Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 3 \\ -7 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} -3 \\ 1 \end{pmatrix}$	CA
$\|\|A-B\|\|$	6	length of AB
$\|\|B-C\|\|$	7.61	length of BC
$\|\|C-A\|\|$	3.16	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} -7 \\ -3 \end{pmatrix}$	AB
c_1	20	
$\mathbf{n_2}$	$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	BC
c_2	-12	
$\mathbf{n_3}$	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	CA
c_3	10	
Area	9	Area of Triangle
$\angle A$	48.36°	Angles
$\angle B$	23.19°	
$\angle C$	108.43°	

TABLE 1: Vectors.

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -1.5, 4.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} -3.5 \\ 1.5 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 5 \\ -4 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -4 \\ -5 \end{pmatrix}$	
c_4	-5	
$\mathbf{m_5}$	$\begin{pmatrix} 0.5 \\ 6.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 6.5 \\ -0.5 \end{pmatrix}$	
c_5	-12	
$\mathbf{m_6}$	$\begin{pmatrix} -5.5 \\ -2.5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} -2.5 \\ 5.5 \end{pmatrix}$	
c_6	17	
$\mathbf{G}$	$\begin{pmatrix} -1.66 \\ 2.33 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

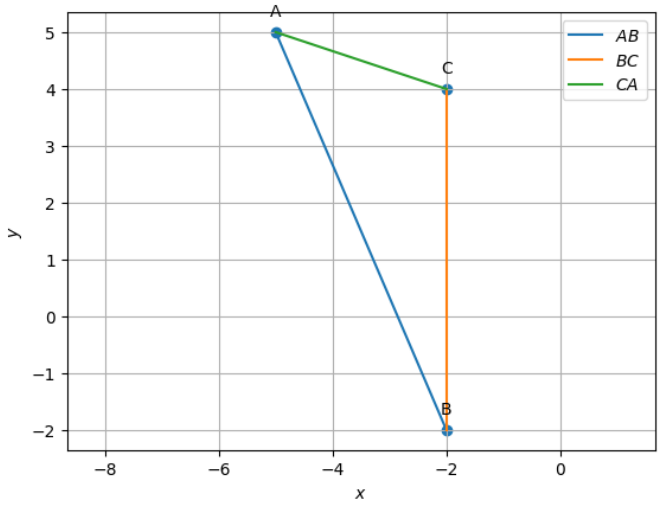

Fig. 1: triangle plotted using python

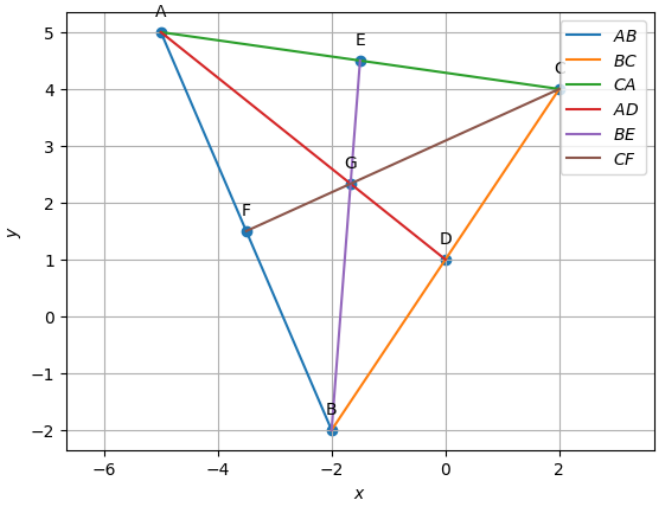

Fig. 2: medians plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}-2\\5\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-0.2\\3.4\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-4.17\\3.06\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}3\\0\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}0\\-3\end{pmatrix}$	
c_7	-15	
$\mathbf{m_8}$	$\begin{pmatrix}1.8\\5.4\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}5.4\\-1.8\end{pmatrix}$	
c_8	-7.2	
$\mathbf{m_9}$	$\begin{pmatrix}-2.17\\-0.93\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}-0.93\\2.17\end{pmatrix}$	
c_9	10.55	
H	$\begin{pmatrix}0.33\\5\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

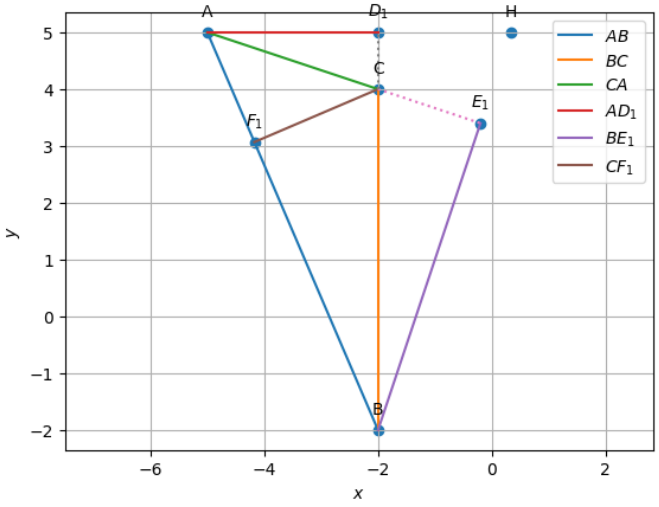

Fig. 3: altitudes plotted using python

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}6\\0\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}0\\6\end{pmatrix}$	
c_{10}	6	
$\mathbf{m_{11}}$	$\begin{pmatrix}-1\\-3\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}3\\-1\end{pmatrix}$	
c_{11}	-15	
$\mathbf{m_{12}}$	$\begin{pmatrix}7\\3\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-3\\7\end{pmatrix}$	
c_{12}	21	
O	$\begin{pmatrix}-4.66\\1\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	4.01	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	96.73°	$\angle BOC = 2\angle BAC$
$\angle BAC$	48.36°	
$\angle AOC$	46.39°	$\angle AOC = 2\angle ABC$
$\angle ABC$	23.19°	
$\angle AOB$	217°	$\angle AOB = 2\angle BCA$
$\angle BCA$	108.43°	

TABLE 4: Perpendicular Bisector.

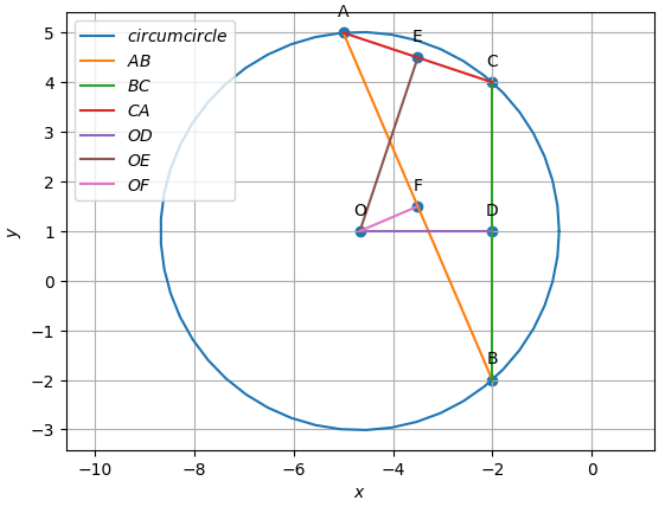

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

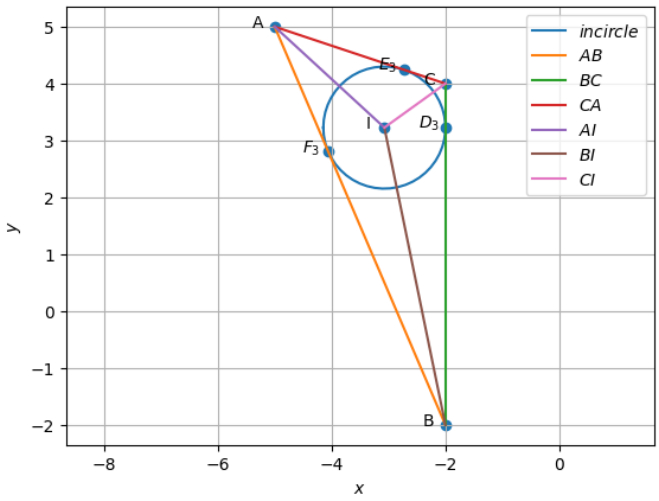

Fig. 5: Angle bisectors plotted using python

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-1.34\\1.23\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}1.23\\1.34\end{pmatrix}$	
c_{13}	0.53	
$\mathbf{m_{14}}$	$\begin{pmatrix}-0.39\\1.91\end{pmatrix}$	BI
$\mathbf{m_{14}}$	$\begin{pmatrix}-1.91\\-0.39\end{pmatrix}$	
c_{14}	4.626	
$\mathbf{m_{15}}$	$\begin{pmatrix}0.94\\0.68\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}-0.68\\-0.94\end{pmatrix}$	
c_{15}	5.16	
$\mathbf{I}$	$\begin{pmatrix}-3.07\\3.22\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}-2\\3.22\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-2.73\\4.24\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}-4.05\\2.8\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	1.07	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	24.18°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	11.59°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	54.21°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.