Practice Problems 9 Solutions

Be sure to provide an answer for each question. You may work with other students, as well as use your notes, the book, and the internet. Do make sure you understand how to solve the problems and answer the questions, as similar ones may appear on the exams.

1. Express each of the following patterns in Sums/Products.
   (e.g.,$\sum_{i=1}^{5} 1 = 1 + 1 + 1 + 1 + 1 = 5$)

   a. $\sum_{i=1}^{5}(2i+3)^2$
   $= (\mathbf{2 \times 1 + 3})^{\mathbf{2}} + (\mathbf{2 \times 2 + 3})^{\mathbf{2}} + (\mathbf{2 \times 3 + 3})^{\mathbf{2}} + (\mathbf{2 \times 4 + 3})^{\mathbf{2}} + (\mathbf{2 \times 5 + 3})^{\mathbf{2}} = \mathbf{445}$

   b. $\sum_{i=1}^{5}(4i^3+3)$ = **$(4\times1^3+3) + (4\times2^3+3) + (4\times3^3+3) + (4\times4^3+3) + (4\times5^3+3) = 915$**

   c. $\prod_{i=1}^{6} i^2$ = **$(1)(2^2)(3^2)(4^2)(5^2)(6^2) = 518{,}400$**

   d. $\sum_{i=1}^{2}\sum_{j=1}^{4}(3ij)$ = $\sum_{i=1}^{2}(3i \times 1 + 3i \times 2 + 3i \times 3 + 3i \times 4)$ = **$(3 \times 1 \times 1 + 3 \times 1 \times 2 + 3 \times 1 \times 3 + 3 \times 1 \times 4) + (3 \times 2 \times 1 + 3 \times 2 \times 2 + 3 \times 2 \times 3 + 3 \times 2 \times 4) = 90$**

2. Use either summation notation or instruction count method to generate an equation expressing the total number of instructions required for the following function.

```
void foo(int arr[], int n){
        int sum = 0;                          →1
        int product = 1;                      →1

        for (int i=0; i<n; i++){              → 1 + (n+1) +1* n
                sum += arr[i];                → 1*n
        }
        for(int i=0; i<n; i++){               → 1 + (n+1) + 1*n
                product *=arr[i];             → 1*n
        }
        cout << sum << ", " << product << endl; →1
}
```

Cost = 1+1+1+(n+1) +1*n +1*n +1 +(n+1)+1*n +1*n +1 =6n+7
Or Cost = $1+1+1+\sum_{i=0}^{n} 1 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} 1 + 1 + \sum_{i=0}^{n} 1 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} 1 + 1$
$= 5 + 2\sum_{i=0}^{n} 1 + 4\sum_{i=0}^{n-1} 1 = 5 + 2(n+1) + 4(n) = 6n+7$

3. Use either summation notation or instruction count method to generate an equation expressing the total number of instructions required for the following function.

```
void unorderedPairs(int arr[], int n){
    for(int i=0; i<n; i++){                  → 1+ (n+1)+ 1*n
        for (int j=0; j<n;j++){              →(1*n)*1 + (1*n )*(n+1) + (1*n)*(1*n)
            cout << arr[i] << endl;          → (1*n)*(1*n)
            cout << arr[j] << endl;          → (1*n)*(1*n)
        }
    }
```

Cost = 1+(n+1)+1*n + (1*n)*1 + (1*n)*(n+1) + (1*n)*(1*n)+ (1*n)*(1*n)+ (1*n)*(1*n)= $4n^2$ + 4n + 2

4. Use either summation notation or instruction count method to generate an equation expressing the total number of instructions required for the following function.

```
void reverse(int arr[], int n){
    for (int i=0; i<n/2; i++){           → 1 + (n/2 + 1) + n/2 * 1
        int other = n-i-1;               → n/2 * 1
        int temp = arr[i];               → n/2 * 1
        arr[i] = arr[other];             → n/2 * 1
        arr[other] =temp;                → n/2 * 1
    }
}
```

Cost = $1 + \left(\frac{n}{2} + 1\right) + \frac{n}{2} * 1 + \frac{n}{2} * 1 + \frac{n}{2} * 1 + \frac{n}{2} * 1 + \frac{n}{2} * 1$ **= 3n+2**