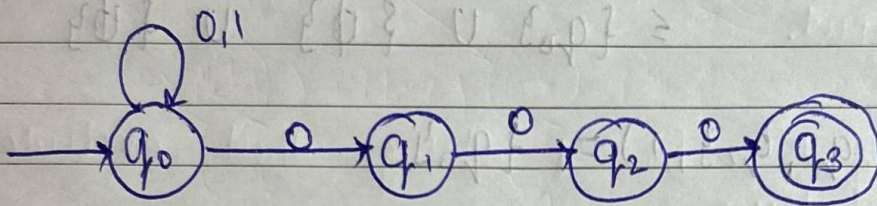## Homework-2

Problem-1:

Given $L = \{w | w$ ends with $000\}$

1. NFA Diagram:

Initial states - $\{q_0\}$
Final state - $\{q_3\}$

2. NFA to DFA

NFA Table :-

	0	1
q_0	$\{q_0, q_1\}$	q_0
q_1	q_2	ϕ
q_2	q_3	ϕ
q_3	ϕ	ϕ

(i) $\hat{\delta}(\{q_0\}, 0) = q_0 \cup q_1$
$\qquad = \{q_0, q_1\}$

$\hat{\delta}(\{q_0\}, 1) = \{q_0\}$

(ii) $\hat{\delta}(\{q_0, q_1\}, 0) =$
$\hat{\delta}(\{q_0\}, 0) \cup$
$\hat{\delta}(\{q_1\}, 0)$
$= \{q_0, q_1\} \cup \{q_2\}$
$\therefore \hat{\delta}(\{q_0, q_1\}, 0) = \{q_0, q_1, q_2\}$

$\hat{\delta}(\{q_0, q_1\}, 1) = \hat{\delta}(\{q_0\}, 1) \cup$
$\hat{\delta}(\{q_1\}, 1)$
$= \{q_0\} \cup \{\phi\}$
$\hat{\delta}(\{q_0, q_1\}, 1) = \{q_0\}$

(iii) $\hat{\delta}(\{q_0, q_1, q_2\}, 0) = \hat{\delta}(\{q_0\}, 0) \cup \hat{\delta}(\{q_1\}, 0) \cup \hat{\delta}(\{q_2\}, 0)$

$$= \{q_0, q_1\} \cup \{q_2\} \cup \{q_3\}$$

$\hat{\delta}(\{q_0, q_1, q_2\}, 0) = \{q_0, q_1, q_2, q_3\}$

$\hat{\delta}(\{q_0, q_1, q_2\}, 1) = \hat{\delta}(\{q_0\}, 1) \cup \hat{\delta}(\{q_1\}, 1) \cup \hat{\delta}(\{q_2\}, 1)$
$$= \{q_0\} \cup \{\phi\} \cup \{\phi\}$$

$\therefore \hat{\delta}(\{q_0, q_1, q_2\}, 1) = \{q_0\}$

(iv) $\hat{\delta}(\{q_0, q_1, q_2, q_3\}, 0) = \hat{\delta}(\{q_0\}, 0) \cup \hat{\delta}(\{q_1\}, 0) \cup \hat{\delta}(\{q_2\}, 0) \cup \hat{\delta}(\{q_3\}, 0)$

$$= \{q_0, q_1\} \cup \{q_2\} \cup \{q_3\} \cup \{\phi\}$$

$\therefore \hat{\delta}(\{q_0, q_1, q_2, q_3\}, 0) = \{q_0, q_1, q_2, q_3\}$

$\hat{\delta}(\{q_0, q_1, q_2, q_3\}, 1) = \hat{\delta}(\{q_0\}, 1) \cup \hat{\delta}(\{q_1\}, 1) \cup \hat{\delta}(\{q_2\}, 1) \cup \hat{\delta}(\{q_3\}, 1)$
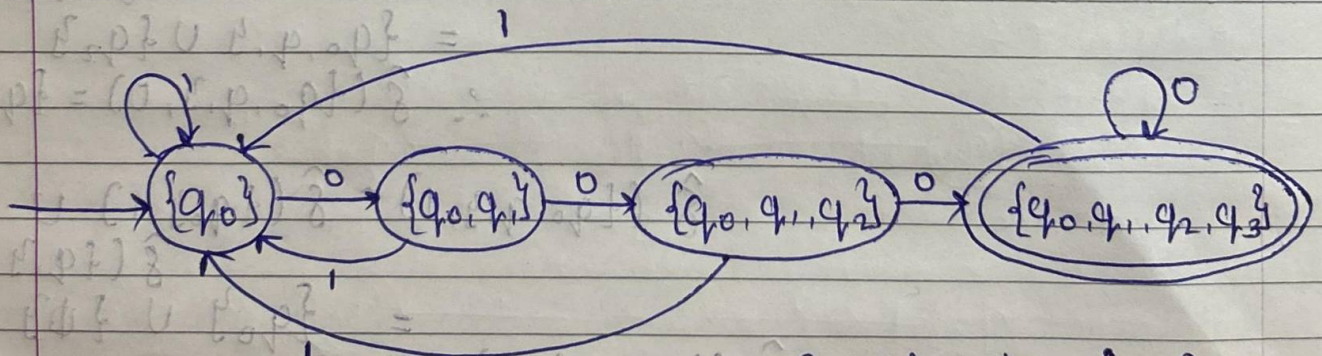$$= \{q_0\} \cup \{\phi\} \cup \{\phi\} \cup \{\phi\}$$

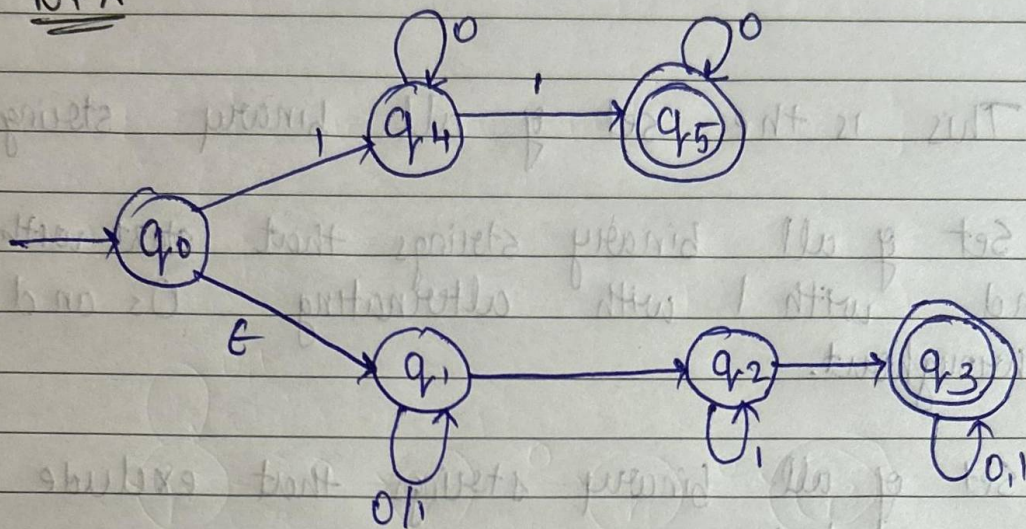$\therefore \hat{\delta}(\{q_0, q_1, q_2, q_3\}, 1) = \{q_0\}$

Initial state : $\{q_0\}$

Final state : $\{q_0, q_1, q_2, q_3\}$

Problem - 2

1. ## NFA

2. ## Regular Expression

$$(1\ 0^*\ 10^*\ 10^*\ )\ \cup\ (1^*\ 01^*\ 0\ (0\cup1)^*)$$

Problem - 3 t

1. $L = \{w \in \Sigma^* | w$ has an odd number of zeros.

   Regular Expression : $1^* 0 (1 \cup (01^* 0))^*$

2. $L = \{w \in \Sigma^* | w$ ends in a double letter$\}$

   Regular Expression: $(0 \cup 1)^* (00 \cup 11)$

3. $L = \{w \in \Sigma^* | w$ contains exactly one double letter$\}$

   Regular expression:

   $$(\varepsilon \cup 1)(01)^* 00 (10)^* (\varepsilon \cup 1) \cup (\varepsilon \cup 0)(10)^* 11 (01)^* (\varepsilon \cup 0)$$

Problem - 4

1.

a) This is the set of all binary strings.

b) Set of all binary strings that start with 0 and end with 1 with alternating 0s and 1s throughout.

c) Set of all binary strings that exclude 010 as a substring

2. **a)** $1^*(\epsilon \cup 0) 1^* (\epsilon \cup 0) 1^* (\epsilon \cup 0) 1^*$

b) $(\epsilon \cup 0 \cup 1)(01 \cup 10)^*$

c) $(1 \cup 01 \cup 001)^* 000 (1 \cup 10 \cup 100)^*$