

Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9, given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution:

RV	description
X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

PMF of the random variables is:

$$P_{X_j}(i) = \begin{cases} \frac{1}{6} & j = 1, 2; 1 \leq i \leq 6 \end{cases} \quad (1)$$

CDF of the random variables is:

$$F_{X_j}(i) = \begin{cases} \frac{i}{6} & j = 1, 2; 1 \leq i \leq 6 \end{cases} \quad (2)$$

1)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5)$$

$$(3)$$

$$= 1 - \Pr(X_2 \leq 4 \mid X_1 = 5)$$

$$(4)$$

$$= 1 - \frac{\Pr(X_2 \leq 4, X_1 = 5)}{\Pr(X_1 = 5)}$$

$$(5)$$

$$= 1 - \Pr(X_2 \leq 4)$$

$$(6)$$

$$= 1 - F_{X_2}(4)$$

$$(7)$$

$$= 1 - \frac{4}{6}$$

$$(8)$$

$$= 1 - \frac{2}{3}$$

$$(9)$$

$$= \frac{1}{3}$$

$$(10)$$

2) considering $X_2 < 4$ as a condition

$$\Rightarrow P_{X_2}(k) = \begin{cases} \frac{1}{3} & j = \{1, 2, 3\} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

thus

$$\Pr(X_1 + X_2 = 8) = \Pr(X_1 = 8 - X_2) \quad (12)$$

$$= E(P_{X_1}(8 - X_2)) \quad (13)$$

$$= \sum_{i=1}^6 P_{X_1}(8 - i) P_{X_2}(i) \quad (14)$$

$$= \frac{1}{3} \sum_{i=1}^3 P_{X_1}(8 - i) \quad (15)$$

$$= \frac{1}{3} \left(0 + \frac{1}{6} + \frac{1}{6} \right) \quad (16)$$

$$= \frac{1}{9} \quad (17)$$