Practical 4 – Particle Swarm Optimization (PSO)

Aim

The goal of this lab is to understand and implement **Particle Swarm Optimization (PSO)** and a few of its variations. You will apply these methods to well-known mathematical test functions and study how different versions of PSO perform.

Problem Statement

PSO is a population-based optimization algorithm inspired by the way birds flock or fish swim together in nature. Each solution (called a particle) moves in the search space by updating its **velocity** and **position**. Updates depend on two guiding memories:

- **pbest**: the particle's own best position found so far.
- **gbest**: the best position discovered by the whole swarm.

By modifying the update rules, we can influence whether particles explore widely or converge quickly. In this lab, you will implement PSO, extend it with variations, and test it on standard benchmark functions.

Equations

- **Velocity Update (Standard PSO):**

$$v_i(t+1) = w \cdot v_i(t) + c_1 \cdot r_1 \cdot (pbest_i - x_i(t)) + c_2 \cdot r_2 \cdot (gbest - x_i(t))$$

- **Position Update:**

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

- **Inertia Weight (linearly decreasing):**

$$w = w_{max} - \frac{(w_{max} - w_{min})}{t_{max}} \cdot t$$

- **Constriction Factor Version:**

$$v_i(t+1) = \chi \cdot [v_i(t) + c_1 \cdot r_1 \cdot (pbest_i - x_i(t)) + c_2 \cdot r_2 \cdot (gbest - x_i(t))]$$

Benchmark Functions

1. **Sphere Function (Simple bowl):**

$$f(x) = \sum_{i=1}^{n} x_i^2, x_i \in [-5.12,5.12]$$

2. **Rastrigin Function (Many local minima):**

$$f(x) = 10n + \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i)], x_i \in [-5.12,5.12]$$

3. **Rosenbrock Function (Curved valley):**

$$f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2], x_i \in [-5,10]$$

Tasks

Part A: Core Implementation

1. Implement two PSO variants:
   - Standard PSO
   - Inertia Weight PSO (linearly decreasing weight)
2. Run both versions on:
   - Sphere Function (30 dimensions)
   - Rastrigin Function (30 dimensions)
3. Use these parameters:
   - Population size = 30
   - Iterations = 300
   - $c_1 = c_2 = 1.5$
   - $w_{max} = 0.9$, $w_{min} = 0.4$
   - Velocity limited to 40% of the search range
4. Record the **best fitness value** each iteration.
5. Plot convergence curves to compare the two variants.

Part B: Extension (Choose One)

Pick one extension to try:

1. **Rosenbrock Test:** Run both Standard and Inertia PSO on Rosenbrock (30D).
2. **Constriction Factor PSO:** Use $\chi = 0.729$, $c_1 = c_2 = 2.05$. Compare it with Inertia PSO on Rastrigin.
3. **Ring-Topology PSO:** Replace global best with a 3-neighbor ring best. Run it on Rastrigin.

Bonus Task (Optional for Extra Credit)

Run your chosen PSO variant on the **Sphere and Rastrigin functions in 100 dimensions**.

- Plot the convergence curves.
- Compare the results with the 30-dimensional experiments.
- Comment on how dimensionality affects convergence speed and accuracy.

Deliverables

- Source code for all PSO variants you implemented.
- Convergence plots for Sphere and Rastrigin.
- Results and plots for your chosen extension.
- (Optional) Results for the Bonus Task.
- A short report (1–2 pages) with observations and conclusions.