

Assignment

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Question: A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- (a) atleast once
- (b) exactly once
- (c) atleast twice?

Solution: Let us define:

Parameter	Value	Description
n	50	number of lotteries
p	0.01	probability of winning a prize
q	0.99	probability of not winning
$\mu = np$	0.5	mean of the distribution
$\sigma^2 = npq$	0.495	variance of the distribution
Y	0,1,2,3,...,50	Number of successes

(a) using Gaussian

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

The CDF of Y :

$$F_Y(y) = 1 - \Pr(Y > y) \quad (2)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{y - \mu}{\sigma}\right) \quad (3)$$

But,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (4)$$

$$(5)$$

the Q-function is defined as:

$$Q(x) = \Pr(Y > x) \quad \forall x \in Y \sim \mathcal{N}(0, 1) \quad (6)$$

therefore the cdf will be:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y - \mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu - y}{\sigma}\right), & y < \mu \end{cases} \quad (7)$$

The probabbility of winning the prize atleast once is given by:

Considering 0.3 as the correction term,

$$\Pr(Y > 0.7) = 1 - F_Y(0.7) \quad (8)$$

$$= Q\left(\frac{0.7 - \mu}{\sigma}\right) \quad \text{from(7)} \quad (9)$$

$$= Q(0.2842) \quad (10)$$

$$= 0.3881 \quad (11)$$

(b) using Gaussian

the gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (12)$$

the probability of the person winning the prize exactly once is given by:

$$p_Y(1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(1-\mu)^2}{2\sigma^2}} \quad (13)$$

$$= 0.44 \quad (14)$$

(c) using Gaussian

the probability of the person winning the prize atleast twice is given by:
considering 0.5 as the correction term,

$$\Pr(Y > 1.5) = 1 - F_Y(1.5) \quad (15)$$

$$= Q\left(\frac{1.5 - \mu}{\sigma}\right) \text{ from (7)} \quad (16)$$

$$= Q\left(\frac{0.1}{\sqrt{0.495}}\right) \quad (17)$$

$$= Q(1.42) \quad (18)$$

$$= 0.0776 \quad (19)$$

Gaussian vs Binomial Table

Y	Gaussian	Binomial
atleast one	0.3881	0.395
exactly one	0.441	0.305
atleast two	0.0776	0.089

Gaussian vs Binomial graph

