Notes · Assignment - 1

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Algebra of Matrices

Practice Examples :—

(a) Test for consistency and solve.

(i) $2x-3y+7z=5$,
$3x+y-3z=13$,
$2x+19y-47z=32$.

Here, $A=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}5\\ 13\\ 32\end{bmatrix}$

Soln :- It can be written in form of $AX=B$. {here, $AX=B$ ∴ non-homogeneous}

$\therefore [A:B]=\begin{bmatrix}2 & -3 & 7 & : & 5\\ 3 & 1 & -3 & : & 13\\ 2 & 19 & -47 & : & 32\end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\approx \begin{bmatrix}3 & 1 & -3 & 13\\ 2 & -3 & 7 & 5\\ 2 & 19 & -47 & 32\end{bmatrix}$

$R_1 \to \frac{1}{3}R_1$

$\begin{bmatrix}1 & \frac{1}{3} & -1 & 13\\ 2 & -3 & 7 & 5\\ 2 & 19 & -47 & 32\end{bmatrix} \xrightarrow{R_2 \to R_2 - R_3} \begin{bmatrix}1 & \frac{1}{3} & -1 & 13\\ 0 & -22 & 54 & -27\\ 2 & 19 & -47 & 32\end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{bmatrix}1 & \frac{1}{3} & -1 & 13\\ 0 & -22 & 54 & -27\\ 0 & \frac{55}{3} & -44 & -7\end{bmatrix}$

$R_3 \to R_3 - 55R_1$

$\begin{bmatrix}1 & \frac{1}{3} & -1 & 13\\ 0 & -22 & 54 & -27\\ -55 & 0 & 11 & -708\end{bmatrix}$

$\therefore [A:B]=\begin{bmatrix}2 & -3 & 7 & : & 5\\ 3 & 1 & -3 & : & 13\\ 2 & 19 & -47 & : & 32\end{bmatrix} \xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{bmatrix}1 & -\frac{3}{2} & \frac{7}{2} & 5\\ 3 & 1 & -3 & 13\\ 2 & 19 & -47 & 32\end{bmatrix}$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 2R_1$

$\begin{bmatrix}1 & \frac{3}{2} & \frac{7}{2} & 5\\ 0 & \frac{1}{2} & -\frac{27}{2} & -2\\ 0 & 22 & -54 & 22\end{bmatrix} \xrightarrow{R_3 \to R_3 - 44R_2} \begin{bmatrix}1 & \frac{3}{2} & \frac{7}{2} & 5\\ 0 & \frac{11}{2} & -\frac{27}{2} & -2\\ 0 & 0 & [illegible] & 3[illegible]\end{bmatrix}$

Here, $\boxed{\rho(A:B)=3}\ \boxed{\rho(A)=2} \neq$ no. of unknown.

$\rho(A) \neq \rho(A:B)$

∴ The given system is inconsistent (No Solution)

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(ii) $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

$-\frac{1}{2} - \frac{5}{6}$

$\frac{-3-5}{6}$

$-\frac{8}{6} = -\frac{3}{2}$

Solve:- Given eq. can be written in form of. $AX = B$.

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

Augmented matrix.

$$[A:B] = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & 4 & 0 \end{bmatrix} \xrightarrow{R_1 \to 1/2 R_1} \begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & 4 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ 0 & 3/2 & 1/2 & 8 \\ 0 & 5/2 & -1/2 & -12 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - \frac{5}{3}R_2} \begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ 0 & 3/2 & 1/2 & 8 \\ 0 & 0 & -4/3 & -\frac{76}{3} \end{bmatrix}$$

Here, Rank, $\rho(A:B) = \rho(A) = 3$

$\therefore$ System is consistent. (Unique sol^n)

Eq^{n}s are:- $\therefore$ $-\frac{4}{3}z = -\frac{76}{3} \Rightarrow \boxed{z = 19}$

$\frac{3}{2}y + \frac{1}{2}z = 8 \Rightarrow \frac{3}{2}y + \frac{19}{2} = 8 \Rightarrow \frac{3}{2}y = -\frac{3}{2} \Rightarrow \boxed{y = -1}$

$x - \frac{1}{2}y + \frac{3}{2}z = 4 \Rightarrow x + \frac{1}{2} + \frac{57}{2} = 4 \Rightarrow 2x + 58 = 8 \Rightarrow 2x = -50 \Rightarrow \boxed{x = -25}$

(iii) $4x - y = 12$,

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

Solve:- Given eq. can be in form of $AX = B$,
$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$$\therefore [A:B] = \begin{bmatrix} 4 & -1 & 0 & : & 12 \\ -1 & 5 & -2 & : & 0 \\ 2 & 0 & 4 & : & 8 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{4}R_1} \begin{bmatrix} 1 & -1/4 & 0 & 3 \\ -1 & 5 & -2 & 0 \\ 2 & 0 & 4 & 8 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 1/4 & 0 & 3 \\ 0 & 19/4 & -2 & 3 \\ 0 & 1/2 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{2}{19}R_2} \begin{bmatrix} 1 & 1/4 & 0 & 3 \\ 0 & 19/4 & -2 & 3 \\ 0 & 0 & 89/19 & 32/19 \end{bmatrix}$$

Here, $\rho(A:B) = \rho(A) = 3 = n$.
$\therefore$ Consistent & Unique

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$(\lambda^2-1)-3(\lambda-1)$
$\lambda^2-1-3\lambda+3 \quad \lambda^2-3\lambda+2.$

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(c) Find for what value of λ the given eq. have a solⁿ and solve them completely in each case.

$x+y+z=1$
$x+2y+4z=\lambda$
$x+4y+10z=\lambda^2$

Soln:- Given eq. in form of $AX=B$, $\begin{bmatrix}1&1&1\\1&2&4\\1&4&10\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\\lambda\\\lambda^2\end{bmatrix}$

$[A:B]=\begin{bmatrix}1&1&1&1\\1&2&4&\lambda\\1&4&10&\lambda^2\end{bmatrix}\xrightarrow[R_3\to R_3-R_1]{R_2\to R_2-R_1}\begin{bmatrix}1&1&1&1\\0&1&3&(\lambda-1)\\0&3&9&(\lambda^2-1)\end{bmatrix}\xrightarrow{R_3\to R_3-3R_2}\begin{bmatrix}1&1&1&1\\0&1&3&(\lambda-1)\\0&0&0&(\lambda^2-3\lambda+2)\end{bmatrix}$

Here, $\rho(A:B)=3$, $\rho(A)=2$.
$\rho(A:B)\neq\rho(A)$ ∴ Inconsistent

In order that given eq. has solⁿ, it should be consistent
$\rho(A:B)$ should $=\rho(A)$

Rank $A=2$, Since rank of $[A:B]$ should also be 2, it is necessary that
$\lambda^2-3\lambda+2=0$
$\Rightarrow \lambda^2-2\lambda-\lambda+2=0 \Rightarrow \lambda(\lambda-2)-1(\lambda-2)$
$\Rightarrow \boxed{\lambda=1,2}$

Case 1:- For $\lambda=1$, $\begin{bmatrix}1&1&1\\0&1&3\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$

$x+y+z=1.$
$y+3z=0$

No. of unknown (3) > Rank of A(2)
∴ Equations have infinite solution and we have to assign $3-2=1$ perimeter say k.

Let $z=k$, ∴ $y+3k=0 \Rightarrow \boxed{y=-3k}$
& $x+y+z=1 \Rightarrow x-3k+k=1 \Rightarrow \boxed{x=2k+1}$.

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Case ②:- For $\lambda = 2$ $\therefore \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\therefore x + y + z = 1.$

$y + 3z = 1.$

Here, also we have to assign $3-2=1$ para. say K.

Let $z = k$.

$\therefore \boxed{y = 1 - 3k}$

$\therefore x + y + z = 1.$

$\Rightarrow x + \cancel{1} - 3k + k = \cancel{1} \Rightarrow \boxed{x = 2k}$

(d) Find the solⁿ. of system of eqⁿ $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$.

Solⁿ:- Here, given eq. are.

$x + 3y - 2z = 0$ it can be expressed in $AX = B$.

$2x - y + 4z = 0$ $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x - 11y + 14z = 0$

$16 - \frac{112}{5}$

$\frac{80-112}{5}$

$\frac{32}{5}$

Here, $AX = 0$ $\therefore$ System is consistent (Unique / Infinite).

Now, $[A]$ = $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 8 \\ 0 & -14 & 16 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{14}{5} R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 8 \\ 0 & 0 & -\frac{32}{5} \end{bmatrix}$

$\rho(A) = 3 =$ no. of unknown

$\therefore$ Consistent and has Unique Solⁿ. Ans.

(e) Find for what values of λ the given eqs $3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$ may posses non-trivial solution, and solve them completely in each case.

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Solun:- Given eqs can be in form of $AX = B$

$3x - y - \lambda z = 0$

$4x - 2y - 3z = 0$

$2\lambda x + 4y + \lambda z = 0$

$$\begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here, $AX = 0$ ($\therefore$ System is consistent (Unique / Infinite)).

Now, $A = \begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{3} R_1} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 4R_1} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 2 & 4 & \lambda \end{bmatrix}$

$\xrightarrow{R_3 \rightarrow R_3 - 2R_1} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & 14/3 & 5\lambda/3 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 + \frac{7}{15} R_2} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & 0 & \left(\frac{11\lambda - 21}{15}\right) \end{bmatrix}$

$\frac{5\lambda}{3} + (-3-2\lambda)\frac{7}{15}$

$\frac{25\lambda}{15} - \frac{7}{5} - \frac{14\lambda}{15}$

$+\frac{11\lambda}{15} - \frac{7}{5}$

$\frac{11\lambda - 21}{15}$

Here, $\rho(A) =$ no. of unknow $= 3$.

$\therefore$ Consisitent & unique.

For, system to possess non-trivial solⁿ. (infinite solⁿ).

$\rho(A) <$ No. of unknow.

$\therefore \frac{11\lambda - 21}{15} = 0 \Rightarrow 11\lambda = 21 \Rightarrow \boxed{\lambda = \frac{21}{11}}$.

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Linear Algebra

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Assignment - 2

Problems :-

Are the following sets of vectors linearly independent or dependent?

1) [1 0 0], [1 1 0], [1 1 1]

Solve:- Let $u_1 = (1,0,0)$, $u_2 = (1,1,0)$, $u_3 = (1,1,1)$

Now, Matrix (A) = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore$ $Det(A) = 1(1-0) - 0 + 0 = 1 \neq 0$

$\therefore$ It is linearly independent. Ans

Note:- Different methods to find LI & LD :-

① Definition check:

- if the only solⁿ is $c_1v_1 + c_2v_2 + c_3v_3 + \dots + c_nv_n = 0$ is $c_1 = c_2 = \dots = c_n = 0$, then LI.

② Matrix Determinant method : [vectors as column].
if $det(A) \neq 0$, then LI.

③ Row Reduction (Gaussian Elimination)
$A:B$ $\left[\begin{matrix} a & b \\ c & d \end{matrix} \;\middle|\; \begin{matrix} 0 \\ 0 \end{matrix}\right]$. $\rho(A)$ ---

④ Rank Nullity Theorem.

If the rank of the matrix formed by the vectors equal no. of vectors, they are linearly independent.

eg:- $v_1 = [1,2]$, $v_2 = [3,4]$, Form matrix & calc. its rank.

$\boxed{\underbrace{\rho(A)}_{\text{Rank}} + \underbrace{N}_{\text{Nullity}} = \underbrace{D}_{\text{Dimension/ no. of col.}}}$

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$.

$\rho(A) = 2$.

Dimension (no. of col) = 2

$\therefore$ $N = D - \rho(A)$
$= 2 - 2$
$= \boxed{0}$

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2. $[7\ -3\ 11\ -6], [-56\ 24\ -88\ 48]$

Solun:- Let $a[7\ -3\ 11\ -6] + b[-56\ 24\ -88\ 48] = 0$.

$\therefore$ $7a - 56b = 0 \rightarrow a = 8b$

$-3a + 24b = 0$

$11a - 88b = 0 \rightarrow$ Put $a = 8b \Rightarrow 11(8b) - 88b = 0 \Rightarrow 0 = 0$.

$-6a + 48b = 0$ (non-trivial soln)

Let $b = k$, then $a = 8k$ $\therefore$ Eigen Space $= k\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

- Also, the second vector is 8 times the First vector.
  $\therefore$ It is Linearly Dependent.

3. $[-1\ 5\ 0], [16\ 8\ -3], [-64, 56\ 9]$

Solu:- Now, $A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$

$\det(A) = -1(72 + 168) - 16(45 - 0) + (-64)(-15 - 0)$

$= -96 - 720 + 960$

$= -816 + 960$

$= \boxed{144} \neq 0$ $\therefore$ It is linearly Independent.

4. $[1\ -1\ 1], [1\ 1\ -1], [-1\ 1\ 1], [0\ 1\ 0]$

Solu:- Let $(A):B = A:B = \left[\begin{array}{cccc:c} 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \rightarrow R_3 + R_2} \left[\begin{array}{cccc:c} 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{array}\right]$

$R_2 \longrightarrow R_2 + R_1$

$\Rightarrow A:B = \left[\begin{array}{cccc:c} 1 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{array}\right]$

Here, $\rho(A) = 3 = \rho(A:B)$

$\therefore$ It is Linearly Independent. [also vectors are not scaler multiple of each other]

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5.) $[2\ -4], [1\ 9], [3\ 5]$.

Solution: Let $a[2\ -4] + b[1, 9] + c[3\ 5] = 0$

$\therefore\ 2a + b + 3c = 0$ — (i) $\Rightarrow b = -3c - 2a$ — (iii).

$-4a + 9b + 5c = 0$ — (ii).

Put b in (ii), $-4a + 9(-3c - 2a) + 5c = 0$

$\Rightarrow -4a - 27c - 18a + 5c = 0$

$\Rightarrow -22a - 22c = 0$

$\Rightarrow \boxed{a = -c}$

So, from (iii), $b = -3a - 2a = -5a$

if $a = k$, then

Now, here we have. $X = \begin{bmatrix} k \\ -5k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$ is a non-trivial solⁿ

Hence, it's a linearly dependent (set of vectors).

6.) $[3\ -2\ 0\ 4], [5\ 0\ 0\ 1], [-6\ 1\ 0\ 1], [2\ 0\ 0\ 3]$.

Solution: Here, $c_1v_1 + c_2v_2 + c_3v_3 = 0$

For scalars c_1, c_2, c_3 where v_1, v_2, v_3 are given vectors.

$\therefore\ c_1\begin{bmatrix} 3 \\ 5 \\ -6 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_4\begin{bmatrix} 4 \\ 1 \\ 1 \\ 3 \end{bmatrix} = 0$

$\therefore\ 3c_1 - 2c_2 + 4c_4 = 0$ — (ii)

$5c_1 + c_4 = 0 \Rightarrow \boxed{c_4 = -5c_1}$

$-6c_1 + c_2 + c_4 = 0$ — (iii)

$2c_1 + 3c_4 = 0$ — (i)

Now, Put c_4 in (i), $2c_1 + 3(-5c_1) = 0 \Rightarrow 2c_1 - 15c_1 = 0 \Rightarrow \boxed{c_1 = 0}$.

$\therefore \boxed{c_4 = 0}$.

Now, from (ii) & (iii), $2\times(3c_1 - 2c_2 + 4c_4) = 0$

$-6c_1 + c_2 + c_4 = 0$

$-c_2 + 5c_4 = 0 \Rightarrow \boxed{c_2 = 5c_4}$.

$\because c_4 = 0 \Rightarrow \boxed{c_2 = 0}$. Since, $c_1 = c_2 = c_3 = c_4 = 0$

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$\therefore$ Linearly Independent

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7) $[3\ 4\ 7], [2\ 0\ 3], [8\ 2\ 3], [5\ 5\ 6]$

Soln:- let $A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$

Now, we perform elementary row operations, to convert into row echlon form.

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 3 & 3 & 1 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_1} \begin{bmatrix} 3 & 2 & 8 & 5 \\ 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 3 & 2 & 8 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 8 & 26 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 8 & 26 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 8R_2} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 0 & 82 & 37 \end{bmatrix}$$

Here By Rank Nullity Theorem,

$\rho(A) + N = D$

$\Rightarrow 3 + N = 4 \Rightarrow \boxed{N = 1}$

Here, no. of unknown $= 4 = n$

But $\rho(A) = 3 \neq n$

$\therefore$ It is infinite solⁿ. and linearly Independent.

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8) [6 0 3 1 4 2] , [0 -1 2 7 0 5] , [12 3 0 -19 8 -11]

Solve:- Let $c_1[6\ 0\ 3\ 1\ 4\ 2] + c_2[0\ -1\ 2\ 7\ 0\ 5] + c_3[12\ 3\ 0\ -19\ 8\ -11] = 0$

(-2) (+3) (1)

① $\therefore\ 6c_1 + 12c_3 = 0 \Rightarrow \boxed{c_1 = -2c_3}$

② $-c_2 + 3c_3 = 0 \Rightarrow \boxed{c_2 = 3c_3}$

③ $3c_1 + 2c_2 = 0 \Rightarrow \boxed{c_2 = \frac{-3c_1}{2}}$

④ $c_1 + 7c_2 - 19c_3 = 0 \Rightarrow c_1 - \frac{21c_1}{2} - 19c_3 = 0 \Rightarrow -\frac{19}{2}c_1 - 19c_3 = 0$

$\Rightarrow -\frac{19}{2}(-2c_3) - 19c_3 = 0 \qquad [\because c_1 = -2c_3]$

$\Rightarrow \frac{38}{2}c_3 - 19c_3 = 0 \Rightarrow 38c_3 - 38c_3 = 0 \Rightarrow \boxed{0 = 0}$

⑤ $4c_1 + 8c_3 = 0 \Rightarrow 4(-2c_3) + 8c_3 = 0 \Rightarrow -8c_3 + 8c_3 = 0$

$\boxed{0 = 0}$

⑥ $2c_1 + 5c_2 - 11c_3 = 0$

$\Rightarrow 2(-2c_3) + 5(3c_3) - 11c_3 = 0$

$\Rightarrow -4c_3 + 15c_3 - 11c_3 = 0$

$\Rightarrow -15c_3 + 15c_3 = 0$

$\Rightarrow \boxed{0 = 0}$

Let $c_3 = k$, then,

$c_1 = -2k, \quad c_2 = 3k$

$$\therefore X = \begin{bmatrix} -2k \\ 3k \\ k \end{bmatrix} = k\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$, this is infinite solⁿ.

$\therefore$ It is linearly Dependent Ans.

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Assignment-3

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$[A][X] = \lambda[X]$

$[A][X] = \lambda[I][X]$

$[A][X] = [\lambda I][X]$.

$[A - \lambda I][X] = 0$

> System of Homogenous Eq.

$|A - \lambda X| = 0$ for Infinite Solⁿ:

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$-1(-12 + 3 - 3\lambda) + 2(12 + 6\lambda + 6) - \lambda((-2-\lambda)(1-\lambda) - 4) = 0$$

$$\underline{12-3} + \underline{3\lambda} + \underline{24} + \underline{12\lambda} + \underline{12} + \underline{2\lambda} - \underline{\lambda^2} - \lambda^3 + \underline{4\lambda} = 0$$

$$45 + 21\lambda - \lambda^2 - \lambda^3 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$(\lambda-3)(\lambda^2-2\lambda-15)=0$

$(\lambda+3)(\lambda+3)(\lambda-5)=0$

$\lambda_1=-3 \quad \lambda_2=-3 \quad \lambda_3=5$

For $\lambda=-3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$\eta(A)=2$

let $x_2=k_1$ & $x_3=k_2$

$x_1+2x_2-3x_3=0$

$x_1=-2k_1+3k_2$

Eigen Vector $X_1=\begin{bmatrix} -2k_1+3k_2 \\ k_1 \\ k_2 \end{bmatrix}$

$$X_1=k_1\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}+k_2\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\eta(A) = 1$

let $x_3 = k$

$-x_1 - 2x_2 - 5x_3 = 0$

$-8x_2 - 16x_3 = 0$

$x_2 = -2k$

$-x_1 = k$

$x_1 = -k$

Eigen Vector $\Rightarrow X = \begin{bmatrix} -k \\ -2k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$[A - \lambda I][X] = 0$

$[A - \lambda I] = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$

$-2(\lambda - 1) + (1-\lambda)\left((4-\lambda)(1-\lambda)\right) = 0$

$(1-\lambda)\left(2 + (4-\lambda)(1-\lambda)\right) = 0$

$(1-\lambda)\left(6 + \lambda^2 - 5\lambda\right) = 0$

$(\lambda - 1)\left(\lambda^2 - 5\lambda + 6\right) = 0$

$(\lambda-1)(\lambda-3)(\lambda-2) = 0$

$\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 2$

$\lambda^2 - 3\lambda - 2\lambda + 6$

For $\lambda = 1$

$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

let $x_2 = k$

$3x_1 + x_3 = 0$

$2x_3 = 0$

$2x_3 = 0$

$\Rightarrow x_1 = 0,\ x_2 = k,\ x_3 = 0$

$$X = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} = k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

<u>For $\lambda = 3$</u>

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$r(A) = 1$

$x_1 + x_3 = 0$

$-2x_2 + 2x_3 = 0$

$\Rightarrow$ let $x_3 = k$

$\Rightarrow x_1 = -k$

$x_2 = k$

$$X = k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda_3 = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$

$-x_2 + x_3 = 0$

let $x_3 = K$

$x_2 = K$

$x_1 = \frac{-K}{2}$

$$X = K \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

3.

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

Its a lower Triangular Matrix.
So, Diagonal Elements will be the Eigen Values.

Hence, $\lambda_1 = 5$, $\lambda_2 = 0$, $\lambda_3 = 3$

For $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & -5 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - 2x_3 = 0$

$-5x_2 + 3x_3 = 0$

$\Rightarrow$ let $x_3 = k$

$x_1 = -2k$

$x_2 = \frac{3k}{5}$

$$\Rightarrow X = k \begin{bmatrix} -2 \\ \frac{3}{5} \\ 1 \end{bmatrix}$$

For $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ -1 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 = 0$$

$$-x_1 + 3x_3 = 0$$

$\Rightarrow$ let $x_2 = k$

$x_1 = 0$ & $x_3 = 0$

$$\Rightarrow X = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$-3x_2 = 0$

$\Rightarrow$ let $x_3 = K$

$x_1 = 0$ & $x_2 = 0$

$\Rightarrow X = K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

Eigen Values $\Rightarrow \lambda_1 = 0, \lambda_2 = 3, \lambda_3 = -2$

For $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_3 = 0$

$3x_2 + 4x_3 = 0$

$\Rightarrow$ let $x_1 = K$

$x_2 = 0$ & $x_3 = 0$

$\Rightarrow X = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

For $\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 = 0$

$4x_3 = 0$

$-5x_3 = 0$

Let $x_2 = K$

$x_1 = 0$

$x_3 = 0$

$$X = K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$5x_2 + 4x_3 = 0$

$\Rightarrow$ Let $x_3 = K$

$x_1 = 0$ & $x_2 = -\frac{4K}{5}$

$$\Rightarrow X = K \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

5.

B

t

5. $$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

Because there are 3 identical rows in this matrix. So, 2 eigen values must be 0

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Assignment - 4

Q.1. Find the rank of the matrix A by reducing in Row reduced echlon Form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Solve:- Performing row operations :

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_4 \to R_4 - 2R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 - R_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, no. of non-zero rows = 3

$\therefore \boxed{\rho(A) = 3}$ Ans

Q.2. Let W be the vector space of all symmetric 2×2 matrices and let $T : W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$. Find the rank and nullity of T.

Solve:- According to Question,

every matrix and vector space W is of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

and,

the linear transformation, $T : W \to P_2$ defined by :

$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

Here, we have three coefficients corresponding to $1, x$ and x^2. (R^3)

$\therefore$ Dimension $= 2+1 = 3$

Now, The standard basis for W is :

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$ ($\because$ W is 2×2 matrix space.

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Also T is defined as $T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$

So, we can represent T in terms of the standard basis elements :

$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Now, these column vectors form the matrix A :

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

Now, we have to convert this into the row-echlon form.
Applying row elementary operation.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Here, $\boxed{\rho(A) = 3}$ linearly independent.

Now, $\rho(A) + N = D \Rightarrow 3 + N = 3 \Rightarrow N = 3-3 \Rightarrow \boxed{N = 0}$

Hence, Rank, $\rho(A) = 3$

Nullity $(N) = 0$

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Q. 3) Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A^{-1} and $A+4I$.

Solⁿ:- To get the eigenvalues we need to solve characteristic equation:

$\det(A - \lambda I) = 0$

$\Rightarrow \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0 \Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$

$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3 = 0$

$\Rightarrow \lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$\Rightarrow \boxed{\lambda = 1, 3}$ Ans. $\longrightarrow$ Eigen values. $\longrightarrow$ for $A^{-1} = \frac{1}{\lambda}$

$\therefore A^{-1} = 1, \frac{1}{3}$

Now, we find eigen space corresponding to $\lambda = 1$, & $\lambda = 3$

- For $\lambda = 1$, $[A - \lambda I]X = [0] \Rightarrow [A - I]X = [0]$

$\Rightarrow \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$

~~Same~~
Same for A^{-1}

$\Rightarrow \left(\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$

$\Rightarrow x - y = 0 \Rightarrow \boxed{x = y}$

& $-x + y = 0$

So, if $x = k$, then $\therefore \boxed{x = y = k}$

$\therefore$ Eigen Space $= \begin{bmatrix} k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $k \in R$ Ans

- For $\lambda = 3$, $[A - 3I][x] = [0]$

$\Rightarrow \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} [x] = [0]$

$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -x - y = 0 \Rightarrow \boxed{-x = y}$

if $x = k$, then, $\boxed{x = -y = -k}$.

Eigen values for $A + 4I$.

$\Rightarrow \lambda_1 + c, \lambda_2 + c$

$\Rightarrow 1 + 4, 3 + 4$

$\Rightarrow \boxed{5, 7}$ Ans

$\therefore$ Eigen Space $= \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ Ans

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Q. 4) Solve by Gauss - Siedel Method Method (Take three iterations)

$$3x + 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4.$$

$$\Rightarrow \begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$.

Solve:- Now, Using Gauss Siedel Method, initial $\boxed{k = 0}$ ($\because k$ = iteration)

$$\therefore x^{(k+1)} = \frac{1}{a_{11}}\left(b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right)$$

$$y^{(k+1)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)}\right)$$

$$z^{(k+1)} = \frac{1}{a_{33}}\left(b_3 - a_{31}x^{(k+1)} - a_{23}y^{(k+1)}\right)$$

Iteration 1st:- $x^{(1)} = \frac{1}{3}(+7.85 + 0.1 \cdot 0 + 0.2 \cdot 0) = \frac{+7.85}{3} = \boxed{2.6167}$

$$y^{(1)} = \frac{1}{7}\left(-19.3 - 0.1x^{(1)} + 0.3 \cdot 0\right) = \frac{1}{7}\left(-19.3 - \frac{0.785}{3}\right) = \frac{56.115}{21} = \boxed{-2.7571}$$

$$z^{(1)} = \frac{1}{10}\left(-71.4 - 0.1 x^{(1)} + 0.3 y^{(1)}\right) = \frac{1}{10}\left(-71.4 - \frac{0.785}{3} + \frac{16.8345}{3}\right)$$

$$= \frac{+198.1505}{30} = \boxed{6.7757}$$

Iteration 2nd:- $x^{(2)} = \frac{1}{3}\left(-7.85 + 0.1 \cdot y^{(1)} + 0.2 \cdot z^{(1)}\right) = \frac{1}{3}\left(-7.85 + 0.1\left(\frac{56.115}{21}\right) + 0.2\left(\frac{-198.1505}{30}\right)\right)$

$$= \boxed{1.7908}$$

$$y^{(2)} = \frac{1}{7}\left(-19.3 - 0.1(-2.6167) + 0.3(6.7757)\right) = \boxed{-3.3377}$$

$$z^{(2)} = \frac{1}{10}\left(71.4 - 0.3(1.7908) + 0.2(-3.3377)\right) = \boxed{7.2991}$$

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Iteration 3rd :- $x^{(3)} = \frac{1}{3}(7.85 + 0.1(-3.3377) + 0.2(7.2991)) = \boxed{1.6792}$

$y^{(3)} = -\frac{1}{7}(-19.3 - 0.1(1.6792) + 0.3(7.2991)) = \boxed{-3.2744}$

$z^{(3)} = \frac{1}{10}(71.4 - 0.3(1.6792) + 0.2(-3.2744)) = \boxed{7.3651}$

Q.5.) Define consistent & inconsistent system of eqⁿ. Hence solve the following system of eqⁿ. if consistent $x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$

Solve:- Here, in the form of $AX = B$,

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(Homogeneous)

Since, $B = 0$ ∴ Our system of eqⁿ is consistent Ans

Now, let's convert this into row-echlon form.

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, $(\rho(A) = 2) < (n = 3)$

∴ This system of eqⁿ has infinite solⁿ Ans

Now, we have, $1x + 3y + 2z = 0$ — ①

$-7y - z = 0 \Rightarrow \boxed{z = 7y}$.

From ①, $1x + 3y + 14y = 0$

$x + 17y = 0$

$\boxed{x = -17y}$

if $y = k$, $x = -17k$, $z = 7k$ $\Rightarrow$ ∴ $X = \begin{bmatrix} -17k \\ k \\ 7k \end{bmatrix} = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$ Ans

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Q.6 Determine whether the Function $T: P_2 \to P_2$ is linear transformation or not. Where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$.

Solve:- To check if T is a linear Transformation, we need to verify two properties

① Additivity : $T(u+v) = T(u) + T(v)$ ($u, v \in$ Polynomial)

② Homogeneity : $T(cu) = cT(u)$ (For any polynomial u and scalar c)

(can take any constant to add. not necessary "1")

let's Check

① Additivity : $T((a_1+b_1x+c_1x^2)+(a_2+b_2x+c_2x^2))$

$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2)$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2 = \text{LHS}$

now, $T(a_1+b_1x+b_2x^2) + T(a_2+b_2x+c_2x^2)$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$= (a_1+a_2+2)+(b_1+b_2+2)x+(c_1+c_2+2)x^2. = \text{RHS}.$

$\because$ LHS $\neq$ RHS, T is not additive.

② Homogeneity : $T(cu) = T(c(a+bx+cx^2)) = (ca+1)+(cb+1)+(cc+1)x^2. = \text{LHS}$

& RHS, $= cT(u) = cT(a+bx+cx^2) = c((a+1)+(b+1)x+(c+1)x^2)$

$= (ca+c)+(cb+c)x+(cc+c)x^2$

$\therefore$ LHS $\neq$ RHS, T is not homogeneous

Therefore, T is not a Linear Transformation.

Q.7 Determine whether the set $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the subspace spanned by S.

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Solve: – To determine if the set S is a basis for $V_3(R)$, we need to check 2 things :

1. Linear Independence : Confirm that none of the vectors in S can be written as a linear combination of the others.

2. Spanning : Verify that the set S spans $V_3(R)$, meaning that any vector in $V_3(R)$ can be expressed as linear combination of vectors from S.

Now, Form a matrix with vectors of S as its column & row reduce to check for linear independence :

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{9}{5}R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $\rho(A) = 2 < (n=3)$

so, it is infinite, linearly dependent, Hence don't span $V_3(R)$ and not basis

Next Let's Determine dimension & basis of subspace spanned by S.

Dimension $\longrightarrow$ no. of linearly independent vectors in S.
From above, we see that the first and second row are L.I but row 3 not.

$\therefore$ Dimension = 2, and basis for the subspace spanned by S is given by L.I. vectors. = $\{(1,2,3), (3,1,0)\}$. Ans

Q.8. Using Jacobi's method (perform 3 iterations), solve.

$3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$,

$$\Rightarrow \begin{bmatrix} 3 & -6 & 2 \\ -4 & 1 & -1 \\ 1 & -3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ -15 \\ 16 \end{bmatrix}$$

with initial values $x_0 = 1, y_0 = 1, z_0 = 1$.

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Solve:- Initilize $x^{(0)} = 1,\ y^{(0)} = 1,\ z^{(0)} = 1$, $\begin{cases} x^{k} = \frac{1}{a_{11}}\left(b_1 + a_{12}y^{(k-1)} + a_{13}z^{(k-1)}\right) \\ y^{k} = \frac{1}{a_{22}}\left(b_2 + a_{21}x^{(k-1)} + a_{23}z^{(k-1)}\right) \\ \text{similarly } z. \end{cases}$

1st Iteration:- $x^{(1)} = \frac{1}{3}(23 + 6\cdot 1 - 2\cdot 1) = \boxed{9}$

$y^{(1)} = \frac{1}{(-1)}(-15+1+1) = \boxed{-10}$

$z^{(1)} = \frac{1}{7}(16+1-3+1) = \boxed{2}$

2nd Iteration:- $x^{(2)} = \frac{1}{3}(23 + 6\cdot(-10) - 2\cdot 2) = \boxed{-36}$

$y^{(2)} = \frac{1}{(-1)}(-15 + 4(9) + 2) = \boxed{26}$ $-3(26)$ -78

$z^{(2)} = \frac{1}{7}(16 - (-36) + 3(-10)) = \boxed{8}$

3rd Iteration:- $x^{(3)} = \frac{1}{3}(23 + 6(26) - 2(8)) = \boxed{47}$

$y^{(3)} = \frac{1}{1}(-15 + 4(-36) + 8) = \boxed{-163}$

$z^{(3)} = \frac{1}{7}(16 - 47 + 3(26)) = \boxed{11}$

After 3 iterations, $\boxed{x = 47}$, $\boxed{y = -163}$, $\boxed{z = 11}$ Ans

Q.9 Explain one application of matrix operation in image processing with example.

Solve:- Application:- Image Transformation through affine transformation.

Affine transformation involve translation, ~~[illegible]~~, rotation, scaling and shearing of images, and these operations can be efficiently represented by using matrices.

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Example:- Scaling Operation :—

Let's take a 2D image represented as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

To scale the image by a factor of s in the x-direction and t in the y-direction, the transformation matrix T is :

$T = \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}$

The resulting transformed image I' is obtained by multiplying the original image matrix I with the transformation matrix T :

$I' = T \cdot I$

This multiplication efficiently applies scaling to each pixel in the image.

For eg:- we have image represented by $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

we want to scale it by a factor of 2 in x-direction and 3 in y-direct

the transformation would be :

$T = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

The resulting transformed image I' would be :

$I' = \begin{bmatrix} 2 & 4 \\ 9 & 12 \end{bmatrix}$

This demonstrates how matrix operations facilitate efficient and systematic manipulation of images in various ways within the field of ~~[illegible]~~ image processing.

Q.10 Give a brief description of Linear Transformation for computer Vision for rotating 2D image.

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Solve:- Linear transformation plays a crucial role in computer vision, particularly in the context of rotating 2D images. A linear transformation can be presented by a matrix that operates on the coordinates of each pixel in the image, producing a transformed image. For rotation specifically, the rotation matrix is employed.

Rotation matrix for 2D images —

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

If I = original image matrix, after multiplying I with R, we obtain the rotated image I', reflecting the rotation.

This linear transformation is fundamental in computer vision application allowing for efficient manipulation and analysis of images, including rotation to correct orientation, or align objects in the image.

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(3)

Solution: — Continue — We know if λ is eigen value of matrix A, then $\frac{1}{\lambda}$ $(\lambda \neq 0)$ is eigen values of A^{-1}

$\therefore$ Eigen values of $A^{-1} = 1, \frac{1}{3}$, [~~Note:~~ for 1 ~~we have already~~ found. ~~eigen vector~~]

- For $\lambda = \frac{1}{3}$, $[A^{-1} - \frac{1}{3}I][x] = [0]$

$\Rightarrow$

Here,

$|A| = \lambda_1 \lambda_2 = (1)(3) = \boxed{3 \neq 0}$ $\therefore A^{-1}$ exist | and $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|}(adj.A) = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$$ // Ans

$\therefore$ Eigen vector :- $\begin{bmatrix} 24/7 & 9 & -3381/74 \\ 2/7 & -2 & 51/148 \\ 1 & 1 & 1 \end{bmatrix}$ Ans

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(3)

Solution:- Continue:- Now for eigen values of $A^{-1} = (1, \frac{1}{3})$.

and $A^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

- Find Eigen vectors:—

(1) For $\lambda = 1$, $[A^{-1} - I][X] = [0]$

$\Rightarrow \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}[X] = [0]$

$\Rightarrow \frac{2}{3}x + \frac{1}{3}y = 0 \Rightarrow x = \frac{-1}{3}y \times \frac{3}{2} \Rightarrow \boxed{x = \frac{-1}{2}y}$

& $\frac{1}{3}x + \frac{2}{3}y = 0 \Rightarrow x = -\frac{2}{3}y \times \frac{3}{1} \Rightarrow \boxed{x = -2y}$.

This is not possible.

Teacher's Signature.........