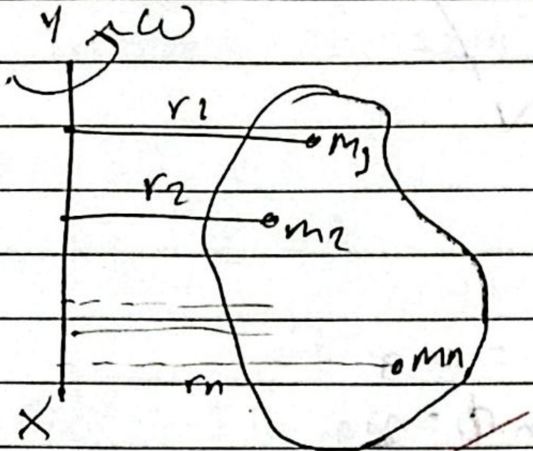
Date
Page

Moment of inertia

→ The inertia in a rotational motion is called moment of inertia. It is denoted by I and given by the product of mass and square of distance of particle from axis of rotation. i.e

$I = mr^2$; m : mass of body/particle
r = ~~squ~~ distance of particle from axis of rotation.

Unit of I = Kgm^2.



$I = m_1 r_1^2 + m_2 r_2^2 + \dots m_n r_n^2$
$= \sum_{i=1}^{n} m_i r_i$

Torque :- It is the turning effect of force.

Mathematically, $\vec{\tau} = \vec{r} \times \vec{F}$
$= rF\sin\theta$

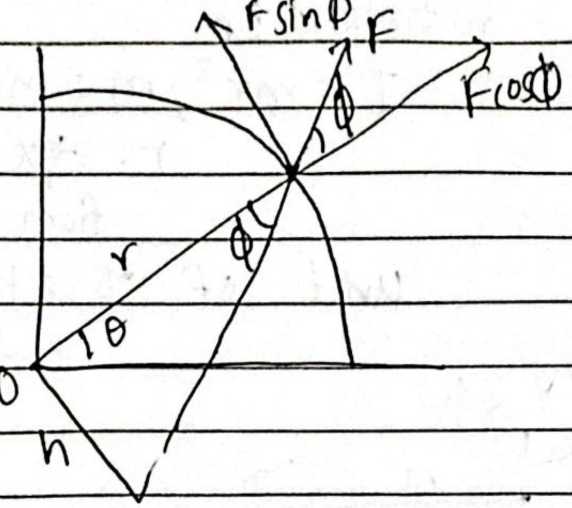
If $\theta = 90°$, $\tau = rF\sin 90$
$= rF$ (max^m torque)

If $\theta = 0, \tau = 0$ (minimum)

Relation between τ and M.I

Q) Show that $\tau = I\alpha$

Q) Establish the relation between torque and inertia



we have,

$$F = ma$$

$$F \sin\phi = ma$$

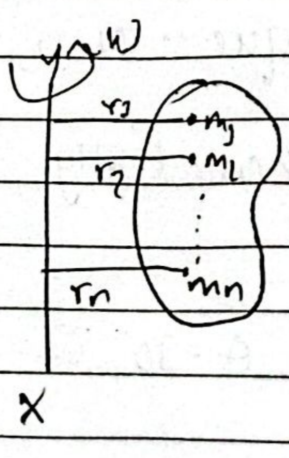
$$F \frac{h}{r} = mr\alpha \quad [a = r\alpha]$$

$$Fh = mr^2\alpha$$

$$\tau = I\alpha$$

Rotational K.E

Let a rigid body be made of n particles of masses $m_1, m_2, \ldots m_n$. at distance $r_1, r_2 \ldots r_n$ respectively from axis of rotation (XY). All particles have same ω.



Rotational $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots + \frac{1}{2} m_n v_n^2$

$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots \frac{1}{2} m_n r_n^2 \omega^2$

$= \frac{1}{2} \omega^2 (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)$

$KE_{rot} = \frac{1}{2} I \omega^2$

Alt method:-

Small amount of work for small displacement $d\theta$

$$dW = \tau \, d\theta$$
$$= I \alpha \, d\theta$$
$$= I \frac{d\omega}{\cancel{dt}} \cdot \omega \cancel{dt}$$
$$dW = I \omega \, d\omega$$

$$W = \int_{\omega_i}^{\omega_f} dW$$
$$= \int_{\omega_i}^{\omega_f} I \omega \, d\omega$$
$$= I \left[\frac{\omega^2}{2} \right]_{\omega_i}^{\omega_f}$$
$$= I \left(\frac{\omega_f^2}{2} - \frac{\omega_i^2}{2} \right)$$
$$= \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$\therefore E_{rot} = \frac{1}{2} I\omega^2$

Angular momentum and its conservation

→ let a particle of mass m with momentum $p = mv$. from Newton's 2nd law.

force = rate of change in momentum

$$f = \frac{d}{dt}(mv)$$

Taking cross product with position vector ($\vec{r}$)

$$r \times \vec{F} = r \times \frac{d}{dt}(mv)$$

$$\Rightarrow \tau = \frac{d}{dt}(r \times mv)$$

$$\Rightarrow \tau = \frac{dr}{dt} \times mv + r \times \frac{d}{dt}(mv)$$

$$\Rightarrow \tau = v \times mv + r \times \frac{d}{dt}(mv)$$

Since cross product of 2 vectors in same direction is zero. i.e $v \times mv = 0$

$$\tau = \frac{d}{dt}(r \times mv)$$

$$\tau = \frac{dL}{dt} \quad [\because r \times mv = L = \text{angular momentum}]$$

- Conservation of L:

In the absence of external torque, the total angular momentum of a system is conserved.

Since $\tau = \frac{dL}{dt}$

If $\tau = 0$, $\frac{dL}{dt} = 0$

$dL = 0$

On integrating

$\int dL = \int 0$

$L = \text{Const}$

$I\omega = \text{const}$

$\boxed{I_1\omega_1 = I_2\omega_2}$

* Power in rotational motion:-

$P = \frac{dW}{dt}$

Since $dW = \tau d\theta$ $\quad$ ∴ Work = Force × dx

$\approx W = \tau\theta$

$dW = \tau d\theta$

Power $= \tau \left(\frac{d\theta}{dt}\right)$

$P = \tau\omega$

Oscillation :- To and fro motion of a body

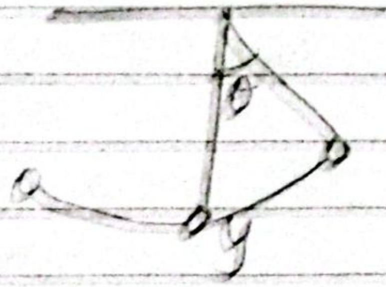
Periodic motion:- If a body/object travels equal distance in equal interval of time then the motion is called periodic motion.

SHM (Simple Harmonic Motion):

→ It is a periodic motion

→ Acceleration is directly proportional to displacement and is always directed towards mean position.

$a \propto (-)y$

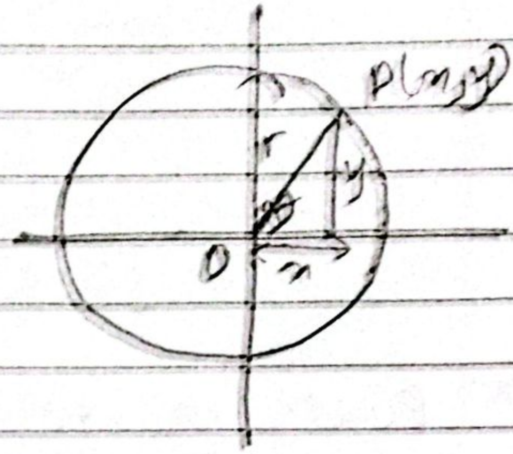


Displacement :-

$\sin\theta = \frac{y}{r}$

$r\sin\theta = y$

$\boxed{y = r\sin\omega t}$ $\boxed{\omega = \frac{\theta}{t}}$



Displacement :

$y = r\sin\omega t$

velocity :-

Differentiating wrt t,

$v = \frac{dy}{dt}$

$v = \frac{d(r\sin\omega t)}{dt}$

$v = r\omega\cos\omega t$

$= r\omega\sqrt{1 - \frac{y^2}{r^2}}$

$= r\omega\sqrt{\frac{r^2 - y^2}{r^2}}$

$= \omega\sqrt{r^2 - y^2}$

velocity : $\boxed{\begin{array}{c} v = r\cos\omega t \\ v = \omega\sqrt{r^2 - y^2} \end{array}}$

Date ______
Page ______

If $y = 0, v = \omega\sqrt{r^2 - 0} = r\omega$

$= v = r\omega$ (maximum)

(extreme position)

If $y = r, v = \omega\sqrt{r^2 - y^2} = \omega\sqrt{r^2 - r^2}$

$= 0$ (minimum)

(lowest point)

Acceleration:

Differentiating wrt t

$$a = \frac{dr}{dt}$$

$$= \frac{d(r\omega \cos \omega t)}{dt}$$

$$= - r\omega^2 \sin \omega t$$

$$= - r\omega^2 \sin \omega t$$

$$= - \omega^2 y$$

$$a \propto -y$$

At extreme position

$y = 0$

$a = -\omega^2 0$

$= 0$

At mean position,

$y = r$

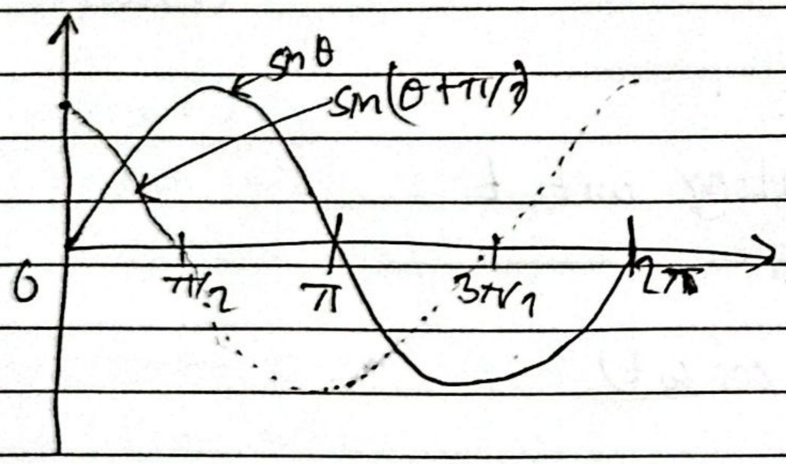
$a = -\omega^2 r$

Time period

$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

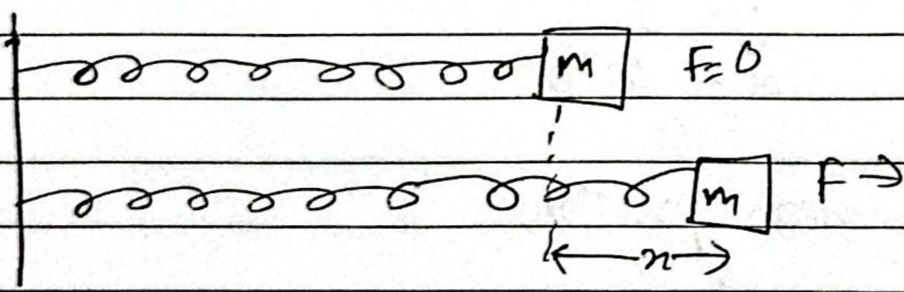
$$\boxed{T = \frac{2\pi}{\omega}}$$



Spring mass system:.

→ Let a body of mass m is attached to a spring of spring constant k.

when a body is pulled with force f then extension (x) is produced in figure.



From Hooke's law,
Extension or compression is directly proportional to applied force.

$$x \propto F$$
$$F = -Kx$$

where, K is force constant.

Also, $F = ma$

$$ma = -Kx$$
$$a = \frac{-K}{m}x$$
$$a = -\frac{K}{m}x \quad \text{—} \; (i)$$

As $a \propto -x$
This is in SHM

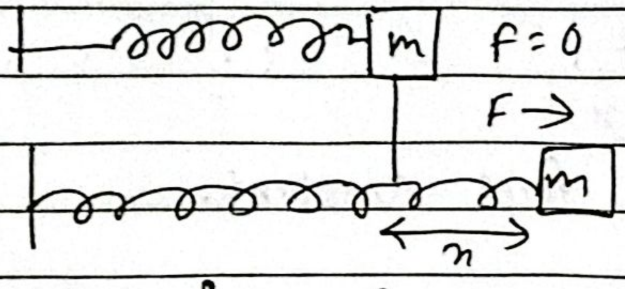
We have,
$$a = -\omega^2 x \quad \text{—} \; (ii)$$

from (i) and (ii)

$$\frac{-Kx}{m} = -\omega^2 x$$
$$\frac{K}{m} = \omega^2$$
$$(2\pi f)^2 = \frac{K}{m}$$
$$\left(\frac{2\pi}{T}\right)^2 = \frac{K}{m}$$
$$T = 2\pi\sqrt{\frac{m}{K}}$$

Energy in SHM :-

Consider a spring mass system as in fig:-



when the force f is applied to pull mass, small displacement produce in the same direction. then.

Work done for small displacement is

$$dW = F dx.$$

Total work,

$$W = \int_0^x F\,dx$$

using, $F = kx$ (Hooke's law)

$$W = k\int_0^x x\,dx$$

$$\therefore W = \frac{1}{2}Kx^2 \text{ (in the form of energy)}$$

$$E_p = PE = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2$$

If spring is initially in its equilibrium position x_1 and is compressed or stretched to position x_2 then, work done.

$$W = K\int_{x_1}^{x_2} x\,dx = \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2 \quad \text{--- (1)}$$

Date
Page

If there is no friction, $ME = KE + PE$

By work-energy theorem,
work done by spring = change in KE

$$\int_{x_1}^{x_2} F\,dx = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

where, v_1 & v_2 are velocity of body at x_1 and x_2

$$\text{i.e.} \; -k\int_{x_1}^{x_2} x\,dx = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad \text{— (II)}$$

$\therefore F = -kx$ -ve sign

From (I) and (II)

$$-\left[\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right] = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Numerical

$$\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2 = \text{constant}$$

This is conservation of energy for spring mass system.

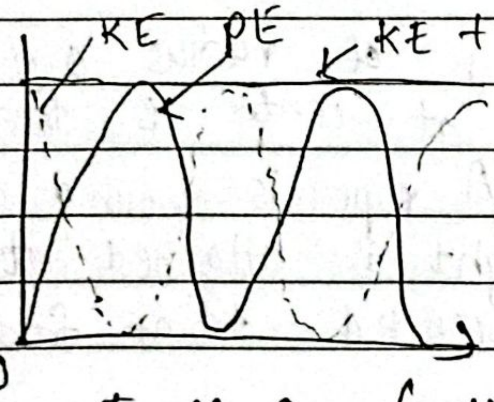


Fig: E as a function of time (t)

Date
Page

Numerical Problems

(8.1)

2) A bicycle wheel of mass 2 kg and radius 0.32m is spinning freely on its axle at 2 rev/sec. When we place our hand against the tire the wheel decelerates uniformly and comes to a stop in 8 sec. Find torque.

→ Solⁿ :-

$m = 2\,Kg$

$I = mr^2 = 2 \times (0.32)^2 = 0.2048\ Kgm^2$

$\omega = 0 \qquad \omega_0 = 2\ rev/sec$

$\tau = I\alpha$

~~Ag~~ $\alpha = \frac{\omega - \omega_0}{t} = \frac{0-2}{8}\ rev/s^2$

$= -0.25\ rev/s^2$

$= -0.25 \times 2\pi\ rad/s^2$

$\tau = I\alpha$

$= 2 \times (0.32)^2 \times (-0.25) \times 2\pi$

$= 0.05 \times 2\pi$

$= 0.314\ Nm$

(1). A large wheel of radius 0.4m & MI 1.2 kgm² pivoted at centre is free to rotate without friction. A rope is wound around it and a 2Kg weight is attached to when the wt has descended 1.5m from starting

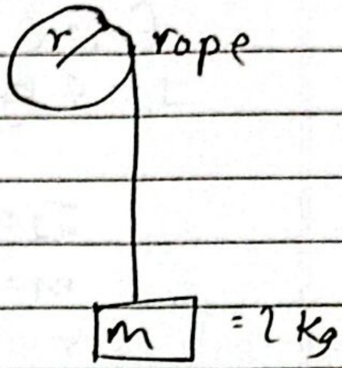
$v = r\omega$

position.

a) what is downward velocity?

b) Rotational velocity of wheel?

Initial $PE = KE_{lin} + KE_{Rot}$



→ $r = 0.4m$

$MI = 1.2\ kgm^2$

$m = 2\ kg$

$h = 1.5m$

Initial $PE = KE_L + KE_{rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2\times1.5\times9.8 = \frac{1}{2}\times2\times v^2 + \frac{1}{2}\times1.2\times\frac{\cancel{(0.4)^2}\,v^2}{(0.4)^2}$$

$$29.4 = v^2(1 + 3.75)$$

$$\sqrt{\frac{29.4}{4.75}} = v$$

$\therefore V = 2.5\ m/s$

Rotational velocity $(v) = \omega r$

$$= \frac{v}{\cancel{r}}\cancel{\times r}$$

$= 2.5\ rad/s.$

3) suppose a ice skater has a moment of inertia $I = 4\ kgm^2$ and her arms have a mass of 5kg each with the center of mass at 0.4m from her body. She starts to turn at 0.5 rev/s on the

point of her skate with of her arms outstretched. She then pulls her arms inward so that their centre of mass is at the axis of the body, r=0. What will be her speed of rotation?

$f_1 = ?$

$I_1 = 4\ kgm^2 \quad I_2 = 4\ kgm^2$

$I_1 = I_2$

$m = 5kg$

$r = 0.4m$

$f_2 = 0.5 rev/s$

$I_1 \omega_1 = I_2 \omega_2$

$\cancel{4 \times 2\pi} f_1 = \cancel{4 \times 2\pi} f_2$

$f_1 = f_2$

$I_1 = I_2$

$(I_{body} + I_{arms})\omega_0 = (I_{body})\omega_f$

$(I + 2mr^2)\omega_0 = 4 \times \omega_f$

$(4 + 2 \times 5 \times 0^2)\omega_0 = 4\omega_f$

$\omega_f = \dfrac{4 + 2 \times 5 \times 0^2 \times 2\pi \times 0.5}{4}$

$= 0.7\ rev/s$

4) A roulette wheel is given an initial rotational velocity of 2 rev/sec. It is observed to be rotating at 1.51 rev/s, 5 sec after it was set in motion.

a) Find angular deceleration of the wheel?
b) How long will it take to stop.

→ Sol:

a) $\omega_0 = 2$ rev/s
$\omega_f = 1.51$ rev/s

$$\alpha = \frac{\omega_f - \omega_0}{t}$$

$$= \frac{1.51 - 2}{5}$$

$$= -0.098$$

-ve sign causes ~~retardat~~ deacceleratio

b) $\omega = \omega_0 + \alpha t$
$0 = 2 + 0.098 \times t$
$t = 20s$

Numerical (Oscillation)

(1) $F = -kx = ma$

spring constant, displacement

(2) $K = m\omega^2$
$\omega = 2\pi f$

(3) $v = r\omega = r \cdot 2\pi f$

(4) $T = 2\pi\sqrt{\frac{m}{K}}$

Date ____
Page ____

① An oscillatory motion of an object is denoted by $y = a\cos(\omega t + \phi)$, where x is displacement in time t, a is amplitude and ω is angular frequency. Show that the motion is SHM.

→ Soln.

we have,

$$y = a\cos(\omega t + \phi)$$

Differentiating wrt t

$$\frac{dy}{dt} = \frac{d}{dt}(a\cos(\omega t + \phi))$$

$$= a\cdot(-\sin(\omega t + \phi))\cdot\omega$$

$$v = -a\omega\cdot\sin(\omega t + \phi)$$

Again, Differentiating wrt t,

$$A = \frac{dv}{dt} = -a\omega\cdot\omega\cos(\omega t + \phi)$$

$$= -\omega^2 a\cdot\cos(\omega t + \phi)$$

$$A = -\omega^2 y$$

we have,

$$A \propto -y$$

Hence, the motion is SHM.

② Find the force constant for a body of mass 100g, which undergoes a SHM of amplitude 10 cm and period 2 sec. Also find the maximum force in Newton.

~~$F=ma$~~

~~$F=2\pi\sqrt{\frac{m}{K}}$~~

→ Soln:-

we have,

$m = 100g = 0.1 kg$

$a = 10 cm = 0.1 m$

$T = 2 sec$

Now,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$\frac{T^2}{4\pi^2} = \frac{m}{K}$$

$$K = \frac{m4\pi^2}{T^2} = \frac{0.1 \times 4 \times \pi^2}{2^2}$$

$$= 0.98 \text{ N/m}$$

Now,

$$A = -\omega^2 a$$

$$= \cancel{-\frac{ma}{K}} - \left(\frac{2\pi}{T}\right)^2 a$$

$$= -\frac{\cancel{4}\pi^2}{\cancel{4}} \cdot 0.1$$

$$= 0.98 \text{ rad/s}^2$$

Now,

$$F = mA$$

$$= 0.1 \times 0.98$$

$$= 0.098 \text{ N}$$

A block of mass 0.250 kg takes 0.199 sec to move between endpoints of the motion which are 4 mm apart.
Calculate the frequency and the force constant of the spring.

Soln.

$m = 0.250$ Kg
$T = 0.149$ sec.

$f = \frac{1}{T} = \frac{1}{0.149} = 6.7$ Hz

Also,

$\omega = 2\pi f$
$= 2\pi \times 6.7$
$= 42.09$ rad/s

$\omega^2 = \frac{m}{k} \frac{K}{m}$

or, $\cancel{m} = \frac{\cancel{m}}{\omega^2} K$

or, $= 0.250$

or, $K = 0.250 \times (42.09)^2$

$\therefore K = 442.89$ N/m

Name :- Aryan Shahi
Section: A
Roll No:- 1405

Assignment 1: Physics.

1) A body is attached to spring of force constant 120 N/m It vibrates with 6.00 Hz. Find the period, angular frequency and mass of body.

→ Soln:-

$$k = 120 \text{ N/m}$$
$$f = 6.00 \text{ Hz}$$

Now,

$$\omega = 2\pi f = 2\pi \times 6 = 12\pi \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{12\pi} = \frac{1}{6} = 0.1667 \text{ sec}$$

We know,

$$K = m\omega^2$$
$$m = \frac{K}{\omega^2}$$
$$= \frac{120}{(12\pi)^2}$$
$$= 0.08 \text{ kg}$$

2) A constant torque of 200 Nm turns a wheel. The MI is 100 Kgm². Calculate the angular velocity gained in 4 sec and KE gained after

20 revolution.

→ Soln:-

Given,

$\tau = 200$ Nm

MI $= 100$ kgm^2

We have,

$\tau = I\alpha$

$\alpha = \frac{200}{100}$

$= 2$ rad/sec^2

Again, In 4 sec

$t = 4$ sec

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 2 \times 4$

$= 8$ rad/sec

Now,

KE gained after 20 rev.

$\theta = 2\pi N$

$= 40\pi$ rad

$\theta = \cancel{\omega_0 t}^{0} + \frac{1}{2}\alpha t^2$

$40\pi = \frac{1}{\cancel{2}} \times \cancel{2}\, t^2$

$\theta = 2\pi n$

$t = \sqrt{40\pi}$
$= 11.2$ sec.

~~Now,~~

$\cancel{KE = \frac{1}{2} I \omega^2}$

$\cancel{= \frac{1}{2} I \cdot \left(\frac{2\pi}{t}\right)^2}$

$\omega = \omega_0 + \alpha t$
$\omega = 2 \times 11.2$
$= 22.4$ rad/s

$KE = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 100 \times (22.4)^2$

$= 25088$ Joules

3) What is phase in SHM? Discuss the variation of KE and PE with time in spring mass system.

→ Phase is the state of a particle in SHM.

We know,
The total kinetic energy of spring mass system is given by

$KE_T = \frac{1}{2} m \omega^2 (r^2 - y^2)$

The total potential energy of the same system is given by

$$PE = W$$
$$= \frac{1}{2} m\omega^2 y^2$$

Now total energy of system is,

$$E_T = KE + PE$$
$$= \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 (r^2 - y^2)$$
$$= \frac{1}{2} m\omega^2 (y^2 + r^2 - y^2)$$
$$= \frac{1}{2} m\omega^2 r^2$$

Hence, the total energy in a system is always constant.

Cases:

At mean position, $y = 0$

$$PE = 0$$
$$KE = \frac{1}{2} m\omega^2 r^2 = E_T$$

At extreme position $y = r$

$$\therefore KE = \frac{1}{2} m\omega^2 (r^2 - r^2) = 0$$
$$\therefore PE = \frac{1}{2} m\omega^2 r^2 = E_T$$

$\therefore$ Maximum PE = Total energy.

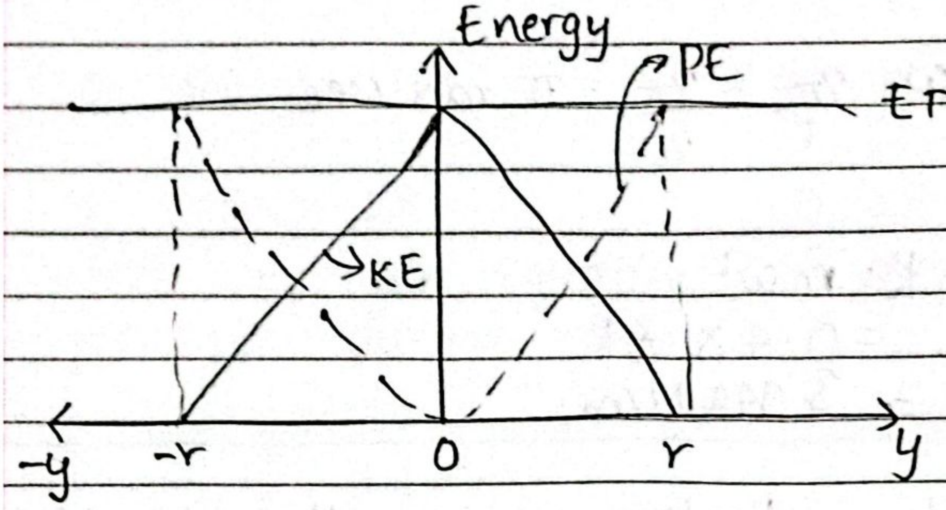


Fig:- Variation of PE & KE with displacement

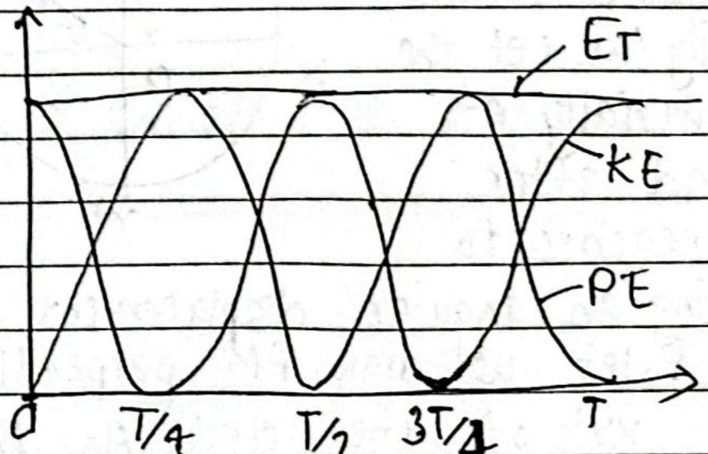


Fig:- Variation of PE & KE with time.

Find the force constant for a body of mass 0.4 Kg, which undergoes a SHM of amplitude 10 cm and period 2 sec.

Soln:-

Here,

$m = 0.4\ Kg$

$a = 10cm = 0.1m$

$t = 2sec$

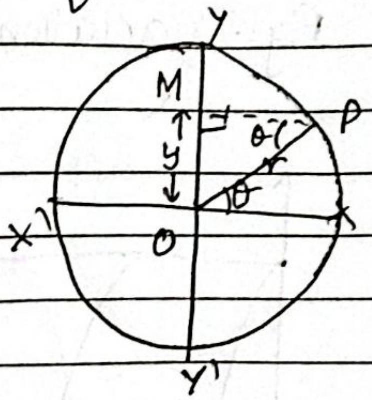
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/sec}$$

Now,

$$K = m\omega^2$$
$$= 0.4 \times \pi^2$$
$$= 3.948 \text{ N/m}$$

Find the displacement equation in SHM.

Let us consider a particle moving in a circle of radius 'r' with uniform velocity 'ω'. Let the particle be initially at point X and after time 't', it reaches to point P covering an angular displacement $\theta = \omega t$.



From point P, let us draw PM perpendicular to the diameter YY' of the circle. As the particle moves along the circle, the projection (M) executes S.H.M along the diameter YY' of the circle.

For displacement,

Displacement is the distance moved by particle from the mean position during SHM. In above figure; let OM = y. Here y is the displacement of the projection (M) from the mean position O.

In Δ POM,

$$\sin\theta = \frac{OM}{OP}$$

or $\sin\theta = \frac{y}{r}$ $\qquad\qquad$ $\because \omega = \frac{\theta}{t}$

or, $y = r\sin\omega t$. $\qquad\qquad$ $\theta = \omega t$

which is the equation of displacement of particle during SHM

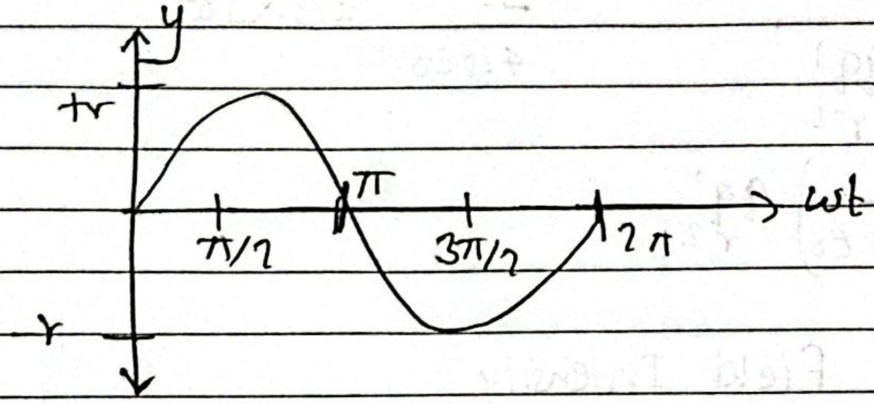


Fig:- Variation of displacement with θ.

Electric Field

* Coulomb's law

$F \propto qq'$

$F \propto \frac{1}{r^2}$

$F \propto \frac{qq'}{r^2}$

$F = \left(\frac{1}{4\pi \epsilon_0}\right) \frac{qq'}{r^2}$

$\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$

* Electric Field Intensity

→ Force experienced by unit positive test charge.

$q' \longrightarrow$ +ve test charge

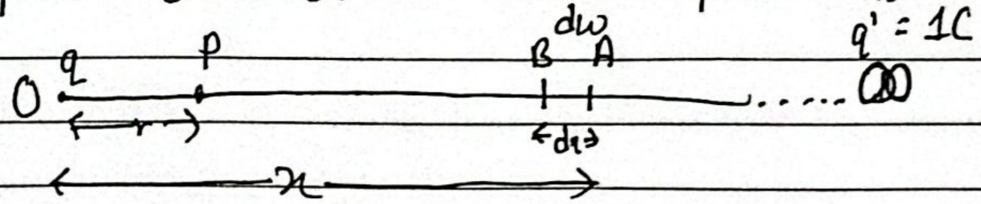
$$E = \frac{F}{q'} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{qq'}{r^2}}{q'}$$

$$= \frac{q}{4\pi\epsilon_0 r^2}$$

Unit = N/C

* Electric Potential.

→ The amount of work done in bringing unit positive test charge from infinity to a point is called electric potential.



Consider a charge q at origin O. Take a point at distance r from O where electric potential is to be measured/calculated.

Small amount of work done for small displacement dx,

$$dW = (-) F\,dx$$
$$= -\frac{1}{4\pi\epsilon_0} \frac{q \times 1}{x^2} dx$$

Total work done to bring charge q from ∞ to P is

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0} \frac{q}{x^2} dx$$
$$= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2} dx$$
$$= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} x^{-2} dx$$
$$= -\frac{q}{4\pi\epsilon_0} \left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$
$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{\infty}\right)$$
$$= \frac{q}{4\pi\epsilon_0 r}$$

$$\boxed{\therefore W = \frac{q}{4\pi\epsilon_0 r}}$$ This much work is in the form of electric potential.

$$\boxed{V = \frac{q}{4\pi\epsilon_0 r}}$$

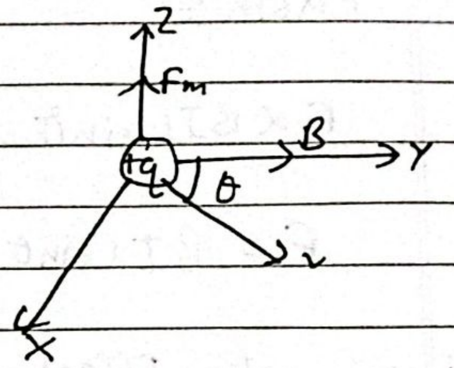
Electric Potential Energy

The amount of work done in bringing positive test charge from ∞ to any point is called electric potential energy.

$$\boxed{U = \frac{qq'}{4\pi\epsilon_0 r}}$$

Magnetic field

(*) Force on moving charge in a magnetic field
$F = Bqv \sin\theta$



Consider a charge +q moving with velocity v in xy-plane which makes angle θ with direction of magnetic field (B). (i.e θ is the angle between v and B).
The magnetic field gives force Fm on the charge.

It is found that magnetic force,

$F \propto B$ —— (i)
$F \propto q$ —— (ii)
$F \propto v$ —— (iii)
$F \propto \sin\theta$ —— (iv)

On combining, $F = KBqv \sin\theta$
$K = 1$, $F = Bqv\sin\theta$, the direction of B is given by fleming left hand rule.

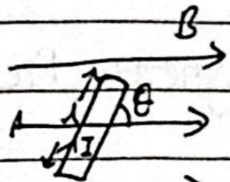
Cases:-
If $\theta = 0°$, then $F = 0$
If $\theta = 90°$, then $F = Bqv$ (max)
$v = 0$ (rest), $F = 0$

$F \propto B$

$F \propto I$

$F \propto l$

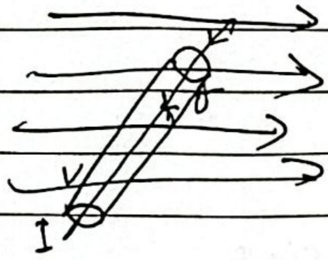
$F \propto \sin\theta$



$F \propto BIl \sin\theta$

$F = BIl \sin\theta$

Magnetic Effect on current carrying conductor.
Let us consider a straight conductor of length 'l' carrying current 'I' placed in a magnetic field intensity 'B' making an angle 'θ' with the direction of B as shown in figure.



Let 'n' be the free electron density and 'v' be the drift velocity of electrons in the conductor. The force on each electron is given by

$$F_e = Bev \sin\theta \quad \text{— (1)}$$

Total number of free electrons in the conductor is

$$N = n \times \text{volume of the conductor},$$

The total force acting is

$$F = NF_e$$
$$= nAl \times Bev\sin\theta$$
$$= B \times venA \times l\sin\theta$$
$$= BIl\sin\theta$$

In vector form,

$$\vec{F} = I(\vec{l} \times \vec{B})$$

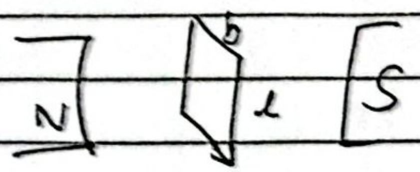
Special cases

→ If $\theta = 0°$ or $180°$, $F = 0N$.

→ If $\theta = 90°$, $F = BIl$ (maximum)

Torque (τ) on coil in field

Let - - -



Torque = Force × perpendicular distance between forces

$$= F \times b\sin\theta$$
$$= IBl \times b\sin\theta$$
$$= BAI\sin\theta$$
$$\tau = BIA\sin\theta$$

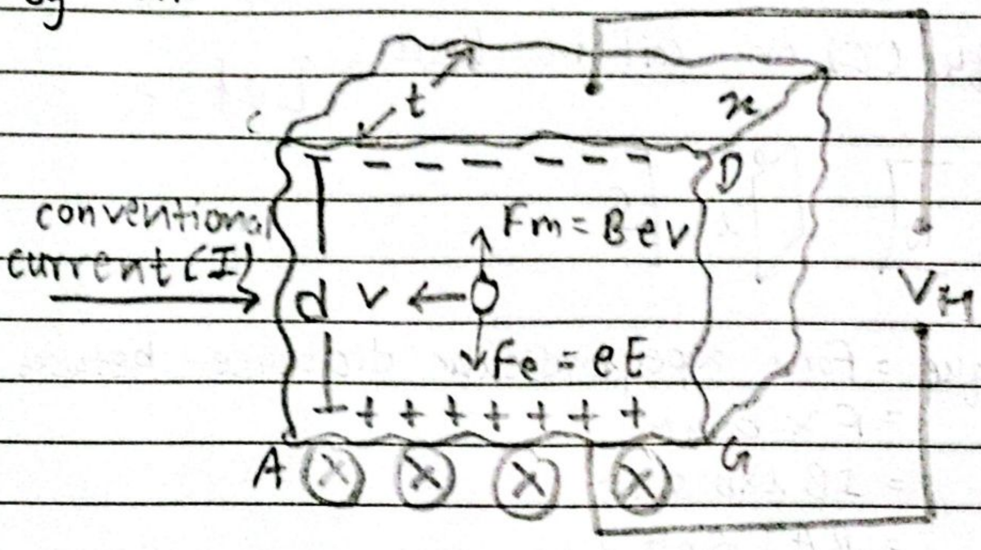
For N coils,

$$\tau = BINA\sin\theta, \text{ for N coils}$$

~~Hall Effect~~

) What is Hall effect? Find the expression of Hall voltage. Discuss the importance of Hall voltage while manufacturing electronic devices.

When a magnetic field is applied to a current carrying conductor, a voltage is developed across the specimen in the direction perpendicular to both the current and the magnetic field. This effect is called Hall effect. The transverse voltage produced in this effect is called Hall voltage and denoted by VH.



Let us consider a current 'I' passsing through a flat strip of a metal in a given direction. A magnetic field 'B' is applied at right angle to the strip. B is perpendicularly inwards.

The Lorentz force experienced by each electron is

$$F_m = Bev \quad \text{—} (1)$$

Due to this Lorentz force, the free electrons shift in upward direction leaving an equal amount of positive charge in downward direction. Due to this force, electrons are accumulated in upward direction and +ve charges are accumulated in downward direction. Due to this difference in charge, an electric fied (E) will be set up on the conductor along Y direction. This field is known as Hall field.

The electronic force is

$$F_e = eE \quad \text{--- (ii)}$$

The accumulation of electrons takes place until an equilibrium is established between the Lorentz force and electrostatic force.

At equilibrium,

$$F_m = F_e$$

$$\text{or, } Bev = eE$$

$$\text{or, } Bv = E$$

$$\text{or, } B \times \frac{I}{neA} = \frac{V_H}{d} \qquad (\because I = venA)$$

$$\text{or, } \frac{BI}{nedt} = \frac{V_H}{d}$$

$$\text{or, } \frac{BI}{net} = V_H$$

$$\therefore V_H = \frac{BI}{net} \quad \text{--- (iii)}$$

This is the required Hall voltage.

Note:-

⇒ We have,

$$V_H = \frac{BI}{net} \rightarrow V_H \propto \frac{1}{n}$$

The density of free electrons in metal ($n \approx 10^{28} m^{-3}$) is greater than that of semiconductor ($n \approx 10^{25} m^{-3}$). So, Hall effect is more measurable in semi-conductor.

$$\text{Hall coefficient } (H_c) = \frac{1}{ne} \text{ or } \frac{1}{nq}$$

⇒ Hall probe is a sensitive device used to measure feeble magnetic flux density (B). It can measure the Earth's magnetic field ($\approx 3.4 \times 10^{-5}$ T)

⇒ Studies of the Hall effect in a material can give us information about the nature of the charge carriers, that is whether they are positive or negative, and their number density. It can also be used to measure the strength of a magnetic field in the order of a Tesla with a Hall probe. It is also used in modern appliences to detect position, speed and other parameters.

Numerical

Three charges $q_1 = 3\times10^{-6}C$, $q_2 = -5\times10^{-6}C$ and $q_3 = -8\times10^{-6}C$ are placed on a straight line. Find PE of the charges.

Soln.:

$$q_1 \xleftrightarrow{4m} q_2 \xleftrightarrow{5m} q_3$$

We know,

Electric Potential Energy, $U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

Then total PE is given by

$$PE = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{4} + \frac{1}{4\pi\epsilon_0}\frac{q_1q_3}{9} + \frac{1}{4\pi\epsilon_0}\frac{q_2q_3}{5}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{3\times10^{-6}\times(-5\times10^{-6}C)}{4} + \frac{3\times10^{-6}\times(-8\times10^{-6})}{9} + \frac{-5\times10^{-6}C\times(-8\times10^{-6})}{5}\right)$$

$= 0.01425J$

$= 1.4\times10^{-2}J$

~~An electron is placed midway between two fixed charges $q_1 = 2.5\times10^{-10}C$ and $q_2 = 5\times10^{-10}C$. If the charges are 100 cm apart, what is the velocity of the electron when it reaches a point 0.1m from q_2?~~

A potential difference of 100V is established between the two plates. A proton of charge $q = 1.6 \times 10^{-19}$ C is released from plate B. What will be the velocity of the proton when it reaches negative plate? The mass of the proton is 1.67×10^{-27} kg.

Soln:-

Potential difference (V) = 100 V
charge (q) = 1.6×10^{-19} C
mass (m) = 1.67×10^{-27} kg

We know,

$$\frac{1}{2} m v^2 = eV$$

$$v = \sqrt{\frac{2qV}{m}}$$

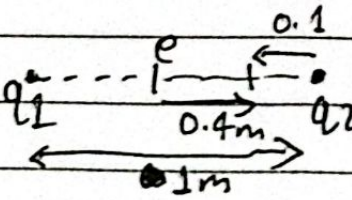
$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{1.67 \times 10^{-27}}}$$

$$= 138425.708 \text{ m/s}$$

An electron is placed midway between two fixed charges $q_1 = 2.5 \times 10^{-10}$ C and $q_2 = 5 \times 10^{-10}$ C. If the charges are 100 cm apart, what is the velocity of the electron when it reaches a point 0.1m from q_2?

Soln.

$q_1 = 2.5 \times 10^{-10}$ C
$q_2 = 5 \times 10^{-10}$ C



Force between q_1 and electron e

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{eq_1}{r_1^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{eq_2}{r_2^2}$$

$$F_{net} = F_2 - F_1 = \frac{1}{4\pi\epsilon_0} e\left[\frac{q_2}{r_2^2} - \frac{q_1}{r_1^2}\right]$$

$$= \cancel{288\ N}\ 144 \times 10^{-18}$$

$$F_{net} = ma$$

$$ma = \cancel{288}\ 1.44 \times 10^{-18}$$

$$a = \frac{\cancel{288}\ 144 \times 10^{-18}}{9.109 \times 10^{-31}}$$

$$= \cancel{1.58 \times 10^{32}}$$

$$= 1.58 \times 10^{12}\ m/s^2$$

Now,

$$v = \sqrt{2as}$$

$$= \sqrt{2 \times 1.58 \times 10^{12} \times 0.4}$$

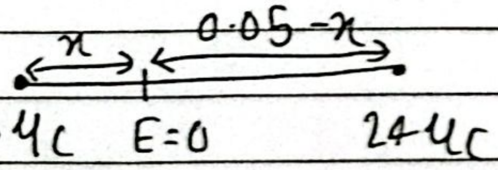
$$= 112427.7546\ m/s$$

---

⑧ Two charges of $6\mu C$ and $24\mu C$ are placed 0.05m apart. Find the location of a point between them where field ~~is~~ strength is zero.

→ Sol^n:

Let x be the distance of the required point from charge $6\mu C$



By question,

$$E_1 = E_2$$

or, $\frac{1}{\cancel{4\pi\epsilon_0}} \frac{q_1}{x^2} = \frac{1}{\cancel{4\pi\epsilon_0}} \frac{q_2}{(0.05-x)^2}$

or, $(0.05 - x)^2 = \frac{q_2}{q_1} x^2$

or, $(0.05)^2 - 2 \times 0.05 \times x + x^2 = \frac{\overset{4}{\cancel{24}} \times \cancel{10^{-6}}}{\cancel{6} \times \cancel{10^{-6}}} \times x^2$

or, $0.0025 - 0.1x + x^2 = 4x^2$

or, $3x^2 + 0.1x - 0.0025 = 0$

on solving, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-0.1 \pm \sqrt{(0.1)^2 - 4 \cdot 3 \cdot (-0.0025)}}{2 \cdot 3}$

$= 0.016 \text{ m}$

$= 0.016 \times 100 \text{ cm}$

$= 1.6 \text{ cm}$

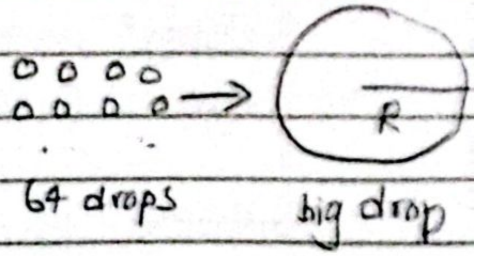
$\therefore$ The point of zero field strength lies 1.6 cm from $6\mu C$ charge and 3.4 cm from $24\mu C$ charge.

64 Identical spherical drops of Hg are charged to a same potential of 10 volt. What will be the potential if all the drops are made to combine to form one large drop?

→ Soln.: Let, r be radius of small drops and R be radius of big drop.

Here,

$V_{small} = 10\,V$



We have

64 small drops = 1 large drop

Vol. of 64 small drop = Vol. of large drop.

$$64 \times \cancel{\frac{4}{3}\pi} r^3 = \cancel{\frac{4}{3}\pi} R^3$$

$$\text{or, } R^3 = 64 r^3$$

$$\text{or, } R^3 = (4r)^3$$

$$\therefore R = 4r$$

Now,

For big drop.

$$Q_{Total} = 64q$$

Now, Potential of big drop,

$$V_{big} = \frac{1}{4\pi\epsilon_0} \frac{Q_{Total}}{R}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\cancel{64}^{16} q}{\cancel{4} r}$$

$$= 16 \cdot \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \right)$$

$$= 16 \cdot V_{small}$$

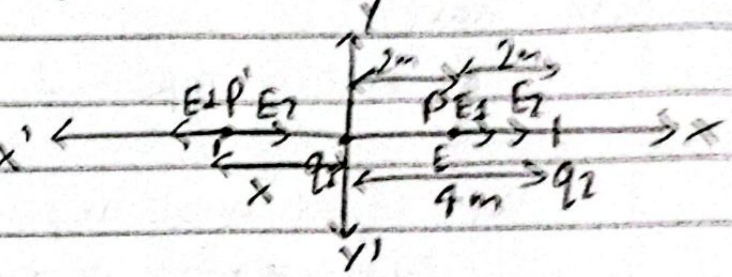
$$= 16 \times 10$$

$$= 160\,V$$

∴ Charge on big drop after combining is 160 V.

Q) q_1 charge of magnitude $3\times10^{-6}C$ is located at the origin of the x-axis. A second charge $q_2 = -5\times10^{-6}C$ is also on the x-axis 4m from the origin in the positive x-direction.

→ ~~Soln.~~



a) Calculate the electric field at the mid-point P of the line joining the two charges.

→ Soln.:-

For electric field at P,

$r_1 = 2m$
$r_2 = 2m$
$q_1 = 3\times10^{-6}C$
$q_2 = -5\times10^{-6}C$

Here, the electric fields act in same direction due to nature of charges involved,

$$E = E_1 + E_2$$
$$= \frac{1}{4\pi\epsilon_0}\cdot\left|\frac{q_1}{r_1^2}\right| + \frac{1}{4\pi\epsilon_0}\cdot\left|\frac{q_2}{r_2^2}\right|$$
$$= \frac{1}{4\pi\epsilon_0}\cdot\left(\frac{3\times10^{-6}}{2^2} + \frac{5\times10^{-6}}{2^2}\right)$$
$$= 18000\ N/C$$

b) At what point P on that field zero?

→ Since q_1 and q_2 are opposite charges, the null point ($E=0$) will be formed outside the charges near the smaller charge as shown in figure.

Let x be the distance of null point from q_1

At null point

$$|E_1| = |E_2|$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}\left|\frac{q_1}{r_1^2}\right| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_2}{r_2^2}\right|}$$

$$\text{or, } \frac{1}{\cancel{4\pi\varepsilon_0}}\left|\frac{q_1}{x^2}\right| = \frac{1}{\cancel{4\pi\varepsilon_0}}\left|\frac{q_2}{(x+4)^2}\right|$$

$$\text{or, } \frac{3\times10^{-8}}{x^2} = \frac{5\times10^{-8}}{(x+4)^2}$$

$$\text{or, } 3(x^2+8x+16) = 5x^2$$

$$\text{or, } 3x^2+24x+48 = 5x^2$$

$$\text{or, } 2x^2-24x-48=0$$

$$\text{or, } x^2-12x-24=0$$

On solving, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-(-12x) \pm \sqrt{(-12)^2 - 4\cdot1\cdot(-24)}}{2\cdot1}$$

$$= \frac{12 \pm \sqrt{144+96}}{2}$$

$$= \frac{12 \pm 15.49}{2}$$

$x = 13.74$ m or, $x = -1.74$m (invalid)

∴ Null point is 13.74 m from q_1 and 17.74m from q_2

Q) 4 charges of equal magnitude are placed at the 4 corners of a square of side 0.5m. Find the electric field at the centre of square?

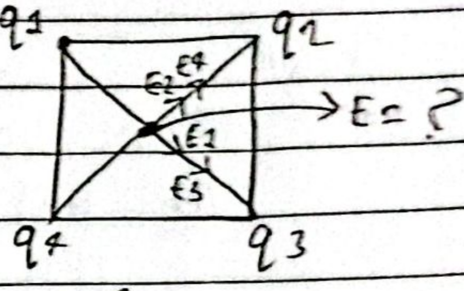
→ Soln:-

$q_1 = +3\times10^{-6}C$

$q_2 = -3\times10^{-6}C$

$q_3 = +3\times10^{-6}C$

$q_4 = -3\times10^{-6}C$



Here, $r = \frac{l}{\sqrt{2}} = \frac{0.5}{\sqrt{2}} = 0.354$

Soln..

Here,

$$E_1 = \frac{1}{4\pi\epsilon_0}\left|\frac{q_1}{r^2}\right|$$

$$= 9\times10^{9}\times\frac{3\times10^{-6}}{(0.354)^2}$$

$$= 2.15\times10^{5}\ N/C$$

Since other charges are same and r is also same,

$$E_1 = E_2 = E_3 = E_4 = 2.15\times10^{5}\ N/C$$

Here,

E_2 and E_4 are in same direction so they add up.

$$E_{24} = E_2 + E_4$$

$$= 2\times2.15\times10^{5}\ N/C$$

$$= 4.3\times10^{5}\ N/C$$

Similar for E_1 and E_3

$$E_{13} = E_1 + E_3$$

$$= 2\times2.15\times10^{5}\ N/C = 4.3\times10^{5}\ N/C$$

Now,

$$E_{net} = \sqrt{E_{24}^2 + E_{13}^2 + 2 \cdot E_{24} \cdot E_{13} \cdot \cos 90^\circ}$$
$$= \sqrt{2\,(4.3 \times 10^5)^2}$$
$$= 608111.8356 \text{ N/C}$$

$$E_{net} = (E_1 - E_3) + (E_2 - E_4)$$
$$= 0$$

Numerical from B Hall effect.

Formula:

$F = BIl \sin\theta$

max value $F = BIl$

$F = Bqv \sin\theta$

Hall voltage $V_H = \frac{BI}{net}$ or, $\frac{BI}{nqt}$

$\tau = BINA$

Magnetic moment $\mu = IA$

Questions:

What force is experienced by a wire of length 8 cm at an angle of 20° to the magnetic field direction carrying a current of 2A in a magnetic field of 1.4 T?

Soln.

$l = 8\,cm = 0.08\,m$

$\theta = 20°$

$I = 2A$

$B = 1.4T$

$F = BIl \sin\theta = 1.4 \times 2 \times 0.08 \sin 20°$

$= 10.02 \times 10^{-2} N$

2) A particle is moving with velocity $v = 3\times10^5 i + 7\times10^5 K$ m/s in a region where there is field $B = 0.4\,j$ T. Find the force felt by the particle.

⇒ Solⁿ:

Here,

$$v = (3\times10^5 i + 7\times10^5 K)\ m/s$$
$$B = 0.4\,j\ T$$

$$F = q(v\times B)$$
$$= e\begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 3\times10^5 & 0 & 7\times10^5 \\ 0 & 0.4 & 0 \end{vmatrix}$$
$$= e\cdot[(-0.4\times7\times10^5)\vec{i} + 3\times10^5\times0.4\,\vec{K}]$$
$$= 1.6\times10^{-19}\,[-280000\,\vec{i} + 120000\,\vec{K}]$$
$$= 1.92\times10^{-14}\vec{K} - 4.48\times10^{-14}\vec{i}$$

The force experienced by particle is
$$(1.92\times10^{-14}\vec{K} - 4.48\times10^{-14}\vec{i})\,N$$

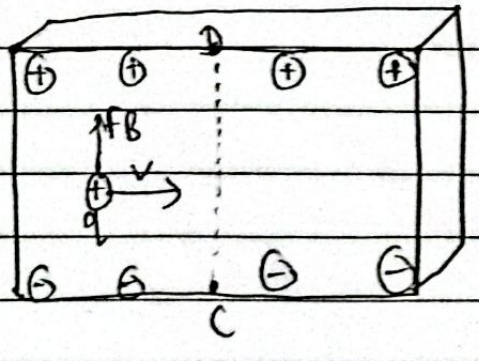
Assignment (2)

Name :- Aryan Shahi
Roll No :- 1405
Section :- A

(1) Define Hall effect. Find the expression of Hall voltage and Hall mobility. Also discuss the importance of Hall effect while manufacturing devices.

→ When a magnetic field is applied to a current carrying conductor, voltage is developed across the conductor (specimen) in a direction perpendicular to both current and magnetic field. This effect is known as Hall effect.

Consider a specimen in the form of rectangular cross section carrying current I_x in x-direction.
If magnetic field B_z is applied in Z-axis, emf (voltage) is developed along y-axis perpendicular to both I_x and B_z. This voltage is known as Hall voltage (V_H)



We have,

$$F_B = Bqv$$
$$F_e = Eq$$
$$F_e = \frac{V_H}{d} q$$

If $F_B = F_E$

$$Bqv = \frac{V_H}{d} q$$

$$V_H = Bvd = \text{Hall voltage.}$$

Now, $I = venA$

$$v = \frac{I}{enA}$$

Then,

$$V_H = \frac{B\,Id}{enA}$$

using $A = dt$, t is thichness, then,

$$V_H = \frac{BI}{net}.$$

This is the required expression of Hall Voltage.

contd....

Date ____
Page ____

Fundamentals of Atomic Theory

Black Body Radiation

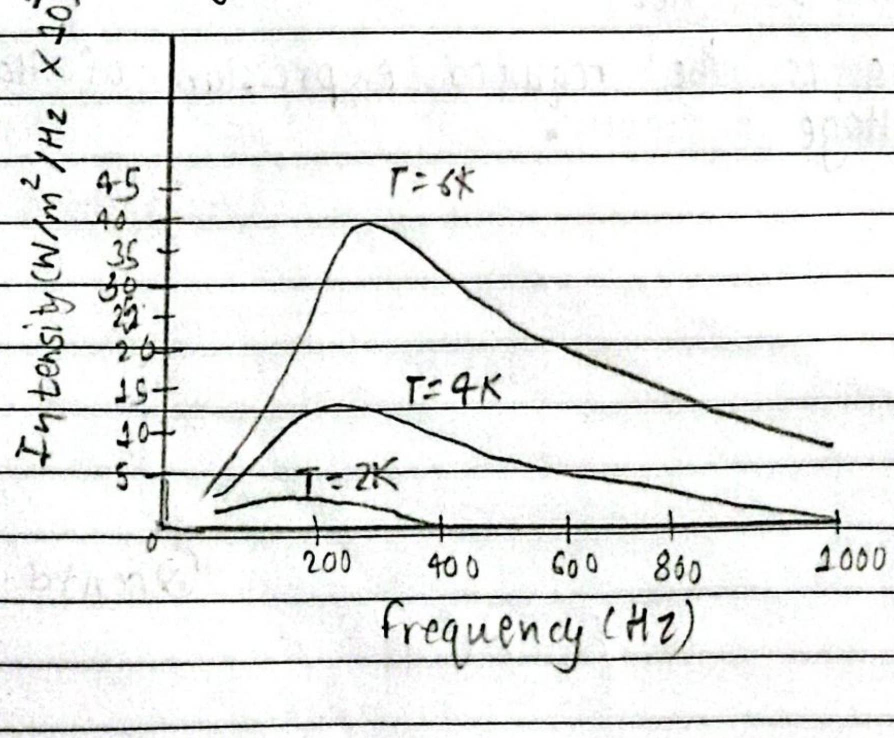
Black Body:-

→ The body whose surface absorbs all the heat radiation incident on it. No body is perfectly black. They do not reflect light and then appear black.

Lampblack or Platinum black - considered as black body since they absorb 96-98% of radiation.

Character of Blackbody ~~Radia~~ Spectrum

1) Spectrum is continuous with a broad maximum as in figure.



Date ____
Page ____

2) The spectrum shifts toward higher frequencies with increasing temperature.

the f_{max}, at which intensity (I) is maximum, increases linearly with temperature.

i.e $f_{max} \propto T$

3) The integral of I over all frequencies represents the energy emitted per unit time per unit area and is found to increase with the 4th power of the temperature.

$$E = \int_0^{\infty} I(f)\, df = \sigma T^4 \quad \text{--- (1)}$$

where, σ is Stefan's constant $= 5.67 \times 10^{-8}$ $W/m^2\text{-}K^4$
$= W/m^2K^4$

Eqⁿ (1) is Stefan-Boltzmann law

....contd (A)

Now, Expression of Hall mobility,

We have,

Hall coefficient $(R_H) = \frac{E}{JB}$

where, $E = vB$ (

J = current density $= \frac{I}{A} = \frac{venA}{A} = ven$

n = number density of electrons

B = magnetic field.

$$R_H = \frac{vB}{venB} = \frac{1}{en}$$

Now, Mobility of hole

$$\mu_H = \frac{v}{E} = \frac{J}{neE} \quad \left[J = ven;\ v = \frac{J}{ne} \right]$$

$$= R_H \cdot \frac{J}{E} \quad \left[\because \frac{1}{ne} = R_H \right]$$

$$= R_H \cdot \sigma \quad \left[\sigma = \frac{J}{E} = \text{conductivity} \right]$$

Importance of Hall effect while manufacturing devices:

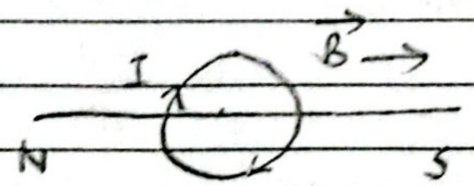
- Studies of the Hall effect in a material can give us information about the nature of the charge carriers ie whether they are positive or negative and their number density. It can also be used to measure the strength of a magnetic field in the order of Tesla with Hall probe.

It can be used to determine whether the sample is metal, semi-conductor, conductor, or Insulator.

- Mobility of electron can be determined.
- No. of free electrons per unit volume can be calculated from Hall coefficients.

2) Define dipole moment Find an expression for torque on a current loop in terms of magnetic field and dipole moment.

→ When current is flowing through a loop of conductor, magnetic field (B) is developed forming a dipole (N-S) as shown in figure.



Thus magnetic dipole moment can be defined as the product of current and area of loop. It is denoted by μ and is given by

$$\mu = IA$$
$$I = \frac{\mu}{A} \quad \text{—(I)}$$

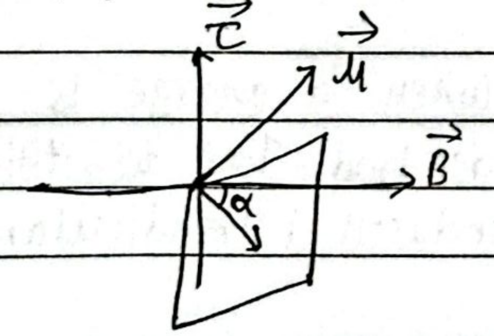
For a current carrying rectangular loop,

$$\tau = BIA\sin\alpha \cdots \text{(II)}$$ where, α = angle made by normal to plane of coil with B.

From (I), $\tau = B\mu\sin\alpha$

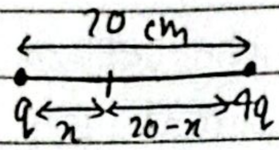
In vector form, $\vec{\tau} = \vec{B} \times \vec{\mu}$ —(III)

Eq^n (III) shows that torque is perpendicular to plane containing $\vec{\mu}$ & $\vec{B}$



3) Two points charges $4q$ and q are 20 cm apart. At what point on the line joining them is the electric field zero?

→ Solⁿ.: Let the required point be x cm from the smaller charge q.



By question,

$$E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{4q}{(20-x)^2}$$

or, $400 - 40x + x^2 = 4x^2$

or, $3x^2 + 40x - 400 = 0$

On solving

we get, $x = 6.67\,cm = 0.667\,m$

$x = -20\,cm$ (X)

∴ The required point of zero electric field is 0.667 m from charge q or 0.133 m from charge $4q$

OR, Write short notes on electromagnetic waves.

→ The waves which are formed when an electric field comes in contact with magnetic field are known as electromagnetic ~~fields~~ waves. They travel with constant velocity of 3×10^8 m/s. They don't need a medium for transmission. They are transverse waves, i.e they travel in parallel to wave progression.

- When a charge is accelerated, then electric field is developed. Due to this electric field, magnetic field is induced perpendicular to the electric field. Due to this magnetic field, electric field is again induced. Continuing this process, a wave is produced which travels in a direction perpendicular to both electric field and magnetic field, known as direction of propagation of wave, which is known as electromagnetic wave. Here, E and B vary sinusoidally with time, i.e

$$\vec{E} = \vec{E_0} \sin(Kx - \omega t)$$

$$\vec{B} = \vec{B_0} \sin(Kx - \omega t)$$

where, E_0 & B_0 are max value of $\vec{E}$ & $\vec{B}$

$\omega \rightarrow$ angular frequency

$K \rightarrow$ wave constant

Here, $\vec{E} \times \vec{B}$ gives the direction of propagation of wave. The electromagnetic wave having varying frequency is electromagnetic spectrum.

γ ray $\lambda = 10^{-13}m$ to $10^{-10}m$

X ray $\lambda = 10^{-11}m$ to $10^{-8}m$

UV ray $\lambda = 10^{-8}m$ to $4 \times 10^{-7}m$

Visible light $\lambda = 4 \times 10^{-7}m$ to $7.8 \times 10^{-7}m$

Infrared ray $\lambda = 7.8 \times 10^{-7}m$ to $10^{-3}m$

microwave $\lambda = 10^{-3}m$ to $0.01m$

radiowave $\lambda = 1m$ to $10^5 m$

Electron in hydrogen is in circular orbit of diameter $10^{-8}cm$ and rotates about the nucleus at the rate of 10^{15} per sec. Find magnetic moment of atom(s)

Soln.:

diameter (d) = $10^{-8}cm$

$= 10^{-10} m$

$f = 10^{15}$ times per sec

Area (A) = $\pi \left(\frac{d}{2}\right)^2$

ime (T) = $\frac{1}{f}$ =

urrent (I) = $\frac{e}{T}$

$= \frac{1.6 \times}{1/}$

$= 1.6$

No

Magnetic Mo