

Date - / - / -

Basic Functional Unit of a Computer

Von Neumann Architecture

In this stored-program concept, programs and data are stored in a separate storage unit called memories and are treated the same.

OR

Defines the programs & operands in same location.

Master dc generator
involves in
Control Unit

CPU

Input
Devices

Control Units

ALU

Output
Devices

Memory
Unit

It is also known as ISA (Instruction set architecture) computer and is having three basic units:

1. The Central Processing Unit (CPU)
2. The Main Memory Unit
3. The Input/Output Device.

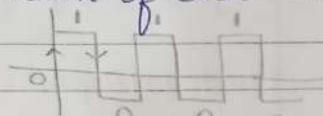
1. Central Processing Unit-

CPU is defined as the it is an electric circuit used for the executing the instruction of computer program.

- It has following major components :
1. Control Unit (CU)
 2. Arithmetic and Logic Unit (ALU)
 3. Variety of Registers.

Control Unit 11 bits generates 2^{11} Location/Address

1. The CU defines the order of the execution of instruction.
2. It also generates the no. of timing signals (clock waveform) that are synchronized with execution of each pulses and every microoperation.



Arithmetic & Logic Unit (ALU)

ALU is a part of the CPU that handles all the calculations the CPU may need, e.g. Addition, Subtraction, Comparisons, It performs Logical Operations, Bit Shifting Operations and Arithmetic operations.

Register - Set of flip flop

Refer to high-speed storage areas in the CPU. The data processed by the CPU are fetched from the registers. There are diff. types of registers used in architecture.

IR

PC (Program Counter)

INPR

MAR (Memory Address Register)

MDR (Memory Data Register)

OUTR (

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1 Mark Signed Number Representation

- Sign-Magnitude
- 1's Complement
- 2's Complement

tve no. are unsigned
-ve no. are signed

{ tve → '0'
-ve → '1' }

→ Sign-Magnitude

Binary of :

MSD Binary of 5

-5 → 10000101

Sign Magnitude.

-15 → 10001111

S M

8-bit format → $2^8 = 256$ Range → -127 to 127 for 8-bit

Decimal	Binary	Sign-Magnitude interpretation
0	00000000	+0
1	00000001	+1
2		
3		
⋮		
127	01111111	+127
128	10000000	-0
129	10000001	-1
⋮		
256	11111111	-127

Generalised Range of Signed Magnitude

$$-2^{n-1} \text{ to } +2^{n-1} - 1$$

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→ 1's Complement Representation

-5

$$+5 \rightarrow 00000101$$

1's Comp. ↓

Binary of -5 → 11111010

1's Complement Interpretation

Decimal	Binary	1's Complement Representation
0	00000000	+0
1	00000001	+1
2	00000010	+2
3	00000011	+3
⋮	⋮	⋮
127	01111111	+127
128	10000000	-127
129	10000001	-126
254	11111110	⋮
255	11111111	-0

Range :- $[-(2^{n-1}-1) \text{ to } +(2^{n-1}-1)]$

→ 2's Complement Representation

-5

$$+5 \rightarrow 00000101$$

1's ↓ Take 1's Comp.

$$-5 \rightarrow 11111010$$

$$\begin{array}{r} + \\ \hline 1 \end{array}$$

Binary of -5, 2's Comp. → 11111011

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Decimal	Binary	2's Comp. Representation
0	00000000	+0
1	00000001	+1
2		+2
3		+3
:		:
127	01111111	+127
128	10000000	-128
129	10000001 ↑ 1 1 1 1 1 1 + 1 1 1 1 1 1 ----- 10000000 → +128	-127
:		:
254		
255	11111111	-1

Range: -128 to +127

Generalised: - 2^{n-1} to $+2^{n-1} - 1$
Range

Signed number Addition & Subtraction
in (2's Complement) form (Default)

Binary Addition

Rules :-

1. $(+5) + (+7)$
 2. $(+5) + (-7)$
 3. $(-5) + (+7)$
 4. $(-5) + (-7)$
1. Determine the binary of the given no.
 2. Perform the addition, if there is a carry out then discard it.
 3. If the result is -ve then determine the 2's Complement of the result except the sign bit.

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Ex-1

Sol.

$$\begin{array}{r} (+5) + (+7) \\ +5 \rightarrow 00000101 \\ +7 \rightarrow 00000111 \\ \hline 00001100 \end{array}$$

$$\begin{array}{l} 0+0=0 \\ 0+1=1 \\ 1+0=1 \\ 1+1=0 \text{ Carry=1} \\ 1+1+1=1 \text{ Carry=1} \end{array}$$

Ex-2 $(+5) + (-7)$

$$\begin{array}{r} +5 \rightarrow 00000101 \\ -7 \rightarrow 11111001 \\ \hline 11111110 \end{array}$$

$$\begin{array}{r} +7 \rightarrow 00000111 \\ \downarrow \\ -7 \rightarrow 11111000 \\ + 1 \\ \hline 11111001 \end{array}$$

\therefore Result is (-ve), take the 2's Comp.

$$\begin{array}{r} - (00000010) \\ \downarrow \quad \swarrow \\ \text{y-bit} \quad \text{7-bit} \end{array}$$

Ex-3 $(-5) + (+7)_{111}$

$$\begin{array}{r} -5 \rightarrow 11111011 \\ +7 \rightarrow 00000111 \\ \hline 100000010 \end{array}$$

$$\begin{array}{r} +5 \rightarrow 00000101 \\ \downarrow \\ -5 \rightarrow 11111010 \\ + 1 \\ \hline 11111011 \end{array}$$

Ex-4 $(-5) + (-7)_{111}$

$$\begin{array}{r} -5 \rightarrow 11111011 \\ -7 \rightarrow 11111001 \\ \hline 11110100 \end{array}$$

$$\Rightarrow - (00001100)$$

$$\begin{array}{r} 11110100 \\ 00001011 \\ + 1 \\ \hline 00001100 \end{array}$$

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Binary Subtraction

Rules :-

A → Minuend
-B → Subtrahend

1. Determine the binary of the given two no.s.
2. Take 2's complement of Subtrahend & add it to the Minuend.
3. Now, Apply the Rules of Binary Addition.

Ex-1 $(+5) - (+7)$

① $+5 \rightarrow 00000101$

$+7 \rightarrow 00000111 \quad +7-$

② Taking 2's complement of subtrahend.

$$\begin{array}{r} 00000111 \\ \xrightarrow{\text{I's}} 11111000 \\ + 1 \\ \hline 11111001 \end{array}$$

③ Add it to the minuend

$$\begin{array}{r} 00000101 \\ + 11111001 \\ \hline 11111110 \xrightarrow{-2} \end{array}$$

$\Rightarrow - (0000010) \text{ Ans}$

$$\begin{array}{r} 111110 \\ \downarrow \text{I's} \\ 00000101 \\ + 1 \\ \hline 0000010 \end{array}$$

Ex-2 $(+5) + (-7)$

$+5 \rightarrow 00000101$

$-7 \rightarrow 11111001$

Taking 2's complement of subtrahend

$$\begin{array}{r} 11111001 \\ \xrightarrow{\text{I's}} 00000110 \\ + 1 \\ \hline 00000111 \end{array}$$

Add it to the minuend

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$$\begin{array}{r}
 \text{111} \\
 00000101 \\
 + 00000111 \\
 \hline
 00001100 \quad \text{Ans}
 \end{array}$$

Ex-3 $(-5) - (+7)$

$$\begin{array}{l}
 -5 \rightarrow 11111011 \\
 +7 \rightarrow 00000111
 \end{array}$$

2's comp. of Subtrahend

$$\begin{array}{r}
 00000111 \quad \text{1's} \\
 + 1 \\
 \hline
 11111001
 \end{array}$$

Add it to the minuend.

$$\begin{array}{r}
 00001100 \\
 + 11111001 \\
 \hline
 100000101
 \end{array}$$

$$\begin{array}{r}
 111111 \\
 11111011 \\
 + 11111001 \\
 \hline
 111110100
 \end{array}$$

$$\begin{array}{r}
 -(00001100) \\
 \cancel{+ 11111001} \\
 \hline
 -12
 \end{array}$$

$$\begin{array}{r}
 00000101 \\
 + 1 \\
 \hline
 11111010
 \end{array}$$

$$\begin{array}{r}
 111110100 \\
 \cancel{+ 00001011} \\
 \hline
 00001100
 \end{array}$$

Ex-3 $(-5) - (-7)$

$$\begin{array}{l}
 -5 \rightarrow 11111011 \\
 -7 \rightarrow 11111001
 \end{array}$$

2's comp. of subtrahend

$$\begin{array}{r}
 11111001 \quad \text{1's} \\
 + 1 \\
 \hline
 00000111
 \end{array}$$

Add it to the minuend

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$$\begin{array}{r}
 111111 \\
 + 00000111 \\
 \hline
 \cancel{100000010}
 \end{array}
 \quad
 \begin{array}{r}
 00000010 \\
 + 11111101 \\
 \hline
 \cancel{11111110}
 \end{array}$$

Overflow Condition

① $(+6) + (+5)$

$$\begin{array}{r}
 \text{Cout} \\
 +6 \rightarrow 10 \\
 +5 \rightarrow 101 \\
 \hline
 101
 \end{array}$$

{IC XOR - 7486}

↪ this 1 shows overflow cond

$$\begin{array}{r}
 \text{Cin} = 0 \\
 \text{Cout} = 1
 \end{array}$$

$$O = C_{in} \oplus C_{out}$$

② ~~Don't~~ $(-6) + (-5)$

$$\begin{array}{r}
 \text{Cout} \\
 -6 \rightarrow 010 \\
 -5 \rightarrow 111 \\
 \hline
 1101
 \end{array}
 \quad
 \begin{array}{r}
 (+6) + (+7) \\
 +6 \rightarrow 110 \\
 +7 \rightarrow 111 \\
 \hline
 1101
 \end{array}
 \quad
 \begin{array}{r}
 110 \\
 +1 \\
 \hline
 010
 \end{array}$$

↪ 3-bit

$$C_{in} = 1$$

$$C_{out} = 1$$

In this case, overflow cond doesn't occur.

③ $+6 \rightarrow 0110$ → 4-bit

$$\begin{array}{r}
 \text{Cout} \\
 +7 \rightarrow 0111 \\
 \hline
 1101
 \end{array}$$

$$\begin{array}{r}
 \text{Cin} = 1 \\
 \text{Cout} = 0
 \end{array}
 \quad
 \{ \text{overflow cond} \}$$

4. $-6 + (-7)$

$$\begin{array}{r} -6 \\ -7 \\ \hline 010 \\ 001 \\ \hline 011 \\ \text{Cin} = 0 \quad \text{if not overflow flow} \\ \text{Cout} = 0 \end{array}$$

5. $(+2) + (+127)111$

$$\begin{array}{r} +2 \\ +127 \\ \hline 00000010 \\ 01111111 \\ \hline 10000001 \\ \text{Cin} = 1 \quad \text{if overflow cond} \\ \text{Cout} = 0 \end{array}$$

Q1. $(-2) + (-127)$
 Q2. $(+2) + (+127)$
 Q3. $(-60) + (-80)$

$$\begin{array}{r} -60 \\ -80 \\ \hline 110000100 \\ 10110000 \\ \hline 101110100 \\ +60 \rightarrow 00011100 \\ \hline 11000100 \\ \text{Cin} = 1 \\ \text{Cout} = 0 \end{array}$$

Rough $\begin{array}{r} -60 \\ -80 \\ \hline 110000100 \\ 10110000 \\ \hline 101110100 \\ +80 \rightarrow 01010000 \\ \hline 00011000 \\ \text{Carryout exist, so discard it} \\ \text{The Result is, } -(0001100) \end{array}$

$\begin{array}{r} 11000100 \\ 101110000 \\ \hline 001110100 \\ \text{Both the bits are diff. or} \\ \text{Cin & Cout are diff.} \\ \text{So, the cond' of overflow} \\ \text{exist.} \end{array}$

$$\begin{array}{r} -60 \\ -80 \\ \hline 11000100 \\ 10111000 \\ \hline 00011000 \\ \text{in 2's complement} \\ \hline 100001100 \end{array}$$

Practice Sheet 1

1. Considering an 8-bit processor, perform the following arithmetic operations & state whether the overflow cond" exist or not.

$$\begin{array}{r}
 \text{Overflow} \\
 (-64) + (-79) = 01101111 \\
 -64 \rightarrow 11000000 \quad 1's \rightarrow 11111111 \\
 -79 \rightarrow 10110001 \qquad \qquad \qquad \underline{+} \qquad \qquad \qquad \\
 \hline
 11000000 \qquad \qquad \qquad 11000000 \\
 + 10110001 \\
 \hline
 001110001 \qquad \qquad \qquad 79 \rightarrow 01001111
 \end{array}$$

Cannot exist, so discard it.
The result is - (000111)

$Cin = L$, $Cout = 0$, $Cin \& Cout$ are diff.
 So, Overflow cond. exist. $\frac{1110001}{1'st} = \underline{0001110} + 1 = 0001111$

$$\begin{array}{r} 125 + 2 \\ \hline 127 \end{array}$$

$$\begin{array}{r}
 0.111110 \\
 + 0000010 \\
 \hline
 0111111
 \end{array}
 \quad \text{Cin & Cout are 0} \\
 \quad \text{: Overflow cond* doesn't exist.}$$

C_{in} & C_{out} are 0
∴ Overflow cond

overflow cond* doesn't exist.

c) $111 - 126$ no overflow 1000111
 $111 \rightarrow 0110111$
 $126 \rightarrow 0111110$

$$\begin{array}{r} \text{Taking 2's complement of subtrahend} \\ 01111110 \xrightarrow{\text{is}} 10000001 \\ + \\ \hline 10000001 \end{array}$$

Add it to the minuend

$G_{in} = 0$ $G_{out} = 0$	$\begin{array}{r} 0110111 \\ + 10000010 \\ \hline 11110001 \end{array}$	$\begin{array}{r} 00001110 \\ - 1111 \\ \hline 111100 \end{array}$
		$\begin{array}{r} + \\ 111100 \\ \hline 11110010 \end{array}$

$$C_{in} = 0 \quad \text{by no overflow}$$

November

$(-100) - (-100)$	$100 \rightarrow 011\ 001\ 000$
$-100 \rightarrow 1\ 001\ 11\ 00$	$\begin{matrix} 1 & 1 \\ 0 & 1 \\ \hline 1 & 1 \end{matrix}$
$-100 \rightarrow 1\ 001\ 11\ 00$	$1\ 001\ 10\ 11$
Takes 2's comp. of subtrahend,	$\overline{1\ 001\ 10\ 11}$
$100111\ 00$	$100111\ 00$
$\begin{matrix} & 1 \\ & + \\ 1 & 00111\ 00 \\ \hline & 1 \end{matrix}$	$100111\ 00$

01100100
1111
→ 10011100
01100100

$C_{in} = 1$ } overflow cond
 $C_{out} = 0$

e) $100 + 100$ Date: 1/1/1
~~Overflow~~ 11001000
 $100 - 01100100$
 $100 - 01100100$
 11001000

Cin =
Cout =

f) $75 - (-55)$ Date: 1/1/1
~~Overflow~~ 01101010
 $75 \rightarrow 01001011$
 $-55 \rightarrow 11001001$ 1's complement of subtrahend
 $\begin{array}{r} 01001011 \\ 00110111 \\ \hline 10000010 \end{array}$
 10000010

Cin =
Cout =

g) $(-56) + 65$ Date: 1/1/1
~~No Overflow~~ 00001001
 $-56 \rightarrow 11001000$
 $65 \rightarrow 01000001$
 100001001

Cin =
Cout =

h) $127 + (-128)$ Date: 1/1/1
~~No Overflow~~ 10000001
 $127 \rightarrow 0111111$
 $128 \rightarrow 10000000$
Take 9's comp. of subtrahend
 $\begin{array}{r} 10000000 \\ 0111111 \\ \hline 10000000 \end{array}$
 0111111
 $+ 10000000$
 1111111

Cin =
Cout =

i) $(-60) + (-80)$ Date: 1/1/1
 $-60 \rightarrow 11000100$
 $-80 \rightarrow 10110000$
 101110100

Cin =
Cout =

$80 \rightarrow 01010000$
 10101111
 10110000

2. Considering a 4-bit processor, perform the following arithmetic operations & state whether the overflow condition exists or not. (-5 to +7)

$$\begin{array}{r}
 a) \quad 6+7 \quad \text{Over}. \quad 1101 \\
 6 - 0 \ 1 \ 1 \ 0 \\
 7 - 0 \ 1 \ 1 \ 1 \\
 \hline
 \quad \quad \quad 1 \ 1 \ 0 \ 1
 \end{array}$$

Cin =

Cout =

$$\begin{array}{r}
 \text{No. of events, } 1110 \\
 (-7) - (-1) \\
 -7 \rightarrow 1001 \\
 -1 \rightarrow 1111 \\
 \hline
 \text{Two's comp. of } -1 \\
 1111 \xrightarrow{\text{1's comp.}} 0000 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 7 \rightarrow 0111 \\
 1'st \\
 \hline
 1000 \\
 + 1 \\
 \hline
 1001 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 7 \rightarrow 0001 \\
 1'st \\
 \hline
 1110 \\
 + 1 \\
 \hline
 1111 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 4 - (-5) \quad \text{out}; 100 \\
 4 \rightarrow 0100 \\
 -5 \rightarrow 1011 \\
 \hline
 \text{Now, } C_{\text{out}} = 2's \text{ of } -5 \\
 \text{out} = 0100 \\
 \hline
 \end{array}$$

PASSION

d) $(-1) + (-8)$

$$\begin{array}{r} -1 \\ -8 \\ \hline 0111 \end{array}$$

$\rightarrow 0001$

Cin =

Cout =

e) $3 - 4 \rightarrow N = 0011, 1001$

$$\begin{array}{r} 3 \\ 4 \\ \hline 0100 \end{array}$$

$\rightarrow 1000$

Two's of 4

$$\begin{array}{r} 0100 \\ 11 \\ \hline 1011 \\ +1 \\ \hline 1100 \end{array}$$

$\Rightarrow 0011$

$$\begin{array}{r} 1100 \\ \hline 1111 \end{array}$$

Cin =

Cout =

Date - / - / -

Floating Point Operation

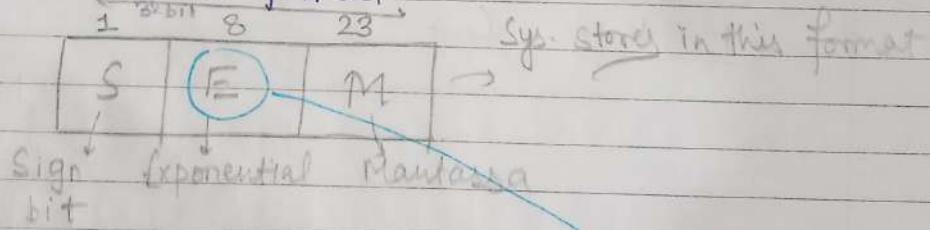
$1.1 \times 10^3 \rightarrow$ Scientific Notation
 $1.1E3 \rightarrow$ Programming Notation

Floating Point Operation $\xrightarrow{\text{represent}} \pm M \times B^E$
M → Mantissa
B → Base
E → Exponent

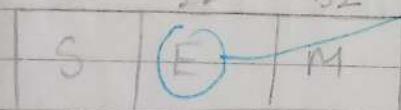
① IEEE 32-bit format :-
(Single Precision format)

② IEEE 64-bit format :-
(Double Precision format)

③ IEEE 32-bit format :-



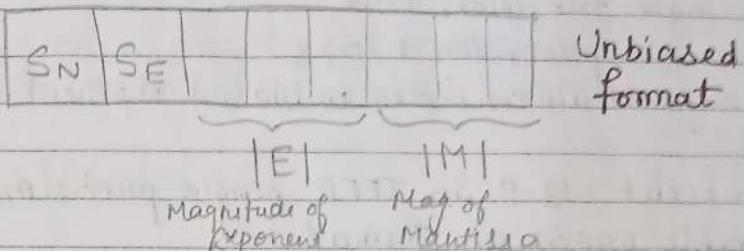
④ IEEE 64-bit format $\xrightarrow{64-bit}$



Biased Exponents
exponents which contain sign with it.

Date / /

→ 8-bit floating point representation with unbiased exponents :-



Ex - $-13.9 \rightarrow (1101.111001)_2$

Normalised Mantissa
 $-(1.101111001 \times 2^3)$

$13 \rightarrow 1101$
 $0.9 \times 2 \rightarrow 1.8$ 1
 $0.8 \times 2 \rightarrow 1.6$ 1
 $0.6 \times 2 \rightarrow 1.2$ 1
 $0.2 \times 2 \rightarrow 0.4$ 0
 $0.4 \times 2 \rightarrow 0.8$ 0

Unbiased

1	0	011	101	
0	1	0	1	

$\frac{4}{0 \rightarrow 2^4-1}$
 $0 \rightarrow 15$ Biased
 -7 -7
 $\underline{-7}$ $\underline{-7}$
 -7 to $+8$ unbiased
 $\frac{15}{2} = 7.5$
 floor of $(7.5) = 7$

$0 - 2^6-1$
 $0 - 255$, $\frac{255}{2} = 127.5$

Biased

$$-127 \downarrow +127$$

To get biased

1	1010	101
---	------	-----

Range of Unbiased: -127 to +127 Date: / /

- Conversion from unbiased to biased exponent.
 - In single precision +127
 - In double precision +1023
 - * By default we consider unbiased exponent
- Q. Represent -13.9 in IEEE single precision & double precision format:-

-13.9

1	8	23
S	E	M

$$-13.9 \rightarrow -(1101.111001)$$

(1.101111001 \times 2^3)

Unbiased exponent = 3
 \therefore Biased Exponent = $3 + 127 = 130$

1	10000010	101111001
1	11	32

In Double Precision

1	100000000010	101111001
1	11	32

\therefore Biased Exp. = $3 + 1023 = 1026$

* Before converting in the biased exponent, first check whether the unbiased exponent is in range or not.

Q. -0.3125
 $-0.3125 \rightarrow -(0.0101)_2$
 $= (0.01 \cdot 01 \times 2^{-2})$

Unbiased exponent = $-2 + 127 = 125$
Biased exponent = $-2 + 127 = 125$

1	11111010	01
1	8	23

2	125
2	62
2	31
2	15
2	7
2	3
1	1

In Double Precision.
Biased exponent = $-2 + 1023 = 1021$

2	1021
2	51

Combinational Logic

digital circuits
 ↳ Combinational logic
 ↳ Sequential logic

→ Half-Adder

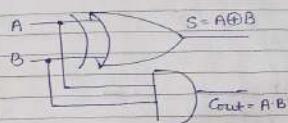
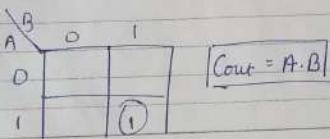
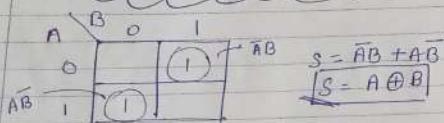
S-1 Design Truth-Table

Inputs		Outputs	
A	B	S	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

S-2 Computation of Boolean func with the help of K-map

(Note: The no. of O/p = no. of Kmap)

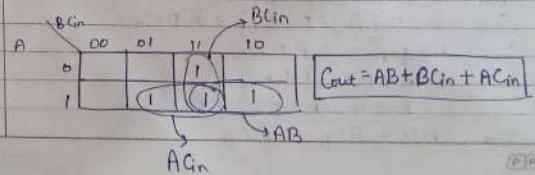
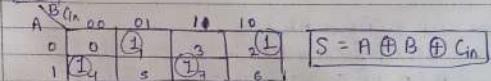
No. of I/P gives no. of variable

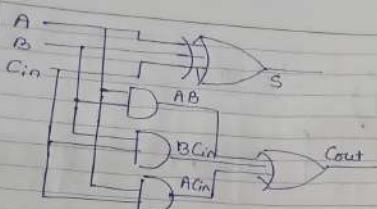


→ Full-Adder

Truth Table				
Input		Output		
A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

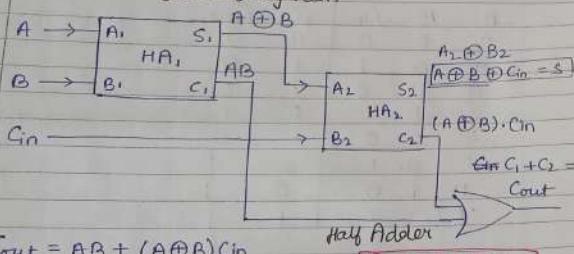
K-map





Implementation of full adder using two half adder.

Block Diagram



$$Cout = AB + (A \oplus B)Cin$$

$$S = A \oplus B \oplus Cin$$

$$Cout = AB + (\bar{A}B + A\bar{B})Cin$$

$$Cout = AB + \bar{A}\bar{B}Cin + A\bar{B}Cin$$

$$Cout = A(B + \bar{B}Cin) + \bar{A}(B\bar{Cin})$$

$$Cout = A(\bar{B} + \bar{B})(B + Cin) + \bar{A}B\bar{Cin}$$

$$Cout = AB + A\bar{Cin} + \bar{A}B\bar{Cin}$$

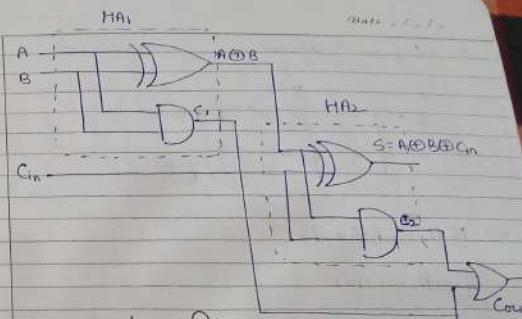
$$Cout = B(A + \bar{A}Cin) + ACin$$

$$= B(A + A)(A + Cin) + ACin$$

$$Cout = \bar{A}B + BCin + ACin$$

$$Cout = AB + BCin + ACin$$

To derive

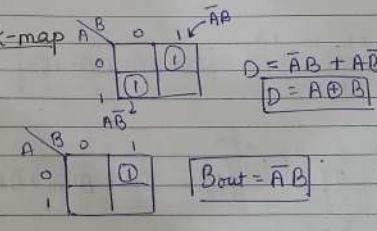


Logic Diagram

⇒ Half-Subtractor {For H's. can + subtract - that's why it's called H's }

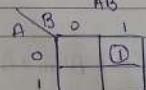
A	B	D	Bout
0	0	0	0
0	1	1	-1
1	0	1	1
1	1	0	0

② K-map



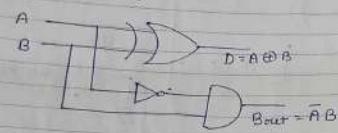
$$D = \bar{A}B + A\bar{B}$$

$$D = A \oplus B$$



$$Bout = \bar{A}B$$

③ Logical Diagram

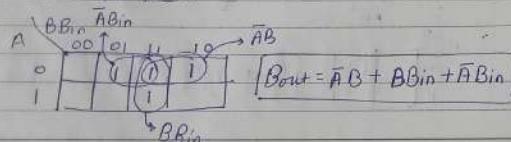


→ Full Subtractor

Truth-Table

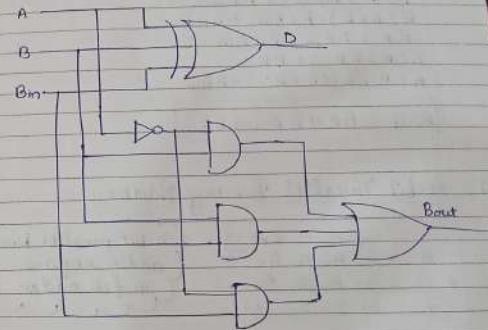
A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

K-map



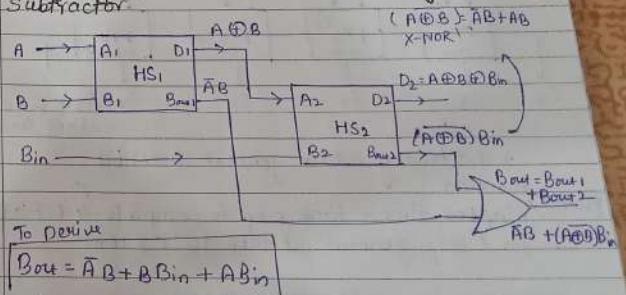
Date - 7-7-18

Logical Diagram



Date - 7-7-18

Designing of a full subtractor using two half subtractor



To Derive

$$B_{out} = \bar{A}B + B\bar{B}_{in} + A\bar{B}_{in}$$

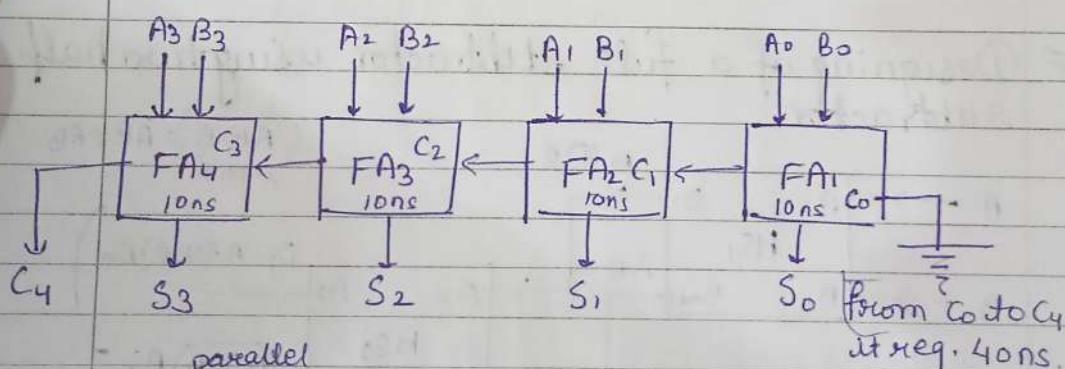
Date - / - / -

$$\begin{aligned}
 & \bar{A}B + (\bar{A}\bar{B} + AB) \text{ Bin} \\
 & \bar{A}B + \bar{A}\bar{B} \text{ Bin} + AB \text{ Bin} \\
 & B(\bar{A} + AB \text{ Bin}) + \bar{A}\bar{B} \text{ Bin} \\
 & \bar{B}(\bar{A} + A)(\bar{A} + B \text{ Bin}) + \bar{A}\bar{B} \text{ Bin} \\
 & \bar{B}A + BB \text{ Bin} + \bar{A}\bar{B} \text{ Bin} \\
 & \bar{A}(B + \bar{B} \text{ Bin}) + BB \text{ Bin} \\
 & \bar{A}(B + \bar{B})(B + B \text{ Bin}) + BB \text{ Bin}
 \end{aligned}$$

$$\boxed{B_{out} = \bar{A}B + \bar{A}B \text{ Bin} + BB \text{ Bin}}$$

4-bit Parallel Binary Adder

$$\begin{array}{rcl}
 & \begin{matrix} c_3 & c_2 & \bar{c}_1 & c_0 \end{matrix} & \downarrow \\
 A & = & A_3 & A_2 & A_1 & A_0 & \stackrel{\text{n-bit parallel binary}}{\equiv} \\
 + B & = & B_3 & B_2 & B_1 & B_0 & \left. \begin{array}{l} \text{adder require} \\ \text{n full adder} \end{array} \right\} \\
 \text{(S4)} & S_3 & S_2 & S_1 & S_0 & &
 \end{array}$$



n -bit binary adder generate $(n+1)$ bit.

propagation delay

elapsed time - time req. to complete 4-bit adder.

→ for 1 output generated, it take 10^{-9} s (10ns)

n -bit binary adder req. $n \times 10$ ns

for n-bit parallel binary adder

Date - / - /

→ Flap time

ntp

tp = 10 ns

↳ tp → Propagation delay

→ Gates responsible for Carrying Propagation delay (10 ns)

2n gate

(Least OR gate & Second
least AND gate)

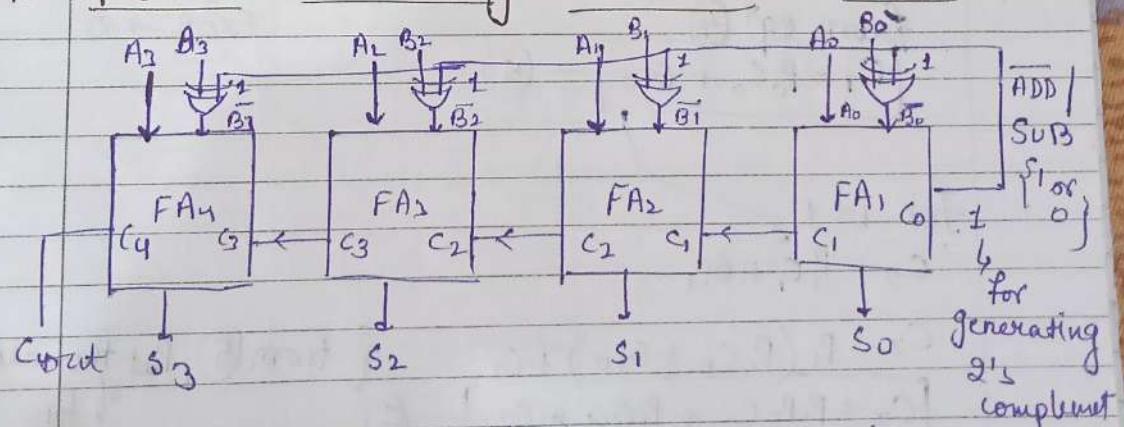
for n-bit parallel binary

adder 2n gate are responsible for propagation
delay.

→ Drawback of 4-bit parallel binary adder.

→ Carrying propagation delay.

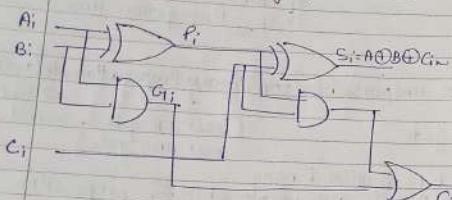
4-bit Parallel binary Adder & Subtractor



$$C_0 \rightarrow 0 \{ \text{Add}^n \}$$

$$C_0 \rightarrow 1 \{ \text{Sub}^n \}$$

Look Ahead Carry Generator



$$P_i = A_i \oplus B_i \quad (1)$$

$$G_i = A_i \cdot B_i \quad (2)$$

$$S_i = P_i \oplus C_i \quad (3)$$

$$C_{i+1} = P_i C_i + G_i \quad (4)$$

for i=0

Subn. eqⁿ (4)

$$C_0 = P_0 C_0 + G_{10} \quad (5)$$

$$\text{for } i=1 \\ C_1 = P_1 C_1 + G_{11}$$

$$C_2 = P_1 (P_0 C_0 + G_{10}) + G_{11} \quad \left. \begin{array}{l} \text{from (5), Put the value of } C_1 \\ C_2 = P_1 P_0 C_0 + P_1 C_{10} + G_{11} \end{array} \right\} - (6)$$

for i=2

$$C_3 = P_2 C_2 + G_{12}$$

$$C_3 = P_2 (P_1 P_0 C_0 + P_1 C_{10} + G_{11}) + G_{12}$$

Extra, Interview Q.
If we have 10 bits except the last bit is not one then the XOR will be odd, XOR → 1
if no. of ones are even, XOR → 0

$$C_3 = P_2 P_1 P_0 C_0 + P_2 P_1 G_{10} + G_1 P_1 G_{12} \quad (7)$$

$$C_4 = P_3 C_3 + G_{13}$$

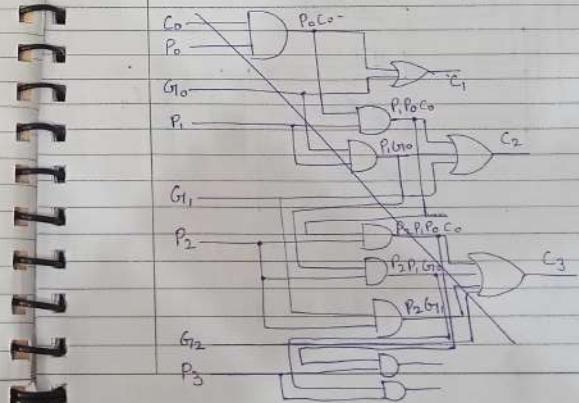
$$C_4 = P_3 (P_2 P_1 P_0 C_0 + P_2 P_1 G_{10} + G_1 P_1 G_{12} + G_{13}) + G_{13} \quad (8)$$

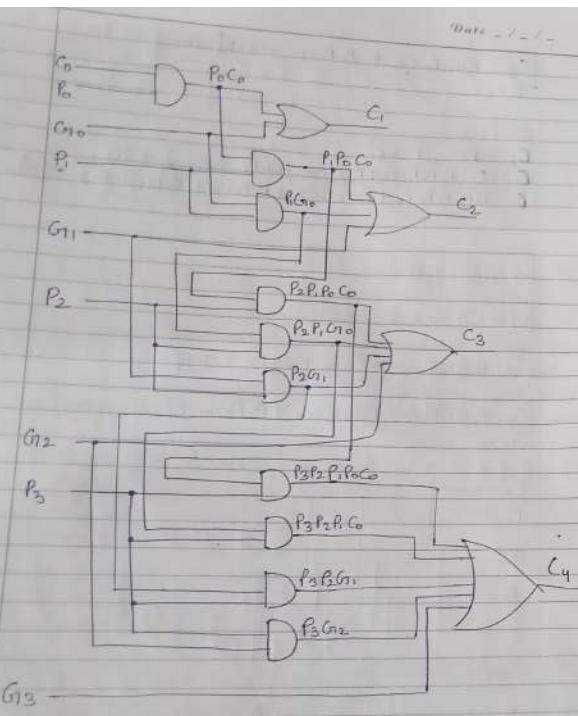
$$\rightarrow C_4 = P_0 C_0 + G_{10}$$

$$C_5 = P_1 P_0 C_0 + P_1 G_{10} + G_{11}$$

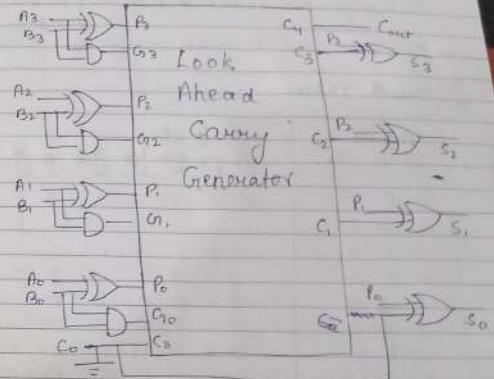
$$C_6 = P_2 P_1 P_0 C_0 + P_2 P_1 G_{10} + G_1 P_2 + G_{12}$$

$$C_7 = P_3 P_2 P_1 P_0 C_0 + P_3 P_2 P_1 G_{10} + P_3 P_2 G_1 + P_3 (G_{12} + G_{13})$$





Look Ahead Carry binary Adder



Binary Decoder (3x8)

32 KB Used to convert their
 32×2^{10} B binary to correspon
 $2^5 \times 2^{10} \times 8$ Bits
 $2^{15} \times 8$ bits
 15×2^{15}
 3×2^3

In General, $m \times 2^n$

3x8 Decoder

- It is the combinational logic circuit that converts a binary input code to one of the possible output.
- Decoder is used to create minterm. + includes all variables

Truth table

Inputs	Outputs
$I_2 \ I_1 \ I_0$	$y_7 \ y_6 \ y_5 \ y_4 \ y_3 \ y_2 \ y_1 \ y_0$
0 0 0	0 0 0 0 0 0 0 1
0 0 1	0 0 0 0 0 0 1 0
0 1 0	0 0 0 0 0 1 0 0
0 1 1	0 0 0 0 1 0 0 0
1 0 0	0 0 0 1 0 0 0 0
1 0 1	0 0 1 0 0 0 0 0
1 1 0	0 1 0 0 0 0 0 0
1 1 1	1 0 0 0 0 0 0 0

$$y_0 = \bar{I}_2 \bar{I}_1 \bar{I}_0$$

$$y_1 = \bar{I}_2 \bar{I}_1 I_0$$

$$y_2 = \bar{I}_2 I_1 \bar{I}_0$$

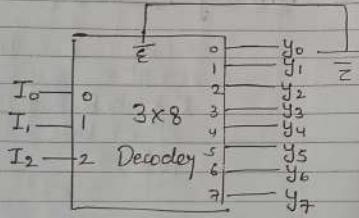
$$y_3 = I_2 \bar{I}_1 \bar{I}_0$$

$$y_4 = I_2 \bar{I}_1 I_0$$

$$y_5 = I_2 I_1 \bar{I}_0$$

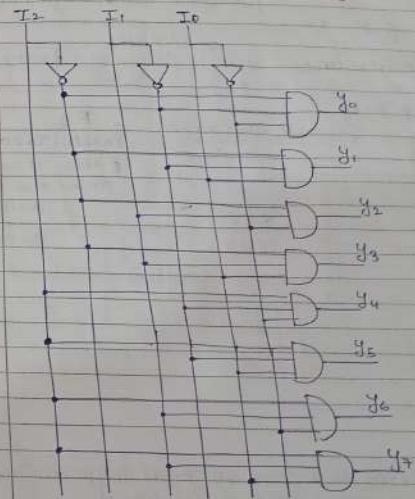
$$y_6 = I_2 I_1 I_0$$

$$y_7 = I_2 \bar{I}_1 \bar{I}_0$$



Block Diagram

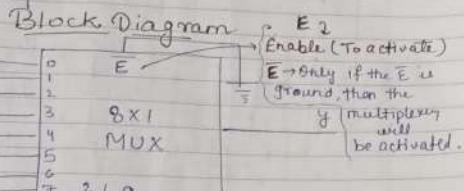
Combinational Logic Circuit diagram



Multiplexer (8x1)

- * Many to One. {Multiple inputs, Single output}

Selection lines (3) (S) {variable used to select}
Size of $\rightarrow (2^3 \times 1)$ Inputs



Inputs to transfer the content of data to the output.

S₂, S₁, S₀

- A multiplexer is a combinational logic circuit that performs many to one function. The selection lines are used to select only one input among multiple inputs to be transferred to the single output line.

Truth-table

Selection lines	Output
S ₂ S ₁ S ₀ 0 0 0	y D ₀
0 0 1	D ₁
0 1 0	D ₂
0 1 1	D ₃
1 0 0	D ₄
1 0 1	D ₅
1 1 0	D ₆
1 1 1	D ₇

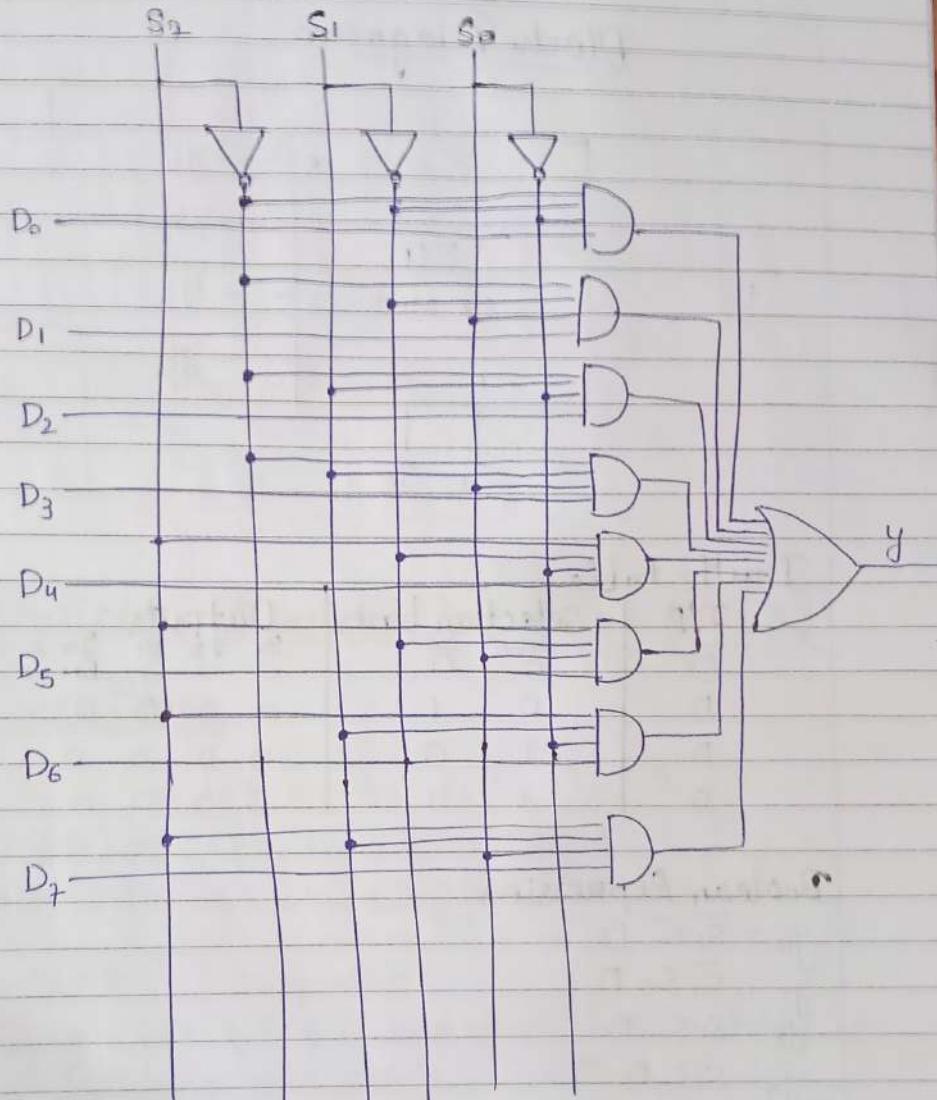
Boolean function

$$y = S_2 \bar{S}_1 \bar{S}_0 D_0 + \bar{S}_2 \bar{S}_1 S_0 D_1 + \bar{S}_2 S_1 \bar{S}_0 D_2 + \bar{S}_2 S_1 S_0 D_3 \\ + S_2 \bar{S}_1 \bar{S}_0 D_4 + S_2 \bar{S}_1 S_0 D_5 + S_2 S_1 \bar{S}_0 D_6 + S_2 S_1 S_0 D_7$$

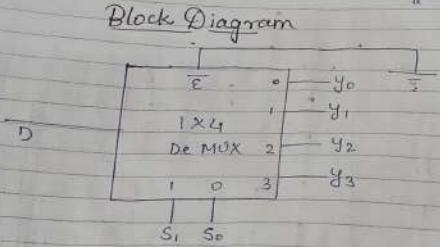
Date - / / -

$$D_{out} = f = f_1 \cdot$$

$$y = \bar{S_2} \bar{S_1} \bar{S_0} D_0 + \bar{S_2} \bar{S_1} S_0 D_1 + \bar{S_2} S_1 \bar{S_0} D_2 + \bar{S_2} S_1 S_0 D_3 + \\ S_2 \bar{S_1} \bar{S_0} D_4 + S_2 \bar{S_1} S_0 D_5 + S_2 S_1 \bar{S_0} D_6 + S_2 S_1 S_0 D_7$$



De multiplexer (De-MUX) (1x4)



Truth-table

I/P	Selection lines	Outputs
D	0 0	y ₀ y ₁ y ₂ y ₃
D	0 1	0 0 D 0
D	1 0	0 D 0 0
D	1 1	D 0 0 0

Boolean Expression

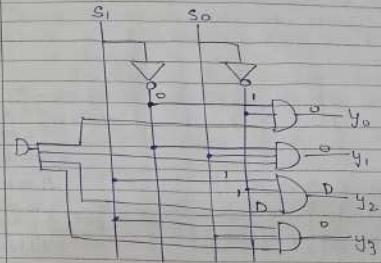
$$y_0 = \bar{S}_1 \bar{S}_0 D$$

$$y_1 = \bar{S}_1 S_0 D$$

$$y_2 = S_1 \bar{S}_0 D$$

$$y_3 = S_1 S_0 D$$

Logical Diagram



Register Transfer Language (RTL)

Overflow

$$\begin{array}{ccccccc} & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ (+70) - & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ (+80) - & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 150 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}$$

Carry in bit Carry out bit

The carry in bit is changing to carry out bit i.e. 0 to 1, then it is overflow cond'. i.e. Cin is diff from Cout bit.

No overflow when we use two diff. sign bit

∴ It is an Overflow

2	+127	128	64	32	16	8	4	2	1	0	R
-127	0	1	1	1	1	1	1	1	1	1	1
-2	-127	0	0	0	0	0	0	0	0	0	1
+2	0	1	0	1	1	1	1	1	1	1	0
-2	1	0	0	0	0	0	0	0	0	0	1
+2	0	0	0	0	0	0	0	0	1	0	1
-2	1	1	1	1	1	1	1	1	1	1	0
C_{in}											
+2	-127	0	0	0	0	0	0	0	0	0	1
-2	-129	1	1	1	1	1	1	1	1	1	0
O_4	1	1	1	1	1	1	1	1	1	1	1
C_{out}	0	0	0	0	0	0	0	0	0	0	1
discussed											
H.	$\therefore C_{in} = 1 \rightarrow \text{Overflow}$										
	$C_{out} = 0$										

Register Transfer Language (RTL)

Register - A storage device which comprises of set of flip flops. Flip flops stores 1-bit at a time.

RTL is a description of a computer process.

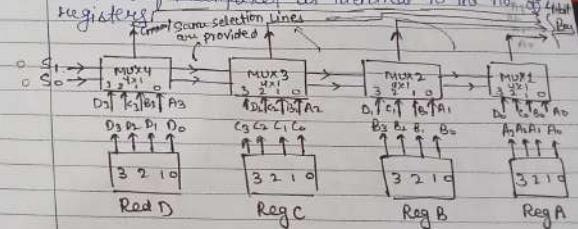
Register transfer in a Common Bus Configuration

There are two mechanism/approach to implement the reg-trans. in a common bus config:-

- 1) MUX
- 2) Tristate Buffers

With the help of Multiplexer (MUX) :-

- The length of register is identical to the no. of multiplexers.
- The size of multiplexer is identical to the no. of registers.



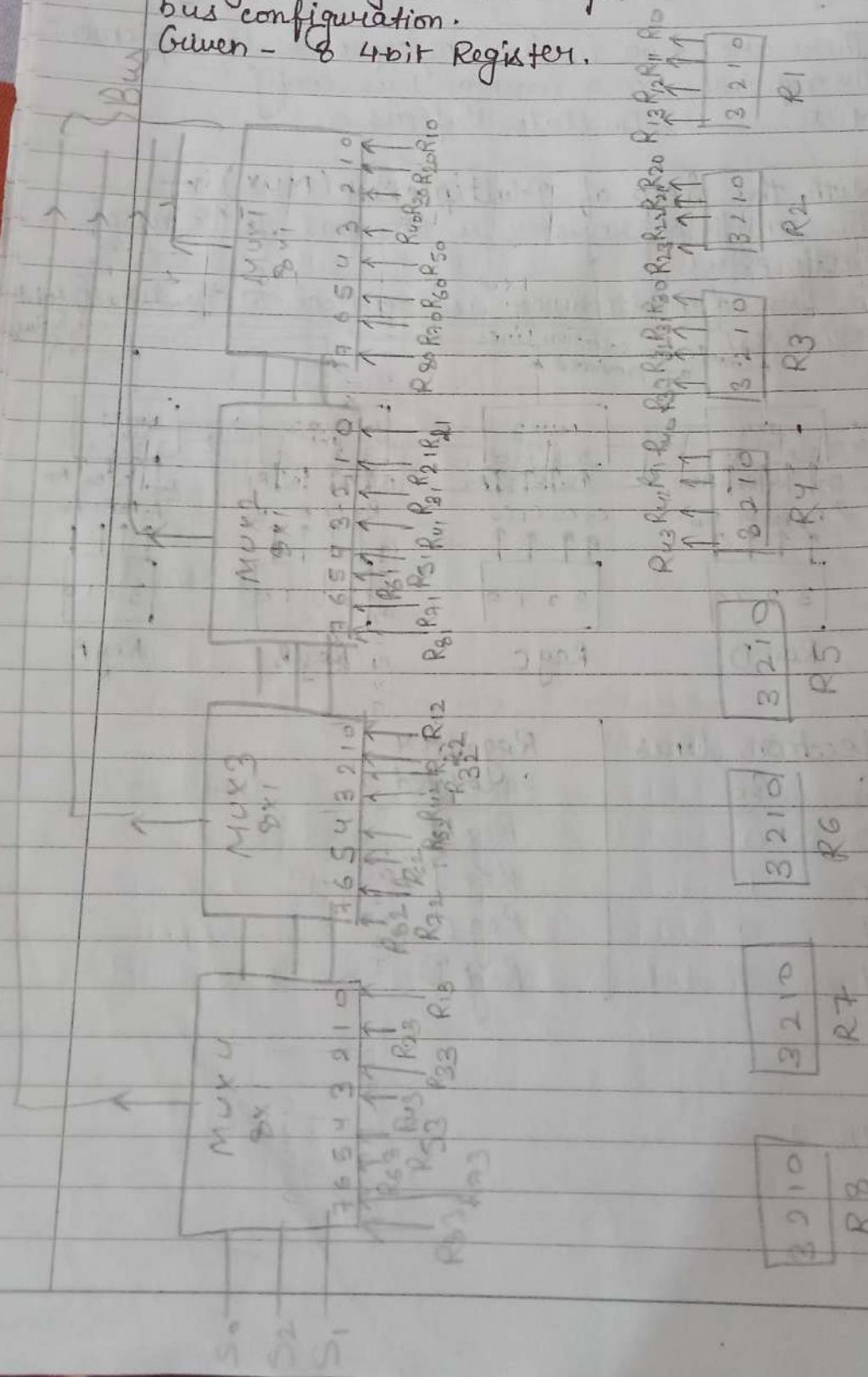
Selection lines		Register selected
S1	S0	
0	0	Reg A
0	1	Reg B
1	0	Reg C
1	1	Reg D

Designs :-

Date - / - / -

- Q. Design a reg. transfer operation in a common bus configuration.

Given - 8 4-bit Register.



Date - / - /

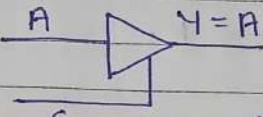
2) With the help of Tri-state Buffer :-

State 1 → '0'

State 2 → '1'

State 3 → 'High Impedance'

Buffer is a logic gate which provide the exact replica of inputs.



$Y = A$, if $C = 1$

High Imped. if $C = 0$

↳ O/P is disconnected
with the O/P.
(No effect)

{ For normal functioning, $C = 1$ }

Truth table

Inputs of decoders Reg. select

$S_1 \quad S_0$

0 0

A

0 1

B

1 0

C

1 1

D

Types of Memory Transfer

Read Operation

Controlling signals → Read : $R_1 \leftarrow M[AR]$
 Address → Memory

Write Operation

Write : $M[AR] \leftarrow R_1$

Types of Microoperation

- Arithmetic microoperation
- Logical microoperation
- Shift "

1. Arithmetic microoperation

Microoperation

Add

Subtract

Add with carry

Subtract with borrow

Transfer

Implement

Decrement

Notation

$R_3 \leftarrow R_1 + R_2$ → 2's complement

$R_3 \leftarrow R_1 + R_2 + 1$

$R_3 \leftarrow R_1 + R_2 + 1$

$R_3 \leftarrow R_1 + R_2 + \{R_1 + R_2 + 1 - k\}$

$R_2 \leftarrow R_1$

$R_2 \leftarrow R_1 + 1$ → 2's complement

$R_2 \leftarrow R_1 - 1$ ($R_1 + T + 1 = R_1 + 0 + 1$)

2. Logical microoperation

Microoperation

Clear

AND

NAND

OR

NOR

EXOR

EXNOR

Complement

Set to 1

Notation

$R_1 \leftarrow 0$

$R_3 \leftarrow R_1 \wedge R_2$

$R_3 \leftarrow \overline{R_1} \wedge R_2$

$R_3 \leftarrow R_1 \wedge \overline{R_2}$

$R_3 \leftarrow R_1 \vee R_2$

$R_3 \leftarrow \overline{R_1} \vee R_2$

$R_3 \leftarrow R_1 \vee \overline{R_2}$

$R_3 \leftarrow \overline{R_1} \vee \overline{R_2}$

$R_3 \leftarrow R_1 \oplus R_2$

$R_3 \leftarrow \overline{R_1} \oplus R_2$

$R_3 \leftarrow \overline{R_1} \oplus \overline{R_2}$

$R_3 \leftarrow \overline{R_1}$

$R_3 \leftarrow 1$

3. Shifting microoperation

Microoperation

Shift left

Shift right

Circular left shift

Circular right shift

Arithmetic shift left

Arithmetic shift right

Notation

$R \leftarrow ShlR$

$R \leftarrow ShrR$

$R \leftarrow ClrR$

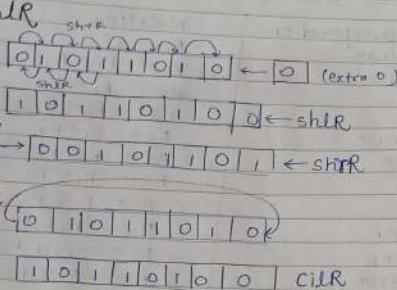
$R \leftarrow CirR$

$R \leftarrow ashlR$

$R \leftarrow ashrR$

8-bit Register

1) shLR



1)

2) shrR
1 0 1 1 0 1 0 0 ← shrR

2)

(extra 0)
0 → 0 0 1 0 1 1 0 1 ← shrR

3)

cilR
0 1 0 1 1 0 1 0

1 0 1 1 0 1 0 0 cilR

4) cirR

0 0 1 0 1 1 0 1 0

0 0 1 0 1 1 0 1 cirR

5) ashR
(divide by

2^n)

0 1 0 1 1 0 1 0

MSB then shift
(constant)

In this case, MSB doesn't change.
Now, shift all the bits starting from MSB.

0 0 1 0 1 1 0 1 ashR

{without changing MSB, just apply shrR}

fx-

$\frac{-4}{2} = -2$

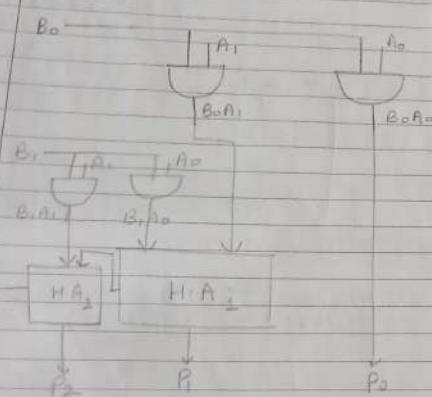
$\frac{-2}{2} = -1$

$\frac{-1}{2} = -2$

$\frac{-2}{2} = -1$

Designing of 2-Bit Binary Multiplier

$$\begin{array}{r}
 A \text{ (multiplicand)} \quad A_1 \ A_0 \ (2) \\
 \times B \text{ (multiplier)} \quad \times B_1 \ B_0 \ (2) \quad 2+2=4 \\
 \hline
 \text{Partial product} \quad B_0 \cdot A_1 \quad B_0 \cdot A_0 \quad 2^4 = 16 \\
 \quad B_1 \cdot A_1 \quad B_1 \cdot A_0 \downarrow \quad \square - 15 \\
 \text{Extra carry} \rightarrow P_3 \quad P_2 \quad P_1 \quad B_0 \cdot A_0 \quad \text{max.} \\
 \hline
 \text{final product}
 \end{array}$$



$$Q. - 65$$

Q. - 65 in 8-bit
 $+65 \rightarrow 00000001$
 $-65 \rightarrow 11011111$

Overflow occurs.

$$\boxed{1\ 0\ 1\ 1\ 1\ 1\ 1\ 1}$$

$$\boxed{0\ 1\ 1\ 1\ 1\ 1\ 1\ 0}$$

Q. - 33

$$\boxed{1\ 1\ 0\ 1\ 1\ 1\ 1\ 1}$$

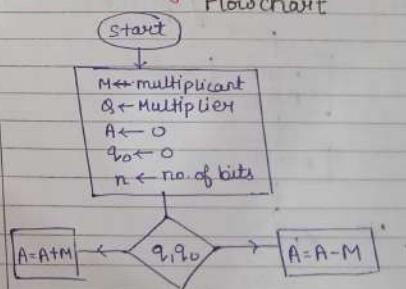
$$\boxed{1\ 0\ 1\ 1\ 1\ 1\ 1\ 0}$$

$$-33 \times 2 = -66$$

Overflow doesn't occur.

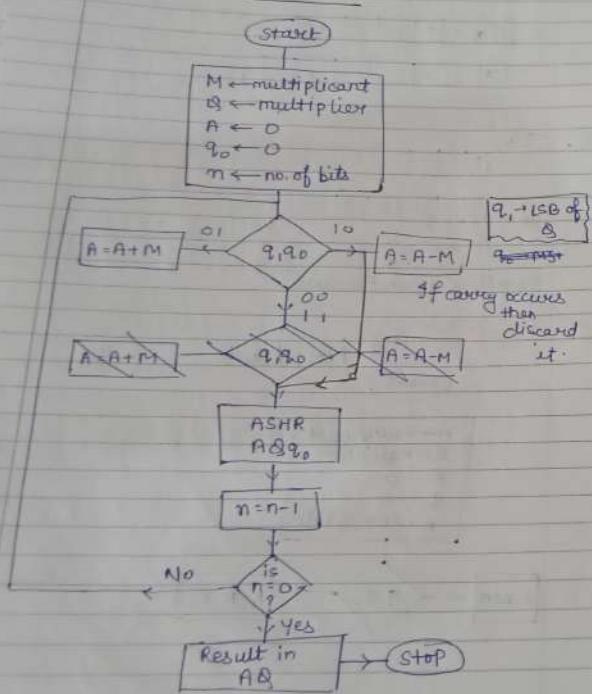
Booth's Algo

Flowchart



Booth's Algo

Flowchart



8. Perform binary multiplication : m = 10 and 2.

Take an extra bit in binary representation

$$M = 10 \rightarrow 10110$$

$$Q = +2 \rightarrow 00010$$

$$A \rightarrow 00000$$

$$q_0 \rightarrow 0$$

$$q_1 \rightarrow 1$$

n	A	Q	q ₀	Action
5	00000	00010	0	Initialisation
4	00000	00001	0	ASHR { ASHR applied }
	01010	00001	0	A = A - M
3	00101	00000	1	ASHR
	11011	00000	1	A = A + M
2	11101	10000	0	ASHR
1	11110	11000	0	ASHR
0	11111	01100	0	ASHR

Result :- 1111101100

Q. Perform the binary multiplication of $-16 \times (-5)$ by using booth algo.

$$-16 \rightarrow 010000$$

$$-5 \rightarrow 111011$$

$$M = -16 \rightarrow 110000$$

$$Q = -5 \rightarrow 111011$$

$$A \rightarrow 000000$$

$$q_0 \rightarrow 0$$

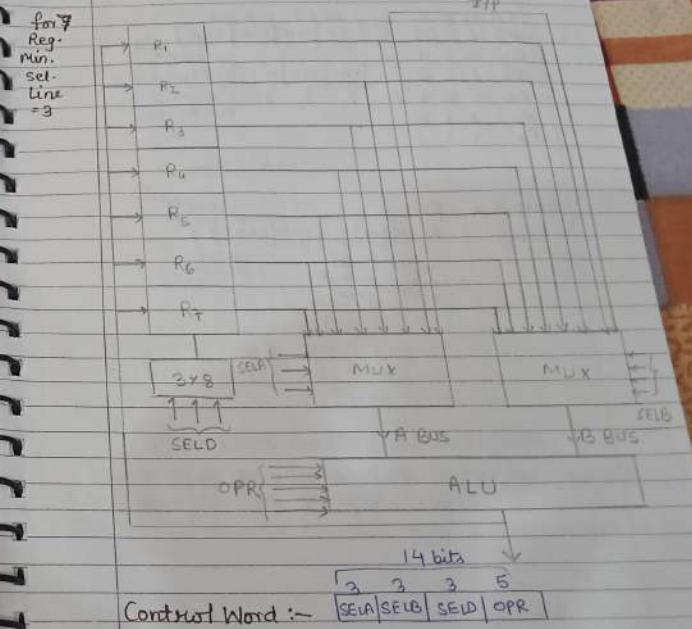
n	A	Q	q_0	Action
6	000000	111011	1	Initialisation
5	010000	111011	0	$A = A - M$
4	001000	011101	1	ASHR
3	000100	001110	1	ASHR
2	110100	001110	1	$A = A + M$
1	111010	000111	0	ASHR
0	001010	000111	0	$A = A + M + 1$
	000101	000011	1	ASHR
	000010	100001	1	ASHR
	000001	010000	1	ASHR

Result :- 000001010000

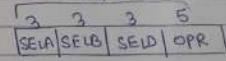
$$-16 \times -5$$

$$= 80$$

General Register Organization (GRO)



Control Word :-



max. no. of micro-operations performed = 85
 Control word generated from control unit to perform the operation $R_3 \leftarrow R_1 + R_2$

Ex - $R_3 \leftarrow R_1 + R_2$

R_1	0 0 0	0 0 0	0 0 0	0 0 0	
R_2	0 0 1	3 3 3	3 3 3	5	
R_3	0 1 1	SEL _A	SEL _B	SEL _C	OPR
R_4	1 0 0	supposed			
R_5	1 0 1	0 0 1	0 1 0	0 1 1	0 0 0 1 0
R_6	1 1 0	R_1	R_2	R_2	$R_3 \leftarrow R_1 + R_2$
R_7	1 1 1				

Ex - $Output \leftarrow R_1$

Same R_1 → $0 0 1$ → $0 0 0$ → $0 0 0 0 1$

R_2 is not given (000) → for output (000)

Output → R_1 → supposed

Stack Organization

- Useful Property in CPU.
- Based on the property of LIFO (Last In First Out)

32 MB
 $32 \times 2^{20} \times 8$ bits
 $2^5 \times 2^{20}$
 $= 2^{25}$

Locations / Address

3 bit in each word.	63
64 Word Stack	
	3
	2
	1 0 0 0 1 1 1 0
	0 1 1 0 1 1 1 0 1

(SP) → Stack Pointer (Address Register)

It holds the address of the top of the element.
 Here 1 is the top of the stack.

If there is 63 then only 6-bit is required

000001

Flag Register

It is always of 1-bit.
 Shows the availability or unavailability of the content of register.

EMPTY FULL

1 → availability (Stack full)

0 → unavailability (Stack empty)

PUSH OPERATION

$SP \leftarrow 0$, $EMPTY \leftarrow 1$, $FULL \leftarrow 0$
 $SP \leftarrow SP + 1$
 $M[SP] \leftarrow DR$
 If ($SP = 0$) then,
 $EMPTY \leftarrow 0$,
 $FULL \leftarrow 1$

{DR \rightarrow Data Register}

POP OPERATION

$DR \leftarrow M[SP]$
 $SP \leftarrow SP - 1$
 If ($SP = 0$) then
 $EMPTY \leftarrow 1$
 $FULL \leftarrow 0$

The initial content of the following registers is given by :

$$R_1 = 10111011$$

$$R_2 = 00111101$$

$$R_3 = 10101100$$

Perform the following micro-operation and write the final content of each register i.e. R_1 , R_2 & R_3 .

(Q.)

- (i) $R_2 \leftarrow R_1 + 1$
- (ii) $R_3 \leftarrow R_2 - 1$
- (iii) $R_1 \leftarrow R_1 \vee R_2$
- (iv) $R_3 \leftarrow \text{ashr } R_3$

$$\begin{array}{r} R_2 \leftarrow R_1 + 1 \\ R_1 = 10111011 \\ R_1 = 01000100 \\ + \quad \quad \quad 1 \\ \hline 01000101 \end{array}$$

$$R_2 \leftarrow 01000101$$

$$\begin{array}{r} R_3 \leftarrow R_2 - 1 \\ R_2 = 01000101 \\ 1 = 00000001 \\ - 1 \rightarrow 11111110 \\ + \quad \quad \quad 1 \\ \hline 11111111 \end{array}$$

$$\begin{array}{r} R_3 \rightarrow 01000101 \\ + 11111111 \\ \hline 01000100 \end{array}$$

$$R_3 \leftarrow 01000100$$

$$\begin{array}{r} iii) R_1 \leftarrow R_1 \vee R_2 \\ R_1 \rightarrow 10111011 \\ R_1 \rightarrow 01000100 \\ R_2 \rightarrow 01000101 \\ R_1 \leftarrow 01000101 \end{array}$$

$$\begin{array}{r} iv) R_3 \leftarrow \text{ashr } R_3 \\ R_3 \leftarrow 01000100 \\ R_3 \leftarrow 00100010 \end{array}$$

- Date - / / -
- Notations for Writing Arithmetic Operation**
1. Infix - $A * B + C * D$
 2. Prefix (Polish Notation) - $\rightarrow * A B$
 3. Postfix (Reverse Polish Notation) - $A B * \rightarrow + C D * +$

Evaluation of Arithmetic Expression using Stack by following Reverse Polish Notation or Postfix notation:

Q1: $A * B + C * D$

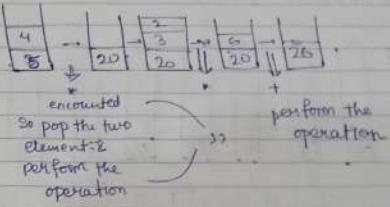
Sol: Postfix
 $A B * C D * +$

S-1 Just consider the variable sized stack.

S-2 Whenever the operand is encountered just push the stack.
 S-3 Now Pop out the operand whether operator is encountered in the stack will perform the operation.

Ex: $5 * 4 + 3 * 2$
 $20 + 6 = 26$

Postfix - $A B * C D * +$
 $5 4 * 3 2 * +$



Q2: $(3+4)[10 * (2+6)+8]$
 Apply Postfix (BODMAS)

$(3 4 +)[10 2 6 + * + 8]$

$(3 4 +)[10 2 6 + * 8 +]$

$3 4 + 10 2 6 + * 8 +$

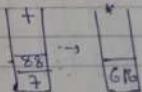
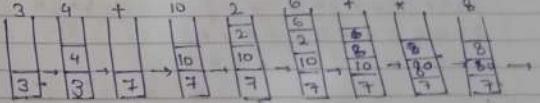
Rough -

$[10 * 8 + 8]$

$[80 + 8]$

$88 + 8$

96



Date: / /

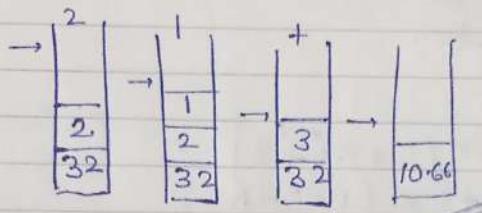
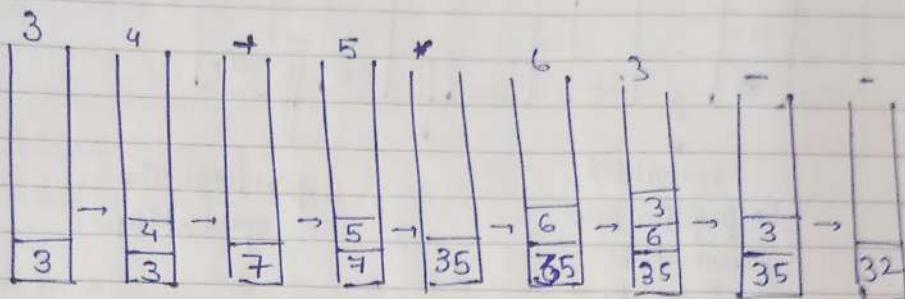
Q3. $(3+4)*5 - (6-3)$
 $(2+1)$

$$7 \times 5 - 3$$

$$\frac{35-3}{3} = \frac{32}{3} = 10.66$$

~~$(3+4)*5 + (34+5*) - (63-)$~~
 $21+$

$34+5*63 - 21+$



Q4. $3+4*[5*6+7*(9-7)]$
 $3+4*[56+797-*$
 $3+4*[56*797-*+]$
 $3+456*797-*+*$
 $3456*797-*+*$

$$7*[30+7*2]$$

$$7*[30+14]$$

$$7*44$$

$$\frac{44}{308}$$

