

9/14/25

Lecture 04

$$\neg(x^2 > 0) \equiv x^2 \leq 0$$

$$x \in \mathbb{Z}^+ \cup \{0\}$$

$$\forall x \forall y \ y > x \equiv F \quad \text{counterexample: } x=2, y=0$$

Think of $\forall x \forall y \ y > x$ as:

| | | y | | |
|---|--|---|-------|-------------|
| | | 0 | 1 | 2 3 |
| x | | 1 | (1,1) | (1,2) (1,3) |
| | | 2 | : | : |
| | | 3 | : | : |

• Check if every combination is true

$$\forall x \exists y \ (y > x) \equiv T$$

$\forall x \exists y \ (y > x) \equiv$ "for every x is there at least one y where $y > x$ "

$\exists y \forall x \ (y > x) \equiv$ "is there one y that is greater than all possible x ?"

$$\exists x \forall y \exists z \ x+y > z \equiv T$$

$$\forall x \forall y \exists z \ x+y > z \equiv F \quad \text{counter example: } \begin{matrix} x \text{ selects } 0 \\ y \text{ selects } 0 \end{matrix}$$

$M(x, y) = x \text{ sent an email to } y$

- Logically express "everyone else"
 $\forall x \forall y ((x \neq y) \rightarrow M(x, y))$

- Logically express "someone else"
 $\forall x \exists y ((x \neq y) \wedge M(x, y))$

- Logically express "exactly one"
 $\exists x (L(x) \wedge \forall y ((x \neq y) \rightarrow \neg L(y)))$

- Logically express "at least 2"

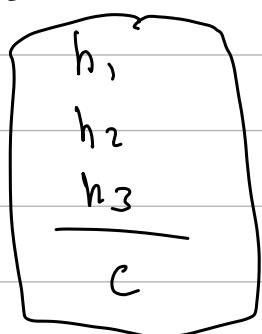
$$\exists x \exists y (x \neq y \wedge M(\text{Tacho}, x) \wedge M(\text{Tacho}, y))$$

- Logically express "exactly 2"

$$\exists x \exists y [x \neq y \wedge M(\text{Tacho}, x) \wedge M(\text{Tacho}, y) \wedge \forall z (z \neq x \wedge z \neq y \rightarrow \neg M(\text{Tacho}, z))]$$



Argument



$$(h_1 \wedge h_2 \wedge h_3) \rightarrow c$$