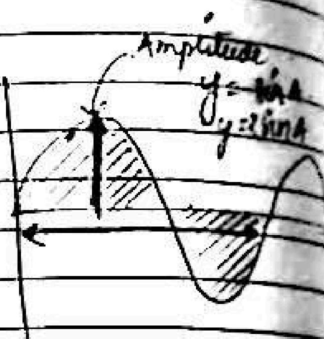
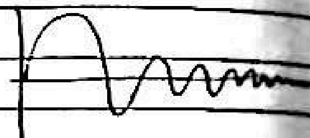
[AC Fundamentals]

Thermal energy (Fuel) → steam → Turbine (Mechanical) → coils moving → magnetic flux change → $d\phi = n N C$ (no. of coils, speed) → Potential change → energy generation



- Average value of current = 0. → (Area enclosed by curve)



- time period should be equal to next repetition (not defined)

- fangent should be equal of each rotation positive/negative half cycle.

* The term AC means 'alternating current'. Such a current reverse its direction periodically. In AC power the voltages and currents vary with time sinusoidly

$$I = I_m \sin \omega t$$

$$V = V_m \sin \omega t.$$

Q.1 A sinusoidal voltage signal is given by $V = 20\sin\omega t$ volts

(a) At what angle will the instantaneous voltage be 10 V

(b) What is the max. value for the voltage and at what angle?

Sol (a) $10 = 20\sin(\omega t)$

$\frac{1}{2} = \sin(\theta) \Rightarrow \boxed{\theta = \frac{\pi}{6}} \boxed{30^\circ}$

(b) $V_{max} = 20\,V$ and at angle $\pi/2$

Q.2 $V = 0.04\sin(2000t + 60^\circ)$ volts. Find f, ω, V (at 160 μs)

Ans: 35.2 mV

Sol $\omega = 2000$

$T = \frac{2\pi}{\omega} = \frac{2 \times \pi}{2000}$ 1000

$\boxed{f = \frac{1000}{\pi}}$

$2000 \times \frac{\pi}{180}$

$V = 0.04\sin(2000 \times 160 \times 10^{-6} + \frac{\pi}{3})$

$= \cancel{0.04\sin\left(\frac{2900\pi}{9} + \frac{\pi}{3}\right)}$

$= \cancel{0.04\sin\left(\frac{103\pi}{9}\right)}$

$= 0.04\sin(320000 \times 10^{-2} + \frac{\pi}{3})$ 60°

$= 0.04\sin(0.32 + \frac{\pi}{3})$ 60°

$\frac{0.32 \times 180}{\pi}$

$= \cancel{0.04\sin(0}$

First convert it into degree (bcz of calculator)

[not in radian] X.

(Mean/DC) value. magnitude.

Average value :-

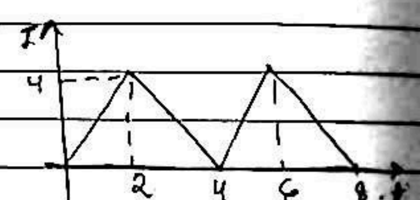
Discrete :-

It is the algebraic sum of all the value divided by the total no. of values

- In a waveform, the continuous variation of the value of a quantity with time t or angle θ repeated after its cycle. The area under the waveform is found by integration of the given wave.

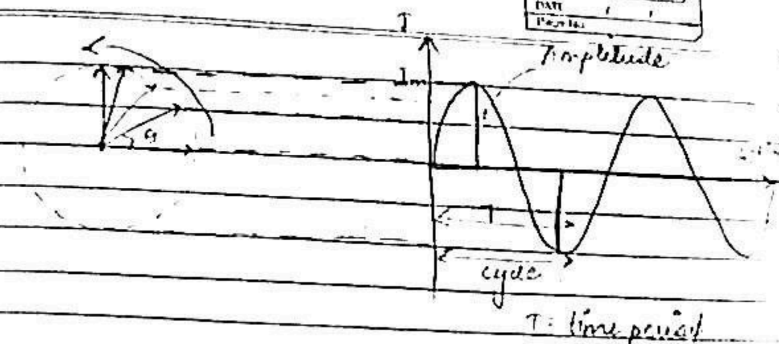
Average value = $\frac{\text{Area under the full cycle.}}{\text{time period. (length of the cycle)}}$

Q. Find average values ?

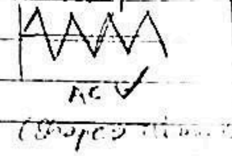


$y - y_1 = m(x - x_1)$

$y = mx + c$.



- Cycle → One complete set of positive and negative value of the function (which goes on repeating) is called a cycle.



- Maximum value / Peak Value → It is the maximum value positive or negative of the quantity. It is also sometimes called the amplitude of the sinusoidal functions.

- Instantaneous value → The value of alternating quantity (voltage, current, emf) at any particular instant is called instantaneous value.

- Time period → The time taken in seconds by an alternating quantity to complete one cycle is known as time period. Denoted by (T) Unit → seconds

- Frequency → The no. of cycles completed per second by an alternating quantity is known as Frequency denoted by "f"

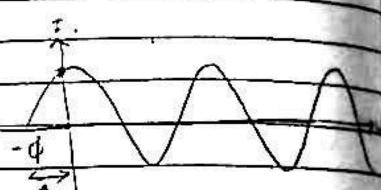
$$\boxed{f = \frac{1}{T}}$$

The Frequency of AC supply in India is 50Hz.
USA is 60Hz

- Angular Frequency (ω) → The values of a sine function repeat after every 2π radians. denoted by (ω).
  $\boxed{\omega = \text{no. of radians covered in 1 sec}}$
  Its unit is radian/seconds.

$$\boxed{\omega = \frac{2\pi}{T}}$$

- PHASE SHIFT →



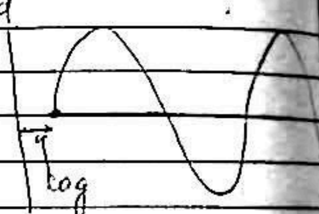
$$I = I_m \sin(\omega t \pm \phi).$$

at $\omega t = 0$

comparison can only be done with respect to anyone lead $t \neq 0$

Ex:- $I_1 = 2\sin(4t - 30)$
$I_2 = 2\sin(4t - 50^\circ)$
$I_3 = 2\sin(4t - 80)$

Lag w.r.t. I_1. should be same.



The initial shift of the waveforms on the time axis or Angular axis is called phase shift.

Angle of lead

→ If ϕ is +ve number ⇒ waveform shift towards left.
→ If ϕ is -ve number ⇒ waveform shift towards right

Angle of lag

(AC value magnitude)

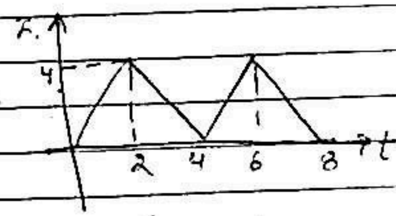
Rms value -: / Effective value.

→ An effective value of an AC current defined on the basis of its heating effect.

Let an AC sinusoidal current $I = I_m \sin\omega t$ flow through a resistor R, our aim is to find that value of the current I which gives the same amount of heating, if it flow through a resistor of the same value R. This would be the effective value of the AC sinusoidal current.

$$I_{eff} / I_{rms} = \sqrt{\frac{1}{T}\int (I(t)^2)\, dt}$$

Q: Find rms value? Ans = $\sqrt{16/3}$



Ans: $I_{rms} = \sqrt[4]{\frac{\int_0^2 4t^2 + \int_2^4 (8-2t)^2 dt}{4}}$ $_0$

RMS.
Root
Mean
Square

$m = \frac{y_2 - y_1}{x_2 - x_1} = 4$

$= (2)$

$(2t)$

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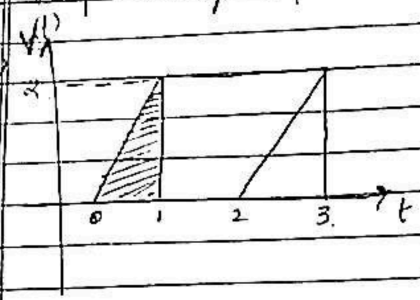
Q-1 Find average and rms values of following —:

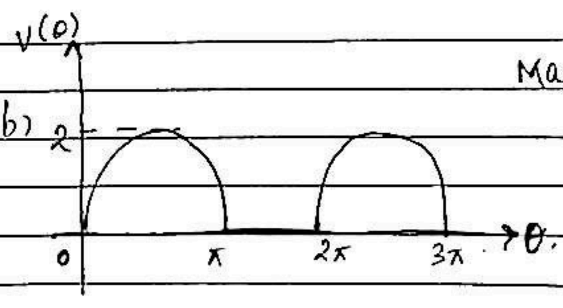
(a)



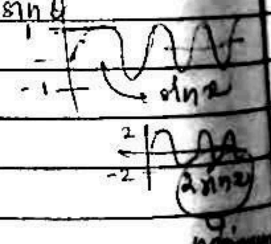
$V_{average} = \frac{1}{2}$

$V_{rms} = \frac{\sqrt{2}}{\sqrt{3}}$.

(b)



Max = 2 sin θ



$$\text{Avg. value} = \frac{\int_0^{2\pi} 2\sin\theta \, d\theta}{2\pi}$$

$$= \frac{\int_0^{\pi} 2\sin\theta + \int_{\pi}^{2\pi} 0 \, d\theta}{2\pi}$$

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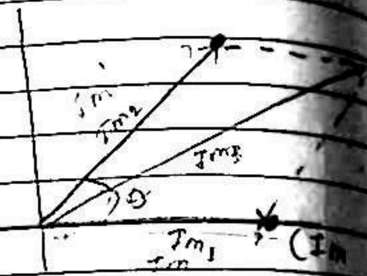
PHASOR ALGEBRA :-

Methods to find net impedance :-

(1) By using concepts of Phasor :-

Resultant of I_{m1} and I_{m2} -



$I_{m1} = I_m \sin(\omega t)$

$I_{m2} = I_m' \sin(\omega t$

Rectangular form / Complex form

$z = x + jy$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Trigonometric to Polar form representation

max. value

$z = r \angle \theta$ ← angle with reference.

$\Rightarrow I_{m1} = I_m \angle \theta (0)$

$I_{m2} = I_m' \angle \theta$

Polar to rectangular form $\Rightarrow$ $r \angle \theta$. (polar)

$\Rightarrow r\cos\theta + j r\sin\theta$ (rectangular)

$I_{m3} = I_{m1} + I_{m2}$

$= I_m + I_m'\cos\theta + j I_m' \sin\theta$.

With example :-

Sine to polar

$i = 2\sin(\omega t + 30^\circ) \longrightarrow 2 \times \angle 30^\circ$ or $2e^{j\ldots}$

Polar to complex / Rectangular

$2 \times \angle \theta \Rightarrow 2\cos\theta + j\,2\sin\theta$

Rectangular to polar

Rect. to Pol.

$x + jy = r\angle$

$r = |x + jy|$

$= \sqrt{x^2 + y^2}$

$\angle\theta = \tan^{-1}$

$2\cos 30^\circ + j\,2\sin 30^\circ = \sqrt{(2\cos 30^\circ)^2 + (2\sin 30^\circ)^2} = 2\angle 30^\circ$

$\theta = \tan^{-1}\left(\dfrac{2\sin 30^\circ}{2\cos 30^\circ}\right)$

$\theta = \tan^{-1}(\tan 30^\circ$

$\boxed{\theta = 30^\circ}$

$z = r \times \theta$
$= 2 \times \angle 30^\circ$

② Addition, Subtraction, Multiplication, Division of two no.s :-

e.g. $z_1 = 2 + 4j$ · Find : $z_1 + z_2$, $z_1 - z_2$, $z_1 \times z_2$, z_1 / z_2

$z_2 = 1 + 3j$

$z_1 = 2 + 4j = \sqrt{2^2 + 4^2} \times \angle \tan^{-1}\left(4/2\right)$

$z_2 = 1 + 3j = \sqrt{1^2 + 3^2} \times \angle \tan^{-1}\left(\dfrac{3}{1}\right)$

(complex form only) For

$z_1 + z_2 = 2 + 4j + 1 + 3j = 3 + 7j \Rightarrow \sqrt{3^2 + 7^2} \times \angle \tan^{-1}\left(7/3\right)$

$z_1 - z_2 = 2 + 4j - 1 - 3j = 1 + j \Rightarrow \sqrt{1^2 + 1^2} \times \angle \tan^{-1}\left(1/1\right)$

(Polar form only)

$z_1 * z_2 =$

Generalised form :-

$x_1 = r_1 \angle \theta_1$

$x_2 = r_2 \angle \theta_2$

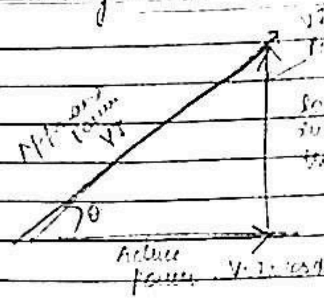
$x_3 = r_3 \angle \theta_3$.

$x_1 \cdot x_2 \cdot x_3 = r_1 \times r_2 \times r_3 \ \angle \theta_1 + \theta_2 + \theta_3$.

$$\frac{x_1 \cdot x_2}{x_3} = \frac{r_1 \cdot r_2 \ \angle(\theta_1 + \theta_2)}{r_3 \angle \theta_3}.$$

$$= \frac{r_1 \cdot r_2}{r_3} \angle \theta_1 + \theta_2 - \theta_3.$$

Single Phase series ac circuits



useful power : $P_{active} = VI\cos\theta$

Reactive power : $P_{reactive} = VI\sin\theta$

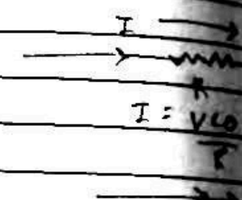
apparent power : $P_{app} = VI$ → Power generated by inductors

Apparent Power = $\sqrt{(\text{Active Power})^2 + (\text{Reactive Power})^2}$

Only Resistive element -

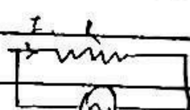
Phase diff = 0

I and V → same direction.



1) Purely resistive circuit -:

$I = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$

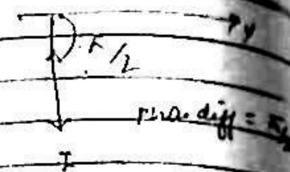


$\boxed{I = I_m \sin\omega t}$

→ Wave form representation -:

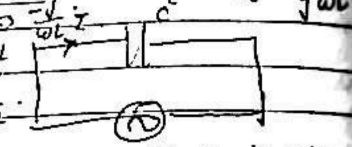
=) Diff. between V_m and I_m is identified by resistance R.

→ Phase representation :-



$$\text{Power} = VI\cos\phi$$
$$= VI\cos \pi/2$$
$$\boxed{P = 0}$$

3) Purely Capacitive circuits :-



$$X_c = \frac{1}{j\omega C}$$

$$\frac{1}{j^2\omega C} = \frac{-j}{\omega C}$$

$$I = \frac{V}{X_c} \qquad X_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

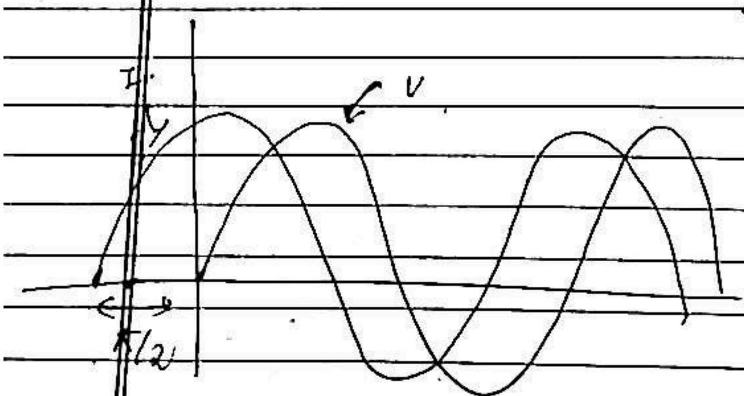
$$|X_c| = \frac{1}{\omega C}$$

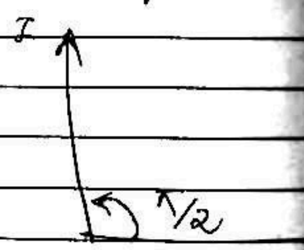
$$I = \frac{V_m \angle 0}{\frac{1}{\omega C}\angle -\pi/2} \qquad \angle X_c = \tan^{-1}\left(\frac{-\frac{1}{\omega C}}{0}\right)$$

$$\boxed{I = I_m \angle \pi/2} \qquad \angle X_c = -\tan^{-1}(\infty) = -\pi/2$$

Leading nature circuit → so called current.

⇒ current lead voltage by an angle of π/2.





Phasor representation -: $\xrightarrow{I} \rightarrow V$

phase diff (ϕ) = 0

Same line voltage & current

In purely resistive, AC circuits, voltage and current are in same phase.

$\xrightarrow{I} \rightarrow V$

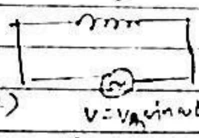
Power = $VI\cos\phi$ → Power factor $= \frac{1}{\sqrt{2}}$

$\boxed{P = VI}$ (I_{rms}, V_{rms})

H.W.R	$I_{rms} = \frac{I_m}{2}$
F.W.R	$I_{rms} = \frac{I_m}{\sqrt{2}}$
	$V_{rms} = \frac{V_m}{2}$
	$V_{rms} = \frac{I_m}{\sqrt{2}}$

Purely Inductive circuits -:

$I = \frac{V}{X_L}$, $X_L = j\omega L$ (Ω)



'Inductive reactance'

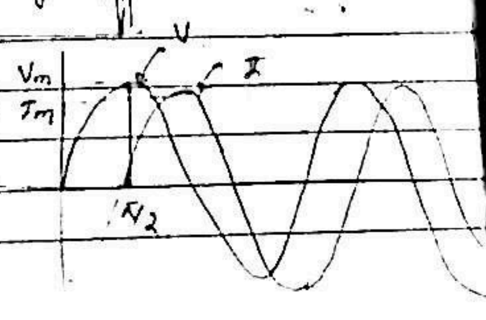
Unit -: (Ω)

Ex: $I = \frac{V}{X_L} = \frac{V_m \sin\omega t}{|j\omega L|} = \frac{V_m \times \angle 0^\circ}{\omega L \times \angle \pi/2} = \frac{V_m}{\omega L}\angle -\pi/2$

(Trigno form) $I = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$

* Current lag w.r.t. voltage by $\pi/2$

→ Wave form representation -



$V = \sqrt{(V_R)^2 + (V_L)^2}$

$I = \frac{V}{R + j\omega L}$; $V = IR + jI\omega L$

$V = (V_R) + j V_L$

$|V| = \sqrt{(V_R)^2 + (V_L)^2}$

5) <u>R-C series circuit</u> :-

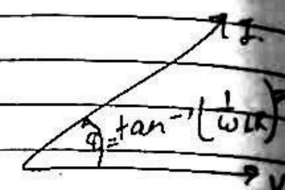
$I = \frac{V}{Z}$; $Z = R - \frac{j}{\omega C}$; $C - X_C = \frac{1}{\omega C}$; $V = V_m \sin \omega t$

$I = \frac{V_m \angle 0^\circ}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{1}{\omega C R}\right)}$

$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\angle Z = -\tan^{-1}\left(\frac{1}{\omega C R}\right)$; $= V_m \angle 0^\circ$

$I = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \angle \tan^{-1}\left(\frac{1}{\omega C R}\right)$.

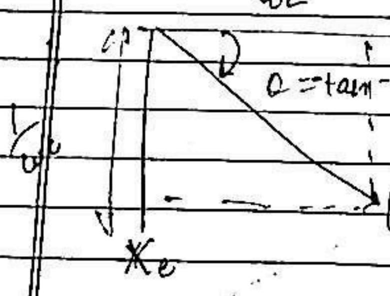


$\Rightarrow$ I lead w.r.t. ~~V~~ to voltage by angle $\tan^{-1}\left(\frac{1}{\omega C R}\right)$

$\cos\phi = \frac{R}{Z}$

$= \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$ (lead).

$Z = R - \frac{j}{\omega C}$ (Impedance Triangle)

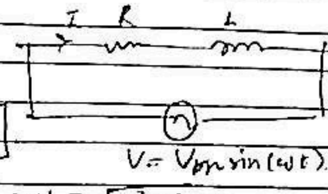


$\theta = \tan^{-1}\left(\frac{1}{\omega C R}\right)$.

$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$.

In AC circuit, V and I cannot be alone represented by magnitude; its showed in phase or complex form.

1) R-L series circuit:



$I = \frac{V}{(R + X_L)}$

$\left[\because Z = R + j\omega L \right]$

$V = V_m \sin(\omega t)$

$I = \frac{V}{Z}$

$|Z| = \sqrt{R^2 + (\omega L)^2}$

$\angle Z = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$I = \frac{V_m \sin \omega t}{\sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$

$\left[\because \text{let } \frac{\omega L}{R} = \theta \right]$

~~$I = V_m \sin \omega t$~~

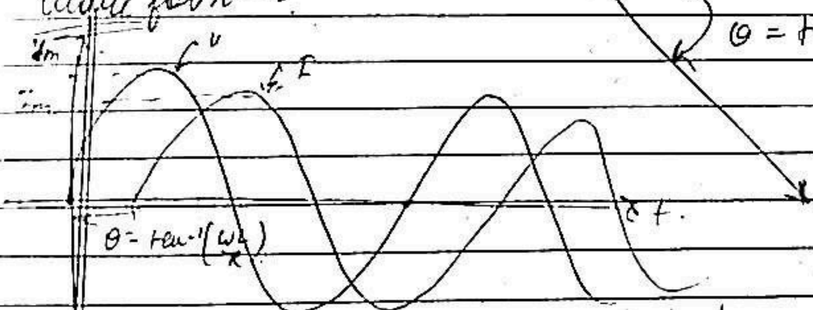
$I = \frac{V_m \angle 0^\circ}{\sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$

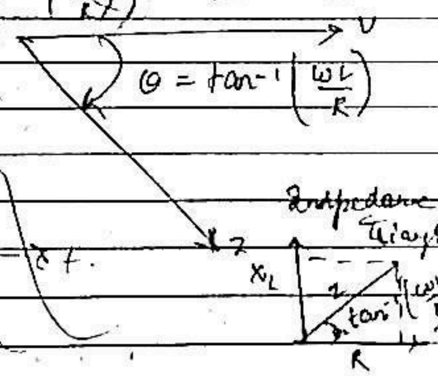
$-\tan^{-1}\left(\frac{\omega L}{R}\right)$

$I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$ (phasor representation)

$(I = I_m \sin(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right))$

Wave form:





In RL series AC circuits, current lag w.r.t. voltage by angle $\tan^{-1}\left(\frac{\omega L}{R}\right)$.

$\tan^{-1}$.

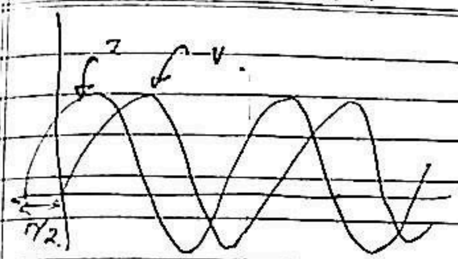
Power $= VI\cos\phi$.

$= VI\cos\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$

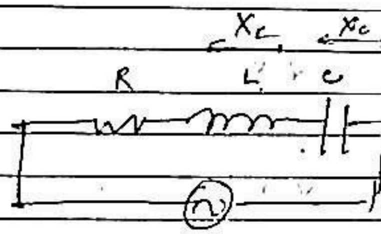
$$\boxed{P = \frac{VI \times R}{\sqrt{R^2 + (\omega L)^2}}}$$

$V = \sqrt{(V_R)^2 + (V_L)^2}$



(6) $I = \frac{V}{Z}$

$= \frac{V_m \angle 0^\circ}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$



$V = V_m \sin \omega t$

$V = V_m \angle 0^\circ$

$I = \frac{V_m \angle 0^\circ}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$

$Z = R + j\omega L + \frac{1}{j\omega C}$

$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$I = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}, \angle -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

$= \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}, \angle -\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

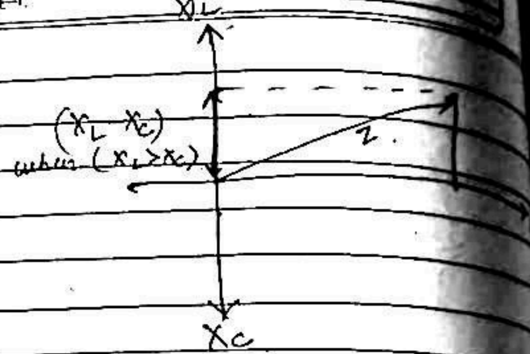
Case-1 :- Purely resistive circuit. (Phase diff. = 0).
where $(X_L = X_C)$

Case-2 :- ~~Purely~~ Inductance circuit. (I lag
where $(X_L > X_C)$. Phase diff. $= -\tan^{-1}\left(\frac{|X_L - X_C|}{R}\right)$

Case-3 :- Capacitance circuit (I lead
where $(X_L > X_C)$. Phase diff $= \tan^{-1}\left(\frac{|X_L - X_C|}{R}\right)$

Impedance Triangle!

Waveform ~~Format~~



'Phasor Diagram'