

Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution:

RV	description
X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

For X_1 (Black Die):

PMF of X_1 : $P_{X_1}(i) = 1/6$ for $i = 1$ to 6

CDF of X_1 : $F_{X_1}(i) = i/6$ for $i = 1$ to 6

For X_2 (Red Die):

PMF for X_2 : $P_{X_2}(j) = 1/6$ for $j = 1$ to 6

CDF for X_2 : $F_{X_2}(j) = j/6$ for $j = 1$ to 6

(a)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5)$$

(1)

$$= 1 - \Pr(X_2 \leq 4 \mid X_1 = 5)$$

(2)

$$= 1 - F_{X_2}(4)$$

(3)

$$= 1 - \frac{4}{6}$$

(4)

$$= 1 - \frac{2}{3}$$

(5)

$$= \frac{1}{3}$$

(6)

(b)

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)}$$

(7)

$$= \frac{\Pr((X_1 > 4), (X_2 < 4))}{\Pr(X_2 < 4)}$$

(8)

As $X_1 > 4, X_2 < 4$ are two independent events hence

$$\Pr(X_1 > 4) = 1 - \Pr(X_1 \leq 4) \quad (9)$$

$$= 1 - F_{X_1}(4) \quad (10)$$

$$= \frac{1}{3} = \frac{1}{3} \quad (11)$$

$$\Pr(X_2 < 4) = F_{X_2}(4) \quad (12)$$

$$= \frac{2}{3} \quad (13)$$

$$\Pr((X_1 > 4), (X_2 < 4)) = \Pr(X_1 > 4) \times \Pr(X_2 < 4) \quad (14)$$

$$= \frac{2}{9} \quad (15)$$

therefore

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\frac{2}{9}}{F_{X_2}(4)} \quad (16)$$

$$= \frac{1}{9} \quad (17)$$