R V COLLEGE OF ENGINEERING
(An autonomous institution affiliated to VTU, Belgaum)
DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MAT211BT)
UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-1

I. Objective type questions:

1. If A is a 3×4 matrix then rank of A cannot exceed _____.

2. Rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is _____.

3. Rank of identity matrix of order 4 is _____ .

4. If the rank of the transpose matrix A is 3 then the rank of matrix A is _____.

5. Rank of singular matrix of order 5 is_____.

II. Find the rank of the following matrices

1) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$
   Answer: rank of A=2

2) $A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 3 \end{bmatrix}$
   Answer: rank of A=2

3) $A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 2 & 0 & 4 & -2 \\ 12 & -3 & 9 & -3 \end{bmatrix}$
   Answer: rank of A=4

4) Find the values of k such that the rank of the matrix A is 3, where

$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$
   Answer: k=1

5) For which value of b the rank of the matrix

$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ is 2

   b=2.

6) Find the rank of A, B, A+B, BA and AB if

$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$

Answer: rank of A=2, rank of B=1, rank of (A+B)=2, rank of (AB)=0, rank of (BA)=1.

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MAT211BT)
UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-2

1. Test the consistency of the following system of equations
   2x+6y=-11
   6x+20y-6z=-3
   6y-18z=-1
   Answer: Inconsistent

2. Test the consistency of the following system and solve if the system is consistent
   $x_1 + 2x_2 + x_3 = 2$
   $3x_1 + x_2 - 2x_3 = 1$
   $4x_1 - 3x_2 - x_3 = 3$
   $2x_1 + 4x_2 + 2x_3 = 4$
   Answer: Consistent. $x_1 = 1, x_2 = 0, x_3 = 1.$

3. Find the value of k such that the following system of equations posses a non-trivial solution. Also find the solution of the system
   $4x_1 + 9x_2 + x_3 = 0$
   $kx_1 + 3x_2 + kx_3 = 0$
   $x_1 + 4x_2 + 2x_3 = 0$
   Answer: $k = 1, x_1 = 2k, y = -k, z = k.$

4. Investigate the values of λ and μ so that the equations
   2x+3y+5z=9
   7x+3y-2z=8
   2x+3y+ λ z= μ
   have (a) Unique solution   (b) Infinite number of Solutions (c) No Solution
   Answer: (a) $\lambda \neq 5$ (b) $\lambda = 5, \mu = 9$ (c) $\lambda = 5, \mu \neq 9$

5. Solve the system of equations by Gauss elimination method
   x-2y+3z=2
   3x-y+4z=4
   2x+y-2z=5
   Answer: $x = \frac{11}{5}, y = -\frac{7}{5}, z = -1$

6. Solve the system of equations by Gauss elimination method
   $6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$
   $12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$
   $3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$
   $-6x_1 + 4x_2 + x_3 - 18x_4 = -34$
   Answer: $x_1 = 3, x_2 = 1, x_3 = -2, x_4 = 1.$

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MAT211BT)
UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-3

1. Solve the following system of equations by Gauss –Jordon method
   $2x+y+z=10$
   $3x+2y+3z=18$
   $x+4y+9z=16$
   Answer: x=7, y=-9, z=5.

2. Find the inverse of a matrix A=$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ using Gauss-Jordan method.

   Answer: $A^{-1} = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

3. Solve the system of equations by Gauss elimination method
   $9x+2y+4z=20$
   $x+10y+4z=6$
   $2x-4y+10z=-15$
   Answer: x=2.7372, y=0.9872, z=-1.6525

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & DIFFERENTIAL EQUATIONS (MAT211BT)
<u>**UNIT-1: ELEMENTARY LINEAR ALGEBRA**</u>
<u>**TUTORIAL SHEET-4**</u>

1. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

   Answer: $\lambda = 0,1,2$ and $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

2. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$.

   Answer: $\lambda = 3,6,9$ and $X_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

3. The sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$ are
   Answer: 0 and 8.

4. If two eigenvalues of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigenvalue is

   _____.
   Answer: 0

5. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then the eigenvalues of A^{-1} are
   Answer: 1 and $\frac{1}{3}$.

1. Find the largest eigenvalue and the corresponding eigenvector of the matrix
   $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by Rayleigh power method. (Perform 5 iterations)

   Answer: $AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$

2. Find the largest eigenvalue and the corresponding eigenvector of the matrix

   $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh power method taking initial eigenvector as

   $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$. (Perform 5 iterations)

   Answer: $AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$