

## Derivation for the Rate constant (K) for Second order Reaction.



$$a \qquad \qquad \qquad 0 \qquad \qquad \qquad t=0$$

$$(a-x) \qquad \qquad \qquad x \qquad \qquad \qquad t=t$$

$$\frac{dx}{dt} \propto [a-x]^2$$

$$\frac{dx}{dt} = k[a-x]^2$$

$$\frac{dx}{[a-x]^2} = k dt$$

on Integration

$$\frac{1}{(a-x)} = kt + c \quad \text{--- (1)}$$

$$\text{When } t=0, x=0 \rightarrow c = \frac{1}{a}$$

on putting the value of  $c$  in eq. (1)

$$\frac{1}{(a-x)} - \frac{1}{a} = kt$$

$$\frac{a-(a-x)}{a(a-x)} = kt$$

$$k = \frac{1}{t} \cdot \frac{x}{(a-x)a}$$

For Half life :-

$$t = t/2, x = \frac{a}{2}$$

$$k = \frac{1}{t_{1/2}} \cdot \frac{a}{2(a - \frac{a}{2})}$$

$$\boxed{t_{1/2} = \frac{1}{ka}}$$

Q The decomposition of a gas is of second order. When the initial concn. of the gas is  $5 \times 10^{-4}$  moles/litre and it is 40% decompose in 50 minutes. Find the Rate constant

$$a = 5 \times 10^{-4}$$

$$x = \frac{40}{100} \times 5 \times 10^{-4} = 2 \times 10^{-4}$$

$$(a - x) = 3 \times 10^{-4}$$

$$k = \frac{1}{50} \times \frac{2 \times 10^{-4}}{(3 \times 10^{-4}) \cdot 5 \times 10^{-4}}$$

$$k = \frac{1}{50} \times \frac{2 \times 10^4}{15} = 26.66$$

Q A second order rxn is 20% complete in 500 seconds. How long will it take for 60% completion of rxn.  
Let  $a = 100$

$$x = \frac{20}{100} \times 100 = 20$$

$$(a - x) = 80$$

$$k = \frac{1}{500} \times \frac{100 \times 100}{40} = \frac{1}{2} \times 10^{-5}$$



For 60% completion

$$a - x = 100 \times \frac{60}{100} = 60$$

$$a - x = 100 - 60 = 40$$

$$k = \frac{1}{t} \times \frac{ax}{(a-x)a} = \frac{1}{t} \times \frac{10^5}{x} = \frac{1}{t} \times \frac{10^5}{40} = \frac{2500}{t}$$

$$t = 0.3 \times 10^4 \text{ sec}$$

Q In a second order of rxn the initial concn. of reactant is given as 0.24 mole/L if the rxn is 30% complete in 60 min. Calculate.

i) Rate constant

ii) Half life

iii) Time req to comp. 75% of rxn.

$$i) a = 0.24$$

$$x = 0.24 \times \frac{30}{100} = 0.072$$

$$a - x = 0.168$$

$$k = \frac{1}{60} \times \frac{0.072}{0.168 \times 0.24} = 29.0 \times 10^{-2}$$

$$ii) T_{1/2} = \frac{1}{ka} = \frac{1}{29.0 \times 10^{-2} \times 0.24} = 13.90 \text{ min}$$

$$iii) x = 0.24 \times \frac{75}{100} = 0.18$$

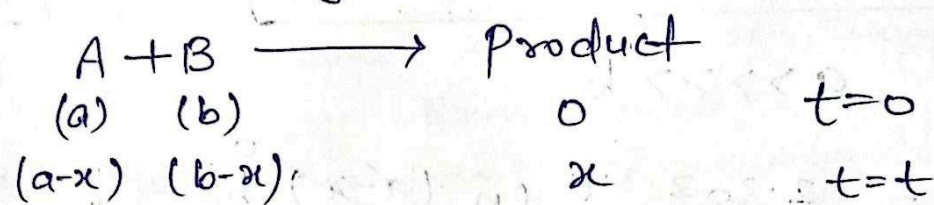
$$a - x = 0.24 - 0.18 = 0.06$$

$$k = 29.0 \times 10^{-2} = \frac{1}{t} \times \frac{0.18}{0.06 \times 0.24}$$

$$t = 42.8 \text{ min}$$

## Derivation for the Rate Constant (K) for Second order Rxn:-

Case II  $\rightarrow$  When both the Reactants are of Different types.



$$\frac{dx}{dt} \propto [a-x][b-x]$$

$$\frac{dx}{dt} = k[a-x][b-x]$$

$$\frac{dx}{[a-x][b-x]} = k dt \quad \text{--- (1)}$$

on splitting it in partial fraction

$$\frac{1}{(a-b)} \left\{ \frac{dx}{b-x} - \frac{dx}{a-x} \right\} = k dt$$

on Integration:-

$$\frac{1}{(a-b)} \left\{ -\log_e(b-x) + \log_e(a-x) \right\} = kt + c \quad \text{--- (2)}$$

$$\text{When } t=0, x=0, \quad c = \frac{1}{a-b} \cdot \ln \frac{a}{b}$$

on putting the value of (c) in eq (2),

$$\frac{1}{a-b} \left\{ \ln \left( \frac{a-x}{b-x} \right) \right\} = kt + \frac{1}{(a-b)} \cdot \ln \frac{a}{b}$$



$$k = \frac{1}{t(a-b)} \left\{ \ln \frac{(a-x)b}{(b-x)a} \right\}$$

$$k = \frac{2.303}{t(a-b)} \left\{ \ln \frac{(a-x) \cdot b}{(b-x) \cdot a} \right\} \rightarrow \text{Second order rxn}$$

$$a \gg \gg \gg b$$

$$k = \frac{2.303}{t(a-b)} \left\{ \ln \frac{(a-x) \cdot b}{(b-x) \cdot a} \right\}$$

$\swarrow$  (very big)

$$k = \frac{2.303}{t} \ln \frac{b}{b-x} \rightarrow \text{First order rxn}$$

### Collision Theory for Reaction Rates:-

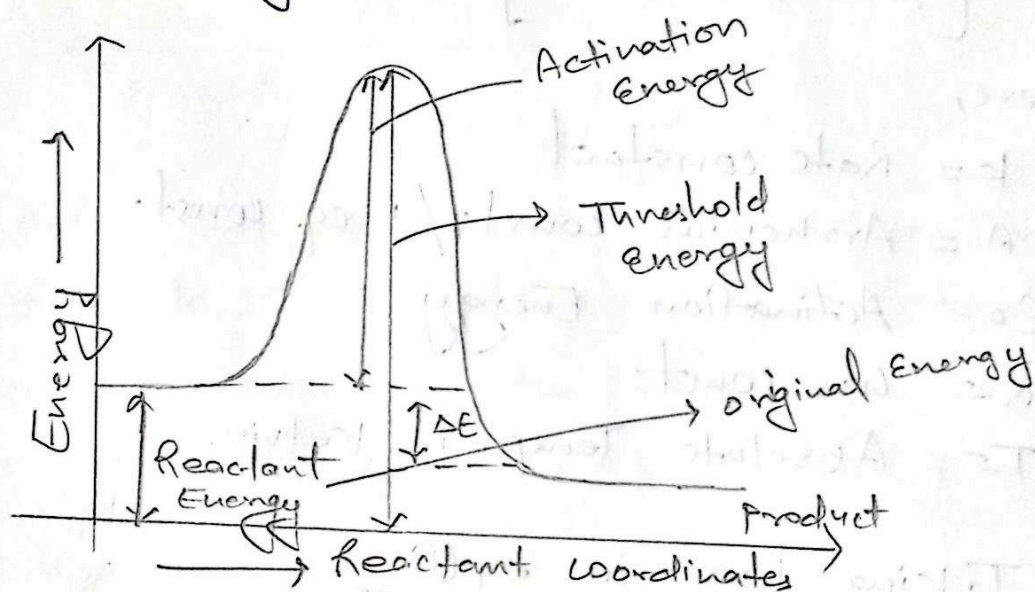
This theory is based on following points.

- i) A chemical rxn occurs only as a result of collision b/w the reactant species.
- ii) All the reactant molecules that collide do not necessarily results the product formation.
- iii) Only those reactant molecules that brings the product formation, their collisions are termed as effective collision.
- iv) The total no. of collisions which occurs among the reacting species / second per unit volume is called as collision frequency (Z).

Two types of Energy barriers are considered imp. for a rxn to occur:-

- i) Energy barrier
- ii) Orientation barrier

i) Energy Barrier :- For any reactant molecule to take part in a chemical rxn it must possess a minimum amount of energy to cross the Energy Barrier and changes to product. This min. amt of energy required to cross the Energy Barrier is called Threshold Energy. And the Extra Energy which the reactant molecules acquire or gain during the collision to ~~cross~~ reach the level of min. required energy is called Activation Energy.



The original Energy which is the reactant molecules possess in the form of K.E is called reactant energy.

### Orientation Barrier

Besides the energy requirement it is also imp. for a reactant molecule that it must collide with



a proper orientation.

A reactant molecules having energy higher than the req. energy, but if it does not collide with proper orientation it will not cross the energy barrier and will bounce back

### Arrhenius eq<sup>n</sup> and Activation Energy concept

Arrhenius give a mathematical relation b/w the rate constant and temperature. This temp. dependent rate const eq<sup>n</sup>. as can be expressed as:-

$$[k = A e^{-E_a/RT}] \quad \text{--- ①}$$

where,

$k$  = Rate constant

$A$  = Arrhenius const / Freq. const.

$E_a$  = Activation Energy

$R$  = Gas const

$T$  = Absolute Temp. in Kelvin.

Taking log in eq ①

$$\left[ \log k = \log A - \frac{E_a}{2.303R} \cdot \frac{1}{T} \right]$$

From the above eq<sup>n</sup> it is clear that as the value of  $E_a$  increase Rate constant / R.O.R decreases and similarly for if the Temp inc, Rate constant Increase.



we can easily calculate the value of Activation Energy for a particular rxn by determining the rate constant values of that rxn at 2 diff Temp.

If  $k_1$  and  $k_2$  are the 2 rate constant for a rxn at 2 diff temp.  $\propto$  of  $T_1$  and  $T_2$

$$\log k_1 = \log A - \frac{E_a}{2.303R} \cdot \frac{1}{T_1} \quad \text{--- (1)}$$

$$\log k_2 = \log A - \frac{E_a}{2.303R} \cdot \frac{1}{T_2} \quad \text{--- (2)}$$

on subtracting eq (1) and eq (2)

$$\left[ \log \left( \frac{k_2}{k_1} \right) = \frac{E_a}{2.303R} \cdot \frac{1}{T_1} - \frac{1}{T_2} \right]$$

Q The R.O.R. Triples on increasing the Temp. from  $17^\circ\text{C}$  to  $57^\circ\text{C}$ . Calculate the Activation Energy of the rxn if the value of Gas const is  $8.314 \text{ J/Kmol}$ .

$$\log \left( \frac{3K}{1K} \right) = \frac{E_a}{2.303 (8.314)} \times \left( \frac{1}{17} - \frac{1}{357} \right)$$

$$0.477 = \frac{E_a}{19.147} \left( \frac{40}{969} \right)$$

$$E_a = \underline{221.24 \text{ cal}}$$