

Problem

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9, given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution:

RV	description
X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

PMF of the random variables is:

$$P_{X_j}(i) = \left\{ \frac{1}{6} \quad j = 1, 2; 1 \leq i \leq 6 \right. \quad (1)$$

CDF of the random variables is:

$$F_{X_j}(i) = \left\{ \frac{i}{6} \quad j = 1, 2; 1 \leq i \leq 6 \right. \quad (2)$$

1)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5) \quad (3)$$

$$= \frac{\Pr(X_2 > 4, X_1 = 5)}{\Pr(X_1 = 5)} \quad (4)$$

$$= \Pr(X_2 > 4) \quad (5)$$

$$= 1 - \Pr(X_2 \leq 4) \quad (6)$$

$$= 1 - F_{X_2}(4) \quad (7)$$

$$= 1 - \frac{4}{6} \quad (8)$$

$$= 1 - \frac{2}{3} \quad (9)$$

$$= \frac{1}{3} \quad (10)$$

2)

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr(X_1 + X_2 = 8, X_2 < 4)}{\Pr(X_2 < 4)} \quad (11)$$

$$= \frac{\sum_{i=1}^3 P_{X_1}(8-i) P_{X_2}(i)}{\Pr(X_2 < 4)} \quad (12)$$

$$= \frac{P_{X_1}(5) P_{X_2}(3) + P_{X_1}(6) P_{X_2}(2)}{\Pr(X_2 < 4)} \quad (13)$$

$$= \frac{\frac{1}{36} + \frac{1}{36}}{\frac{3}{6}} \quad (14)$$

$$= \frac{1}{9} \quad (15)$$