Exercise:

1) Evaluate $\iint_R x \sin(xy)\, dA$ over the region $R = \{(x,y) \mid 0 \le x \le \pi,\ 1 \le y \le 2\}$ Ans: 0

2) Find the volume V of solid S that is bounded by elliptic paraboloid $2x^2 + y^2 + z = 27$ over the region $R = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le 3\}$ Ans: $\int_{y=0}^{3}\int_{x=0}^{3} (27 - 2x^2 - y^2)\, dx\, dy =$ 162 cubic units.

3) Evaluate $\iint_R (2 - 3x^2 + y^2)\, dA$ where $R = \{(x,y) \mid 3 \le x \le 5,\ -3 \le y \le 2\}$ Ans: $-\frac{1340}{3}$

4) Evaluate $\iint_D (x+y)\, dA$ where D is bounded by $y = \sqrt{x}$ and $y = x^2$ Ans: $3/10$

Determine the value of given integrals by change of order of integration:

1) $\int_0^\infty \int_0^\infty e^{-xy} \sin px\, dx\, dy$ and hence S.T. $\int_0^\infty \frac{\sin px}{x}\, dx = \frac{\pi}{2}$

Ans:- $\frac{\pi}{2}$

2) $\int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dy\, dx$ Ans: $e - 1$

3) $\int_0^1 \int_x^1 \sin y^2\, dy\, dx$ Ans: $\frac{1}{2}\{1 - \cos 1\}$

4) $\int_0^a \int_{\sqrt{ax}}^{a} \frac{y^2\, dx\, dy}{\sqrt{(y^4 - a^2 x^2)}}$ Ans: $\frac{\pi a^2}{6}$

Evaluate the following integrals by change of variables:

1) $\iint_D xy \, dx\, dy$ where D is the portion of circle with centre 0, radius 1 that lies in first quadrant

Ans: $\frac{1}{8}$

2) Let D be the region in the first quadrant bounded by $xy = 1$, $xy = 9$ and the lines $y = x$ and $y = 4x$

Evaluate $\iint_D \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right) dx\, dy$ Ans: $8 + \frac{52}{3} \ln 2$

3) $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} \, dx\, dy$ by changing to polar coordinates

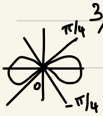
Ans: $\frac{\pi}{4}$

4) Evaluate $\iint_R (3x + 4y^2)\, dA$, where R is the region in the upper half plane bounded by the circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$. Ans: $\frac{15\pi}{2}$

Problems on Area enclosed by the curves

1) Find area enclosed by one loop of the four leaved rose $r = \cos 2\theta$ (Hint: $D = \{(r,\theta) \mid -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}, \; 0 \le r \le \cos 2\theta\}$) Ans: $\frac{\pi}{8}$ sq. units

2) Find the area lying inside the circle $r = a \sin\theta$ and outside the cardioid $r = a(1+\cos\theta)$ Ans: $a^2\left(1 - \frac{\pi}{4}\right)$ sq. units



3) Find the area bounded by lemniscate $r^2 = a^2 \cos 2\theta$

(Hint: $\int_{\theta=-\pi/4}^{\pi/4}\int_{r=0}^{a\sqrt{\cos 2\theta}} r \, dr\, d\theta$) Ans: a^2 sq. units

4) Find the area common to the circles $r = a\cos\theta$, $r = a\sin\theta$ Ans: $\frac{a^2}{4}$

5) Find area enclosed by the cardioid $r = a(1+\cos\theta)$ b/w $\theta = 0$ and $\theta = \pi$. Ans: $\frac{3\pi a^2}{4}$