Aryan Shah Assignment 03

19110179

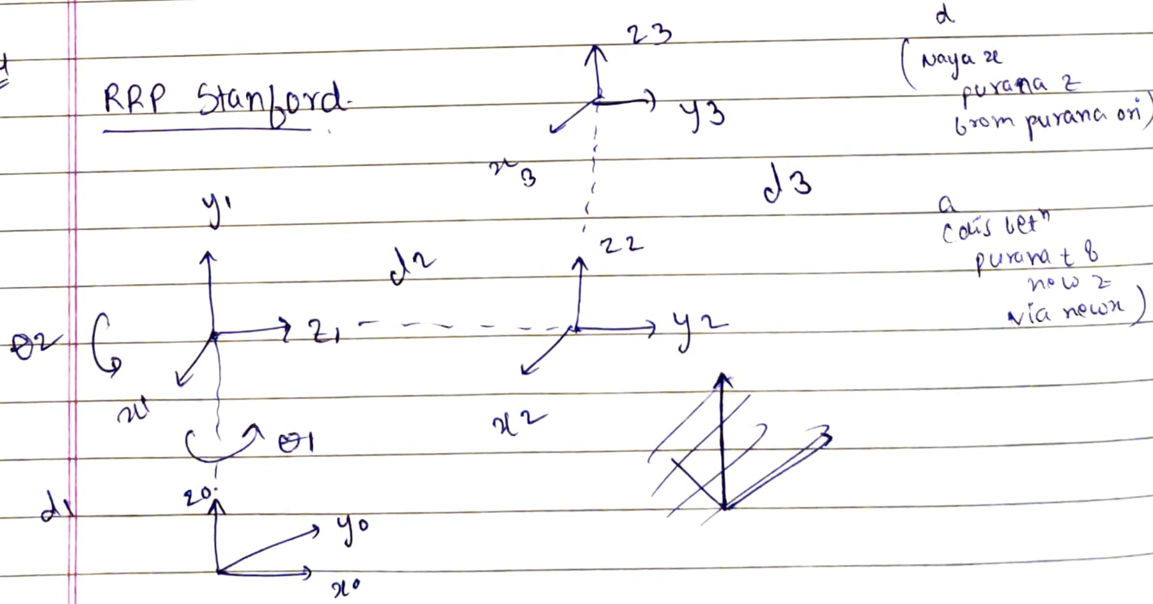
Q1

In simple terms manipulator singularity is a configuration in which the end effector becomes blocked in certain directions. At a singularity robotic arm loses one or more degrees of freedom. the cartesian velocity is a function of a multiplication of joint velocities and a jacobian matrix. Jacobian matrix being a function of q and geometery of robot. so when this matrix becomes singular (linearly dependent rows columns / rank decreases) these are the cases when the velocity components of end effector becomes ~~the~~ linearly dependent on each other. and hence determining them beomes impossible.

In other words a robot is said to be close to singularity when the determinant of Jacobian matrix is close to zero.

Q4

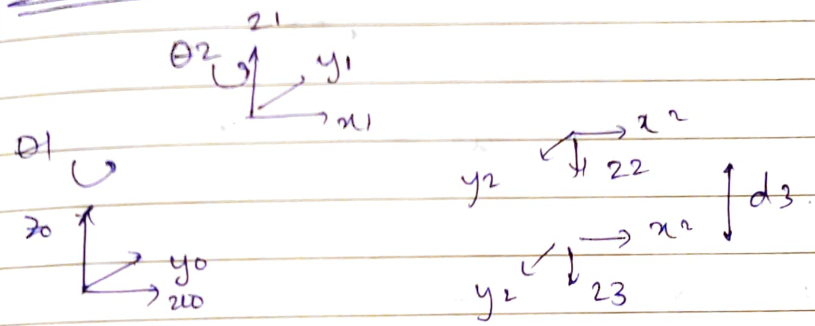
RRP Stanford



DH table

	θ	d	a	α
1	θ_1	d_1	0	-90
2	θ_2	d_2	0	90
3	0		d_3	0

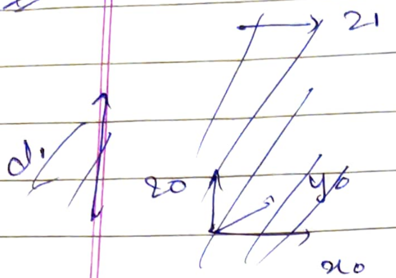
RRP Scara



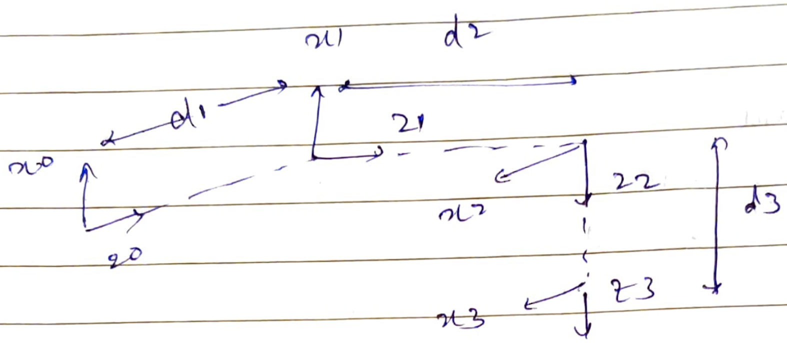
DH table

	θ	d	a	α
1	θ_1	0	d_1	0
2	θ_2	0	d_2	180
3	0	d_3	0	0

Q4



Q5



	a	α	d	θ
1	0	-90	d_1	0
2	0	90	d_2	90
3	0	0	d_3	-90

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T = A_1 A_2 A_3$$

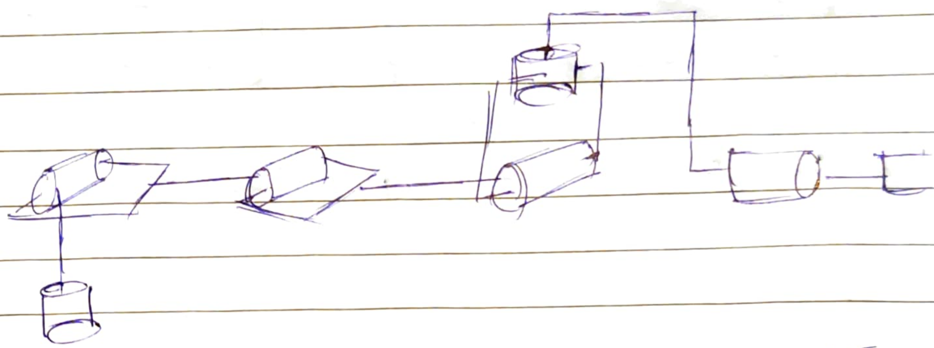
$$= \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

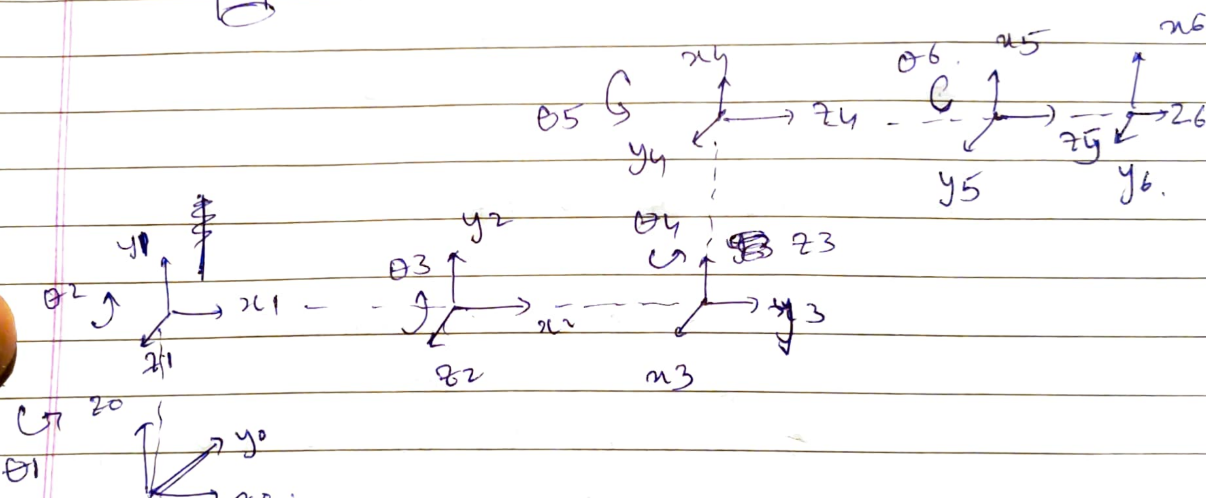
d_1, d_2, d_3 length of links.

let $d_1 = 3$ $d_2 = 2$ $d_3 = 1$

$$T = \begin{bmatrix} 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q6





DH table.

	a	α	d	θ
1	0	90°	0	θ_1
2	d_2	0	0	θ_2
3	d_3	0	0	θ_3
4	0	-90°	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6.

$$A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & d_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & d_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & d_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & d_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$$

= [since the resultant matrix is quite complex lets assume $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ to be zero a stationary config]

$$= \begin{bmatrix} 0 & 0 & 1 & d_3 + d_6 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

take $d_1 = 1$ $d_2 = 2$ $d_3 = 3$ $d_6 = 6$.

compare!

we get the same ans

Page
Date

Q7 2R Manipulator types.

Direct drive

- Has no gear component so motor is directly attached to joint.
- gives a quicker response.
- since it has no gearbox for an high torque requiring task we need to choose a low rpm high torque motor.
- cheaper

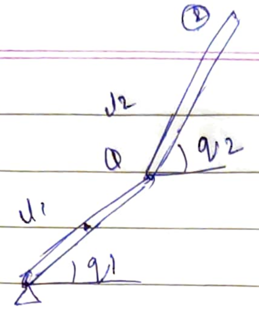
Remote driven joint

- Motor has an gearbox so as to ~~to~~ use for high torque operations.
- since a gearbox is added onto the motor the resulting structure is quite heavy but since the motor is attached at the base (base frame).

5 bar parallelogram.

- As the name suggests it ~~used~~ uses a four bar mechanism that forms a parallelogram and a fifth open link.
- The combination of these can be assumed as a 2R manipulator.
- The motors are used at the base of the parallelogram
- can be used for high torque applications

Q8



$x = \frac{l_1}{2} \cos q_1$
$y = \frac{l_1}{2} \sin q_1$

velocity of link ① & link ② COM points can be determined by just finding derivative of their position vectors.

$$\therefore V_{c1} = \begin{bmatrix} -l_1/2 \, s q_1 \\ l_1/2 \, c q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$V_{c2} = \begin{bmatrix} -l_1 s q_1 & -l_2/2 \, s q_2 \\ l_1 c q_1 & l_2/2 \, c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

we know,

$$KE = \frac{1}{2} \sum_{i=1}^{2} (m_i V_{ci}^T V_{ci}) + (\omega_i^T I_i \omega_i)$$

$$= \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

$$\omega_i = \dot{q}_i \hat{k} \qquad D(q) = \begin{bmatrix} I_1 + \frac{m_1 l_1^2}{4} + m_2 l_1^2 & \frac{m_2 l_1 l_2}{2} c(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} c(q_2 - q_1) & I_2 + \frac{m_2 l_2^2}{4} \end{bmatrix}$$

$$PE = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g l_1 \sin q_1 + m_2 g \frac{l_2}{2} \sin q_2$$

Also generalized form.

$\cancel{\sum D(q)\ddot{q}}$

① $\tau_k = \sum_j d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q)$

$\phi_1 = \frac{d(PE)}{dq_1} = \frac{m_1 g l_1 \cos q_1}{2} + m_2 g l_1 \cos q_1$

$\phi_2 = \frac{d(PE)}{dq_2} = m_2 g \frac{l_2}{2} \cos q_2$

$c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)$

we found,

$C_{111} = 0$

Illy $C_{121}, C_{211}, C_{212}, C_{122}, C_{222} = 0$.

&

$C_{221} = -\frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}$ $\cancel{C_{112}} =$

$C_{112} = \frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}$

Substituting this in ①

$\tau_1 = \left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 + \frac{m_2 l_1 l_2}{2}\cos(q_2 - q_1)\ddot{q}_2$

$- \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)\dot{q}_2^2 + m_1 g \frac{l_1}{2}\cos q_1$

$+ m_2 g l_1 \cos q_1$

$\tau_2 = \left(\frac{m_2 l_2^2}{4} + I_2\right)\ddot{q}_2 + \frac{m_2 l_1 l_2}{2}\cos(q_2 - q_1)\ddot{q}_1$

$+ \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)\dot{q}_1^2 + \frac{m_2 g l_2}{2}\cos q_2$

same results

Q10

$$KE = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j \qquad (d_{ij} = d_{ji})$$

$$PE = V(q)$$

we know, $L = KE - PE$
(V)

$$\tau_k = \frac{d}{dt}\left(\frac{dL}{d\dot{q}_k}\right) - \frac{dL}{dq_k} \quad — ①$$

$$\frac{dL}{d\dot{q}_k} = \sum_{j}^{n} d_{kj}\, \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_k}\right) = \sum_j d_{kj}\, \ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}\, \dot{q}_j$$

$$= \sum_j d_{kj}\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i}\, \dot{q}_j \dot{q}_i$$

$$\frac{dL}{dq_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k}\, \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

using ①,

$$\tau_k = \sum_j d_{kj}\, \ddot{q}_j + \sum_{i,j} \left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right] \dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k}$$

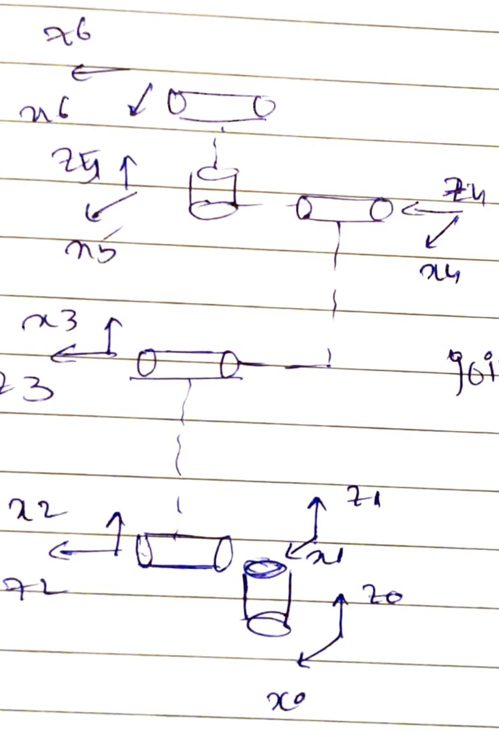
d being symmetric

$$\sum_{ij}\left(\frac{d_{kj}}{dq_i}\right)\dot{q}_i\dot{q}_j = \frac{1}{2}\left[\sum_{ij}\left(\frac{\partial d_{kj}}{\partial q_i}+\frac{\partial d_{ki}}{\partial q_j}\right)\right]\dot{q}_i\dot{q}_j$$

$\therefore$ the equation becomes

$$\tau_k = \sum_j d_{kj}\ddot{q}_j \underbrace{+\frac{1}{2}\sum_{ij}\left(\frac{\partial d_{kj}}{\partial q_i}+\frac{\partial d_{ki}}{\partial q_j}-\frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i\dot{q}_j}_{\sum_{ij} c_{ijk}\dot{q}_i\dot{q}_j} + \frac{\partial V}{\partial q_k}$$

Q12



Joints / Links - 6

Type - All revolute

	a	α	d	θ
1	0	0	d_1	θ_1
2	d_2	90	0	θ_2
3	d_3	0	0	θ_3
4	0	0	d_4	θ_4
5	0	90	d_5	θ_5
6	0	-90	d_6	θ_6