Algorithmic Machine Learning, Fall 2024
Homework 1

Due date: 9/29/24 at 11:50PM

1. The discrete random variables X_0, X_1, and X_2 have the following joint distribution along with values that each can attain:

For $X_0 = 1$:

	X_1		
X_2	1	3	5
3	0.08	0.04	0.02
4	0.03	0.01	0.01
5	0	0.06	0

For $X_0 = 3$:

	X_1		
X_2	1	3	5
3	0.02	0.05	0.02
4	0.03	0.05	0.02
5	0.15	0.06	0.06

For $X_0 = 4$:

	X_1		
X_2	1	3	5
3	0.02	0.03	0.02
4	0.02	0.02	0.01
5	0.05	0.08	0.04

where the top table is the joint pmf values of the random vector $X = (X_0, X_1, X_2)$ for $X_0 = 1$, the middle table is the joint pmf values of X for $X_0 = 4$ and the bottom table is the joint pmf of X for $X_0 = 4$.

Answer the following questions using Python. Justify your answers using notebook markup. All logarithms are in base 2.

1b) Which if any of the pairs (X_0, X_1), (X_0, X_2), and (X_1, X_2) are independent

Part 1. Checking if (X_0, X_1) are independent

$P(X_0=1)$ Marginal Probability for X_0

$0.08+0.04+0.02+0.03+0.01+0.01+0+0.06+0 = 0.25$

Marginal Probability for X_0

$P(X_0=3) = 0.02+0.05+0.02+0.03+0.05+0.02+0.15+0.06+0.06 = 0.46$

Marginal Probability for X_0

$P(X_0=4) = 0.02+0.03+0.02+0.02+0.02+0.01+0.05+0.08+0.04 = 0.29$

Calculating Marginal Probabilities for X_1

$P(X_2=1) = 0.08+0.03+0+0.02+0.03+0.15+0.02+0.02+0.05 = \boxed{0.40}$

$P(X_1=3) = 0.04+0.01+0.06+0.05+0.05+0.06+0.03+0.02+0.08 = 0.40$

$P(X_1=5) = 0.02+0.01+0+0.02+0.02+0.06+0.02+0.01+0.04 = 0.20$

Checking example to test if they are independent
if they are independent joint probability will = product of marginal

Example 1 = $P(X_0=1, X_1=1)$

$P(X_0=1, X_1=1) = 0.08+0.03+0 = 0.11$

Product of marginals $P(X_0=1) \times P(X_1=1) = 0.1$
0.25×0.40

Since $0.11 \neq 0.1$ X_0, X_1 are not independent

Checking independence between (X_0, X_2)
we already have marginal probabities of
X_0, $P(X_0=1)=0.25$, $P(X_0=3)=0.46$,
$P(X_0=4)=0.29$

but still need to find marginal probabilities for X_2

$P(X_2=3) = 0.08+0.04+0.02+0.02+0.05+0.02+0.02+0.03+0.02$
$= \boxed{0.30}$

$P(X_2=4) = 0.03+0.01+0.01+0.03+0.05+0.02$
$+0.02+0.02+0.01 = \boxed{0.2}$

$P(X_2=5) = 0+0.06+0+0.15+0.06+0.06+0.05+0.08+0.04$
$= \boxed{0.50}$

Check independence using $P(X_0=1, X_2=3)$

$P(X_0=1, X_2=3) = 0.08+0.04+0.02 = \underline{0.14}$

$P(X_0=1) = 0.25 \quad P(X_2=3) = 0.30$

$0.25 \times 0.30 = \underline{0.075} \quad 0.14 \neq 0.07$

Since $P(X_0=1, X_2=3) \neq P(X_0=1) * P(X_2=3)$,

(X_0, X_2) are not independent

Checking if (X_1, X_2) are independent

since we have all marginal probabilities we will check example using

$P(X_1=1, X_2=3) = 0.08+0.02+0.02 = 0.12$

$P(X_1=1) = 0.40 \times 0.30 = 0.12$

$P(X_2=3) = 0.30$

Since $\underline{P(X_1=1, X_2=3) = P(X_1=1) * P(X_3=3)}$

(X_1, X_2) are independent