

Lecture 16

Recursive algorithms & Divide and Conquer

1. Divide

2. Conquer the smaller "subproblem"

3. Combine

Analyzing Time Complexity Given pseudocode

Merge Sort

MS(A, P, R) $\rightarrow T(n)$

1 if ($P < R$)

2 $Q = \lfloor (P+R)/2 \rfloor$

3 MS(A, P, Q) $\rightarrow T(\frac{n}{2})$

4 MS($A, Q+1, R$) $\rightarrow T(\frac{n}{2})$

5 Merge $[A, P, Q, R]$ $\rightarrow n$

So $T(n)$ of this is $T(n) = 2T(\frac{n}{2}) + n + 1$

Binary Search

Binary Search ($A, v, \text{low}, \text{mid}$)

- 1 if $\text{low} \leq \text{high}$
- 2 $\text{mid} = \lfloor (\text{low} + \text{high}) / 2 \rfloor$
- 3 if $v == A[\text{mid}]$
- 4 return mid
- 5 else if $v < A[\text{mid}]$
- 6 return Binary Search ($A, v, \text{low}, \text{mid}-1$)
- 7 else return Binary Search ($A, v, \text{mid}+1, \text{high}$)
- 8 return NIL

Line 1-5 : 1

Line 6: $T\left(\frac{n}{2}\right)$

Line 7: $T\left(\frac{n}{2}\right)$

8 : 1

$$\begin{aligned} T(n) &= 1 + T\left(\frac{n}{2}\right) + 1 && \leftarrow \text{only } T\left(\frac{n}{2}\right) \text{ and not } 2T\left(\frac{n}{2}\right) \text{ because} \\ &= T\left(\frac{n}{2}\right) + 1 && \text{Line 6 or 7 runs, not both.} \end{aligned}$$

- Always assume that the function was initially called on an input size of n .
- $T\left(\frac{n}{2}-1\right)$ can be simplified to $T\left(\frac{n}{2}\right)$
- $T(n-1)$ cannot be simplified, it stays $T(n-1)$