Problem

EE22BTECH11010 - Aryan Bubna

question: A fair coin is tossed four times and a person win Re 1 for each head and lose Re 1.5 for each tail that turns up.from the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:

RV	values	description
X	0,1,2,3,4	No.of heads

TABLE 0: Random variable X declaration

X	profit(Rs)
0	-6
1	-3.5
2	-1
3	1.5
4	4

TABLE 0: profits obtained

we know that n=4,$p = \frac{1}{2}$,$q = \frac{1}{2}$

RV	Value	Description
$\mu = np$	2	Mean
$\sigma^2 = npq$	1	Varience

TABLE 0: mean and variance of binomial distribution

$$\mu = np \tag{1}$$
$$= 2 \tag{2}$$
$$\sigma = npq \tag{3}$$
$$= 1 \tag{4}$$

now we will solve this question using PDF and CDF

1) using PDF

$$P_X(x) = \frac{1}{\sigma \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{x-\mu}{\sigma}\right)^2} \tag{5}$$
$$P_X(0) = \frac{1}{1 \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{0-2}{1}\right)^2} \tag{6}$$
$$= 0.0215 \tag{7}$$
$$P_X(1) = \frac{1}{1 \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{1-2}{1}\right)^2} \tag{8}$$
$$= 0.096 \tag{9}$$
$$P_X(2) = \frac{1}{1 \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{2-2}{1}\right)^2} \tag{10}$$
$$= 0.159 \tag{11}$$
$$P_X(3) = \frac{1}{1 \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{3-2}{1}\right)^2} \tag{12}$$
$$= 0.096 \tag{13}$$
$$P_X(4) = \frac{1}{1 \times 2\pi} \times e^{\frac{-1}{2} \times \left(\frac{4-2}{1}\right)^2} \tag{14}$$
$$= 0.0215 \tag{15}$$

2) using CDF

$$X \sim \mathcal{N}(np, npq) \tag{16}$$

We need to find

$$P_X(x) = \begin{cases} F_X(0), & x = 0 \\ F_X(x) - F_X(x-1), & x = 1, 2, 3, 4 \end{cases} \tag{17}$$

After corrections to make the values more accurate, we need to find

$$\Pr(X \leq x) = F_X(x) \tag{18}$$

then CDF of X is:$F_X(x) = \Pr(X \leq x)$
Q function is defined

$$Q(x) = \Pr(X > x) \ \forall x \in X \sim \mathcal{N}(0, 1) \tag{19}$$

From above , $F_X(x) = 1 - Q(x)$

hence

$$P_X(0) = 1 - Q(0) \tag{20}$$
$$= 0.5 \tag{21}$$
$$P_X(1) = Q(0) - Q(1) \tag{22}$$
$$= 0.341 \tag{23}$$
$$P_X(2) = Q(1) - Q(2) \tag{24}$$
$$= 0.135 \tag{25}$$
$$P_X(3) = Q(2) - Q(3) \tag{26}$$
$$= 0.021 \tag{27}$$
$$P_X(3) = Q(3) - Q(4) \tag{28}$$
$$= 0.001 \tag{29}$$