H.W.

① Prove that $f(r)\,\vec{r}$ is irrotational.

② Find the divergence and curl of the vectors

$$\vec{R} = (x^2 + yz)\,\hat{i} + (y^2 + zx)\,\hat{j} + (z^2 + xy)\,\hat{k}.$$

Ans. (1) $2(x+y+z)$; $\vec{0}$

③ Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$ is irrotional. ~~If~~ If $\vec{F} = grad\,\phi$, ~~show that $\phi =$~~ find ϕ.

Ans. $\dfrac{x^2}{2} - \dfrac{3y^2}{2} + z^2 + 2xy + 4xz - yz + c$