Problem

EE22BTECH11010 - Aryan Bubna

question: A black and a red dice are rolled.

(a) find the conditional probability of obtaining a sum greater than 9,given that the black dice resulted in a 5.

(b) find the conditional probability of obtaining the sum 8,given that the red die resulted in a number less than 4.

Solution:

RV	description
X_1	Black die
X_2	Red die

TABLE 0: random variables of X_1 and X_2

PMF of the random variables is:

$$p_{X_j}(i) = \begin{cases} \frac{1}{6} & j = 1,2; 1 \le i \le 6 \end{cases} \tag{1}$$

CDF of the random variables is:

$$F_{X_j}(i) = \begin{cases} \frac{i}{6} & j = 1,2; 1 \le i \le 6 \end{cases} \tag{2}$$

1)

$$\Pr(X_1 + X_2 > 9 \mid X_1 = 5) = \Pr(X_2 > 4 \mid X_1 = 5) \tag{3}$$

$$= \frac{\Pr(X_2 > 4, X_1 = 5)}{\Pr(X_1 = 5)} \tag{4}$$

$$= \Pr(X_2 > 4) \tag{5}$$

$$= 1 - \Pr(X_2 \le 4) \tag{6}$$

$$= 1 - F_{X_2}(4) \tag{7}$$

$$= 1 - \frac{4}{6} \tag{8}$$

$$= 1 - \frac{2}{3} \tag{9}$$

$$= \frac{1}{3} \tag{10}$$

2)

$$\Pr(X_1 + X_2 = 8 | X_2 < 4) = \frac{\Pr(X_1 + X_2 = 8, X_2 < 4)}{\Pr(X_2 < 4)} \tag{11}$$

$$= \frac{\sum_{i=1}^{3} p_{X_1}(8-i)\, p_{X_2}(i)}{\Pr(X_2 < 4)} \tag{12}$$

$$= \frac{p_{X_1}(5)\, p_{X_2}(3) + p_{X_1}(6)\, p_{X_2}(2)}{\Pr(X_2 < 4)} \tag{13}$$

$$= \frac{\frac{1}{36} + \frac{1}{36}}{\frac{3}{6}} \tag{14}$$

$$= \frac{1}{9} \tag{15}$$