Solution to 12.13.4.3

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Question: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are the possible values of X? Also find the Probability distribution of X.

Solution:

It is given that the coin is tossed 6 times.

Let H be a random variable which denotes the number of heads,

$$H = \{0, 1, 2, 3, 4, 5, 6\} \tag{1}$$

Let T be a random variable which denotes the number of tails,

$$T = 6 - H \tag{2}$$
$$= \{6, 5, 4, 3, 2, 1, 0\} \tag{3}$$

Let X be a random variable which denotes the absolute value of the difference between the number of heads and number of tails,

$$X = |H - T| \tag{4}$$
$$= |H - (6 - H)| \tag{5}$$
$$= |2H - 6| \tag{6}$$

Therefore, X can take values from the set {0,2,4,6}.

Now we will find the probability distribution of X using the CDF approach,

The CDF of H is given by:

$$F_H(k) = \sum_{i=0}^{k} \frac{{}^6C_i}{2^6} \tag{7}$$

First we will do the analysis for k = 2,4 and 6

So the CDF of X will be:

$$F_X(k) = \Pr(X \le k) \tag{8}$$
$$= \Pr(|2H - 6| \le k) \tag{9}$$
$$= \Pr(-k \le 2H - 6 \le k) \tag{10}$$
$$= \Pr\left(\frac{6-k}{2} \le H \le \frac{6+k}{2}\right) \tag{11}$$
$$= F_H\left(3 + \frac{k}{2}\right) - F_H\left((3 - \frac{k}{2}) - 1\right) \tag{12}$$
$$= F_H\left(3 + \frac{k}{2}\right) - F_H\left(2 - \frac{k}{2}\right) \tag{13}$$

So the probability distribution of X would be given by:

$$P_X(k) = F_X(k) - F_X(k-1) \tag{14}$$

$$= F_H\left(3 + \frac{k}{2}\right) - F_H\left(2 - \frac{k}{2}\right) - F_H\left(2.5 + \frac{k}{2}\right) + F_H\left(3.5 - \frac{k}{2}\right) \tag{15}$$

$$= \left(\sum_{i=0}^{3+\frac{k}{2}} \frac{{}^6C_i}{2^6} - \sum_{i=0}^{2.5+\frac{k}{2}} \frac{{}^6C_i}{2^6}\right) + \left(\sum_{i=0}^{3.5-\frac{k}{2}} \frac{{}^6C_i}{2^6} - \sum_{i=0}^{2-\frac{k}{2}} \frac{{}^6C_i}{2^6}\right) \tag{16}$$

$$= \frac{{}^6C_k}{2^6} + \frac{{}^6C_{6-k}}{2^6} \tag{17}$$

$$= \frac{{}^6C_k}{2^5} \tag{18}$$

Now we will do the analysis for k = 0,
CDF of X for will be

$$F_H(k) = F_H\left(\left[3 + \frac{k}{2}\right]\right) - F_H\left(\left[3 - \frac{k}{2}\right]\right) \tag{19}$$

So the probability distribution of X would be given by:

$$P_X(k) = F_X(k) - F_X(k-1) \tag{20}$$

$$= F_H\left(\left[3 + \frac{k}{2}\right]\right) - F_H\left(\left[3 - \frac{k}{2}\right]\right) - F_H\left(\left[2.5 + \frac{k}{2}\right]\right) + F_H\left(\left[3.5 - \frac{k}{2}\right]\right) \tag{21}$$

$$= F_H\left(3 + \left[\frac{k}{2}\right]\right) - F_H\left(3 - \left[\frac{k}{2}\right]\right) - F_H\left(2 + \left[\frac{k}{2}\right]\right) + F_H\left(3 - \left[\frac{k}{2}\right]\right) \tag{22}$$

$$= F_H\left(3 + \left[\frac{k}{2}\right]\right) - F_H\left(2 + \left[\frac{k}{2}\right]\right) \tag{23}$$

$$= \sum_{i=0}^{3+\left[\frac{k}{2}\right]} \frac{{}^6C_i}{2^6} - \sum_{i=0}^{2+\left[\frac{k}{2}\right]} \frac{{}^6C_i}{2^6} \tag{24}$$