

Logic  $\rightarrow$  study of formal reasoning

- Proposition  $\rightarrow$  Statements that are either true or false
- $\rightarrow$  typically declarative sentences
  - $\rightarrow$  "Are you awake?" is NOT a proposition
  - $\rightarrow$  Commands are NOT propositions

A Propositions Truth Value  $\rightarrow$  the value indicating whether the proposition is true or false

$\rightarrow$  VALUES can be true, false, unknown or a matter of opinion

Proposition Variables ( $p, q, r$ )  $\rightarrow$  denote arbitrary propositions

Compound Proposition  $\rightarrow$  connecting individual propositions with logic operations

$\rightarrow$  " $\wedge$ " means AND

Truth Table  $\rightarrow$  shows all scenarios of a compound proposition

Disjunction Operation  $\rightarrow$  " $\vee$ " means INCLUSIVE OR

Exclusive OR  $\rightarrow$  Exactly 1 proposition is true but NOT BOTH

$\rightarrow$  Symbol " $\oplus$ "

Negation Operation  $\rightarrow$  acts on only one proposition

$\rightarrow$  read as "not  $p$ "

$\rightarrow$  symbol " $\neg$ "

## Order of operations without parentheses

1.  $\neg$  (not)

2.  $\wedge$  (and)

3.  $\vee$  (or)

Conditional Operation  $\rightarrow$  "if  $p$  then  $q$ "

$\rightarrow$  symbol " $p \rightarrow q$ "

$\rightarrow$  where  $p$  is the hypothesis and  $q$  is the conclusion

$\rightarrow$  if the hypothesis is false then the conditional statement is true regardless

Converse, contrapositive, and inverse of  $p \rightarrow q$

Converse:  $q \rightarrow p$

contrapositive:  $\neg q \rightarrow \neg p$

Inverse:  $\neg p \rightarrow \neg q$

Biconditional Operation: " $p$  if and only if  $q$ "

Symbol: " $p \leftrightarrow q$ "

- this is only true if  $p$  and  $q$  have the same truth value

Tautology  $\rightarrow$  a compound proposition that is always true

Contradiction  $\rightarrow$  a compound proposition that is always false

Logically Equivalent  $\rightarrow$  Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions.

Notation:  $s \equiv r$

De Morgan's Laws : logical equivalences that show how to correctly distribute a negation operation inside parenthesized expressions.

$$1^{\text{st}} \text{ Law: } \neg(p \vee q) \equiv (\neg p \wedge \neg q)$$

$$2^{\text{nd}} \text{ Law: } \neg(p \wedge q) \equiv (\neg p \vee \neg q)$$