

Lecture 16

Recursive algorithms + Divide and Conquer

1. Divide
2. Conquer the smaller "subproblem"
3. Combine

Analyzing Time Complexity Given pseudocode

Merge Sort

$MS(A, P, R) \quad \text{--- } T(n)$

1	if ($P < R$)	1
2	$Q = \lfloor (P+R)/2 \rfloor$	
3	$MS(A, P, Q)$	$T(\frac{n}{2})$
4	$MS(A, Q+1, R)$	$T(\frac{n}{2})$
5	Merge $[A, P, Q, R]$	n

So $T(n)$ of this is $T(n) = 2T(\frac{n}{2}) + n + 1$

Binary Search

BinarySearch(A, v, low, mid)

```
1  if low ≤ high
2      mid = ⌊(low + high) / 2⌋
3      if v == A[mid]
4          return mid
5      else if v < A[mid]
6          return BinarySearch(A, v, low, mid - 1)
7      else return BinarySearch(A, v, mid + 1, high)
8  return NIL
```

Line 1-5 : 1

Line 6 : $T(\frac{n}{2})$

Line 7 : $T(\frac{n}{2})$

8 : 1

$$\begin{aligned} T(n) &= 1 + T(\frac{n}{2}) + 1 \\ &= T(\frac{n}{2}) + 1 \end{aligned}$$

← only $T(\frac{n}{2})$ and not $2T(\frac{n}{2})$ because
line 6 or 7 runs, not both.

• Always Assume that the function was initially called on an input size of n .

• $T(\frac{n}{2} - 1)$ can be simplified to $T(\frac{n}{2})$

• $T(n-1)$ cannot be simplified, it stays $T(n-1)$