# Homework 1 CSCE 421

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## 1

### 1: Calculate the gradient of the function $f(x, y) = x^2 + ln(y) + xy + y^3$.

We need to calculate the derivative of the function with respect to x and y.
$\frac{df}{dx} = 2x + y$
$\frac{df}{dy} = \frac{1}{y} + x + 3y^2$
$\nabla f(x, y) = (2x + y)\mathbf{i} + (\frac{1}{y} + x + 3y^2)\mathbf{j}$

**What is the gradient value for $(x, y) = (10, -10)$?**

Just plug in x=10 and y = -10 into the gradient found above: $\nabla f(x, y) = (2(10) + (-10))\mathbf{i} + (\frac{1}{-10} + (10) + 3(-10)^2)\mathbf{j}$
$\nabla f(x, y) = 10\mathbf{i} + 309.9\mathbf{j}$

### 2: Calculate the gradient of the function $f(x, y, z) = \tanh(x^3y^3) + sin(z^2)$.

We need to calculate the derivative of the function with respect to x, y, and z.
$\frac{df}{dx} = 3x^2y^3sech^2(x^3y^3)$
$\frac{df}{dy} = 3x^3y^2sech^2(x^3y^3)$
$\frac{df}{dz} = 2zcos(z^2)$
$\nabla f(x, y, z) = (3x^2y^3sech^2(x^3y^3))\mathbf{i} + (3x^3y^2sech^2(x^3y^3))\mathbf{j} + (2zcos(z^2))\mathbf{k}$

**What is the gradient value for $(x, y, z) = (-1, 0, \frac{\pi}{2})$?**

Plug in the values $(-1, 0, \frac{\pi}{2})$ for x, y and z

$\nabla f(x, y, z) = (3(-1)^2(0)^3 sech^2((-1)^3(0)^3))\mathbf{i}+(3(-1)^3(0)^2 sech^2((-1)^3(0)^3))\mathbf{j}+(2(\frac{\pi}{2})cos((\frac{\pi}{2})^2))\mathbf{k}$

$\nabla f(x, y, z) = 0\mathbf{i} + 0\mathbf{j} - 2.452\mathbf{k}$

$\nabla f(x, y, z) = -2.452\mathbf{k}$

## 2) Multiply the following matrices

### 1

$$\begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} * \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

Multiply the rows of the first matrix with the columns of the second

$$\begin{bmatrix} 10*0 & 10*3 & 10*0 & 10*1 \\ -5*0 & -5*3 & -5*0 & -5*1 \\ 2*0 & 2*3 & 2*0 & 2*1 \\ 8*0 & 8*3 & 8*0 & 8*1 \end{bmatrix} == \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

### 2

$$\begin{bmatrix} 7 & -3 & 1 & 9 \end{bmatrix} * \begin{bmatrix} -3 \\ -4 \\ 6 \\ 0 \end{bmatrix}$$

Multiply rows of first matrix with columns of the second

$\begin{bmatrix} 7*-3+-3*-4+1*6+9*0 \end{bmatrix} == \begin{bmatrix} -3 \end{bmatrix}$

### 3

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1\cdot 6+(-1)\cdot 0+6(-3)+7\cdot 3 & 1\cdot 2+(-1)(-1)+6\cdot 0+7\cdot 4 & 1\cdot 0+(-1)\cdot 1+6\cdot 4+7\cdot 7 \\ 9\cdot 6+0\cdot 0+8(-3)+1\cdot 3 & 9\cdot 2+0\cdot(-1)+8\cdot 0+1\cdot 4 & 9\cdot 0+0\cdot 1+8\cdot 4+1\cdot 7 \\ (-8)\cdot 6+1\cdot 0+2(-3)+3\cdot 3 & (-8)\cdot 2+1\cdot(-1)+2\cdot 0+3\cdot 4 & (-8)\cdot 0+1\cdot 1+2\cdot 4+3\cdot 7 \\ 10\cdot 6+4\cdot 0+0\cdot(-3)+1\cdot 3 & 10\cdot 2+4(-1)+0\cdot 0+1\cdot 4 & 10\cdot 0+4\cdot 1+0\cdot 4+1\cdot 7 \end{bmatrix}$$

$$== \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$

# 3

Calculate the distance between the two vectors using the following norms

$a = \begin{bmatrix} 7 \\ 0 \\ -1 \end{bmatrix}, b = \begin{bmatrix} 7 \\ 9 \\ -5 \end{bmatrix}$

## 1: L0

$||a||_0 =$ the number of nonzero elements, so it's 2
$||b||_0 =$ the number of nonzero elements, so it's 3

$$||b - a||_0 = 3 - 2 = 1$$

## 2: L1

$||a - b||_1 = \sum_{i=1}^{n} |b_i - a_i| = |7 - 7| + |9 - 0| + |-5 - (-1)| = 13$

## 3: L2

$||a - b||_2 = \sum_{i=1}^{n} \sqrt{(b_i - a_i)^2} = 0 + \sqrt{91} + \sqrt{16} = 13$

## 4: L∞

$||a||_\infty = max|a_n| = max[|7|, |0|, |-1|] = 7$

$||b||_\infty = max|b_n| = max[|7|, |9|, |-5|] = 9$

$$||b - a||_\infty = |9 - 7| = 2$$

# 4

Consider a problem where we are rolling 2 dices where each dice has 6 faces numbered from 1 to 6. Answer the following questions:

## 1) What is the sample space?

The sample space is every possible outcome of rolling the two dice.
(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

## 2) If the event we are interested in is the sum being 10, what would be the probability of observing such an event?

Probability of rolling a sum of 10 = number of possible outcomes of rolling a 10/ total number of possible outcomes.

Rolling a 10 sample space = {(4,6), (5,5) , (6,4) }

Probability = 3/36 = 1/12

## 3) If the event we are interested in is the sum being 6, what would be the probability of observing such an event?

Rolling a 10 sample space = {(1,5), (2,4) , (3,3), (4,2), (5,1) }
Probability of rolling a sum of 6 = 5/36

Probability = 3/36 = 1/12

# 5

$$f(x) = \begin{cases} \frac{1}{b-a} & if a \leq x \leq b \\ 0 & otherwise \end{cases}$$

## 1) What is the mean of X?

$E[X] = \int_a^b x * f(x)dx$
$= \int_a^b \frac{x}{b-a}$
$\frac{x^2}{2(b-a)}$ from a to b
$\frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)}$
$E[x] = \frac{1}{2}(b+a)$

## 2) What is the standard deviation of X?

$\sigma = \sqrt{(E(X^2) - [E(X)]^2)}$
We need to find $E(X^2)$

$E[X^2] = \int_a^b x^2 * f(x)dx$
$= \int_a^b \frac{x^2}{b-a}$
$\frac{x^3}{3(b-a)}$ from a to b
$\frac{b^3}{3(b-a)} - \frac{a^3}{3(b-a)}$
$E[x^2] = \frac{b^2+ba+a^2}{3}$

$\sigma = \sqrt{\frac{b^2+ba+a^2}{3} - (\frac{1}{2}(b+a))^2}$
$= \sqrt{\frac{b^2-2ba+a^2}{12}}$
$\sigma = \frac{b-a}{\sqrt{12}}$

## 6

| | | ground truth | |
|---|---|---|---|
| | | avocado | no avocado |
| Avocado detector | avocado | 37 | 23 |
| | no avocado | 45 | 55 |

### 1)What is the accuracy of the detector?

Accuracy $A = \frac{TP+TN}{TP+TN+FP+FN}$
$A = \frac{37+55}{37+55+23+45}$
$A = \frac{92}{160}$
Accuracy = 57.5 %

### 2)What is the balanced accuracy of the detector?

Balanced Accuracy $= \frac{TP}{2(TP+FN)} + \frac{TN}{2(TN+FP)}$
BA $= \frac{37}{2(37+45)} + \frac{55}{2(55+23)}$

BA $= 0.2256 + 0.3525$
Balanced Accuracy = 57.8 %

### 3)What is the precision of the detector?

Precision $= \frac{TP}{TP+FP}$
Precision $= \frac{37}{37+23}$

Precision = 0.617

### 4) What is the recall of the detector?

Recall $= \frac{TP}{TP+FN}$
Recall $= \frac{37}{37+45}$

Recall = 0.451

### 5) What is the F1-measure of the detector?

$F_1 = \frac{2TP}{2TP+FP+FN}$
$F_1 = \frac{2*37}{2*37+23+45}$

F1 = 0.521

## 7

In Problem 6, assume that their microwave avocado detector does not give a binary output regarding the existence of avocados inside the taco. Alternatively, it outputs a probability of such an event. Jose, a CS sophomore who wants to put his knowledge to practice, wants to approximate the AUROC of the detector using 5 points as candidate thresholds: $\{0, 0.25, 0.5, 0.75, 1\}$. In a few tests that they ran, the probabilities and their corresponding ground truths were as follows:

| predicted | ground truth |
|---|---|
| 10% | 0 |
| 5% | 0 |
| 70% | 1 |
| 50% | 0 |
| 90% | 1 |
| 65% | 1 |
| 35% | 1 |
| 60% | 0 |
| 15% | 1 |
| 20% | 0 |

### 1) What would be the ROC value for threshold 0?

TPR $= \frac{TP}{TP+FN}$
TPR $= \frac{5}{5+0} = 1$

FPR $= \frac{FP}{TN+FP}$
FPR $= \frac{5}{0+5} = 1$

### 2) What would be the ROC value for threshold .25?

TPR $= \frac{TP}{TP+FN}$
TPR $= \frac{4}{4+1} = 0.8$

FPR $= \frac{FP}{FP+TN}$
FPR $= \frac{2}{2+3} = 0.4$

### 3) What would be the ROC value for threshold .5?

TPR $= \frac{TP}{TP+FN}$
TPR $= \frac{3}{3+2} = 0.6$

FPR $= \frac{FP}{FP+TN}$
FPR $= \frac{2}{2+3} = 0.4$

### 4) What would be the ROC value for threshold .75?

TPR $= \frac{TP}{TP+FN}$
TPR $= \frac{1}{1+4} = 0.2$

FPR $= \frac{FP}{FP+TN}$
FPR $= \frac{0}{0+5} = 0$

### 5) What would be the ROC value for threshold 1?

TPR $= \frac{TP}{TP+FN}$
TPR $= \frac{0}{0+5} = 0$

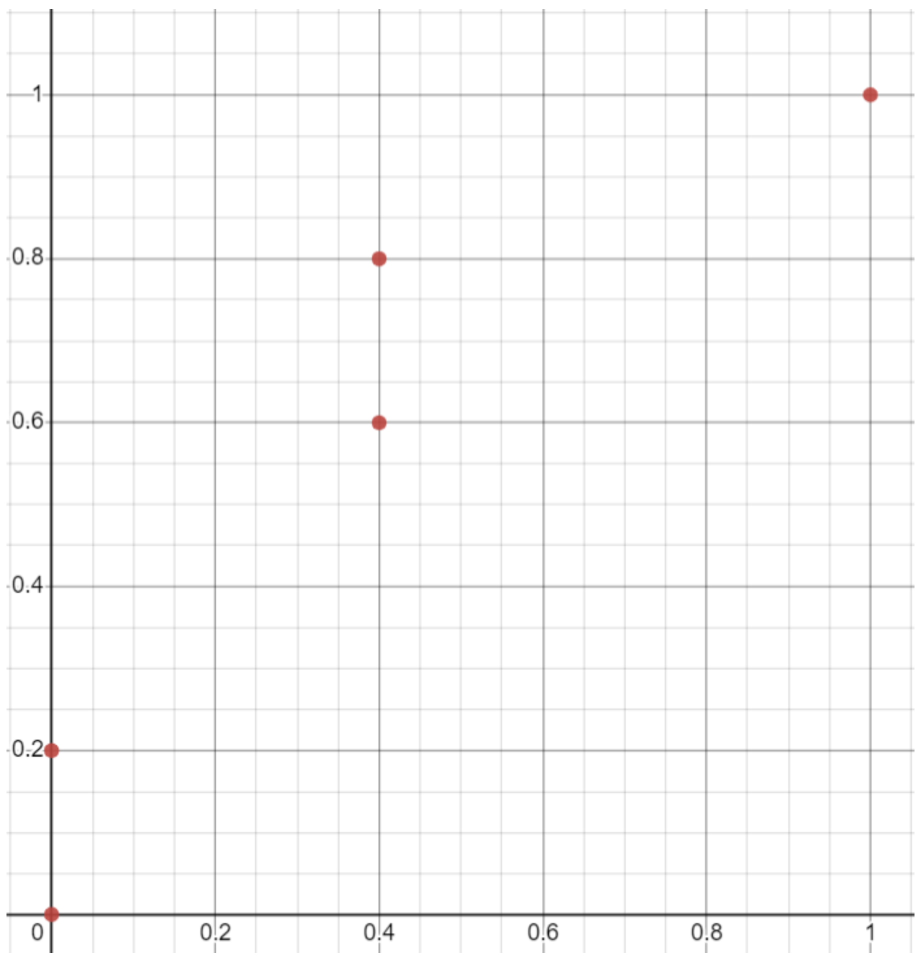
FPR $= \frac{FP}{FP+TN}$
FPR $= \frac{0}{0+0} = 0$

### 6) What would be the AUROC approximation using the above results (HINT: remember Riemann sum)

Trapazoidal Riemann Sum:
$\sum_{i=1}^{4}(x_{i+1} - x_i)(\frac{y_{i+1}+y_i}{2})$

$= ((0-0)(\frac{0+0.2}{2})) + ((0.4-0)(\frac{0.6+0.2}{2})) + ((0.4-0.4)(\frac{0.6+0.8}{2})) + ((1-0.4)(\frac{1+0.8}{2}))$

$= 0.7$
AUROC Approximation is 0.7


1
0.8
0.6
0.4
0.2
0
0.2
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0.8
1