H.W.

① If $\cos^{-1}\left(\frac{y}{b}\right) = \log_e\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + 2n^2 y_n = 0$$

② If $x = \sin\left(\frac{\log_e y}{a}\right)$, prove that

$$(1-x^2) y_{n+2} - (2n+1)x y_{n+1} - (n^2+a^2) y_n = 0$$

③ If $y = (\sin^{-1}x)^2$, prove that

(i) $(1-x^2)y_2 - xy_1 - 2 = 0$

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

④ Find y_n

ⓐ $y = \dfrac{1}{(x+2)(x-1)}$ Ans. $\dfrac{(-1)^n n!}{3}\left[\dfrac{1}{(x-1)^{n+1}} - \dfrac{1}{(x+2)^{n+1}}\right]$

ⓑ $y = \dfrac{x^2}{(x+2)(2x+3)}$ Ans. $\dfrac{(-1)^n n!}{2}\left[\dfrac{9 \cdot 2^n}{(2x+3)^{n+1}} - \dfrac{8}{(x+2)^{n+1}}\right]$

ⓒ $y = \dfrac{5x+12}{x^2+5x+6}$ Ans. $(-1)^n n!\left[\dfrac{2}{(x+2)^{n+1}} + \dfrac{3}{(x+3)^{n+1}}\right]$