Assignment

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Question: A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

(a) atleast once
(b) exactly once
(c) atleast twice?

Solution: Let us define:

Parameter	Value	Description
n	50	number of lotteries
p	0.01	probability of winning a prize
q	0.99	probability of not winning
$\mu = np$	0.5	mean of the distribution
$\sigma^2 = npq$	0.495	variance of the distribution
Y	0,1,2,3,...,50	Number of successes

(a) **using Gaussian**

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

The CDF of Y:

$$F_Y(y) = 1 - \Pr\left(Y > y\right) \tag{2}$$

$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{y-\mu}{\sigma}\right) \tag{3}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}\left(0, 1\right) \tag{4}$$

$$\implies F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{5}$$

The probabbility of winning the prize atleast once is given by:
Considering 0.5 as the correction term,

$$\Pr\left(Y > 0.5\right) = 1 - F_Y\left(0.5\right) \tag{6}$$

$$= Q\left(\frac{0.5-\mu}{\sigma}\right) \quad from(5) \tag{7}$$

$$= Q\left(0\right) \tag{8}$$

$$= 0.5 \tag{9}$$

(b) **using Gaussian**
the gaussian distribution function is defined as:

$$p_Y\left(x\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{10}$$

the probability of the person winning the prize exactly once is given by:

$$p_Y(1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(1-\mu)^2}{2\sigma^2}} \tag{11}$$

$$= 0.44 \tag{12}$$

(c) **using Gaussian**
the probability of the person winning the prize atleast twice is given by:
considering 0.5 as the correction term,

$$\Pr(Y > 1.5) = 1 - F_Y(1.5) \tag{13}$$

$$= Q\left(\frac{1.5-\mu}{\sigma}\right) \quad from(5) \tag{14}$$

$$= Q\left(\frac{1}{\sqrt{0.495}}\right) \tag{15}$$

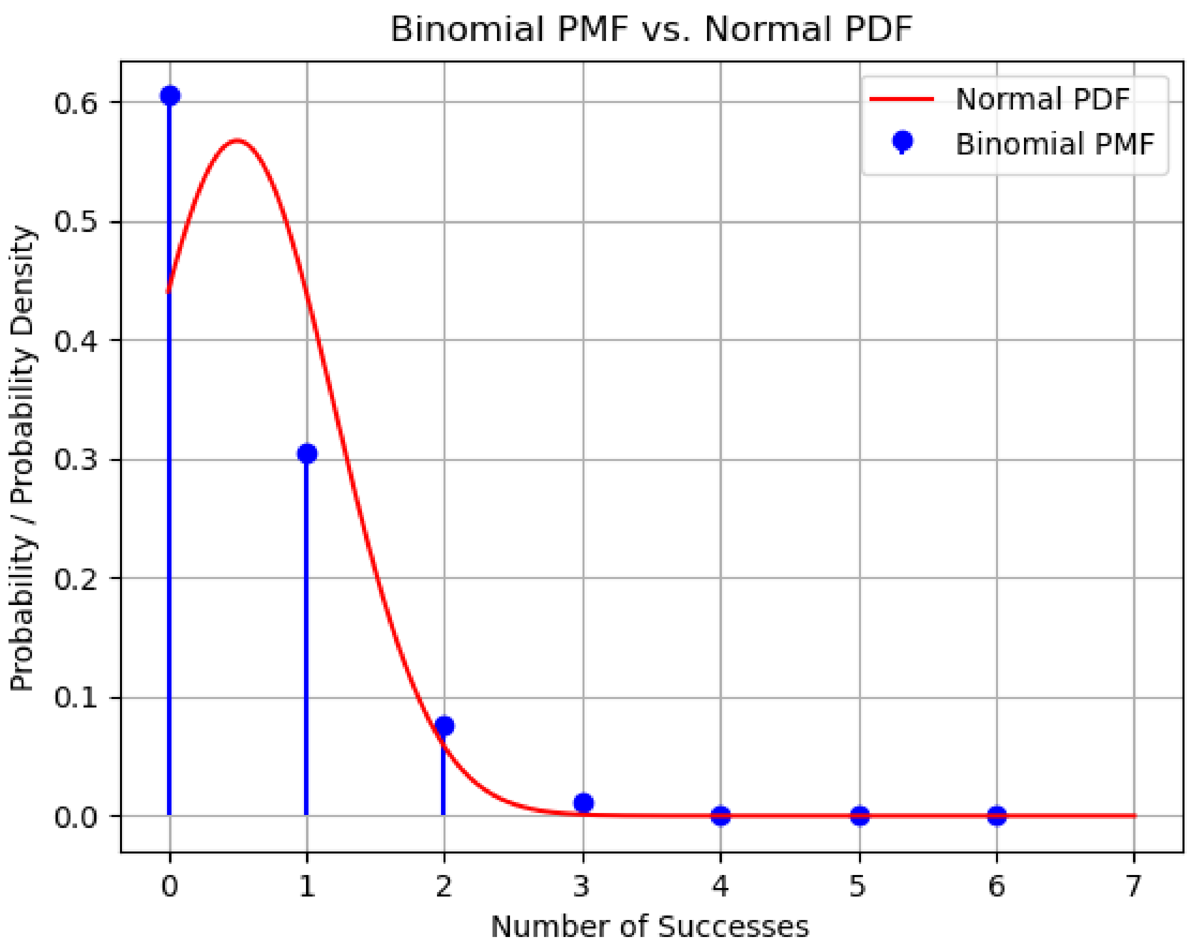
$$= Q(1.42) \tag{16}$$

$$= 0.0776 \tag{17}$$

Gaussian vs Binomial Table

Y	Gaussian	Binomial
atleast one	0.5	0.395
exactly one	0.441	0.305
atleast two	0.0776	0.089

Gaussian vs Binomial graph


Binomial PMF vs. Normal PDF
Normal PDF
Binomial PMF
Probability / Probability Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7
Number of Successes