Homework 1 CSCE 633

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Gradient Calculations

1) Calculate the gradient of the function $f(x, y) = x^2 + ln(y) + xy + y^3$. What is the gradient value for $(x, y) = (10, -10)$?

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + \ln(y) + xy + y^3)$$

$$\frac{\partial f}{\partial x} = 2x + y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 + \ln(y) + xy + y^3)$$

$$\frac{\partial f}{\partial y} = \frac{1}{y} + x + 3y^2$$

The gradient of f is:

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

$$\nabla f(x, y) = (2x + y, \frac{1}{y} + x + 3y^2)$$

The gradient at the point $(x, y) = (10, -10)$:

$$\left.\frac{\partial f}{\partial x}\right|_{(10,-10)} = 2 * (10) + (-10) = 20 - 10 = 10$$

$$\left.\frac{\partial f}{\partial y}\right|_{(10,-10)} = \frac{1}{-10} + 10 + 3(-10)^2 = -0.1 + 10 + 300 = 309.9$$

The gradient at $(x, y) = (10, -10)$ is:

$$\nabla f(10, -10) = (10, 309.9)$$

Calculate the gradient of the function $f(x, y, z) = tanh(x^3y^3) + sin(z^2)$. What is the gradient value for $(x, y, z) = (-1, 0, \frac{\pi}{2})$?

$$\frac{\partial f}{\partial x} = 3x^2y^3sech^2(x^3y^3)$$

$$\frac{\partial f}{\partial y} = 3x^3y^2sech^2(x^3y^3)$$

$$\frac{\partial f}{\partial z} = 2zcos(z^2)$$

The gradient of f is:

$$\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\nabla f(x, y, z) = (3x^2y^3sech^2(x^3y^3), 3x^3y^2sech^2(x^3y^3), 2zcos(z^2))$$

The gradient value for $(x, y, z) = (-1, 0, \pi/2)$

$$\left.\frac{\partial f}{\partial x}\right|_{(-1,0,\pi/2)} = 3(-1)^2(0)^3sech^2((-1)^3(0)^3) = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_{(-1,0,\pi/2)} = 3(-1)^3(0)^2sech^2((-1)^3(0)^3) = 0$$

$$\left.\frac{\partial f}{\partial z}\right|_{(-1,0,\pi/2)} = 2(\frac{\pi}{2})cos((\frac{\pi}{2})^2) = \pi cos(\frac{\pi^2}{4})$$

The gradient at $(x, y, z) = (-1, 0, \frac{\pi}{2})$ is:

$$\nabla f(-1, 0, \frac{\pi}{2}) = (0, 0, \pi cos(\frac{\pi^2}{4})$$

2) Matrix Multiplication

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$$\begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 \cdot 0 & 10 \cdot 3 & 10 \cdot 0 & 10 \cdot 1 \\ -5 \cdot 0 & -5 \cdot 3 & -5 \cdot 0 & -5 \cdot 1 \\ 2 \cdot 0 & 2 \cdot 3 & 2 \cdot 0 & 2 \cdot 1 \\ 8 \cdot 0 & 8 \cdot 3 & 8 \cdot 0 & 8 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$