

9/4/25

# Lecture 04

$$\neg(x^2 > 0) \equiv x^2 \leq 0$$

$$x \in \mathbb{Z}^+ \cup \{0\}$$

$$\forall x \forall y \quad y > x \equiv F \quad \text{counter example: } x=2, y=0$$

Think of  $\forall x \forall y \quad y > x$  as:

		y			
		0	1	2	3
x	1	(1,1)	(1,2)	(1,3)	
	2	:	:	:	
	3				
	3				

• Check if every combination is true

$$\forall x \exists y \quad (y > x) \equiv T$$

$\forall x \exists y \quad (y > x) \equiv$  "for every  $x$  is there at least one  $y$  where  $y > x$ "

$\exists y \forall x \quad (y > x) \equiv$  "is there one  $y$  that is greater than all possible  $x$ "

$$\exists x \forall y \exists z \quad x+y > z \equiv T$$

$$\forall x \forall y \exists z \quad x+y > z \equiv F \quad \text{counter example: } x \text{ selects } 0, y \text{ selects } 0$$

$M(x, y) = x$  sent an email to  $y$

• Logically express "everyone else"  
 $\forall x \forall y ((x \neq y) \rightarrow M(x, y))$

• Logically express "someone else"  
 $\forall x \exists y ((x \neq y) \wedge M(x, y))$

• Logically express "exactly one"  
 $\exists x (L(x) \wedge \forall y ((x \neq y) \rightarrow \neg L(y)))$

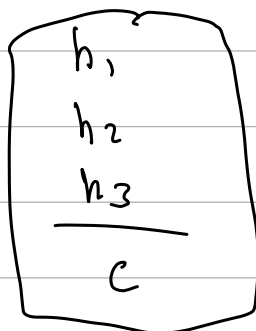
• Logically express "at least 2"

$$\exists x \exists y (x \neq y \wedge M(\text{Tuebo}, x) \wedge M(\text{Tuebo}, y))$$

• Logically express "exactly 2"

$$\exists x \exists y [x \neq y \wedge M(\text{Tuebo}, x) \wedge M(\text{Tuebo}, y) \wedge \forall z (z \neq x \wedge z \neq y \rightarrow \neg M(\text{Tuebo}, z))]$$

Argument



$$(h_1 \wedge h_2 \wedge h_3) \rightarrow c$$