)\, dt \tag{2}$$

$$= \int_x^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}\, dt \quad from(1) \tag{3}$$

The CDF of Y is given by:

$$F_Y(x) = \int_{-\infty}^x P_Y(t)\, dt \tag{4}$$

$$= 1 - \int_x^\infty P_Y(t)\, dt \tag{5}$$

$$= 1 - Q(x) \tag{6}$$

The probabbility of winning the prize atleast once is given by:

$$\Pr(Y \geq 1) = 1 - F_Y(0) \tag{7}$$

$$= Q(0) \quad from(6) \tag{8}$$

$$= 0.5 \tag{9}$$

(b) **using Gaussian**
the probability of the person winning the prize exactly once is given by:

$$p_Y(1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(1-\mu)^2}{2\sigma^2}} \quad from(1) \tag{10}$$

$$= 0.44 \tag{11}$$

(c) **using Gaussian**
the probability of the person winning the prize atleast twice is given by:

$$\Pr(Y \geq 2) = 1 - F_Y(1) \tag{12}$$
$$= Q(1) \quad from(6) \tag{13}$$
$$= 0.158 \tag{14}$$

Gaussian vs Binomial Table

Y	Gaussian	Binomial
atleast one	0.5	0.395
exactly one	0.441	0.305
atleast two	0.158	0.089

Gaussian vs Binomial graph