Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9,given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8,given that the red die resulted in a number less than 4.

Solution:

X_1	Black die
X_2	Red die

we know that

$$\Pr(X_1 = 5) = \frac{1}{6} \tag{1}$$

$$\Pr(X_2 < 4) = \frac{1}{2} \tag{2}$$

(a) The sum of X_1 and X_2 is greater than 9 such that $X_1 = 5$ is for only two cases,i.e (5,6),(5,5) respectively.

Therefore

$$\Pr((X_1 + X_2 > 9),(X_1 = 5)) = \frac{2}{36} \tag{3}$$

$$= \frac{1}{18} \tag{4}$$

Hence

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \frac{\Pr((X_1 + X_2 > 9),(X_1 = 5))}{\Pr(X_1 = 5)} \tag{5}$$

$$= \frac{\Pr((X_2 > 4),(X_1 = 5))}{\Pr(X_1 = 5)} \tag{6}$$

$$= \frac{\frac{1}{18}}{\frac{1}{6}} \tag{7}$$

$$= \frac{1}{3} \tag{8}$$

(b)

The sum of $X_1 + X_2 = 8$such that $X_2 < 4$ is possible for two cases (5,3),(6,2) out of 36 cases.

Therefore

$$\Pr((X_1 + X_2 = 8),(X_2 < 4)) = \frac{2}{36} \tag{9}$$

$$= \frac{1}{18} \tag{10}$$

Hence

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8),(X_2 < 4))}{\Pr(X_2 < 4)} \tag{11}$$

$$= \frac{\frac{1}{18}}{\frac{1}{2}} \tag{12}$$

$$= \frac{1}{9} \tag{13}$$