2023 CHANTILLY MATH COMPETITION
MIDDLE SCHOOL DIVISION

DO NOT OPEN THIS PACKET UNTIL YOU ARE INSTRUCTED TO DO SO

Participant Information

(a) Participant Name KEY

(b) Participant Grade Level –

(c) Participant Email Address –

(d) School Name –

RULES/INFORMATION

- Participants will have 90 minutes for the exam.
- Outside resources such as calculators, mobile devices, textbooks are not allowed.
- Collaboration is not allowed.
- This exam consists of 25 free response questions.
- The problems will be in order of increasing difficulty in each section, but you may occasionally find some later questions easier, depending on experience.
- Each problem in each section will be worth 5, 5, 6, 6, and 8 points respectively.
- The advanced problems are worth 8 points.
- The answers to the problems are guaranteed to be integers.

ALGEBRA

Question 1

Let $y = \frac{3}{32}x^2 - 6$. What is the distance between one of the x-intercepts and y-intercept of the function?

KEY

10

Question 2

A hand glider descends from a 8 km cliff at an initial horizontal velocity of 20 km/hr and a constant vertical descent velocity of 5km/hr.

(a) How many minutes will it take until the glider descends to the ground?

~~24~~ 96 minutes

(b) How far will the glider travel horizontally by the time the glider lands?

32 km

Question 3

The height of a ball thrown upwards is given by $h = 16t - 4t^2$, where h is measured in meters and t in seconds. At what time $t > 0$ will the ball reach its peak height of 16 meter before falling back down?

2 seconds

Question 4

Timmy rides a roller coaster that follows the path $y = -t^3 + 6t^2 - 9t + 4$. Assuming the roller coaster starts at time $t = 0$, how long will Timmy stay above ground until he goes **underground** for the first time? Consider $y > 0$ to be above ground and $y < 0$ to be below ground.

4 seconds

Question 5 continued on next page...

Question 5

Calculate the six roots of the polynomial $x^6 + 9x^3 + 8 = 0$.

(a) What are the two real roots of the polynomial? (2pts)

-2, -1

(b) Two of the remaining four roots of the polynomial will be of the form $a \pm i\sqrt{b}$ once simplified. What is $|a| + |b|$? (3pts)

4

(c) Two of the remaining four roots of the polynomial will be of the form $\frac{a \pm i\sqrt{b}}{c}$ once simplified. What is $|a| + |b| + |c|$? (3pts)

6

GEOMETRY

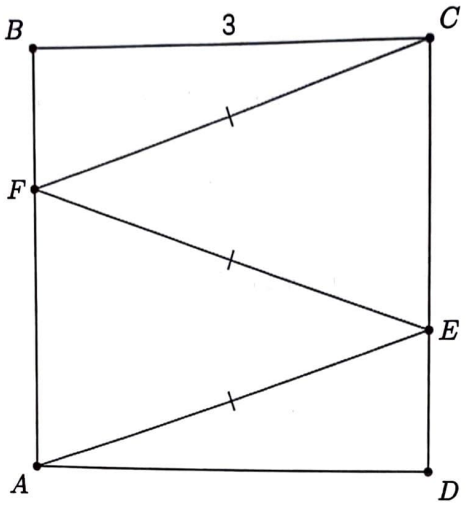
Question 6

In a cafeteria, there two circular and four rectangular tables. The circular tables have a radius of 2 feet and the rectangular tables have dimensions of 2 x 9 feet. What is the total surface area of all the tables? Use the approximation $\pi = 3$ when evaluating the areas for the circular tables.

96

Question 7

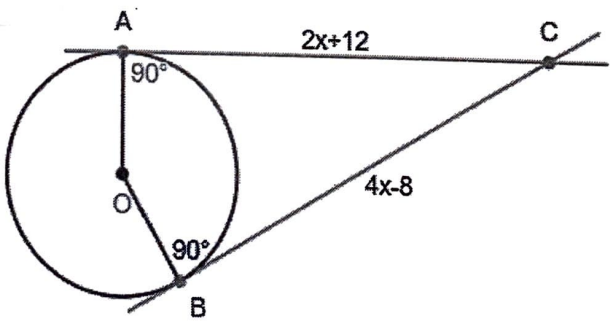
Given the square ABCD with side length 3 and three congruent segments AE, EF, and FC as shown below, the total length of the red line segment can be written as $a\sqrt{b}$, where a and b are integers. Find $a+b$.



13

Question 8

Circle O is tangent to $\overline{AC}$ and $\overline{BC}$. If the distance of $\overline{AC}$ is $2x+12$ and that of $\overline{BC}$ is $4x-8$, find x.

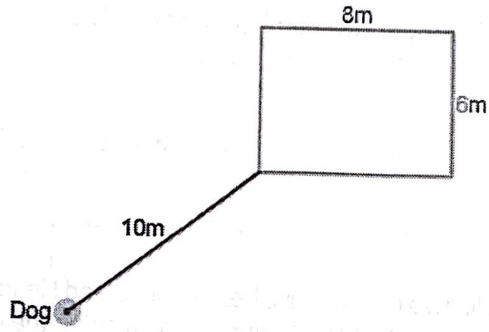


10

Question 9 continued on next page...

Question 9

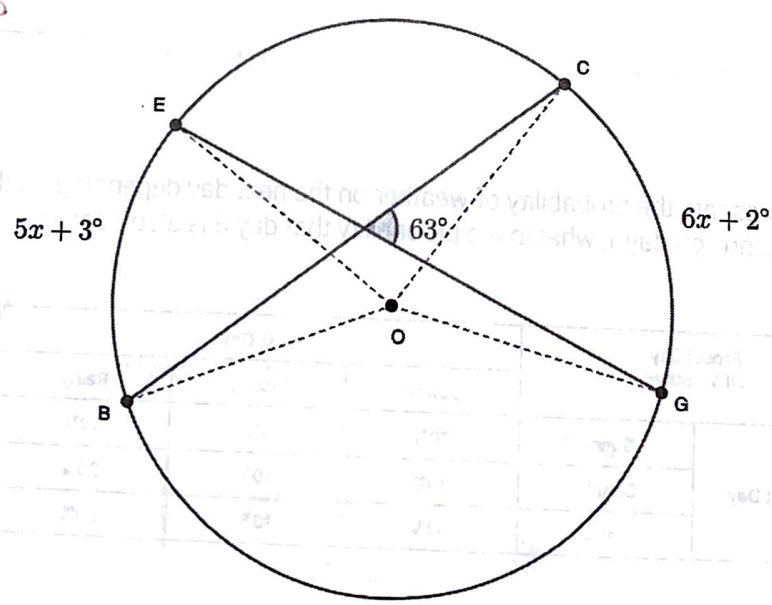
A dog is tied to an inelastic string attached to the corner of the rectangular fence so that the dog cannot move into the fence. The string is 10 meters and the fence has width of 6 meters and length of 8 meters. What is the area in which the dog can move freely? Use the approximation $\pi = 3$.



240 m^2

Question 10

What is the value of x where the given values for AB and CD are the respective angles of the arcs of the circle with center O.



11

PROBABILITY & STATISTICS

Question 11

Sam rolls two fair dice and multiplies the two numbers from each dice. What is the probability the product of is even?

75 %

Question 12

Laura flips a fair coin until she rolls two consecutive heads (Head and Head) or tails followed by a head (Head and Tail). What is the probability that Laura stops flipping the coin because she rolls two consecutive heads?

0.5

25 %

Question 13

If the average of 5 consecutive numbers is n and the average of the 5 consecutive numbers individually squared is $n^2 + a$, what is a?

2

Question 14

Below is a table represents the probability of weather on the next day depending on the weather on that day. If it is sunny on day 1, what is the probability that day 3 is also sunny?

Probability Of Weather		Next Day		
		Sunny	Cloudy	Rainy
Current Day	Sunny	70%	20%	10%
	Cloudy	30%	40%	30%
	Rainy	40%	10%	50%

11

59 %

Question 15 continued on next page...

Question 15

Suppose that S is a finite set of positive integers.

- If the least integer in S is removed from S, then the average value of the integers remaining is 29.
- If the second least integer in S is also removed, then the average value of the integers remaining is 30.
- If the greatest integer in S is also removed, then the average value of the integers remaining is 29.
- If the second least integer is then returned to the set, the average value of the integers remaining becomes 28.

If the difference between the greatest integer and the second least integer is 18, what is the average value of all the integers in the original set S?

28

NUMBER THEORY

Question 16

What is the difference between least common multiple and greatest common factor of 72, 84 and 90?

2514

Question 17

What is the value of the following expression?

$$\left(\frac{3^9}{\frac{2^3(52623^0 + 242^1 + 0^{52623})}{3^6}} \right)^{-52623}$$

$$\frac{2^6 + 2^3}{}$$

1

Question 18 continued on next page...

Question 18

How many positive integers less than 1,000 exist that have exactly 3 factors?

11

Question 19

How many possible non-zero integers n exist such that

$$\frac{n^2+1}{n+1}$$

is an integer? For example, when $n=0$, $\frac{n^2+1}{n+1}=\frac{0^2+1}{0+1}=\frac{1}{1}=1$ is an integer.

3

Question 20

Find the number of positive integers $n<20$ such that n^3+n is divisible by 3.

6

ADVANCED PROBLEMS

Question 21

Let x, y, z be positive real numbers less than 5. First, find the probability that $x+2y+3z \leq 9$. Your answer will be in the form $\frac{p}{q}$. Calculate $p+q$ after reducing $\frac{p}{q}$ to lowest terms if necessary.

1033

Question 22

The surface area of a a box is 216 square meters. What dimensions of the box length ℓ, width w, and height h will maximize the volume of the box?

$$V_{\text{box}} = \ell w h$$
$$A_{\text{surface of box}} = 2(\ell w + \ell h + h w)$$

(a) *What is the value of ℓ?*

6

(b) *What is the value of w?*

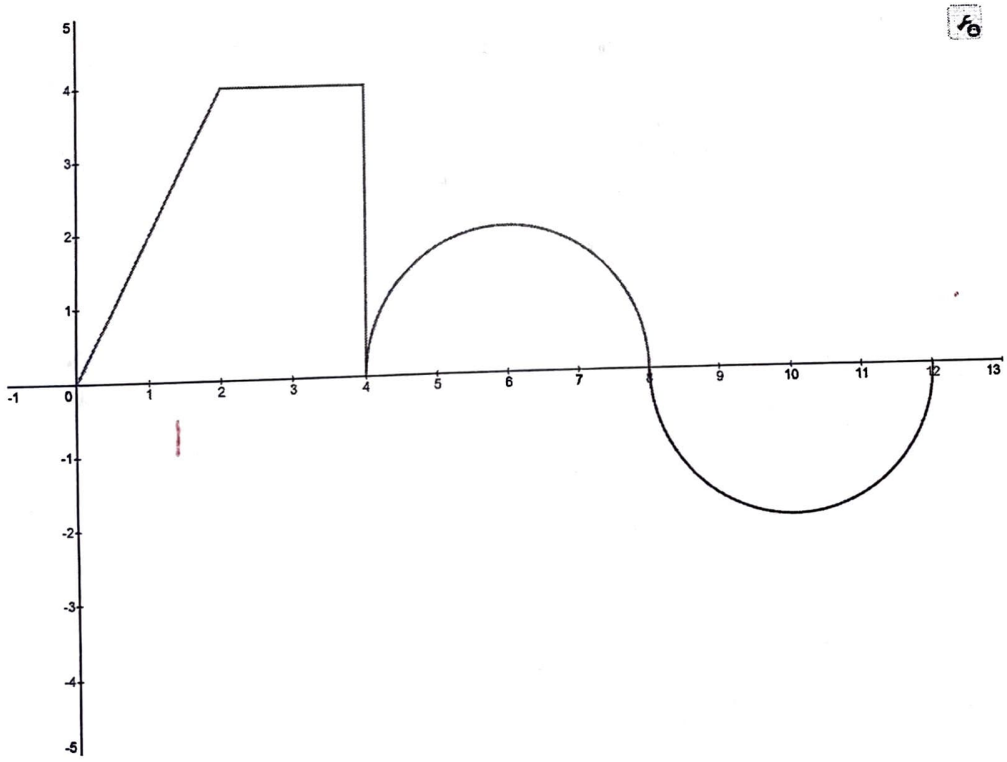
6

(c) *What is the value of h?*

6

Question 23

Calculate the area between the curve and the x-axis shown below on the domain $0 \leq x \leq 12$. Consider the area underneath the x-axis to be negative and the area above the x-axis to be positive.



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Question 24

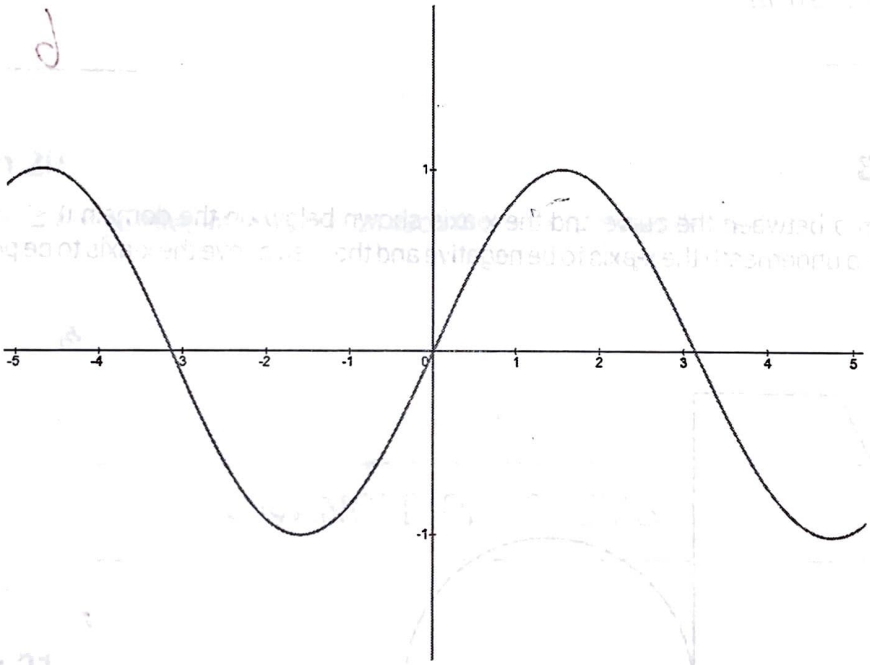
Find $x^3 + y^3$ given the following (you don't necessarily need to find x and y individually):

$$x^2 + y^2 = 12$$
$$x + y = 4$$

40

Question 25

The graph of $y = \sin x$ is shown below. Calculate the slope of the line tangent to graph at $x = 0$.



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