(4)

H.W.

① Using the transformation $x+y=u$, $y=uv$, prove that $\iint xy(1-x-y)^{1/2}\,dx\,dy = \frac{2\pi}{105}$ taken over the area of triangle bounded by the lines $x=0, y=0, x+y=1$.

② Evaluate the integral $\iint_R (x-y)^2 \cos^2(x+y)\,dx\,dy$, where R is the rhombus with vertices at $(\pi,0)$, $(2\pi,\pi)$, $(\pi,2\pi)$ and $(0,\pi)$. Ans. $\frac{\pi^4}{3}$

③ Using the transformation ~~$x+y=u$, $y=uv$~~, $u=x-y$, $v=x+y$, prove that $\iint_R \cos\left(\frac{x-y}{x+y}\right)dx\,dy = \frac{1}{2}\sin 1$, where R is bounded by $x=0, y=0$, $x+y=1$.

④ Evaluate $\iint_D (y-x)\,dx\,dy$ where D is the region in $xy$-plane bounded by the straight lines $y=x+1$, $y=x-3$, $y=-\frac{x}{3}+\frac{7}{3}$, $y=-\frac{x}{3}+5$.