

Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9, given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution: Let X_1 denotes the outcome of black die, X_2 denote the outcome of red die

(a) We need to get the conditional probability of obtaining a sum greater than 9 and given that black dice resulted in a 5, i.e $\Pr(X_1 + X_2 > 9 | X_1 = 5)$

The sum of X_1 and X_2 is greater than 9 such that $X_1 = 5$ is for only two cases, i.e (5,6), (5,5) respectively. Therefore

$$\Pr((X_1 + X_2 > 9), (X_1 = 5)) = \frac{2}{36} \quad (1)$$

$$= \frac{1}{18} \quad (2)$$

We know that $\Pr(X_1 = 5) = \frac{1}{6}$ Hence

$$\Pr(X_1 + X_2 > 9 | X_1 = 5) = \frac{\Pr((X_1 + X_2 > 9), (X_1 = 5))}{\Pr(X_1 = 5)} \quad (3)$$

$$= \frac{\Pr((X_2 > 4), (X_1 = 5))}{\Pr(X_1 = 5)} \quad (4)$$

$$= \frac{\frac{1}{18}}{\frac{1}{6}} \quad (5)$$

$$= \frac{1}{3} \quad (6)$$

(b) We need to get the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4, i.e $\Pr(X_1 + X_2 = 8 | X_2 < 4)$.

The sum of $X_1 + X_2 = 8$ such that $X_2 < 4$ is possible for two cases (5,3), (6,2) out of 36 cases.

Therefore

$$\Pr((X_1 + X_2 = 8), (X_2 < 4)) = \frac{2}{36} \quad (7)$$

$$= \frac{1}{18} \quad (8)$$

We know that $\Pr(X_2 < 4) = \frac{1}{2}$

Hence

$$\Pr(X_1 + X_2 = 8 | X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (9)$$

$$= \frac{\frac{1}{18}}{\frac{1}{2}} \quad (10)$$

$$= \frac{1}{9} \quad (11)$$