Problem 12.13.1.10

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9,given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8,given that the red die resulted in a number less than 4.

Solution:

X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

Let's calculate the PMF and CDF for both X_1 (black die) and X_2 (red die):

For X_1 (Black Die):

The sample space for the black die is 1, 2, 3, 4, 5, 6.

Since the die is fair, the probabilities for each outcome are the same: $\Pr(X_1 = i) = 1/6$ for i = 1 to 6.

PMF of X_1: $\Pr(X_1 = i) = 1/6$ for i = 1 to 6

CDF of X_1: $F(X_1 = i) = \Pr(X_1 \le 1)$

$F(X_1 = i) = \sum \Pr(X_1 = k)$ for k=1 to i

$F(X_1 = i) = i/6$ for i = 1 to 6

For X_2 (Red Die):

The sample space for the red die is 1, 2, 3, 4, 5, 6.

Since the die is fair, the probabilities for each outcome are the same: $\Pr(X_2 = j) = 1/6$ for j = 1 to 6.

PMF for X_2: $\Pr(X_2 = j) = 1/6$ for j = 1 to 6

CDF for X_2: $F(X_2 = j) = \Pr(X_2 \le j)$

$F(X_2 = j) = \sum \Pr(x2 = k)$ for k = 1 to j

$F(x2 = j) = j/6$ for j = 1 to 6

(a)The possible outcomes when the black die is 5 are:

Black die $(X_1) = 5$, Red die $(X_2) = 1, 2, 3, 4, 5, 6$

therefore

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5) \tag{1}$$

$$= 1 - \Pr(X_2 \le 4 \mid X_1 = 5) \tag{2}$$

$$= 1 - F(X_2 = 4) \tag{3}$$

$$= 1 - \frac{4}{6} \tag{4}$$

$$= 1 - \frac{2}{3} \tag{5}$$

$$= \frac{1}{3} \tag{6}$$

(b)

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)} \tag{7}$$

$$= \frac{\Pr((X_1 > 4), (X_2 < 4))}{\Pr(X_2 < 4)} \tag{8}$$

As $X_1 > 4$,$X_2 < 4$ are two independent events hence

$$\Pr(X_1 > 4) = F(X_1 = 2) \tag{9}$$

$$= \frac{1}{3} \tag{10}$$

$$\Pr(X_2 < 4) = F(X_2 = 4) \tag{11}$$

$$= \frac{2}{3} \tag{12}$$

$$\Pr((X_1 > 4), (X_2 < 4)) = \Pr(X_1 > 4) \times \Pr(X_2 < 4) \tag{13}$$

$$= \frac{2}{9} \tag{14}$$

therefore

$$\Pr(X_1 + X_2 = 8 \mid X_2 < 4) = \frac{\frac{2}{9}}{F(X_2 = 4)} \tag{15}$$

$$= \frac{1}{9} \tag{16}$$