Practice Problems 5 Solutions

Be sure to provide an answer for each question. You may work with other students in your recitation, as well as use your notes, the book, and the internet. Do make sure you understand how to solve the problems and answer the questions, as similar ones may appear on the exams.

Identify propositions within each sentence, assign a single-letter variables name for each proposition, provide compound propositions equivalent to each sentence using these variables, and use truth tables to exhaustively explore truth values of these compound propositions.

1. Roses are red and violets are blue.
   R = roses are red
   V = violets are blue
   R ∧ V

R	V	R ∧ V
T	T	T
T	F	F
F	T	F
F	F	F

2. Roses are red and either violets are blue or sugar is sweet
   R = roses are red
   V = violets are blue
   S = sugar is sweet
   R ∧ (V ∨ S)

R	V	S	(V ∨ S)	R ∧ (V ∨ S)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

3. Either it will rain or snow, but not both
   R = it will rain
   S = it will snow
   R ⊕ S
   OR
   (R ∧ ¬S) ∨ (¬R ∧ S)

R	S	¬R	¬S	(R ∧ ¬S)	(¬R ∧ S)	(R ∧ ¬S) ∨ (¬R ∧ S)
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

4. If Jane wins or if Jane loses, then Jane will be tired.
   W = Jane wins
   L = Jane loses
   T = Jane is tired
   (W ∨ L) -> T

W	L	T	(W ∨ L)	(W ∨ L) -> T
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

5. If the program halted, then the message was corrupted or the connection was ended.
H = the program halted
M = the message was corrupted
C = the connection was ended
H -> (M ∨ C)

H	M	C	(M ∨ C)	H -> (M ∨ C)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

Use propositional logic to prove that the following arguments are valid. Do not use truth tables.

1. $(A \rightarrow C) \wedge (C \rightarrow \neg B) \wedge B \therefore \neg A$
   1) A → C   Hypothesis
   2) C → ¬B   Hypothesis
   3) B   Hypothesis
   4) ¬¬B   Double Negation 3
   5) ¬C   Modus Tollens 2,4
   6) ¬A   Modus Tollens 1,5

2. $(P \rightarrow (Q \wedge R)) \wedge P \therefore (P \wedge Q)$
   1) P → (Q ∧ R)   Hypothesis
   2) P   Hypothesis
   3) Q ∧ R   Modus Ponens 1,2
   4) Q   Simplification 3
   5) P ∧ Q   Conjunction 2,4

3. $Z \therefore (Z \wedge Z)$
   1) Z   Hypothesis
   2) Z ∧ Z   Idempotence 1

4. $A \therefore (A \vee B) \wedge (A \vee C)$
   1) A   Hypothesis
   2) A ∨ B   Addition 1
   3) A ∨ C   Addition 1
   4) (A ∨ B) ∧ (A ∨ C) Conjunction 2,3

5. $(I \rightarrow H) \land (G \lor \lnot H) \land I \therefore G$

    1) $I \rightarrow H$ **Hypothesis**
    2) $G \lor \lnot H$ **Hypothesis**
    3) I   **Hypothesis**
    4) $\lnot H \lor G$ **Commutativity 2**
    5) $H \rightarrow G$ **Conditional Identities 4**
    6) $I \rightarrow G$ **Hypothetical Syllogism 1,5**
    7) G   **Modus Ponens 3,6**