

# Logic → Study of formal reasoning

Proposition → Statements that are either true or false

→ typically declarative sentences

→ "Are you awake?" is NOT a proposition

→ Commands are NOT propositions

A Proposition's Truth Value → the value indicating whether the proposition is true or false

→ Values can be true, false, unknown or a matter of opinion

Proposition Variables ( $p, q, r$ ) → denote arbitrary propositions

Compound Proposition → connecting individual propositions with logic operations

→ " $\wedge$ " means AND

Truth Table → shows all scenarios of a compound proposition

Disjunction Operation → " $\vee$ " means INCLUSIVE OR

Exclusive OR → Exactly 1 proposition is true but NOT BOTH

→ Symbol " $\oplus$ "

Negation Operation → acts on only one proposition

→ read as "not  $p$ "

→ Symbol " $\neg$ "

## Order of operations without parentheses

1.  $\neg$  (not)

2.  $\wedge$  (and)

3.  $\vee$  (or)

Conditional Operation  $\rightarrow$  "if  $p$  then  $q$ "

$\rightarrow$  symbol " $p \rightarrow q$ "

$\Rightarrow$  where  $p$  is the hypothesis and  $q$  is the conclusion

$\rightarrow$  if the hypothesis is false then the conditional statement is true regardless

Converse, contrapositive, and inverse of  $p \rightarrow q$

Converse:  $q \rightarrow p$

contrapositive:  $\neg q \rightarrow \neg p$

Inverse:  $\neg p \rightarrow \neg q$

Biconditional Operation: " $p$  if and only if  $q$ "

Symbol: " $p \leftrightarrow q$ "

• this is only true if  $p$  and  $q$  have the same truth value

Tautology  $\rightarrow$  a compound proposition that is always true

Contradiction  $\rightarrow$  a compound proposition that is always false

Logically Equivalent  $\rightarrow$  Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions.

Notation:  $S \equiv r$

De Morgan's Laws: logical equivalences that show how to correctly distribute a negation operation inside parenthesized expressions.

$$\text{1st Law: } \neg(p \vee q) \equiv (\neg p \wedge \neg q)$$

$$\text{2nd Law: } \neg(p \wedge q) \equiv (\neg p \vee \neg q)$$