

Numeric

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Chapter Number System :-

- (1) Natural Numbers :- $(1, 2, 3, \dots)$ pd divisible (E)
- (2) Whole Numbers :- $(0, 1, 2, 3, \dots)$ bni to mre k
- (3) Rational Numbers :- $(\frac{p}{q}, q \neq 0)$ PES valgavas
- (4) Integers :- $(\dots, -3, -2, -1, 0, 1, 2, 3, \dots)$
- (5) Real Numbers
- (6) Imaginary Numbers

example . N = 235

place value	face value
$PV(2) = 2 \times 100 = 200$	$\therefore fV(2) = 2$
$PV(3) = 3 \times 10 = 30$	$\therefore fV(3) = 3$
$PV(5) = 5 \times 1^0 = 5$	$\therefore fV(5) = 5$

- (7) Prime Number (example :- 2, 3, 5, 7, ...)
- (8) Composite Number (example :- 4, 6, 8, ...)
- (9) Co-prime Number :- (whose H.C.F is one) is called Co-prime example :- (i) (2, 3) (ii) (3, 7)
- (10) Twin-prime Number. (Two prime numbers whose difference is 2 are called Twin Prime Numbers. example :- (i) (3, 5) (ii) (5, 7) (iii) (11, 13))

Divisibility Test :-

- (1) divisible by 2 :-

→ Unit place must be 0, 2, 4, 6, 8

- (2) divisible by 3 :-

→ Sum of individual digits must be divisible by 3.

example: $123 = 1+2+3 = 6$

(3) divisible by 9 :-

→ sum of individual digits divisible by 9.

example: 279.

(4) divisible by 4 :-

→ last 2 digits must be divisible by 4.

example: (1) 496138 :- 38 is not divisible by 4.

(2) 498104 :- 04 is divisible by 4.

(5) divisible by 8 :-

→ last 3 digits must be divisible by 8.

example: 167352.

(6) divisible by 16 :-

→ last 4 digits must be divisible by 16.

(7) divisible by 10 :-

→ Unit place must be 0.

(8) divisible by 5 :-

→ Unit place must be 0 or 5.

(9) divisible by 11 :-

→ A Number is divisible by 11, if difference between sum of digits at odd place and sum of digits at even place is either 0 or divisible by 11.

example:- 294354

$$\rightarrow \text{sum of odd place} = 7 + 4 + 3 + 9$$

$$\rightarrow \text{sum of even place} = 1 + 5 + 4 + 2$$

$$\rightarrow \text{difference} = 23 - 12 = 11 \text{ is divisible by 11.}$$

(10) divisible by 25 :-

→ last two digits must be 00 or divisible by 25.

(11) divisible by 7 OR 13 :-

example:-

4537792

$$4 \Big| 537 \quad | 792$$

$$= \text{sum of odd places} - \text{sum of even places}$$

$$= (792 + 4) - (537)$$

$$= 259 \text{ is divisible by 7.}$$

579488

$$579 \Big| 488$$

$$= 488 - 579$$

= 89 is divisible by

+ 17 and 13

Note:- If Number is divisible by p as well as q, where p and q are co-primes; then given number is divisible by $p \times q$.

example:- $80 = 16 \times 5$.

$$15 = 5 \times 3, \text{ it checks } \Rightarrow 3 \times 5 = 15$$

$$p+q+p = 9 + 9 + 1 = 19 \text{ is not divisible}$$

The sum of place value of 3 in number 503535 is

$$5+0+3+1 = 9 \text{ is not divisible}$$

answer:-

$$3000 + 301 = 3030$$

example find Multiple of 11 in following.

(a) 112144

(b) 44735500

(c) 969756

(d) 978626

answer:- (a) $6-7 = -1 \times$

(b) $12-16 = -4 \times$

(c) $19-22 = -3 \times$

(d) $19-19 = 0 \checkmark$

example:- $\begin{array}{r} 4 \alpha 3 \\ + 984 \\ \hline \end{array}$

$\begin{array}{r} 1367 \\ + 984 \\ \hline \end{array}$

1367 is divisible by 11. (SPF)

$a+b=?$

$(7+3)-(b+1)=0$ [if Must be zero because two digit number is not allowed]

$10-b-1=0$

$9-b=0 \quad b=9$

$a=1$

$a+b=10$

example The sum of 4 consecutive even numbers addition is 180 then what is the sum of next consecutive even number.

answer :- example :- assume 2, 4, 6, 8 then Next even, 10, 12, 14, 16

8 difference

$$\begin{array}{l} \text{sum of 'Next even} = 8(4) + 180 \\ \text{4 Numbers} \quad = 32 + 180 = 212. \end{array}$$

example 5 P, 9

3 2 7

2 9 8

1 1 1 4

find Maximum possible
Value of 9.

answer :-

$\boxed{9=7}$

$9+7+8=24$ (2 carry goes to next step)

$5+3+2=10$ (but there is 11 so one carry come from previous step).

$2+P+2+9=11$, $P+9=7$, $P=0$, $9=7$

example Institute runs a corporate training programme. At the end of running first programme, it's total taking is 38950. There were more than 45 and less than 100 participants, then what was the participants fee for the programme.

(a) 410

(b) 450

(c) 500

(d) 510

answer :- (a) 38950 = 95 (b) $\frac{38950}{450} = 86.55$ X

(Note:- logic is Number of Participants is integer)

$$(c) \frac{38950}{500} = 77.9 \quad (\text{Ans}) \quad \frac{38950}{510} = 76.37 \quad (\text{Ans})$$

example A young girl counted on her left hand as

thumb: 1

index: 2

Middle: 3

ring: 4

little: 5

ring: 6

Middle: 7

index: 8

Thumb: 9

index: 10

Middle: 11

so on, she counted upto 1994, she ended on which position.

answer: $T_n = a + (n-1)d$ (a = first term, d = difference)

Thumb: 1, 9, 17, ..., $8n-7$

1994 can not be expressed as $8n+1$.

index: 2, 8, 10, 16, ..., $8n-6$, $8n$

$$1994 = 8n - 6 \quad \text{OR} \quad 8n = 1994$$

$$n = 250$$

→ so, she ended on index finger.

Digit At Unit's Place :-

Note:- In below all,
right side of equal to
is only unit digit of
(that Number)

(1) $0, 1, 5, 6 \rightarrow$ raise to any power then unit digit is same. ($0^k = 0, 1^k = 1, 5^k = 5, 6^k = 6$)

(2) $4 \rightarrow 4^1 = 4$ }
 $\rightarrow 4^3 = 4$ } 4 raise to any odd power then
 $\rightarrow 4^5 = 4$ } unit digit is 4.

$\rightarrow 4^2 = 6$ }
 $\rightarrow 4^4 = 6$ } 4 raise to any even power
 $\rightarrow 4^6 = 6$ } then unit digit is 6.

(3) $9 \rightarrow 9^1 = 9$ }
 $\rightarrow 9^3 = 9$ } 9 raise to any odd power then
 $\rightarrow 9^5 = 9$ } unit digit is 9.

$\rightarrow 9^2 = 1$ }
 $\rightarrow 9^4 = 1$ } 9 raise to any even power
then unit digit is 1.

(4) $2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 6$
 $2^5 = 2$ } and then cycle starts.

(5)

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 3 \quad \text{--- (and cycle starts)}$$

(6)

$$7^1 = 7$$

$$7^2 = 49$$

$$7^3 = 3$$

$$7^4 = 1$$

$$7^5 = 7 \quad \text{--- (and cycle starts)}$$

(7)

$$8^1 = 8$$

$$8^2 = 4$$

$$8^3 = 2$$

$$8^4 = 6$$

$$8^5 = 8 \quad \text{--- (and cycle starts)}$$

example:- find digits at unit place of $(123)^{99}$

Answer:-

$$99 = 96 + 3 = 96 \quad \text{is divisible by 4.} \quad (A)$$

$$(123)^{99} = (123)^{96} \cdot (123)^3$$

$$= (3) \cdot (3)^3$$

$$= 7 \text{ at Unit place.}$$

$$\left. \begin{array}{l} 3^{4k} = 1 \\ 3^1 = 1 \\ 3^2 = 9 \\ 3^3 = 7 \end{array} \right\}$$

example The last digit in decimal representation of

$$\left(\frac{1}{5}\right)^{2000}$$

$$\left(\frac{1}{5}\right)^{2000} = (0.2)^{2000}$$

answer $\rightarrow 2000/4$ is divisible then answer is 6. ($2^{4K}=6$)

example The last digit of

$$(898)^{898} \times (7977)^{73} \times (232)^{112} \\ (333)^{87} \times (89)^{12345} \times (84)^{807}$$

answer :- Answer is 6.

example $(897) \times (222) \times (333) \times (666)$ find digit at unit place.

answer :-

$$\begin{aligned} & \rightarrow (555) \rightarrow (5) \\ & \rightarrow (222) \rightarrow 2 \quad \left(\begin{array}{l} 5 \times 2 = 10 \\ \text{So } 0 \times \text{ Any Number} \end{array} \right) \\ & \rightarrow (898) \rightarrow 8 \quad \left(\begin{array}{l} 0 + 0 + 0 = 0 \\ \text{So } 0 \end{array} \right) \end{aligned}$$

example $(2464) \times (615) \times (131)$ find digit at unit place?

answer :-

$$(4) \text{ odd} \times (5) \text{ Any} \times (1) \text{ Any}$$

$$4 \times 5 \times 1 \\ = [0]$$

example :- $81 \times 82 \times 83 \times \dots \times 89 = ?$
Unit place?

answer :- 0.

$$82 \times 85 = \dots [0]$$

(Note:- (i) first check there is pair of (2,5) or (6,5) in the question, because if those pair present then Unit digit must be zero.)

example $(2467)^{153} \times (341)^{72}$

Unit digit place.

answer :- $= 7.$

$$\left\{ \times (7)^{15^2} \times (7)^1 \times (1)^{72} = 7 \right\}$$

example $(264)^{102} + (264)^{103}$

Unit digit place

answer :- $0 \times (008) \times \left\{ (4)^{10^2} + (4)^{10^3} \Rightarrow 6 + 4 = 10 \right\}$

example find Total Number of prime factors in expansion of $(4)^{11} \times (7)^5 \times (11)^2$.

answer :- $\rightarrow (4) \times (7)^5 \times (11)^2$

$\rightarrow (2)^{22} \times (7)^5 \times (11)^2$

$\rightarrow 2, 7, 11$ is prime factors (Numbers).

$\rightarrow 22 + 5 + 2 = 29$. (There are total 29. Prime factors.)

example what is the Number of zeros at the end of product of Numbers from 1 to 100!

answer :- $1 \times 2 \times 3 \times \dots \times 100$

$\rightarrow \text{Number of zeros} = \min \left(\frac{\text{number of } 5's}{5}, \frac{\text{number of } 2's}{2} \right)$

number of
(divisible by)
 $2^3 = \left[\frac{100}{2} \right] = 50$
 $\left[\frac{50}{2} \right] = 25$
 $\left[\frac{25}{2} \right] = 12$
 $\left[\frac{12}{2} \right] = 6$

$\left[\frac{5}{2} \right] = 3$
 $\left[\frac{3}{2} \right] = 1$
 $\left[\frac{1}{2} \right] = 0$

number of 5's $= \left[\frac{100}{5} \right] = 20$
 $\left[\frac{20}{5} \right] = 4$
total = 24.

Number of zeros = 24.

example find Number of zeros.

$$10 \times 20 \times 30 \times 40 \times 50 \times \dots \times 2000$$

answr:

$$(10 \times 20 \times 30 \times 40 \times 50 \times \dots \times 2000)$$

$$\rightarrow (1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 200) \times (10^{200})$$

$$\text{Number of zeros} = \min(2^{\text{'s}} \text{ divisible}, 5^{\text{'s}} \text{ divisible})$$

(as we know minimum come from 5's divisible)

$$\left[\frac{200}{5} \right] = 40 \quad \left[\frac{8}{5} \right] = (1) \times (1) \times (1) \quad \text{total} = 40 + 8 + 1$$

$$\left[\frac{40}{5} \right] = 8 \quad \left[\frac{1}{5} \right] = 0 \quad = 49$$

$$\text{total zeros} = 200 + 49$$

$$= 249$$

LCM & HCF

- LCM : Least Common Multiple.
- HCF : Highest Common Factor.

example find LCM of 2, 4

answer

$$\begin{array}{|c|c|c|} \hline 2 & 2 & 4 \\ \hline 2 & 1 & 2 \\ \hline \end{array} \quad \text{LCM} = 2 \times 2 = 4$$

example find HCF and LCM of 15, 30, 45

answer

$$\begin{aligned} 15 &= \underline{\underline{3}} \times \underline{\underline{5}} \\ 30 &= \underline{\underline{2}} \times \underline{\underline{3}} \times \underline{\underline{5}} \\ 45 &= \underline{\underline{3}} \times \underline{\underline{3}} \times \underline{\underline{5}} \end{aligned}$$

$$\therefore \text{HCF} = 3 \times 5 = 15$$

$$\text{LCM} = 15 \times 2 \times 3 = 90$$

Note : H.C.F. \times L.C.M. = Product of Numbers
(only for two Numbers)

→ HCF & LCM of fractions:

$$\bullet \text{HCF} \left(\frac{3}{2}, \frac{5}{4}, \frac{7}{8} \right) = \frac{\text{H.C.F.}(3, 5, 7)}{\text{LCM}(2, 4, 8)} = \text{HCF of Numerator}$$

$$\bullet \text{LCM} \left(\frac{3}{2}, \frac{5}{4}, \frac{7}{8} \right) = \frac{\text{LCM of Numerator}}{\text{HCF of denominator}}$$

$$= \frac{\text{LCM}(3, 5, 7)}{\text{HCF}(2, 4, 8)}$$

example

answer:

Method :- 1

$$= \text{HCF} \left(\frac{63}{100}, \frac{105}{100}, \frac{21}{10} \right)$$

$$= \frac{\text{HCF}(63, 105, 21)}{\text{LCM}(100, 100, 10)}$$

$$= [0.21]$$

Ignore
two decimal
Point

method :- 2

$$= \text{HCF}(0.63, 1.05, 2.1)$$

$$= \text{HCF}(0.63, 1.05, 2.10)$$

$$= \text{HCF}(63, 105, 210)$$

$$= [21]$$

$$= [0.21]$$

Put ignored
decimal
Point for
final answer

- LCM is factor of HCF :- False.
- HCF is factor of LCM :- True.
- LCM is multiple of HCF :- True.
- G.C.D. = H.C.F. (GCD = Greatest Common divisor)

example L.C.M. of three Numbers is 120 then which of the following cannot be their H.C.F.?

(a) 8 (b) 12 (c) 24 (d) 35

answer 35 (d) (H.C.F. is factor of L.C.M.)

example A, B, C are three Numbers.

$$\text{LCM}(A, B) = B$$

$$\text{LCM}(B, C) = C$$

$$\text{LCM}(A, B, C) = ?$$

answer :-

$$= \text{LCM}(A, B, C)$$

$$= \text{LCM}(B, C) \quad (\text{LCM}(A, B) = B)$$

$$= C \quad (d = (a, b))$$

$$x \in (d, c) \quad (c = d + n)$$

example Three Numbers are in ratio 1:2:3 & their HCF is 12 and the Numbers are.

answer -

- first Number = x
- Second Number = $2x$
- Third Number = $3x$
- HCF = $x = 12$
- Numbers = 12, 24, 36.

example If Sum of Two Numbers is 36 and their HCF and LCM are 3 and 105 then find sum of reciprocal of numbers.

answer $\rightarrow \text{HCF} \times \text{LCM} = ab$ $a+b=36$ $ab=3 \times 105$

$$\begin{aligned}\rightarrow \frac{1}{a} + \frac{1}{b} &= \frac{a+b}{ab} = \frac{36}{3 \times 105} \\ &= \frac{12}{105} = \frac{4 \times 3}{105} \\ &= \frac{4}{35}\end{aligned}$$

example The sum of two Numbers is 528, and their HCF is 33 then Number of pair of Numbers satisfying above condition

answer:

$$\therefore a+b = 528 \quad (a, b = \text{Numbers})$$

\because HCF is 33 then HCF is factor of both Numbers.

$$\therefore 33x + 33y = 528 \quad (\text{where } x, y = \text{co-prime Numbers})$$

$$\therefore x+y = 16$$

\rightarrow we have choose those pair where both are coprimes Number and their sum is 16.

$$\rightarrow (1, 15), (3, 13), (5, 11), (7, 9)$$

\rightarrow so, there are 4 pairs who satisfying these conditions.

example HCF of 2 Number is 23 and other factors of their LCM are 13 and 14 then find larger of two Number.

answer - $\rightarrow \text{HCF} = 23$

$\rightarrow \text{LCM} = \text{HCF} \times 13 \times 14$

$\text{LCM} = 23 \times 13 \times 14$

then bigger Number $= 23 \times 14 = 322$.

Smaller Number $= 23 \times 13 = 299$.

example HCF of 3240, 3600, b is 36.

$\text{LCM} = 2^4 \times 3^5 \times 5^2 \times 7^2$. find Third Number (b).

answer

$3600 = 2 \times 3 \times 2 \times 3 \times 2 \times 2 \times 5 \times 5 = 2^4 \times 3^2 \times 5^2$

$3240 = 2 \times 5 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^3 \times 3^4 \times 5$

$b = 3^2 \times 2^2 \times 3^3 \times 7^2 = [3^5 \times 2^2 \times 7^2]$

$\text{HCF} = 2^2 \times 3^2$

$\text{LCM} = 2^4 \times 3^5 \times 5^2 \times 7^2$

Third Number $= 3^5 \times 2^2 \times 7^2$

example 21 Mango Tree, 42 apple Tree, 56 orange tree have to planted in row such that each row contains same number of one variety. find Minimum Number of rows in which trees planted?

answer - How many tree in one row $= \text{HCF}(21, 42, 56) = 7$

∴ How many rows $= \max(21, 42, 56) / 7 = 56 / 7 = 8$

example find the least Number which is exactly divisible by
32, 36, 45, 60 & 80.

answer: (divisible by \therefore LCM) \rightarrow or divided by
divided by \therefore HCF

\rightarrow for Exactly divisible by we have to find

$$\text{LCM } (32, 36, 45, 60, 80)$$

$$\therefore 32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$\therefore 36 = 2 \times 2 \times 3 \times 3$$

$$\therefore 45 = 5 \times 3 \times 3$$

$$\therefore 60 = 5 \times 2 \times 3 \times 2$$

$$\therefore 80 = 5 \times 2 \times 2 \times 2 \times 2$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 1440.$$

example find the least number which can divided by
6, 7, 8, 9, 12, leaves the same remainder 1 in each case.

answer:

$$\rightarrow \text{LCM } (6, 7, 8, 9, 12)$$

$$\therefore 6 = 2 \times 3$$

$$\therefore 7 = 7 \times 1$$

$$\therefore 8 = 2 \times 2 \times 2$$

$$\therefore 9 = 3 \times 3$$

$$\therefore 12 = 2 \times 2 \times 3$$

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 504$$

$\therefore 504$ is exactly divisible by 6, 7, 8, 9, 12 but here it
leaves one remainder so answer is $= 504 + 1 = 505$

example find the greatest number of \downarrow digits which is divisible by 15, 21, 36.

answer:-

$$\text{LCM}(15, 21, 36)$$

$$15 = 5 \times 3$$

$$21 = 7 \times 3$$

$$36 = 12 \times 3$$

$$\text{LCM} = 5 \times 7 \times 12 \times 3$$

$$\text{LCM} = 35 \times 36$$

$$\boxed{\text{LCM} = 1260}$$

\rightarrow Greatest Number = 99,999

$$\begin{array}{r}
 & 78 \\
 & \boxed{99999} \\
 1260 & \overline{)8820} \\
 & \underline{8820} \\
 & \overline{11799} \\
 & \underline{11340} \\
 & \overline{459}
 \end{array}$$

\rightarrow Answer = 99999 - 459

$$\boxed{= 99540}$$

example find the smallest Number of 5 digits which is exactly divisible by 16, 24, 36, 54.

answer

$$\rightarrow \text{LCM}(16, 24, 36, 54)$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$8 \times 2 = 16$$

$$24 = 2 \times 2 \times 3 \times 2$$

$$8 \times 3 = 24$$

$$36 = 2 \times 3 \times 2 \times 3$$

$$8 \times 9 = 72$$

$$54 = 2 \times 3 \times 3 \times 3$$

$$8 \times 27 = 216$$

$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 3 \times 2$$

$$= 48 \times 9$$

$$02 \cdot 21 = 192$$

$$= 432$$

FIND THE REQUIRED NUMBER

23

432	10,000 ← smallest five digit Number
	864
	1360
	1296
	64

$$\therefore \text{Answer} = 10000 + (432 - 64)$$

$$= 10368$$

example find a greatest number of 4 digits which when divided by 4, 5, 6, 7, 8 leaves remainder as 1, 2, 3, 4, 5.

answer :-

$$\text{LCM}(4, 5, 6, 7, 8) :$$

$$4 = 2 \times 2 \quad (d+n)(d-n) = d^2 - n^2$$

$$5 = 5 \quad (d+n)d^2 + d^2n^2 - d^2 - n^2 = (d-n)$$

$$6 = 3 \times 2 \quad (d-n)d^2 - d^2n^2$$

$$7 = 7 \quad (d+n)^2 - (d-n)^2 = (d+n) = d + n$$

$$8 = 2 \times 2 \times 2 \quad (d+n)^2 + (d-n)^2 = d^2 + n^2$$

$$\text{LCM} = 2 \times 2 \times 15 \times 14 = (2+4)(2+6) = (2+6+8)$$

$$\text{LCM} = (60 \times 14) = 2 \times 2 \times 3 \times 5 \times 7 = 840$$

$$\boxed{\text{LCM} = 840}$$

Greatest 4 digit Number = 9999

$$\begin{array}{r}
 & \overline{1} \\
 840 & \overline{9999} \\
 & \overline{840} \\
 & \overline{1599} \\
 & \overline{840} \\
 & \overline{759}
 \end{array}$$

E.F.O (i)

greatest 4 digit number

$$\text{divisible by } 9999 - 759$$

$$= 9240$$

→ Number of 4 digits divided

$$\begin{aligned}
 & \text{by } 4, 5, 6, 7, 8 \\
 & \text{leaves remainder as } 1, 2, 3, 4, 5
 \end{aligned}
 = 9240 - 3$$

$$= 9237$$

why 3!

$$\rightarrow 4-1=3$$

$$\rightarrow 5-2=3$$

$$\rightarrow 6-3=3$$

$$\rightarrow 7-4=3$$

$$8-5=3$$

$$\rightarrow (a+b)^2 = a^2 + b^2 + 2ab$$

$$\rightarrow (a-b)^2 = a^2 + b^2 - 2ab$$

$$\rightarrow a^2 - b^2 = (a-b)(a+b)$$

$$\rightarrow (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$= a^3 + b^3 + 3ab(a+b)$$

$$\rightarrow (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$$

$$= a^3 - b^3 - 3ab(a-b)$$

$$\rightarrow a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\rightarrow a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\rightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$\rightarrow a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

(If $a+b+c=0$, $a^3 + b^3 + c^3 = 3abc$)

(If $a^3 + b^3 + c^3 = 3abc$ then $a+b+c=0$)

example

Convert the following in Vulgar fraction.

(i) 0.75

answer

$$\begin{array}{r} 75 \\ \hline 100 \\ = \boxed{\frac{3}{4}} \end{array}$$

$$(ii) 5172.49 + 378.352 + (?) = 9318.678$$

find (?)

 \rightarrow

$$\begin{array}{r}
 5172.49 \\
 + 378.352 \\
 \hline
 5550.842
 \end{array}$$

(iii) $0.5 \times 5.6 \div 0.5 \times 12$

answer

$$= 0.5 \times 5.6 \div 0.5 \times 12$$

$$= (0.5) \times (5.6 \div 0.5) \times 12$$

$$= (0.5) \times (5.6 \div 5) \times 12$$

$$= (0.5 \times 11.2 \times 12)$$

$$= 11.2 \times 6$$

$$= \boxed{67.2}$$

(iv) $0.01 \times 0.1 - 0.001 \div 10 + 0.01$

$$= 0.01 \times 0.1 - 0.0001 + 0.01$$

$$= 0.001 - 0.0001 + 0.01$$

$$= 0.0009 + 0.01$$

$$= \boxed{0.0109}$$

(5) If $\frac{1}{3.718} = 0.2689$ then find

value of $\frac{1}{0.0003718}$

answer

$$\boxed{2689}$$

(6) The Product of two numbers is 0.008 and one number is $(\frac{1}{5})^{\text{th}}$ of other then find Numbers.

answer

both Numbers can be (x) $\frac{x}{5}$

$$(x) \left(\frac{x}{5}\right) = 0.008$$

$$x^2 = \frac{8 \times 5}{1000} \times 2.0$$

$$x = \frac{40}{1000}$$

$$x = \frac{4}{100}$$

$$x = 0.2$$

both Numbers are $= 0.2, \frac{0.2}{5}$

$$= 0.2, 0.04$$

(7) Express into Vulgar fraction, $0.\overline{37}$ (or express in p/q form)

answer

$$x = 0.\overline{37}$$

$$x = 0.373737$$

$$x = \frac{37}{99} \rightarrow \text{shortcut}$$

(8) express into Vulgar fraction, $0.\overline{053}$

answer :- $= \boxed{\frac{53}{999}}$

(9) express into vulgar fraction $3.\overline{142857}$

answer:-

$$= 3 \frac{142857}{999999} \rightarrow \text{Mixed fraction form}$$

(10) simplify :-

$$\frac{0.009 \times 0.036 \times 0.016 \times 0.08}{0.002 \times 0.0008 \times 0.0002}$$

answer:-

$$= \frac{3 \times 6 \times 4}{2}$$

$$= 36$$

Hint :- Count decimal places and then calculate it

(11) find Value of

$$0.21 \times 0.21 \times 0.21 + 0.021 \times 0.021 \times 0.021$$

$$0.63 \times 0.63 \times 0.63 + 0.063 \times 0.063 \times 0.063$$

answer :-

$$\frac{(21 \times 21 \times 21) \times 10 + (21 \times 21 \times 21)}{(63 \times 63 \times 63) \times 10 + (63 \times 63 \times 63)}$$

$$= \boxed{\frac{1}{27}}$$

(12) Simplify :-

$$\frac{2.75 \times 2.75 \times 2.75 - 2.25 \times 2.25 \times 2.25}{2.75 \times 2.75 + 2.75 \times 2.25 + 2.25 \times 2.25}$$

answer

$$a = 2.75$$

$$b = 2.25$$

$$= \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$= 0.5$$

(13) Simplify :-

$$\frac{(0.01)^2 + (0.22)^2 + (0.333)^2}{(0.001)^2 + (0.022)^2 + (0.0333)^2}$$

answer :-

$$a = 0.01, b = 0.22, c = 0.333$$

$$= \frac{a^2 + b^2 + c^2}{(a/10)^2 + (b/10)^2 + (c/10)^2}$$

$$= 100$$

$$(14) \frac{5}{8}, \frac{7}{12}, \frac{13}{16}, \frac{16}{29}, \frac{3}{4}$$

Set in ascending Order.

Answer:-

LCM of $(8, 12, 16, 29, 4)$:

$$8 = \underline{2} \times \underline{2} \times \underline{2}$$

$$12 = \underline{3} \times \underline{2} \times \underline{2}$$

$$16 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$$

$$29 = \underline{29}$$

$$4 = \underline{2} \times \underline{2}$$

$$\text{LCM} = 29 \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3} \times \underline{2}$$

$$\text{LCM} = 29 \times 48$$

$$(A-E)m + (E-D)m + (D-A)m = (80A)m$$

$$\therefore \frac{5 \times 29 \times 6}{48 \times 29}, \frac{7 \times 29 \times 4}{48 \times 29}, \frac{13 \times 3 \times 29}{48 \times 29}, \frac{16 \times 48}{29 \times 48},$$

$$\frac{3 \times 29 \times 12}{48 \times 39}$$

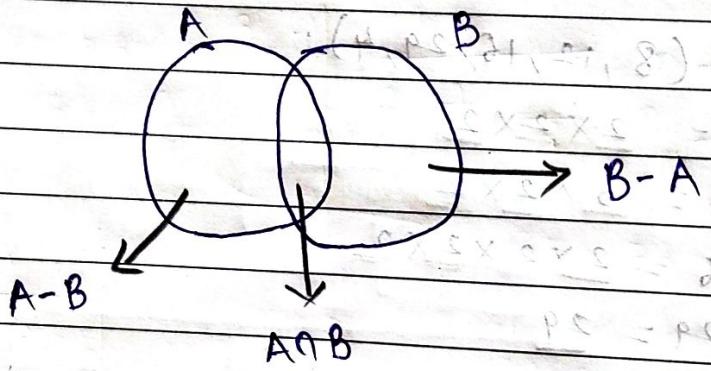
$$\therefore \frac{30 \times 29}{16 \times 48}, \frac{28 \times 29}{16 \times 48}, \frac{39 \times 29}{16 \times 48}, \frac{16 \times 48}{16 \times 48}, \frac{36 \times 29}{16 \times 48}$$

$$= \frac{16}{29}, \frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}$$

$\rightarrow (), \{ \}, []$

Priority Order

\rightarrow Set :-



$$\rightarrow n(A - B) + n(A \cap B) = n(A)$$

$$\rightarrow n(B - A) + n(A \cap B) = n(B)$$

$$\rightarrow n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$

$$\rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

example

Simlify :-

$$1 \div [1 + 1 \div \{1 + 1 \div [1 + 1 \div (1 + 1 \div 2)]\}] + 1$$

answer :-

13
/ 8

example Simplify :-

$$\left[3\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6} \right) \right\} \right]$$

answer :- (first solve bar →)

$$= \boxed{78}$$

example If $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$

then find $\frac{c^2-a^2}{c^2+a^2}$

answer

$$\therefore \frac{c^2-a^2}{c^2+a^2} = \frac{\left(\frac{c}{a}\right)^2 - 1}{\left(\frac{c}{a}\right)^2 + 1}$$

$$\left(\frac{b}{c} \times \frac{a}{b} = \frac{4}{5} \times \frac{15}{16} \right)$$

$$\frac{a}{c} = \frac{3}{4}, \quad \frac{c}{a} = \frac{4}{3}$$

$$\therefore \frac{c^2-a^2}{c^2+a^2} = \frac{\frac{16}{9}-1}{\frac{16}{9}+1} = \frac{16-9}{16+9} = \boxed{\frac{7}{25}}$$

example find Value of

$$4 - \frac{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}{5}$$

answer :-

$$= \boxed{\frac{1}{8}}$$

example If $4x + 5y = 83$ & $\frac{3x}{2y} = \frac{21}{22}$

find $y-x = ?$

answer :-

$$\frac{3x}{2y} = \frac{21}{22}$$

$$\therefore \frac{x}{y} = \frac{7}{11}$$

$$\therefore \frac{y}{x} = \frac{11}{7}$$

$$\therefore \frac{y-x}{x} = \frac{4}{7}$$

$$\therefore \frac{y-x}{7} = \frac{4}{7}$$

$$\therefore y-x = 4$$

$$1. 4x + 5y = 83$$

divided by x .

$$\therefore 4 + 5\left(\frac{y}{x}\right) = \frac{83}{x}$$

$$\therefore 4 + 5\left(\frac{11}{7}\right) = \frac{83}{x}$$

$$\therefore \frac{28 + 55}{7} = \frac{83}{x}$$

$$\therefore x = 7$$

example

find value of

$$\left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{100}\right)$$

answer :-

$$\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{99}{100}$$

$$= \frac{2}{100} = 2\%$$

$$= \boxed{\frac{1}{50}} = 2\%$$

example

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{9 \times 10}$$

answer :-

$$= \left(\frac{3-2}{2 \times 3}\right) + \left(\frac{4-3}{3 \times 4}\right) + \dots + \left(\frac{10-9}{9 \times 10}\right)$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{9} - \frac{1}{10}$$

$$= \frac{1}{2} - \frac{1}{10}$$

$$= \frac{10-2}{2 \times 10}$$

$$= \frac{8}{2 \times 10}$$

$$= \boxed{\frac{2}{5}}$$

example The boy was asked to multiple certain number by 53. he multiplied it by 35 and got his answer less than correct one by 1206. find the number to be multiplied.

answer :-

$$\therefore x \times 53 = n$$

$$53 = \frac{n}{x}$$

$$x \times 35 = n - 1206$$

$$35 = \frac{n}{x} - \frac{1206}{x}$$

$$35 = 53 - \frac{1206}{x}$$

where

x = Number original,

n = original answer.

$$\frac{1206}{x} = 18$$

$$x = 67$$

example $\frac{4}{15}$ of $\frac{5}{7}$ of a number is greater than

$\frac{4}{9}$ of $\frac{2}{5}$ of same number by 8.

what is the half of that number.

answer

$$\frac{4}{15} \left(\frac{5}{7}(a) \right) - 8 = \frac{4}{9} \left(\frac{2}{5}(a) \right) \quad (a = \text{original number})$$

$$\frac{4 \times 5 \times a}{15 \times 7} - 8 = \frac{4 \times 2 \times a}{9 \times 5}$$

$$a = 630$$

$$\frac{a}{2} = 315 \rightarrow \text{Answer.}$$

example

$$\text{If } (x-4)^3 + (x-9)^3 + (x-8)^3 = 3(x-4)(x-9)(x-8)$$

$$\text{then } x=?$$

answer

$$\therefore (x-4) + (x-9) + (x-8) = 0$$

$$3x = 21$$

$$x = 7$$

(Identity: $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$)

example Rupees 6500 were divided equally among the certain number of persons, had there been 15 more persons then each would have got 30 rupees less. find original number of persons.

answer $\rightarrow x = \text{original Number of persons}, m = \text{rupees/person}$

$$\frac{6500}{x} = n$$

$$\frac{6500}{x+15} = n-30$$

$$\therefore \frac{6500}{x} = 30 + \frac{6500}{x+15}$$

$$6500 = nx - 30x + 15n - 450$$

~~$$\therefore 6500 = 30x + 2x^2$$~~

$$6500 = 6500 - 30x + 15n - 450$$

~~$$\therefore 3225 = 15n + x^2$$~~

$$450 = 15n - 30x$$

~~$$\therefore x^2 + 15x - 3250 = 0$$~~

$$\therefore 30 = n - 2x$$

~~$$\therefore x^2 + 65x - 50x - 3250 = 0$$~~

~~$$\therefore x(x+65) - 50(x+65) = 0$$~~

$$x = 50 \text{ or } -65$$

$$x = 50$$

example

$$x + \frac{1}{x} = \frac{17}{4} \quad \text{find } x - \frac{1}{x}$$

answer :-

$$\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4$$

$$\therefore \left(x - \frac{1}{x}\right)^2 = \frac{289 - 64}{64}$$

$$\therefore \left(x - \frac{1}{x}\right)^2 = \frac{225}{64}, \quad \boxed{\left(x - \frac{1}{x}\right) = \frac{15}{4}}$$

example

In objective examination of go exam, 5 marks allocated for right answer and 2 marks deducted for wrong answer. After attempting all 90 questions, students get 387 marks. find wrong question attempted.

answer

$x \rightarrow$ right

$y \rightarrow$ wrong

Total questions

$$x + y = 90$$

$$2x + 2y = 180$$

$$\therefore 2y = 180 - 2x$$

$$5x - 2y = 387 \quad \leftarrow \text{got 387 marks}$$

$$5x - 180 + 2x = 387$$

$$7x = 387 + 180$$

$$\therefore 7x = 567$$

$$\boxed{x = 81}$$

$$\therefore x + y = 90$$

$$\therefore y = 90 - 81 = 9 \rightarrow \text{wrong questions}$$

example In Group of 70 people, 37 likes coffee, 52 likes tea and each person likes at least one of the two drinks. Find the number of people who like both?

answer

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\therefore 70 = 37 + 52 - n(A \cap B)$$

$$\therefore n(A \cap B) = 19 \rightarrow (19 \text{ people likes both coffee and tea})$$

example Arun and Sejal are two friends. Arun gives 30 rupees to Sejal then Sejal will have twice the money left with Arun, but Sejal gives 10 rupees to Arun then Arun will have twice as much as left with Sejal. How much money does each have?

answer
Sejal $\rightarrow x$ rupees
Arun $\rightarrow y$ rupees.

$$2(y - 30) = x + 30$$

$$\therefore 2y - 60 = x + 30$$

$$\therefore 2y - 90 = x$$

$$(d+1w) = d + 1 + (d-w)$$

$$2.5x = 2x + 1 + (d-w)$$

$$2x = d - w$$

$$3(x - 10) = y + 10$$

$$3x - 30 = y + 10$$

$$3(2y - 90) - 30 = y + 10$$

$$6y - 270 - 30 = y + 10$$

$$6y - 300 = y + 10$$

$$5y = 310$$

$$y = 62$$

$y = 62 \rightarrow$ Arun's money

$$x = 34 \rightarrow$$
 Sejal's money

example A Zoo keeper counted heads of animal in zoo and found it be 80, when he count legs then he got 260. if zoo has pigeons and horses then How many Horses in zoo?

answer

$x \rightarrow$ pigeons

$y \rightarrow$ Horses

(Easiest)

(Solve)

Total head count

$$\therefore x + y = 80$$

$2x + 4y = 260 \rightarrow$ Total leg count

$$2(80 - y) + 4y = 260$$

$$160 - 2y + 4y = 260$$

$$2y = 100$$

$$y = 50$$

(50 Horses in zoo.)

example

$$\text{IF } a^2 + b^2 = 117, ab = 54,$$

$$\frac{a+b}{a-b} = ?$$

$$a^2 + b^2 = (a-b)^2 + 2ab$$

$$\therefore (a+b)^2 - 2ab = 117$$

$$(a+b)^2 - 108 = 117$$

$$a+b = 15$$

$$(a-b)^2 + 4ab = (a+b)^2$$

$$(a-b)^2 + 216 = 225$$

$$a-b = 3$$

$$\therefore \frac{a+b}{a-b} = \frac{15}{3} = 5$$

example

There are certain number of benches in classroom.

Now if 4 student can sit on each bench then 3 bench remain unoccupied. If 3 student sit on each bench then 3 student remain standing in the class, so find the number of students in the class?

answer

$$\rightarrow \text{students} = x$$

$$\rightarrow \text{benches} = y$$

$$\begin{array}{l} \text{occupied} \\ \text{bench} \times 4 \\ \text{= Number} \\ \text{of student} \end{array}$$

$$\therefore (y-3)4 = x$$

$$\therefore (x-3-9)4 = 3x$$

$$4x - 48 = 3x$$

$$\therefore x = 48$$

*total numbers
of students can
sit*

$$\therefore 3y = x-3$$

$$\therefore y = \frac{x-3}{3}$$

*total
students
on bench*

Chapter

Square Root And Cube Roots

Page No.

Date:

Method 1 to

find Square
root :-

$$\sqrt{6084}$$

Method 1 (Prime factorization)

Method 2
find square root of

$$1471369013$$

$$\begin{array}{r} 2 \\ \sqrt{6084} \end{array}$$

$$\begin{array}{r} 2 \\ \sqrt{3042} \end{array}$$

$$\begin{array}{r} 3 \\ \sqrt{1521} \end{array}$$

$$\begin{array}{r} 3 \\ \sqrt{507} \end{array}$$

$$\begin{array}{r} 13 \\ \sqrt{169} \end{array}$$

$$\begin{array}{r} 13 \\ \sqrt{13} \end{array}$$

$$\begin{array}{r} 1 \\ \sqrt{1} \end{array}$$

$$= \sqrt{2^2 \times 3^2 \times 13^2}$$

$$= 2 \times 3 \times 13$$

$$= 78$$

$$\begin{array}{r} 1 \\ \sqrt{1471369013} \\ \quad + 1 \\ \hline 22 \\ x = p + t - 2 \\ \quad .44 \\ 241 \\ \hline 1 \\ 1471369013 \\ \quad - 1471369 \\ \hline 047 \\ \quad - 047 \\ \hline 0 \\ 241 \\ \hline 7269 \\ 7269 \\ \hline 0 \end{array}$$

Square root is 1213.

example find Square root of $\sqrt{175.2976}$
answer

$$\begin{array}{r} 1324 \\ \sqrt{1752976} \\ \quad + 1 \\ \hline 23 \\ \quad 075 \\ \quad 69 \\ \hline 629 \\ \quad 524 \\ \hline 10576 \\ \quad 10576 \\ \hline 00000 \end{array}$$

(ignore points)

$$= \frac{1324}{100}$$

(divide by 100 for Points)

$$1324 = \boxed{13.24} + 14.87 + 14.87$$

Method to
find cube
root

Method 1: Prime factorisation method

$$\sqrt[3]{343} = \boxed{7}$$

$$\begin{array}{r|rr} 7 & 343 \\ \hline 7 & 49 \\ 7 & 7 \\ \hline & 1 \end{array}$$

$$\therefore = \sqrt[3]{343}$$

(soil estimation \rightarrow find H)

$$\begin{aligned} 3E_1 - E_1 - E_{11} &= \sqrt[3]{7 \times 7 \times 7} \\ &= \sqrt[3]{7^3} \\ &= \boxed{7} \end{aligned}$$

$$\frac{E_1 + E_{11}}{E_1 - E_{11}} = \frac{E_1 + 2010}{E_1 - 2010} = \frac{1 + 2010}{1 - 2010} = \frac{2011}{-2009}$$

$$\frac{(E_1 + E_{11})}{(E_1 - E_{11})} = \frac{E_1 + 2010}{E_1 - 2010} = \frac{1 + 2010}{1 - 2010} = \frac{2011}{-2009}$$

example

If $\sqrt{841} = 29$, find value of

answer

$$\sqrt{841} + \sqrt{841} + \sqrt{0.0841} + \sqrt{0.000841}$$

$$= 29 + \frac{29}{100} + \frac{29}{1000} + \frac{29}{10000}$$

$$= 29 + 2.9 + 0.29 + 0.029$$

$$= 32.219$$

example

$$\frac{1}{\sqrt{100} - \sqrt{99}} - \frac{1}{\sqrt{99} - \sqrt{98}} + \frac{1}{\sqrt{98} - \sqrt{97}} \\ + \frac{1}{\sqrt{97} - \sqrt{96}}$$

answer

(Hint:- Rationalisation)

$$(\sqrt{100} + \sqrt{99}) - \sqrt{99} - \sqrt{98} + \sqrt{98} + \sqrt{97} - \sqrt{97} - \sqrt{96}$$

$$+ \sqrt{2} + \sqrt{1}$$

$$= 10 + 1$$

$$= 11$$

Hint :- $\frac{1}{\sqrt{100} - \sqrt{99}} \times \frac{\sqrt{100} + \sqrt{99}}{\sqrt{100} + \sqrt{99}} - \frac{1}{\sqrt{99} - \sqrt{98}} \times \frac{\sqrt{99} + \sqrt{98}}{\sqrt{99} + \sqrt{98}}$

$$= \frac{\sqrt{100} + \sqrt{99}}{100 - 99} - \frac{(\sqrt{99} + \sqrt{98})}{99 - 98}$$

$$\text{example } \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{\sqrt{3}} + \frac{1}{3 + \sqrt{3}} - \frac{1}{3 - \sqrt{3}}$$

$$\text{answer} \quad \frac{\frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}-3-\sqrt{3}}{6}}{\left(\begin{array}{l} \text{soothing xito} \\ \text{island} \end{array} \right)} \left(\begin{array}{l} \text{calm} \\ \text{island} \end{array} \right) = \left(3 + \sqrt{3} \right) \left(3 - \sqrt{3} \right) = 6$$

$$= \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}$$

@ 18.

PPP	PP
-----	----

C

example find least square number which is exactly divisible by 10, 12, 15, 18.

Answer :-

$$\text{LCM}(10, 12, 15, 18) = \frac{10}{2} \times \frac{10}{5} \times \frac{12}{3} \times \frac{12}{4} \times \frac{15}{3} \times \frac{15}{5} = 2 \times 5 \times 3 \times 2 \times 3 \times 5 = 360$$

$10 = 2 \times 5$ LCM in perfect form in P.P.P.P.P.P.

$12 = 2 \times 2 \times 3$ LCM in reduced form in P.P.P.P.P.P.

$15 = 3 \times 5$ LCM in reduced form in P.P.P.P.P.P.

$18 = 2 \times 3 \times 3$ LCM in reduced form in P.P.P.P.P.P.

$\text{LCM} = 2 \times 3 \times 5 \times 2 \times 3 = 360$

$$\text{LCM} = 5 \times 2 \times 3 \times 2 \times 3$$

$$\text{LCM} = 180 \text{ days sediment follows salt bridge}$$

$$\therefore \text{Least Square Number} = 2^2 \times 3^2 \times 5^2$$

$$= 180 \times 5$$

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example find the greatest Number of 5 digits which is a perfect square.

answer

$99,999 \rightarrow$ greatest 5 digit number

$$\approx (31)^2 - (30)^2 = 61$$

$$\begin{array}{r}
 & 316 \\
 \overline{-} & 99,999 \\
 3 & \overline{9} \\
 + 3 & \overline{099} \\
 \hline
 61 & \overline{3899} \\
 + 1 & \overline{3756} \\
 \hline
 626 & \overline{0143}
 \end{array}$$

$$\frac{1}{2} + \frac{1}{2} + 8 =$$

$$\boxed{8}$$

$$= (81, 81, 81, 01) HCD$$

\rightarrow 99,999 is not Perfect Square

\rightarrow it leaves remainder 143.

Answer:- $99999 - 143$

$$= \boxed{99856}$$

example find the smallest Number that must be added to 1780 to make it perfect square.

answer:

$$\begin{array}{r}
 & 42 \\
 \overline{-} & 1780 \\
 4 & \overline{16} \\
 \hline
 82 & \overline{160} \\
 2 & \overline{164} \\
 \hline
 84 & \overline{16}
 \end{array}$$

\rightarrow 1780 is not a perfect square.

\rightarrow It leaves remainder 16 and next perfect square is $(42+1)=43$.

2. Number to be added = $(43)^2 - 1780$
 $= \boxed{69}$

example If $x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$, $y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

∴ find (i) $x^2 y^2$ (ii) $x^2 + y^2$

answer:

$$(i) (x^2 y^2) = (xy)^2$$

$$= (1)^2$$

$$\left(xy = \left(\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \right) \times \left(\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \right) = 1 \right)$$

$$(ii) x^2 + y^2 = (x+y)^2 - 2xy$$

$$= \left(\frac{8+8}{2} \right)^2 - 2(1)$$

$$= \boxed{62}$$

$$x+y = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$= (\sqrt{5} + \sqrt{3})^2 + (\sqrt{5} - \sqrt{3})^2$$

$$= (\sqrt{5})^2 - (\sqrt{3})^2$$

example find Value of $\sqrt{6} + \sqrt{6} + \sqrt{6} \dots \dots$

answer:

$$x = \sqrt{6} + \sqrt{6} + \sqrt{6} \dots \dots$$

$$x = \sqrt{6} + x$$

$$x^2 = 6 + x$$

$$\therefore x^2 - x - 6 = 0$$

$$\therefore x^2 - 3x + 2x - 6 = 0$$

$$\therefore [x = 3, -2]$$

x (cancel)

$$\boxed{x = 3}$$

Example cube root of 27449

answer

$$\begin{array}{r}
 2 \quad | \quad 2744 \\
 2 \quad | \quad 1372 \\
 2 \quad | \quad 686 \\
 7 \quad | \quad 343 \\
 7 \quad | \quad 49
 \end{array}
 \quad
 \begin{array}{r}
 3 \sqrt[3]{2744} = \sqrt[3]{2^3 \times 7^3} \\
 2 - 2 = 14
 \end{array}$$

example By what least number 4320 be multiplied to obtain a number which is a perfect cube? (ii)

anyway:

		$\therefore 4320 = 3^3 \times 2^2 \times 2^2 \times 5 \times 2$
		$\therefore 4320 = 3^3 \times 2^3 \times 2^2 \times 5$
		\therefore
3	4320	
3	1440	
3	480	
4	160	
4	40	Number = 2×5^2 b. m.f to multiple = 50
5	10	
2	2	

\rightarrow (Number to be divided to make 4320 is perfect cube
 $15 = 2^2 \times 5 = 20$)

→ $\sqrt[n]{x}$ → surds

$$\rightarrow \sqrt[n]{a} = (a)^{\frac{1}{n}}$$

→ Indices = Power = exponent

$$\therefore (x)^a \xrightarrow[\text{base}]{\text{Power}} (\text{SF})$$

$$(i) x^a \times x^b = x^{a+b}$$

$$(ii) \frac{x^a}{x^b} = x^{a-b}$$

$$(iii) (x^a)^b = x^{ab}$$

$$(iv) x^0 = 1$$

$$(v) x^1 = x$$

$$\rightarrow (a) \sqrt{xy} = (\sqrt{x})(\sqrt{y})$$

$$(b) \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$(c) \sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$$

example Solve

$$8.6 \quad 3.9 \quad 4.4 \quad 3.9 \quad 8.6 \\ 9 \times 8 \times 72 \times 9 \times 8 = (72)^x$$

answer

$$\therefore (9^{8.6+3.9}) (8^{3.9+4.4}) (72)^{3.9} = (72)^x$$

$$\therefore (72)^{16.9} = (72)^x \quad \text{(i)}$$

$$\boxed{x = 16.9} \quad \text{(ii)}$$

Answer (ii)

example

$$(?)^{\frac{1}{4}} = \frac{48}{(?)^{\frac{3}{4}}} \quad \text{find (?)}$$

answer

$$(x)^{\frac{1}{4}} = \frac{48}{(x)^{\frac{3}{4}}}$$

$$(x)^{\frac{1}{4}} (x)^{\frac{3}{4}} = 48 \quad (x)^{\frac{1}{4}} = 48$$

$$\boxed{x = 48}$$

example $2^{x-1} + 2^{x+1} = 1280$ find (x)

answer $\therefore 2^x \left(\frac{1}{2} + 2 \right) = 1280$

$$\therefore 2^x = \frac{1280 \times 2}{5}$$

$$\therefore 2^x = 256 \times 2$$

$$\therefore 2^x = 2^8 \times 2$$

$$\therefore 2^x = 2^9$$

$$\boxed{x = 9}$$

example If $8^x \cdot 2^y = 512$ and $3^{3x+2y} = 9^6$
find x, y.

answer $\therefore 8^x \cdot 2^y = 512$ $\therefore 3^{3x+2y} = (3^2)^6$
 $\therefore 2^{3x+y} = 2^9$ $\therefore 3x+2y = 12$
 $\therefore 3x+y = 9$ $\therefore 9-y+2y = 12$
 $y = 9-3x$ $\therefore \boxed{y = 3}$
 $\therefore 3 = 9-3x$
 $\boxed{x=2}$

example $(2^{\frac{1}{4}} - 1)(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1)$

$$= (2^{\frac{1}{4}} - 1)(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1)$$

$$= (2^{\frac{1}{4}} - 1)(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1) = 2^{\frac{1}{4}} + 2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} - 2^{\frac{3}{4}} - 2^{\frac{1}{2}} - 2^{\frac{1}{4}} - 1$$

$$(2^{\frac{1}{4}} - 1) 8 = \boxed{1} = 2^{\frac{1}{4}} + 2^{\frac{3}{4}} - 2^{\frac{1}{2}} - 2^{\frac{1}{4}} - 1$$

example If $2^x = 3^y = 6^{-z}$ find $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

answer

$$\begin{aligned} 2^x &= K, & 3^y &= K, & 6^{-z} &= K + \frac{1}{K} \\ x &= \log_2 K, & y &= \log_3 K, & -z &= \log_6 K \\ &&&&& z = \log_6 \left(\frac{1}{K}\right) \\ &&&&& z = -\log_6 K \end{aligned}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{\log_2 K} + \frac{1}{\log_3 K} + \frac{1}{\log_6 K}$$

$$\frac{\log 2 + \log 3 - \log 6}{\log K}$$

example If $t = 2 + 3\sqrt[3]{4} + 3\sqrt[3]{2}$

then find $t^3 - 6t^2 + 6t - 2 = ?$

answer

$$t = 2 + 3\sqrt[3]{4} + 3\sqrt[3]{2}$$

$$t - 2 = 3\sqrt[3]{4} + 3\sqrt[3]{2}$$

$$(t-2)^3 = (3\sqrt[3]{4} + 3\sqrt[3]{2})^3$$

$$(t-2)(t^2 + t^2 + 2t^2 - 2t^2 + 4t^2 + 4t^2)$$

$$t^3 - 8 - 6t(t-2) = 4 + 2 + 3(2)^{\frac{2}{3}}(2)^{\frac{1}{3}}(3)\sqrt[3]{4} + 3\sqrt[3]{2}$$

$$t^3 - 8 - 6t^2 + 12t = 6 + 6(3\sqrt[3]{4} + 3\sqrt[3]{2})$$

$$x^3 - 6x^2 + 6x - 2 + 6x - 6 = 6 + 6(3\sqrt{4} + 3\sqrt{2})$$

\therefore

(\therefore n = wanted answer)

$$x = 12 + 6\sqrt[3]{4} + (3\sqrt{2})6 = 12 - 6\sqrt[3]{4} - 6\sqrt[3]{2}$$

$$\boxed{x = 0 \text{ (zero)}}$$

$$x^3 - 6x^2 + 6x - 2 = 0.$$

example

$$\left(\frac{x^a}{x^b} \right) \times \left(\frac{x^b}{x^c} \right) \times \left(\frac{x^c}{x^a} \right)$$

answer

$$\frac{x^{a+b} + a^2 b}{x^{b+a} + b^2 + ab} \times \frac{x^{b+c} + b^2 c + b c^2}{x^{c+b} + c^2 b + b c^2} \times \frac{x^{c+a} + c^2 a + a c^2}{x^{a+c} + a^2 c + a c^2}$$

$$\begin{aligned} & x^0 \\ & = \boxed{1} \end{aligned}$$

example If $x = y^a$, $y = z^b$, $z = x^c$

then find abc

answ

$$\log_y x = a, \log_z y = b, \log_x z = c$$

$$abc = (\log_y x) (\log_z y) (\log_x z)$$

$$\boxed{abc = 1}$$

base changing

theorem

$$\log_b a = \frac{\log b}{\log a}$$

$$\hookrightarrow = \log_y x \cdot \log_z y \cdot \log_x z$$

$$= \frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}$$

$$= \boxed{1}$$

example

find a Number such that when 15 is subtracted from 7 times the number the result is 10 more than twice number.

answer:- Number = x .

\therefore when 15 is subtracted from 7 times the number
 $=$ 10 more than twice Number

$$\therefore 7x - 15 = 2x + 10$$

$$x = 5$$

example The sum of two numbers is 184. if one third of one exceeds $\frac{1}{7}$ th of others by 8 then find smaller number

answer:

number = x, y

$$\rightarrow \text{sum of Numbers} = 184$$

$$\therefore x + y = 184$$

$$\therefore 3x + 3y = (184)(3)$$

$$\frac{x}{3} = \frac{y}{7} + 8$$

$$\therefore \frac{7x}{3} = y + 56$$

$$\therefore 7x = 3y + 56 \times 3$$

$$7x = (184)(3) - 3x + (56)(3)$$

$$10x = 240 \times 3$$

$$x = 72$$

\rightarrow smaller Number is 72.

example If sum of numbers is 42 and their product is 437 then find absolute difference between them?

answer : $x+y = 42$, $xy = 437$, Number = x, y

$$\therefore (x-y)^2 = (x+y)^2 - 4xy$$

$$\therefore (x-y)^2 = (42)^2 - 4(437)$$

$$\therefore (x-y)^2 = 16$$

$$\therefore x-y = 4$$

(we have to find absolute difference so ignore -4 answer)

example If sum of numbers is 10 and sum of their reciprocal is $\frac{5}{12}$ then find numbers.

answer

$$\text{Given } a+b = 10$$

$$\frac{1}{a} + \frac{1}{b} = \frac{5}{12}$$

$$\therefore \frac{ab}{ab} = \frac{5}{12}$$

$$\therefore \frac{10}{ab} = \frac{5}{12}$$

$$\therefore ab = 24$$

$$\therefore a+b = 10, ab = 24$$

then numbers = 6, 4.

EXERCISE

$$EF = ?$$

• ST is midpoint of BC

Example Sum of 7 natural consecutive numbers is 1617

then How many of this is prime? if it is prime then how many of them are prime?

Answer:

$$\therefore (x) + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6)$$

$$= 7x + 21$$

$$7x + 21 = 1617$$

$$\therefore x = 228$$

∴ Numbers = 228, 229, 230, 231, 232, 233, 234

∴ Prime Numbers = 229, 233

∴ Number of Prime Numbers = 2

Example The sum of square of three consecutive odd numbers is 2531. Find numbers?

Answer

∴ Numbers = $a-2, a, a+2$

$$(a-2)^2 + a^2 + (a+2)^2 = 2531$$

$$\therefore a^2 - 4a + 4 + a^2 + a^2 + 4a + 4 = 2531$$

$$\therefore 3a^2 = 2531 - 8$$

$$\therefore a^2 = 841$$

$$\therefore a = 29$$

∴ Number = 27, 29, 31

example

Ratio between two digit number and sum of digits of their number $4:1$. Digit in unit place is 3 more than digit at 10 place. find number.

answer

\rightarrow Number = ab . \rightarrow Ratio between number and sum of digits

\rightarrow Digit in unit place is 3 more than digit at tens place

$$\therefore b = a + 3$$

$$\therefore 2a = a + 3$$

$$\therefore a = 3$$

$$\therefore b = 6$$

$$\text{Number} = 36$$

$$\frac{10a+b}{a+b} = 4$$

$$\therefore 10a+b = 4a+4b$$

$$\therefore 6a = 3b$$

$$\therefore 2a = b$$

example

If the digits of two digit are interchanged then number form is greater than original number by 45. If difference between is 5, find Number

answer

$$\text{number} = ab$$

$$b-a=5$$

 \therefore

$$\therefore a+5-a=5$$

$$\therefore 5=5$$

$$10a+b+45 = 10b+a$$

$$\therefore 9a+45 = 9b$$

$$\therefore a+5=b$$

answer = (\because so Number cannot determine)

$b > a$ because when number is interchanged then it is bigger than original number)

example 50 is divided into two parts such that sum of their reciprocals is $\frac{1}{12}$. find two parts of 50.

answer :-

$$50 \rightarrow a \\ 50 \rightarrow b$$

$$\therefore a+b = 50$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{12}$$

$$\therefore \frac{a+b}{ab} = \frac{1}{12}$$

$$12 \times 50 = ab$$

$$1600 = ab$$

$$\therefore \text{numbers} = 20, 30$$

(options for above question is
 (a) 10, 40 (b) 20, 30 , (c) 15, 35 , (d) 25, 25)

example If three numbers are added in pair their sums = 10, 19, 21. Find Numbers.

answer :- number = a, b, c

$$\therefore a+b = 10, \quad b+c = 19, \quad c+a = 21$$

: method :- 2

$$\text{numbers} = a, b, c$$

$$\therefore a+b = 10, \quad b+c = 19,$$

$$c+a = 21.$$

$$b+21-a = 19$$

$$b-a = -2$$

add all three,

$$2(a+b+c) = 50$$

$$\therefore a+b+c = 25$$

$$2(a+b+c) - (a+b) = 25 - 10$$

$$\therefore c = 15$$

$$2(a+b+c) - (b+c) = 25 - 19$$

$$\therefore a = 6, \text{ similarly } b = 4$$

$$\therefore \text{Numbers} = 4, 6, 15.$$

example fraction becomes the $\left(\frac{2}{3}\right)$ when one is added both Numerator and denominator. And becomes $\left(\frac{1}{2}\right)$ one is subtracted from both. find fraction.

answer: \therefore fraction: $\frac{x}{y}$

$$\frac{2(x+1)}{3(y+1)} = \frac{x}{y}$$

$$\frac{1(x-1)}{2(y-1)} = \frac{x}{y}$$

$$\therefore (2x+2)y = 3xy + 3x$$

$$\therefore 2xy + 2y = 3xy + 3x$$

$$\therefore 2y = xy + 3x$$

$$\therefore \boxed{2y = xy + 3x}$$

$$\therefore (x-1)y = 2xy - 2x$$

$$\therefore xy - y = 2xy - 2x$$

$$\therefore 2x = y + xy$$

$$y = \frac{2x}{x+1}$$

$$\therefore 2\left(\frac{2x}{x+1}\right) = x\left(\frac{2x}{x+1} + 3x\right)$$

$$\therefore 4x = 4x + 3x^2 + 3x$$

$$\therefore \text{fraction} = \frac{x}{y}$$

$$\therefore \frac{x+1}{y+1} = \frac{2}{3}$$

$$\therefore \frac{x-1}{y-1} = \frac{1}{2}$$

$$\text{I.e. } 3x+3 = 2y+2$$

$$\therefore 3x+1 = 2y \quad \text{---(i)}$$

$$\therefore 2x-2 = y-1$$

$$\therefore 2x = y+1 \quad \text{---(ii)}$$

Solve equation (i) and (ii), we get $x=3, y=5$

$$\therefore \text{fraction} = \frac{x}{y} = \frac{3}{5}$$

→ Average = Sum of Observations

of below result / number of observations

→ Average speed = Total Distance
/ Total time.

where $x \rightarrow$ and then Avg. speed = $\frac{2xy}{x+y}$ km/h

(x, y = speed and D is constant distance for both cases)

example There are 6 Numbers 30, 72, 53, 68, x , 87. Avg. of numbers is 60.

$$30 + 72 + 53 + 68 + x + 87 = 6 \times 60$$

$$\therefore x = 50$$

example A man bought 5 shirts at 450 ₹ each. 4 trouser at 750 ₹ each and 12 pairs of shoes at 750 ₹ each. Find avg. expense.

$$\text{Avg. Expense} = \frac{(5)(450) + (4)(750) + 12(750)}{21}$$

$$\text{Avg. Expense} = \frac{(450)(5) + 16(750)}{21}$$

$$\boxed{\text{Avg. Expense}} = 678.57$$

example 13 chairs and 5 table bought for ₹ 8280,
If avg. cost of table 1227 ₹ . then what is
the avg. cost of chair.

answer

$$\therefore 13(\text{avg. of chair}) + 5(1227) = 8280$$

$$\therefore \boxed{\text{avg. of chair} = 165 \text{ ₹}}$$

example Avg. Monthly expenditure of family was rupees 2200 ₹ for first three months, 2250 ₹ for next 4 months, 3120 ₹ for next 5 months. If Total saving during year is 1260 ₹ then find avg monthly income of family?

answer

$$\therefore (2200)(3) + (2250)(4) + (3120)(5) + 1260$$

$$= 6600 + 9000 + 15600 + 1260$$

$$= \boxed{2705}$$

∴ Monthly income
of family

example Avg. of 5 numbers is 58. Avg. of first two numbers is 48.5 and Avg. of last two numbers is 53.5 then what is third number?

answ: numbers = a, b, c, d, e

$$\angle a+b+c+d+e = (58)(5) \quad \frac{a+b}{2} = 48.5$$

$$\therefore 2(48.5) + c + (53.5) 2 = (58)(5) \quad \frac{c+d}{2} = 53.5$$

$$\therefore c + 2(102) = (58)(5)$$

$$\angle c + 204 = 290$$

$$\boxed{c = 86}$$

\hookrightarrow third Number

example Distance between Two station is 778 km. Train covers journey A to B at 84 km/h, and return back to A with uniform speed of 56 km/h. And avg. speed of train during whole journey?

answ

$$\therefore s_1 = 84 \text{ km/h}, s_2 = 56 \text{ km/h}$$

$$\therefore \text{avg. speed} = \frac{2s_1 s_2}{s_1 + s_2}$$

$$\text{avg. speed} = \frac{(2)(84)(56)}{84 + 56}$$

$$= \boxed{67.2 \text{ km/h}}$$

Average speed of train

During Whole journey.

example The batting avg. of 40 innings of a cricket player is 50 runs. His highest score in any innings exceeds his lowest score by 172 runs. If this two innings are excluded then avg. score of remaining 38 innings is 48 runs. find (Highest Score) + (Lowest Score)

answr:

$$\begin{aligned} \rightarrow \text{lowest score} &= x : (2.82) + 2 + (2.84) \\ \rightarrow \text{Highest score} &= x + 172 \\ \text{avg of } 50 \text{ innings} &= \frac{40 \times 50 - x - x - 172}{38} = 48 \end{aligned}$$

Highest and lowest score excluded

$$\rightarrow \text{Highest score} = 174$$

Example In a class there are 50 students. Their Avg. weight is 45 kg. When a student leaves the class Avg. reduces by 100 gram. Find the weight of student who left the class?

answr $\rightarrow x = \text{weight of student who leaves.}$

avg. weight of 50 students \leftarrow

$$\frac{50 \times 45 - x}{49} = 45 - 0.1$$

avg. reduces by 100 gram
(100 gram = 0.1 kg)

your way should be like

$x = 49.9 \text{ kg}$

$$\rightarrow \log_a x = y$$

$$\boxed{a^y = x}$$

$$(a, x > 0) \\ a \neq 1$$

$$y = \log_a x$$

$$(1, 0)$$

$$x=1, y=0$$

$$x=\infty, y=\infty$$

$$x=0, y=-\infty$$

$$\rightarrow e^x$$

$$(0, 1)$$

$$\boxed{e^x = x}$$

$$\rightarrow (1) \log_a x + \log_a y = \log_a xy$$

$$(2) \log_a x - \log_a y = \log_a (x/y)$$

$$(3) \log_a x^m = m (\log_a x)$$

$$(4) \log_a b = \frac{\log_m b}{\log_m a}$$

$$(5) \text{Natural log} = \ln x = \log_e x$$

$$(6) \log_a a = 1, \log_a 1 = 0$$

example If $\log_2 (\log_3 (\log_2 x)) = 1$

find x

answer

$$\therefore x = 2^9$$

$$\boxed{x = 512}$$

$$\boxed{x = 512}$$

$$(0 < x < 1 + \infty)$$

example $\log_3 3 + \log_{10} (4x+1) = \log_{10} (x+1) + 1$

find x .

answer

$$\therefore 12x + 3 = 10x + 10$$

$$\therefore 2x = 7$$

$$\boxed{x = \frac{7}{2}}$$

$$(1, 0)$$

example If $\log_{10} (x^2 - 6x + 45) = 2$, find x ?

answer:

$$\therefore x^2 - 6x + 45 = 100$$

$$\therefore x^2 - 6x - 55 = 0$$

$$\therefore x^2 - 11x + 5x - 55 = 0$$

$$\boxed{x = 11, -5}$$

example Simplify :-

$$\left(\frac{1}{\log_{xy} (xyz)} + \frac{1}{\log_{yz} (xyz)} + \frac{1}{\log_{zx} (xyz)} \right) \quad (E)$$

answer

$$\boxed{2}$$

example $\log_{\sqrt{8}} x = 3\frac{1}{3}$, & finds $x = 8^{10/3} = 32$. 3/7/2020

answer $\log_{\sqrt{8}} x = \frac{10}{3}$ es (i) to enter diff
e.g. calc (ii)

$$x = 2^{\frac{3}{2} \cdot \frac{10}{3}} = (ii) \quad \text{es part (i)}$$

$$x = 2^{\frac{5}{2}} = \boxed{32} \quad (\frac{5}{2}) \text{ part}$$

example If $\log_a b = \frac{1}{2}$, $\log_b c = \frac{1}{3}$, $\log_c a = \frac{k}{5}$, $k = ?$

answer $\boxed{k = 30}$ (part - (i) part - (ii) part)

$$\left(\frac{\log b}{\log a} \times \frac{\log c}{\log b} = \frac{1}{6}, \log_a c = \frac{1}{6} \right)$$

example If $\log_{10} 2 = 0.30103$, $\log_{10} 50 = ?$

answer $\log_{10} 50 = \log_{10} (10 \cdot 2)$ (i) part 3/7/2020

$$= \log_{10} 100 - \log_{10} 2$$

$$= 2 - 0.30103$$

$$= \boxed{1.69}$$

- (a) 1.5 (b) 1.79 (c) 1.69 (d) 1.59

example $\log 2 = 0.3010$ & $\log 3 = 0.4771$

find value of (i) $\log 25$

(ii) $\log 4.5$

answer

(i) $\log 25$

$$= \log\left(\frac{100}{4}\right)$$

$$= \log(100) - \log(4)$$

$$= 2(\log 10 - \log 2)$$

$$= 2(1 - 0.3010)$$

$$= 1.398$$

(ii) $\log 4.5$

$$= \log\left(\frac{45}{10}\right)$$

$$= \log 45 - 1$$

$$= \log\left(\frac{90}{2}\right) - 1$$

$$= \log(9) - \log 2 - 1$$

$$= \log(9) + 1 - \log 2 - 1$$

$$= 2(\log 3) - \log 2$$

$$= 0.6532$$

example $\log\left(\frac{1}{18}\right)$

answer

$$= \log\left(\frac{1}{18}\right)$$

$$= \frac{\log 3 + \frac{1}{2} \log 2}{-}$$

$$= -\log(18) (2)$$

$$= \log 9 + \log 2$$

$$= -2$$

example

$$\log \frac{0.00001}{0.01}$$

$$\log \frac{\frac{1}{100000}}{\frac{1}{100}} = \log \left(\frac{1}{1000} \right) =$$

$$= \boxed{\frac{5}{2}}$$

answer

All 0's + add 5 = since follow 1 to zero

example simplify:

$$\log \frac{75}{15} = 2 \log \left(\frac{5}{3} \right) + \log \frac{32}{243}$$

$$\text{anyway: } = \log \left(\frac{75}{15} \cdot \frac{81}{25} \cdot \frac{32}{243} \right)$$

$$= \log \left(\frac{3 \times 2}{3} \right)$$

$$(2) = \boxed{\log 2}, \text{ or } 9.6021990$$

example

$$\left[\log_{10} (5 \log_{10} 100) \right]^2$$

answer

$$= (2)^2 = \boxed{1}$$

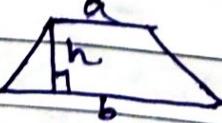
example

$$\log \frac{(125)(625)}{5^2}$$

answer

$$\log_5 5$$

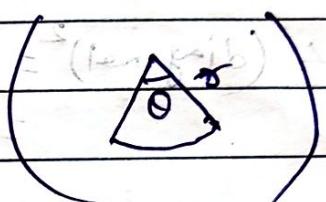
$$= \boxed{5}$$

- rectangle = length × width
- perimeter of rectangle = $2(\text{length} + \text{width})$
- area of square = $(\text{length})^2$
 $= \frac{(\text{Diagonal})^2}{2}$
- area of 4 wall of room = $2bh + 2lh$
 $(h \rightarrow \text{height}, l \rightarrow \text{length}, b \rightarrow \text{breadth})$
- area of triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$ (for equilateral)
 $= \frac{1}{2} \times \text{base} \times \text{height}$ (for right angle)
 $= \sqrt{s(s-a)(s-b)(s-c)}$
 $\text{where } s = \frac{a+b+c}{2}$
 $\text{(applicable for all triangle)}$
- Area of parallelogram: $b \times h$
- (Trapezium →   ← parallelogram)
- Area of rhombus: $\frac{1}{2} \times \text{product of diagonal}$
- Area of trapezium: $\frac{1}{2} (\text{sum of parallel sides}) (\text{perpendicular distance})$
 $= \frac{1}{2} (a+b)(h)$

→ Area of Circle = πr^2

→ Area of Sector = $\frac{\theta}{360} \pi r^2$ (where θ is the angle subtended by the sector at the center)

→ Circumference of circle = $2\pi r$ (where $r = d/2$)

→ Area of Sector = $\frac{\theta}{360} \pi r^2$ (Minor Sector) 

→ Area of Major sector = $\pi r^2 - \frac{\theta}{360} \pi r^2$

→ Segment :-

Area = Area of circle - Area of triangle
Minor sector



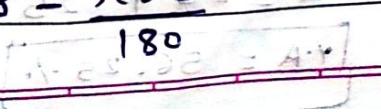
$$\text{Area} = \frac{\theta}{360} \pi r^2 - \text{Area of triangle}$$

Minor Segment

(longest chord of circle = diameter)

→ Area of Major Segment = $\pi r^2 - \text{area of minor segment}$

→ Minor Arc length = $\frac{\theta}{360} \times 2\pi r$ 

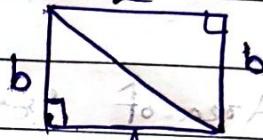
→ Major Arc length = $2\pi r - \frac{\theta}{360} \times 2\pi r$ 

example A rectangular carpet has an area of 120 square meter and perimeter of 46 meter. find length of diagonal.

answer

$$\therefore l+b = \frac{46}{2}, \quad lb = 120$$

$$\begin{aligned} \therefore (\text{diagonal})^2 &= l^2 + b^2 \\ &= (l+b)^2 - 2lb \\ &= \left(\frac{46}{2}\right)^2 - 2(120) \\ &= 289 \\ \boxed{\text{diagonal} = 17} \end{aligned}$$



example The diagonal two square are in ratio $\frac{2}{5}$. find Ratio of their area.

answer

$$\text{Area of Square} \propto (\text{diagonal})^2$$

$$\text{Ratio of Square} = (\text{Ratio of diagonals})^2$$

$$= \left(\frac{2}{5}\right)^2$$

$$= \boxed{\frac{4}{25}}$$

example If each side of Square is increased by 25%. then find Percent change in Area.

answer :-

$$\%A = l\% + b\% + \frac{(l)(b)}{100}$$

$$\%A = 25 + 25 + \frac{(25)(25)}{100}$$

$$\boxed{\%A = 56.25 \%}$$

(Note:- Refer QA subject)

example Square park is surrounded by path of uniform width of 2 meter all round it. The area of path is 288 square meter. find per meter of park.

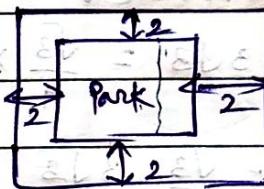
answer → Let side of park is (x) meter.

→ Then side of path is $(x+4)$ meter

$$\rightarrow (x+4)^2 = 288$$

$$x^2 + 8x + 16 - 288 = 0$$

$$\therefore x^2 + 8x - 272 = 0$$



example The Dimensions of Room are 12.5 meters by 9m by 7m. There are two doors and 4 windows in the room. Each door measures 2.5 meter by 1.2 meter.

Each window Dimensions 1.5 m by 1 m. find Cost of painting the walls at Rupees 36.50 per square meter.

answer

∴ Area of

$$\text{wall} = 2(7)(12.5 + 9) - 2(2.5 \times 1.2) - 4(1.5 \times 1)$$

$$= (14)(21.5) - 6 - 6$$

$$= 289 \text{ per square meter}$$

$$\rightarrow 1 = 36.50 \text{ Rupees}$$

$$289 = (?)$$

$$\rightarrow 289 \times 36.5$$

$$= 10,548.5 \text{ Rupees}$$

example: Find length of Altitude of an equilateral triangle of side $3\sqrt{3}$ cm.

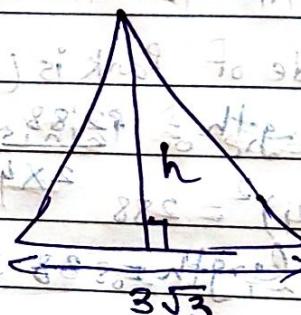
answer:

$$\text{Area} = \text{Area}.$$

$$\frac{1}{2} \times h \times 3\sqrt{3} = \frac{\sqrt{3}}{4} \times (3\sqrt{3})^2$$

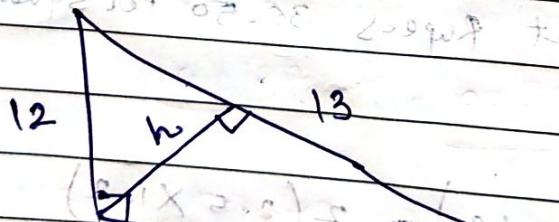
$$h = \frac{\sqrt{3} \times 3\sqrt{3}}{2}$$

$$\boxed{h = \frac{9}{2}}$$



example: Base and Altitude of a right angle triangle are 12 cm and 5 cm. find perpendicular distance of its hypotenuse.

answer:



$$\text{hypotenuse} = \sqrt{12^2 + 5^2} = 13$$

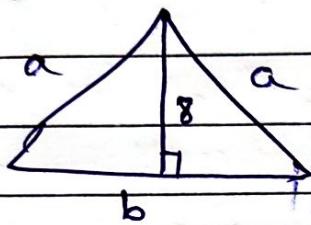
$$\text{Area} = \text{Area}$$

$$\frac{1}{2} \times 12 \times 5 = \frac{1}{2} \times h \times 13$$

$$\therefore \boxed{\frac{60}{13} = h}$$

example Altitude drawn to the base of isosceles triangle is 8 cm. Perimeter = 32 cm. Find Area of triangle.

answer :-



$$2a + b = 32$$

$$b = 32 - 2a$$

$$\boxed{b = 12}$$

$$a^2 = 64 + \left(\frac{b}{2}\right)^2$$

$$a^2 = 64 + \left(\frac{32-2a}{2}\right)^2$$

$$a^2 = 64 + 256 + a^2 - 32a$$

$$32a = 320$$

$$\boxed{a = 10}$$

$$\text{Area} = \frac{1}{2} \times 8 \times b$$

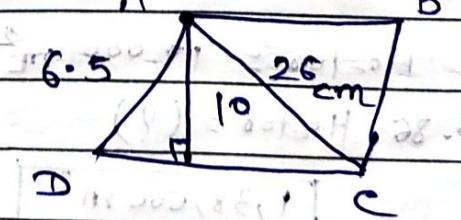
$$= \frac{1}{2} \times 8 \times 12$$

$$\boxed{= 48 \text{ cm}^2}$$

example Length of one side is 6.5 cm, Altitude is 10 cm.

If length of one of the diagonal is 26 cm. Find length of another diagonal.

answer :-



\angle Area of Rhombus = base \times Altitude = $\frac{1}{2} \times d_1 \times d_2$

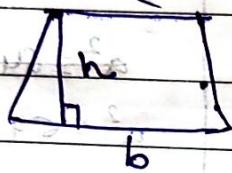
$$(6.5) \times (10) = \frac{1}{2} \times 26 \times d_2$$

$$\boxed{d_2 = 5 \text{ cm}}$$

example

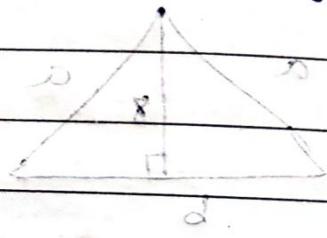
Difference between two parallel side of trapezium is 4 cm
The perpendicular distance between them is 19.

Area of Trapezium is 475 cm^2 then find length of parallel sides

answer

$$\frac{1}{2}(a+b)h = 475$$

$$h = 19$$



$$\therefore \text{Area} = 475$$

$$\therefore \frac{1}{2}(a+b)(19) = 475$$

$$a-b = 4$$

$$a+b = 50$$

$$2a = 54$$

$$a = 27$$

$$b = 23$$

example

Area of circular field is $38.13.86$ hectare. finds cost of fencing at rate of 4.4 rupees/meter.

answer

$$1 \text{ Hectare} = 10,000 \text{ m}^2$$

$$13.86 \text{ Hectare} = (?)$$

$$1,38,600 \text{ m}^2$$

$$\pi r^2 = 1,38,600$$

$$\frac{22}{7} r^2 = 1,38,600$$

$$r^2 = 44100$$

$$r = 210 \text{ m}$$

$$\therefore 2\pi r = 2 \times 22 \times 210$$

$$= 60 \times 22$$

$$= 1320 \text{ meter}$$

(H.E = 13) + 10%.

$$\therefore 1 \text{ meter} = 4.41 \times 11.8 \times 5 = 60$$

$$1320 \text{ meter} = (?)$$

$$= 5808 \text{ m}$$

example The ratio of circumference of two circle is $\frac{2}{3}$. What is ratio of area.

→ Ratio of circumference of two circles is $\frac{2}{3}$.

$$\text{Ratio of area} \propto r^2$$

$$\text{Ratio of area} = \left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}$$

example The radii of three concentric circles are $1:2:3$. Find the ratio of areas between two inner circle to two outer circle.

$$\text{answer: } r_1 = k, r_2 = 2k, r_3 = 3k$$

$$= \frac{4k^2 - k^2}{9k^2 - 4k^2}$$

$$\text{Ratio} = \boxed{\frac{3}{5}}$$

example The minute of an clock is 1.5 cm long. What is the distance travelled by its stick during interval of 40 minutes. ($\pi = 3.14$)

answer

$$60 = 2 \times 3.14 \times 1.5$$

$$40 = (9)$$

$$\frac{40 \times 2 \times 3.14 \times 1.5}{60}$$

so answer is

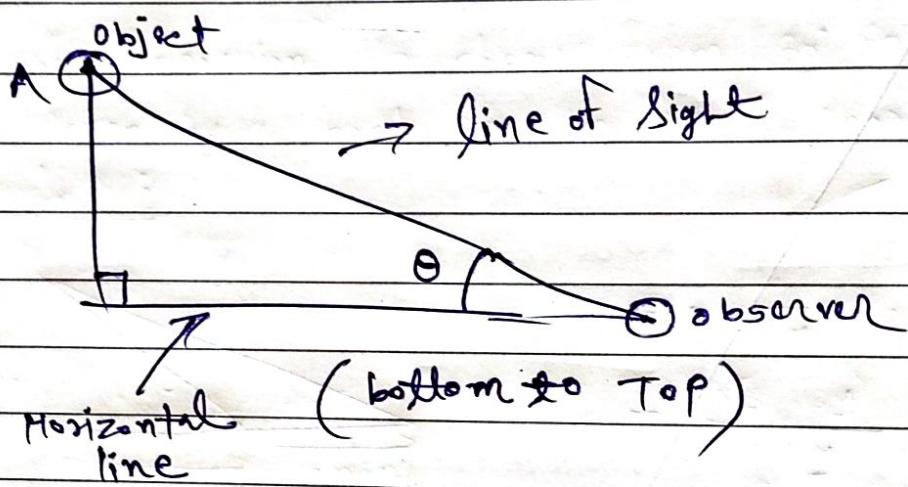
$$= 6.28 \text{ cm}$$

example If radius of circle decreased by 50% then find decrease in Area

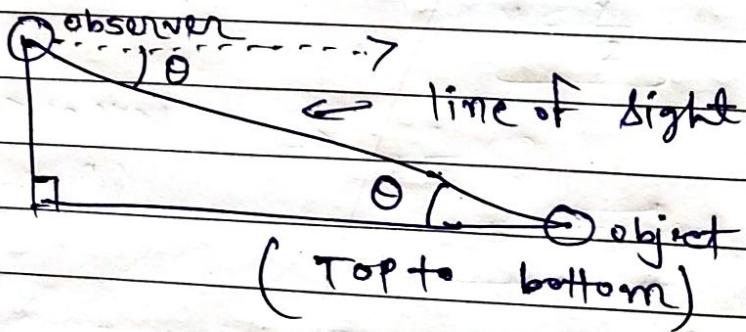
answer:

$$A = -75\%$$

→ Angle of elevation:-

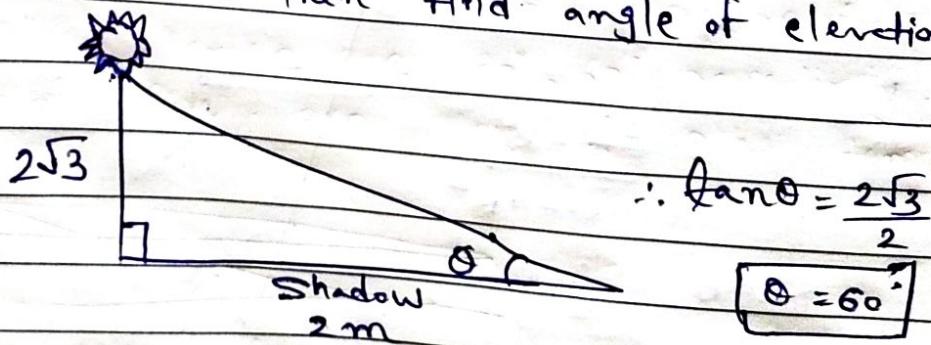


→ Angle of depression:-



example If the height of pole is $2\sqrt{3}$ m and length of its shadow is 2m then find angle of elevation of sun.

answer :-

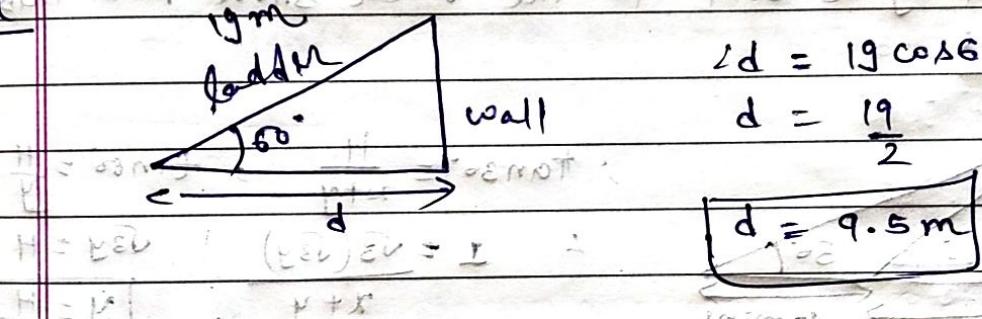


$$\begin{aligned}\sqrt{3} &= 1.73 \\ \sqrt{2} &= 1.41\end{aligned}$$

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example A ladder leaning against a wall makes angle of 60° with the ground. If length of ladder is 19 m. find the distance of foot of ladder from wall.

answer It is given that angle between ground and ladder is 60° .

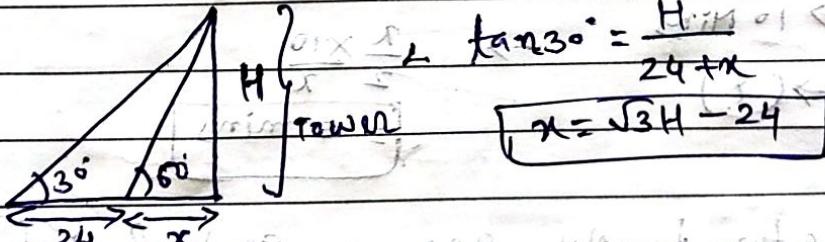


$$d = 19 \cos 60^\circ$$

$$d = \frac{19}{2}$$

example The angle of elevation of top of tower at point on the ground is 30° . On walking 24 meter towards tower the angle becomes 60° . Find height of tower.

answer :-



$$\tan 30^\circ = \frac{H}{24+x}$$

$$x = \sqrt{3}H - 24$$

$$\tan 60^\circ = \frac{H}{x}$$

$$\therefore \sqrt{3} = \frac{H}{\sqrt{3}H - 24}$$

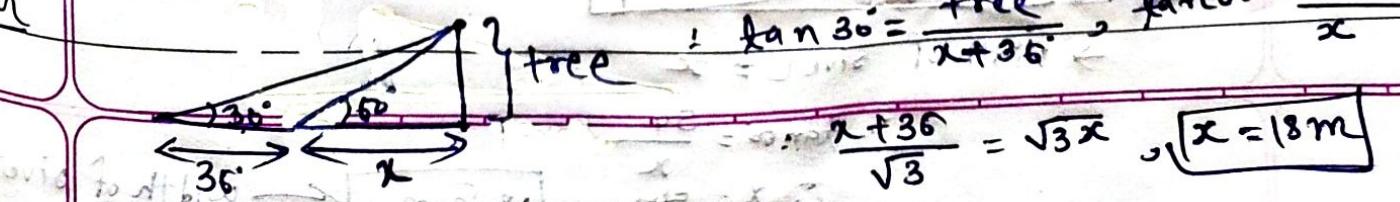
$$\sqrt{3}H - 24\sqrt{3} = H$$

$$2H = 24\sqrt{3}$$

$$H = 12\sqrt{3}$$

example A man standing on a bank of river observes that angle subtended by a tree on the opposite bank is 60° . When he retires 36 m from bank, he finds angle to be 30° . Find breadth of river.

answer

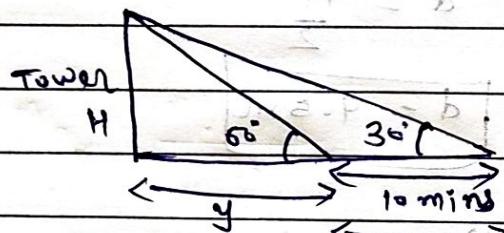


$$\tan 30^\circ = \frac{\text{tree}}{x+36}, \tan 60^\circ = \frac{\text{tree}}{x}$$

$$\frac{x+36}{\sqrt{3}} = \sqrt{3}x, x = 18m$$

example A Man On Top of tower standing on the see saw find that a boat coming towards him. Takes 10 min for the angle of depression to change from 30° to 60° then find taken by a boat to the see shore from this position

anywhⁿ



$$\therefore \tan 30^\circ = \frac{H}{x+y}, \tan 60^\circ = \frac{H}{y}$$

$$L \quad I = \frac{\sqrt{3}(\sqrt{3}y)}{x+y} \quad \sqrt{3}y = H$$

$$y = \frac{H}{\sqrt{3}}$$

$$\text{Ratio of } x \text{ to } y \text{ is } 2 : 3 \text{ and } x + y = 3y$$

$$\text{constant return to scale} \Rightarrow K=2Y \quad \boxed{K=2Y}$$

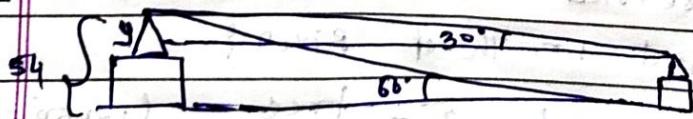
~~start to think what we wanted~~

$$y = \frac{x}{2}$$

$$\frac{3}{\cancel{3}} \times 10$$

example There are two temples, one on each bank of River, Just opposite to each other. One Temple is 54 meter High, from Top of this temple Angle of depression of Top and Foot of other Temple are 30° , 60° respectively. find width of River & and Height of other temple.

anywhere



\rightarrow width of $\text{several} = x$

$$\tan 60^\circ = \frac{54}{x}$$

$$\therefore x = \frac{54}{\sqrt{3}} = [18\sqrt{3} \text{ m}] \longleftrightarrow \text{width of river}$$

$$\tan 30^\circ = \frac{y}{18\sqrt{3}}$$

$$\therefore y = 18$$

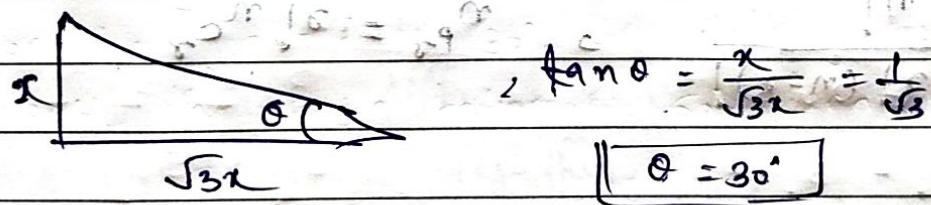
∴ Height of other temple is $= 54 - y$

$$= 54 - 18$$

$$= 36 \text{ m}$$

example If shadow of tower is $\sqrt{3}$ times its height then the angle of elevation of sun.

answer:



$$\tan \theta = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

→ Permutations :-

↳ arrangement

→ Combinations:- (if n objects are to be selected from n objects)

↳ selection

$$\rightarrow n_{P_r} = \frac{n!}{(n-r)!}$$

if r objects are selected from n objects then n_{P_r} ways to arrange them

$$\rightarrow n_{C_r} = \frac{n!}{r!(n-r)!} \Rightarrow n_{P_r} = r! \cdot n_{C_r}$$

$$\rightarrow n_{C_0} = 1$$

$$\rightarrow n_{C_1} = n$$

$$\rightarrow n_{C_r} = n_{C_{n-r}}$$

$$\rightarrow n! = l^n$$

example

$$100_{C_{98}}$$

answer

$$100 \times 99$$

$\frac{2}{=}$

$$= 50 \times 99$$

$$\boxed{\rightarrow 4950}$$

example In How many different ways can letter of word 'FIGHT' be arranged?

→ Number of letter = 5

$$= 5!$$

$$= [120]$$

example In How many different ways can letter of word 'PRESENT' be arranged?

$$= 7!$$

$$= 2!$$

example In How many different can letter of word 'DESIGN' be arranged, are vowels come together.

N D S G (E I)

$$= 5! \times 2!$$

$$= 120 \times 2$$

$$= [240]$$

example 'DESIGN' can be arranged such that vowels are at two ends.

O — — — O

$$[48] = 4! \times 2!$$

example 'DIRECTOR' can be arranged such that vowels are together

! begins to 'THREE'

answer

EIO

D R C T R

$$= 6! \times 3!$$

2!

now

$$= 6! \times 3!$$

7,260

is settled to reduce to

12 =

6512

example 'DIGEST' vowels are never together.

answer

method:-1

$$= 6! - 5! \times 2!$$

$$= 720 - 240$$

= 480

method:-2

$$= 4! \times 5!$$

$$= 4! \times \frac{5!}{3!}$$

$$= 4 \times 120 = 480$$

example 'DETAIL', vowels occupy only odd position

answer

○ □ ○ □ ○ □

$$= 3! \times 3!$$

$$= 36$$

$$12 \times 12 = 144$$

example 'CRICKET' team of 11 player to be chosen Out of 14 player?

answer

$$= 14C_{11} = 14C_3$$

$$= \frac{14 \times 13}{3 \times 2 \times 1}$$

$$= 28 \times 13$$

$$= 364$$

example In How many ways committee of 6 members be selected from 7 Men, 5 ladies consisting of 4 Men and 2 ladies

answer

$$= 7C_4 * 5C_2$$

$$= \frac{7 \times 6}{3 \times 2} \times \frac{5 \times 4}{2}$$

$$= 350$$

example Out of 5 Women and 4 MEN, Committee of 3 member is formed in such way atleast one member is woman.
In How many different ways can it be done.

answer:-

$$4C_2 * 5C_1 + 5C_2 * 4C_1 + 5C_3 + 4C_0$$

$$\therefore 30 + 40 + 10$$

$$= 80$$

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Example 10 $\frac{75_{P_2}}{75_{C_2}}$ equal to _____

Answer $\frac{75_{P_2}}{75_{C_2}}$

$$= \frac{75_{P_2}}{75_{C_2}}$$

$$= \frac{75!}{(75 - 2)!} = \frac{75!}{73!} = 75 \times 74$$

$$= 75_{C_2}$$

$$= \frac{75 \times 74}{2}$$

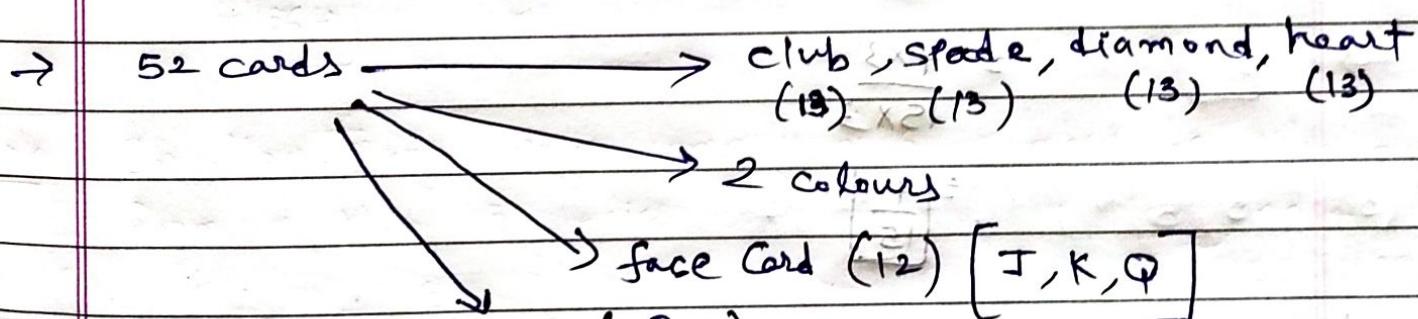
$$= \boxed{2775}$$

$$\rightarrow P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$$

$$\rightarrow (i) 0 \leq P(E) \leq 1$$

$\rightarrow P(E) = 0$, Impossible event (sun rises from west)

$\rightarrow P(E) = 1$, 100% possible event, (sun rises from east)
sure event



→ 2 coin:-

(HH, HT, TH, TT)

→ 3 coin:-

H H H

H H T

H T H

H T T

T H H

T H T

T T H

T T T

2^n possible outcomes for n coin flip at time.

6^n possible outcomes for n dice flip at time.

example A Bag contains 6 white balls and 4 black balls. 2 balls are drawn at random. find probability that they are of same colour.

answer

$$6C_2 + 4C_2$$

$$1 \geq (3)9 \geq 0 \quad (i)$$

$$10C_2$$

$$= 15 + 6$$

$$45$$

$$= 7 \times 3$$

$$= 15 \times 3 (ii)$$

$$\left[\frac{7}{15} \right]$$

example 2 cards drawn at random from a pack of 52 cards, what is probability that both are black or both are queen.

answer

$$26C_2 + 4C_2 - 1$$

$$P(A) + P(B) - P(A \cap B)$$

$$52C_2$$

$$= \frac{26 \times 25}{2} + \frac{4 \times 3}{2} - 1$$

$$\frac{52 \times 51}{2}$$

$$= \boxed{\frac{330}{1326}} \rightarrow \boxed{\frac{55}{221}}$$

example 52 cards \rightarrow one queen is removed, then find probability of king of black colour.

answer:

$$= \frac{2}{51}$$

as, QS, KS, QH, QD, KH, KD (a)

QS, KS, QH, QD, KH, KD (c)

as, QS, KS, QH, QD, KH, KD (d)

as, QS, KS, QH, QD, KH, KD (e)

as, QS, KS, QH, QD, KH, KD (f)

example find Odd Man Out | Odd Number | Wrong Number
values should be from 1 to

(1) $3, 5, 7, 12, 17, 19$

→ 12

(2) $10, 14, 16, 18, 21, 24, 26$

→ 21

(3) $3, 5, 9, 11, 14, 17, 21$

→ 14

(4) $1, 4, 9, 16, 23, 25, 36$

→ 23

(5) $6, 9, 15, 21, 24, 28, 30$

→ 28

(6) $41, 43, 47, 53, 61, 71, 73, 81$

→ 81

(7) 16, 25, 36, 72, 144, 196, 225

→ [72] [Left] ←

(8) 10, 25, 45, 54, 60, 75, 80

→ [54] [Left] ←

(9) 1, 4, 9, 16, 20, 36, 49

→ [20] [Right] ←

(10) 8, 27, 64, 100, 125, 216, 343

→ [100] [Left] ←

(11) 1, 5, 14, 30, 50, 55, 91

→ [50] [Left] ←

(12) 385, 462, 572, 396, 427, 671, 264

→ [427] [Left] ←

Q. → (13) 835, 734, 642, 751, 853, 981, 532

→ [751] ← [SF]

→ (14) 331, 482, 551, 263, 383, 242, 111

→ [383] ← [F2]

(15) 2, 5, 10, 17, 26, 37, 50, 64

→ [64] ← [LCS]

(16) 253, 136, 352, 460, 324, 631, 244

→ [324] ← [001]

(17) 2, 5, 10, 50, 500, 5000

→ [5000] ← [02]

(18) 4, 5, 7, 10, 14, 18, 25, 32

→ [18] ← [LCS]

(19) 582, 605, 588, 611, 634, 617, 680

→ 634

(20) $46080, 3840, 384, 48, 24, 2, 1$ (ECS, EAP, P)

→ | 24

(21) 56, 72, 90, 110, 132, 150, 168, 182, 195, 202, 208 (c)

$$\rightarrow \boxed{150}$$

$$(22) \quad 8, 13, 21, 32, 47, 63, 83$$

→ 37

(22) example - Insert Missing Number

$$(1) \quad 4, -8, 16, -32, 64, \underline{\quad}$$

→ -128

$$(2) \quad 5, 10, 13, 26, 29, 58, 61, \dots$$

122

Q. (3) 8, 7, 11, 12, 14, 17, 17, 22, —

→ 20

(4) 9, 11, 15, 23, 39, —

→ 71

(5) 264, 262, 271, 243, 308, —

→ 182

Volume & Surface Area

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Shape

CSA

(Curved Surface Area)

TSA

(Total Surface Area)

$$= \text{CSA} + \text{Area of}$$

Top + Area of bottom

Volume

= Cross sectional Area \times Height

(1) Cuboid

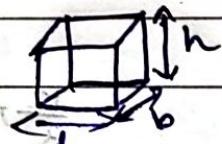
$$2lh + 2bh$$

$$2lh + 2bh +$$

$$2lb$$

$$= (lb)h$$

$$= lbh$$



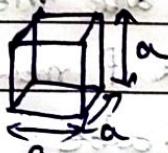
$$\text{diagonal } d = \sqrt{l^2 + b^2 + h^2}$$

(2) Cube

$$4a^2$$

$$6a^2$$

$$a^3$$



$$l = b = h = a$$

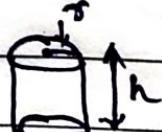
$$\text{cube's diagonal } d = \sqrt{3}a$$

(3) Cylinder

$$2\pi rh$$

$$= 2\pi rh + 2\pi r^2$$

$$= \pi r^2 h$$



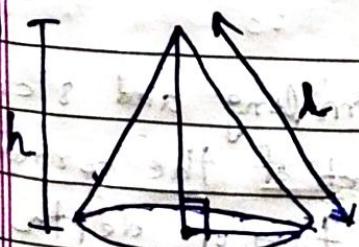
$$S.C.A = 2\pi r(h+r)$$

(4) Cone

$$\pi rl$$

$$= \pi rl + \pi r^2$$

$$\frac{1}{3}\pi r^2 h$$



Cone: $l = \text{Slant height}$

$$l^2 = r^2 + h^2$$

Shape

CSA

TSA

Volume

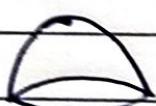
(5) Sphere



$$4\pi r^2$$

$$\frac{4}{3}\pi r^3$$

(6) Hemisphere



$$2\pi r^2$$

$$3\pi r^2$$

$$\frac{2}{3}\pi r^3$$

example 3 solid cube of side 1cm, 2cm, 8cm are melted to form a new cube then find surface area of new cube.

answer.

volume of old = volume of new cube

$$1 + 216 + 512 = a^3$$

$$\therefore a = 9$$

$$\text{Surface Area} = 6(a)^2$$

$$= 6 \times 81$$

$$= 486 \text{ cm}^2$$

example Rectangular sheet of paper 10cm long and 8cm wide has squares of side 2cm but each of the corners. The sheet is folded to form a tray of depth 2cm. find volume of this tray.

Answer: Total perimeter of the given figure is 36 cm.

After performing factor substitution analysis = added to model

$$1 \times 21 \times 0.5 = 21 \times 21 \times 21$$

exactas

example A room is 12m long, 9m broad & 8m high. Find length of longest bamboo pole that can be placed in it.

answer

\therefore Maximum length = diagonal of Cuboid

$$d = \sqrt{1^2 + b^2 + h^2} \text{ meters per second}$$

Example Two cubes have volume in ratio $\frac{1}{27}$. Then find ratio of surface areas.

Answer :-

$$\frac{(\text{to})}{\text{e}} \rightarrow$$

right hand & F often in case need to switch hands

Introduces the idea of a function + its definition in simple terms.

Reference 13. Truss (vibro) & survey

$$\left(\frac{dx}{dt} \right) \left(\frac{\partial \phi}{\partial x} \right) = \text{either small or}$$

$$1000 \text{ cm}^3 = 1 \text{ litre}$$

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example Cube of edge 15 cm is emerged completely into rectangular vessel containing water. If the dimension of base of vessel are 20 cm \times 15 cm. find rise in water level.

answer

volume of cube = volume rectangular vessel containing water

$$15 \times 15 \times 15 = 20 \times 15 \times h$$

$$5 \times 3 \times 5 \times 3$$

After brief shift we get $5 \times 4 = h$ (and now A is more)

$h = 11.25 \text{ cm}$ if solid topped top) is

→ rise in height of water

biscuit if required surface

example How many meter of cloth 5 m wide will be required to make conical tent the radius of base is 7 m and height is 24 meter

answer:

If Height of two cone in ratio $7:3$ and their diameter are in ratio $6:7$. What is ratio of volumes?

answer Δ volume $\propto (\text{radius})^2 \text{ Height}$ OR $\frac{(\text{diameter})^2}{2} (\text{Height})$

$$\text{volume ratio} = \left(\frac{36}{49}\right) \left(\frac{7}{3}\right)$$

$$= \frac{12}{7}$$

example Three sphere of radii 3 cm, 4 cm, 5 cm are melted to form a sphere then find radius of new sphere.

answer:

$$\frac{4}{3}\pi((3)^3 + (4)^3 + (5)^3) = \frac{4}{3}\pi(r^3)$$

$$\therefore 27 + 64 + 125 = r^3$$

$$\therefore r = 6 \text{ cm}$$

example How many spherical bullets can be made out of lead cylinder 28 cm High and base radius 6 cm. Each bullet 1.5 cm in diameter.

answer:

→ n is number of bullets.

$$\pi(6)^2(28) = n\left(\frac{4}{3}\right)(\pi)(0.75)^3$$

$$\therefore n = 1792$$

(equalling total volume)

example Cylindrical container of radius 16 cm and height 15 cm filled with ice cream. Now whole ice cream have to distributed to 10 children in equally cones, with hemispherical top if height of conical portion is 4 times the radius of its base then find radius of ice cream.

answer:

$$\therefore \pi r^2 h = 10 \left(\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \right)$$

$$\therefore (6)^2(15) = 10 \left(\frac{1}{3} (\gamma^2)(4\gamma) + \frac{2}{3} \gamma^3 \right)$$

$$\therefore \frac{36 \times 3 \times 15}{10} = 6\gamma^3$$

$$\gamma^3 = \frac{36 \times 3 \times 15}{10 \times 6} \quad \boxed{\gamma = 3}$$