Assignment

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Question: Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

1) $\frac{1}{2}$
2) $\frac{1}{3}$
3) $\frac{2}{3}$
4) $\frac{4}{7}$

Solution:

Let X_0, X_1, X_2 be the random variables which denotes the three children, where X_0 is the eldest child and X_2 is the youngest child.

RV	Value	Description
X_0	0	eldest one is boy
	1	eldest one is girl
X_1	0	middle one is boy
	1	middle one is girl
X_2	0	youngest one is boy
	1	youngest one is girl

TABLE 4
RV DESCRIPTION TABLE

so the required probability is,

$$\Pr\left(X_0 = 1 \mid X_0 + X_1 + X_2 \geq 1\right) = \frac{\Pr\left(X_0 = 1\right)}{\Pr\left(X_0 + X_1 + X_2 \geq 1\right)} \tag{1}$$

$$= \frac{\Pr\left(X_0 = 1\right)}{1 - \Pr\left(X_0 + X_1 + X_2 = 0\right)} \tag{2}$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{8}} \tag{3}$$

$$= \frac{4}{7} \tag{4}$$

Therefore, the probability that the eldest child is a girl given that the family has atleast one girl is $\frac{4}{7}$