

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad (1)$$

1 VECTORS

parameters	values	description
\mathbf{m}_1	$\begin{pmatrix} 3 \\ -7 \end{pmatrix}$	AB
\mathbf{m}_2	$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$	BC
\mathbf{m}_3	$\begin{pmatrix} -3 \\ 1 \end{pmatrix}$	CA
$\ A - B\ $	6	length of AB
$\ B - C\ $	7.61	length of BC
$\ C - A\ $	3.16	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
\mathbf{n}_1	$\begin{pmatrix} -7 \\ -3 \end{pmatrix}$	AB
c_1	20	
\mathbf{n}_2	$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	BC
c_2	-12	
\mathbf{n}_3	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	CA
c_3	10	
Area	9	Area of Triangle
$\angle A$	48.36°	Angles
$\angle B$	23.19°	
$\angle C$	108.43°	

TABLE 1: Vectors.

2 MEDIAN

parameters	value	description
\mathbf{D}	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	BC midpoint
\mathbf{E}	$\begin{pmatrix} -1.5 \\ 4.5 \end{pmatrix}$	CA midpoint
\mathbf{F}	$\begin{pmatrix} -3.5 \\ 1.5 \end{pmatrix}$	AB midpoint
\mathbf{m}_4	$\begin{pmatrix} 5 \\ -4 \end{pmatrix}$	AD
\mathbf{n}_4	$\begin{pmatrix} -4 \\ -5 \end{pmatrix}$	
c_4	-5	
\mathbf{m}_5	$\begin{pmatrix} 0.5 \\ 6.5 \end{pmatrix}$	BE
\mathbf{n}_5	$\begin{pmatrix} 6.5 \\ -0.5 \end{pmatrix}$	
c_5	-12	
\mathbf{m}_6	$\begin{pmatrix} -5.5 \\ -2.5 \end{pmatrix}$	CF
\mathbf{n}_6	$\begin{pmatrix} -2.5 \\ 5.5 \end{pmatrix}$	
c_6	17	
\mathbf{G}	$\begin{pmatrix} -1.66 \\ 2.33 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by \mathbf{G}
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

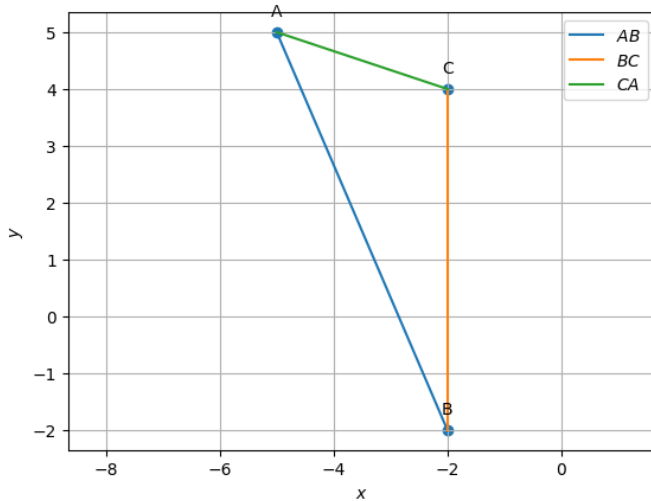


Fig. 1: triangle plotted using python

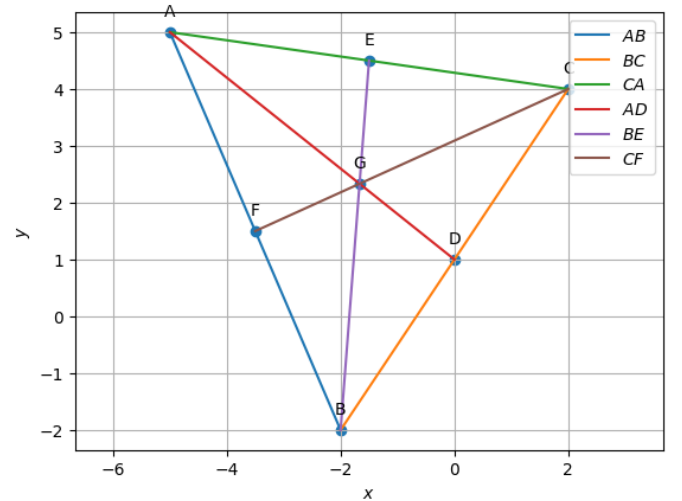


Fig. 2: medians plotted using python

3 ALTITUDE

parameters	value	description
D_1	$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	Foot of altitude from A
E_1	$\begin{pmatrix} -0.2 \\ 3.4 \end{pmatrix}$	Foot of altitude from B
F_1	$\begin{pmatrix} -4.17 \\ 3.06 \end{pmatrix}$	Foot of altitude from C
m_7	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	AD_1
n_7	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	
c_7	-15	
m_8	$\begin{pmatrix} 1.8 \\ 5.4 \end{pmatrix}$	BE_1
n_8	$\begin{pmatrix} 5.4 \\ -1.8 \end{pmatrix}$	
c_8	-7.2	
m_9	$\begin{pmatrix} -2.17 \\ -0.93 \end{pmatrix}$	CF_1
n_9	$\begin{pmatrix} -0.93 \\ 2.17 \end{pmatrix}$	
c_9	10.55	
H	$\begin{pmatrix} 0.33 \\ 5 \end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 PERPENDICULAR BISECTOR

parameters	value	description
m_{10}	$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	AD_1
n_{10}	$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$	
c_{10}	6	
m_{11}	$\begin{pmatrix} -1 \\ -3 \end{pmatrix}$	BE_1
n_{11}	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	
c_{11}	-15	
m_{12}	$\begin{pmatrix} 7 \\ 3 \end{pmatrix}$	CF_1
n_{12}	$\begin{pmatrix} -3 \\ 7 \end{pmatrix}$	
c_{12}	21	
O	$\begin{pmatrix} -4.66 \\ 1 \end{pmatrix}$	Circumcentre
$\ O - A\ $	4.01	$OA = OB = OC = R$
$\ O - B\ $		
$\ O - C\ $		
R		
$\angle BOC$	96.73°	$\angle BOC = 2\angle BAC$
$\angle BAC$	48.36°	
$\angle AOC$	46.39°	$\angle AOC = 2\angle ABC$
$\angle ABC$	23.19°	
$\angle AOB$	217°	$\angle AOB = 2\angle BCA$
$\angle BCA$	108.43°	

TABLE 4: Perpendicular Bisector.

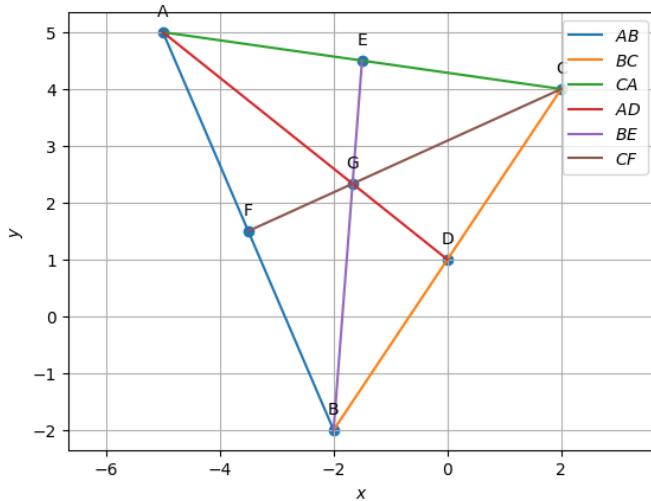


Fig. 3: altitudes plotted using python

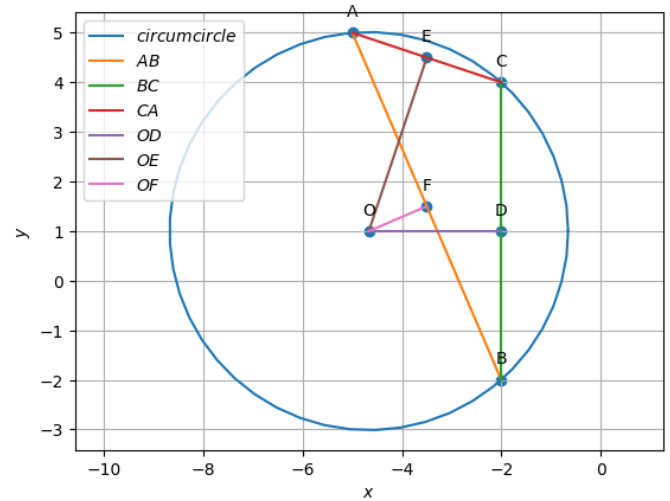


Fig. 4: perpendicular bisectors plotted using python

5 ANGLE BISECTOR

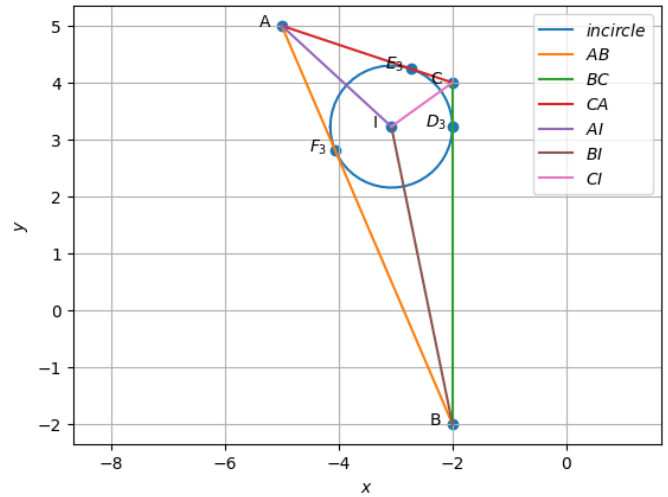


Fig. 5: Angle bisectors plotted using python

parameters	value	description
\mathbf{m}_{13}	$\begin{pmatrix} -1.34 \\ 1.23 \end{pmatrix}$	AI
\mathbf{n}_{13}	$\begin{pmatrix} 1.23 \\ 1.34 \end{pmatrix}$	
c_{13}	0.53	
\mathbf{m}_{14}	$\begin{pmatrix} -0.39 \\ 1.91 \end{pmatrix}$	BI
\mathbf{m}_{14}	$\begin{pmatrix} -1.91 \\ -0.39 \end{pmatrix}$	
c_{14}	4.626	
\mathbf{m}_{15}	$\begin{pmatrix} 0.94 \\ 0.68 \end{pmatrix}$	CI
\mathbf{n}_{15}	$\begin{pmatrix} -0.68 \\ -0.94 \end{pmatrix}$	
c_{15}	5.16	
\mathbf{I}	$\begin{pmatrix} -3.07 \\ 3.22 \end{pmatrix}$	Incentre
\mathbf{D}_3	$\begin{pmatrix} -2 \\ 3.22 \end{pmatrix}$	Point of contact with BC
\mathbf{E}_3	$\begin{pmatrix} -2.73 \\ 4.24 \end{pmatrix}$	Point of contact with AC
\mathbf{F}_3	$\begin{pmatrix} -4.05 \\ 2.8 \end{pmatrix}$	Point of contact with AB
$\ \mathbf{I} - \mathbf{D}_3\ $	1.07	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $		
$\ \mathbf{I} - \mathbf{F}_3\ $		
r		
$\angle BAI$	24.18°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	11.59°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	54.21°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.