

# Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9, given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

**Solution:**

RV	description
$X_1$	Black die
$X_2$	Red die

TABLE 0: random variables of  $X_1$  and  $X_2$

For  $X_1$  (Black Die): (1)

PMF of  $X_1: P_{X_1}(i) = 1/6$  for  $i = 1$  to  $6$  (2)

CDF of  $X_1: F_{X_1}(i) = i/6$  for  $i = 1$  to  $6$  (3)

For  $X_2$  (Red Die): (4)

PMF for  $X_2: P_{X_2}(j) = 1/6$  for  $j = 1$  to  $6$  (5)

CDF for  $X_2: F_{X_2}(j) = j/6$  for  $j = 1$  to  $6$  (6)

(7)

2)

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (14)$$

$$= \frac{\Pr((X_1 > 4), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (15)$$

As  $X_1 > 4, X_2 < 4$  are two independent events hence

$$\Pr(X_1 > 4) = 1 - \Pr(X_1 \leq 4) \quad (16)$$

$$= 1 - F_{X_1}(4) \quad (17)$$

$$= \frac{1}{3} = \frac{1}{3} \quad (18)$$

$$\Pr(X_2 < 4) = F_{X_2}(4) \quad (19)$$

$$= \frac{2}{3} \quad (20)$$

$$\Pr((X_1 > 4), (X_2 < 4)) = \Pr(X_1 > 4) \times \Pr(X_2 < 4) \quad (21)$$

$$= \frac{2}{9} \quad (22)$$

therefore

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\frac{2}{9}}{F_{X_2}(4)} \quad (23)$$

$$= \frac{1}{9} \quad (24)$$

1)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5) \quad (8)$$

$$= 1 - \Pr(X_2 \leq 4 \mid X_1 = 5) \quad (9)$$

$$= 1 - F_{X_2}(4) \quad (10)$$

$$= 1 - \frac{4}{6} \quad (11)$$

$$= 1 - \frac{2}{3} \quad (12)$$

$$= \frac{1}{3} \quad (13)$$