Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9,given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8,given that the red die resulted in a number less than 4.

Solution:

RV	description
X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

PMF of the random variables is:

$$P_{X_j}(i) = \begin{cases} \frac{1}{6} & j = 1, 2; 1 \le i \le 6 \end{cases} \quad (1)$$

CDF of the random variables is:

$$F_{X_j}(i) = \begin{cases} \frac{i}{6} & j = 1, 2; 1 \le i \le 6 \end{cases} \quad (2)$$

1)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5) \quad (3)$$

$$= 1 - \Pr(X_2 \le 4 \mid X_1 = 5) \quad (4)$$

$$= 1 - F_{X_2}(4) \quad (5)$$

$$= 1 - \frac{4}{6} \quad (6)$$

$$= 1 - \frac{2}{3} \quad (7)$$

$$= \frac{1}{3} \quad (8)$$

2)

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (9)$$

$$= \frac{\Pr((X_1 > 4), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (10)$$

As $X_1 > 4, X_2 < 4$ are two independent events hence

$$\Pr(X_1 > 4) = 1 - \Pr(X_1 \le 4) \quad (11)$$

$$= 1 - F_{X_1}(4) \quad (12)$$

$$= \frac{1}{3} \quad = \frac{1}{3} \quad (13)$$

$$\Pr(X_2 < 4) = F_{X_2}(4) \quad (14)$$

$$= \frac{2}{3} \quad (15)$$

$$\Pr((X_1 > 4), (X_2 < 4)) = \Pr(X_1 > 4) \times \Pr(X_2 < 4) \quad (16)$$

$$= \frac{2}{9} \quad (17)$$

therefore

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\frac{2}{9}}{F_{X_2}(4)} \quad (18)$$

$$= \frac{1}{9} \quad (19)$$