Problem 1.1.5

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question:

The normal form of the equation of AB is

$$\mathbf{n}^{\mathsf{T}} \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{1}$$

where

$$\mathbf{n}^{\mathsf{T}}\mathbf{m} = \mathbf{n}^{\mathsf{T}} \left(\mathbf{B} - \mathbf{A} \right) = 0 \tag{2}$$

or,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{3}$$

find the normal form of the equations of $AB\ BC$ and CA

solution:

The normal equation for the side BC is

$$\mathbf{n}^{\mathsf{T}} \left(\mathbf{x} - \mathbf{B} \right) = 0 \tag{4}$$

$$\mathbf{n}^{\mathsf{T}}\mathbf{x} = \mathbf{n}^{\mathsf{T}}\mathbf{B} \tag{5}$$

Now our task is to find the \boldsymbol{n} so that we can find $\boldsymbol{n}^{\scriptscriptstyle \top}$

As given in the question

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{6}$$

here $\mathbf{m} = \mathbf{C} - \mathbf{B}$ for side \mathbf{BC}

$$\implies \mathbf{m} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{m} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{8}$$

now we as we have obtained vector \mathbf{m} we can use this to obtain vector \mathbf{n}

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{9}$$

$$\mathbf{n} = \begin{pmatrix} -11\\ -1 \end{pmatrix} \tag{10}$$

The transpose of \mathbf{n} is

$$\mathbf{n}^{\mathsf{T}} = \begin{pmatrix} -11 & -1 \end{pmatrix} \tag{11}$$

hence the normal equation of side BC is

$$\begin{pmatrix} -11 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -11 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} -11 & -1 \end{pmatrix} \mathbf{x} = 38 \tag{13}$$