Lecture 12

Partial Orders

- A relation is a partial order if its:
    1. reflexive
    2. transitive
    3. anti-symmetric
- If aRb for a partial order R, we say $a \preceq b$.
- Partially ordered set: $(A, \preceq)$ is called a poset.
- if $x \preceq y$, then x, y are comparable
- Total order: A $\preceq$ is a total order if every two elements in the domain are comparable.
- x is minimal if there is no y such that $y \preceq x$.
- x is maximal if there is no y such that $x \preceq y$.

Strict Order

- a relation is a strict order if its:
    1. transative
    2. anti-symmetric.

Algorithms

- An algorithm is a finite set of precise instructions for solving a problem.
- A problem is defined by a pair (input, desired output).

The Growth of Functions

- We use "time complexity" to evaluate algorithms.
- Big O notation:
  - When f's growth rate is less than or equal to g's growth rate.

$$f(x) = \boxed{3x^2} + 2x + 4 = O(x^3)$$

analyze dominant part and drop constants

$$f(x) = 10x^2 \log(x) + x + 4 = O(x^2 \log x)$$

$$f(x) \text{ is } \begin{cases} \text{faster than } g(x) & \text{if } \lim_{x \to \infty} \frac{f(x)}{g(x)} = +\infty \\ \text{as fast as } g(x) & \text{if } \lim_{x \to \infty} \frac{f(x)}{g(x)} = \text{some constant} \\ \text{slower than } g(x) & \text{if } \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \end{cases}$$

Example

$$4x^3 = O(x^3)$$
$$4x^3 = O(x^4)$$
$$4x^3 = O(x^{3.5})$$
$$4x^3 \neq O(x^{2.5})$$

- Little - Oh : "less than and not equal to"
- Big - theta : "Exactly equal to"
- Big - Omega : " Greater than or equal to "
- Little - Omega : " Greater than and not equal to "

Formal / Mathematical Approach

- f is $O(g)$ if :

$$\exists c, n_0 \quad \forall n > n_0 : f(n) \le c \cdot g(n)$$

$\boxed{\text{Example}}$

$$3n + 7 = O(n)$$

$$f(n) \le c \cdot g(n)$$

n	f(n)	g(n)	$\left\lceil \frac{f(n)}{g(n)} \right\rceil$
1	10	1	10
10	37	10	4
100	307	100	4

ceiling function

4 is potential candidate for c

so $n > 10 : f(n) \le 4 \cdot g(n)$

so $n > 10$

$$3n + n > 3n + 10$$

$$4n > 3n + 10 > 3n + 7$$

$$c \cdot g(n) > f(n)$$