B565 Homework 2
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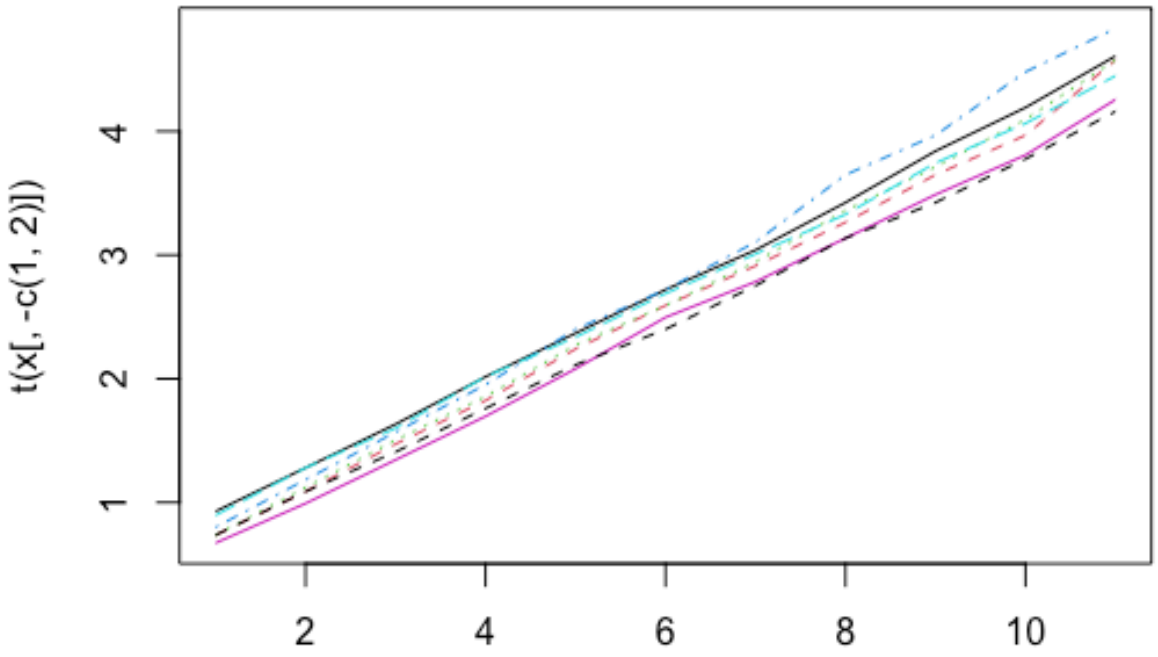
Q1.
A)

```
x = as.matrix(read.csv("rachmaninov_pc2_onset.csv"))
matplot(t(x[, -c(1,2)]), type="l")
```



b)

```
#with R function
auto_cov <- cov(x)
```

	X	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
X	4.6666	-0.048(	-0.090(	-0.101:	-0.138(	-0.144	-0.170(	-0.176(	-0.176(	-0.192(	-0.256(	-0.266(	-0.352(
V1	-0.048	0.0032	0.0050:	0.0056:	0.0051:	0.0063	0.0050:	0.0049(	0.0048:	0.0039:	0.0059:	0.0055	0.0043(

V2	-0.090	0.0050	0.0088	0.0098	0.0090	0.0110	0.0089	0.0085	0.0087	0.0085	0.0114	0.0118	0.0085
V3	-0.101	0.0056	0.0098	0.0114	0.0107	0.0130	0.0111	0.0101	0.0108	0.0114	0.0143	0.0156	0.0118
V4	-0.138	0.0051	0.0090	0.0107	0.0109	0.0129	0.0122	0.0112	0.0122	0.0139	0.0168	0.0191	0.0165
V5	-0.144	0.0063	0.0110	0.0130	0.0129	0.0156	0.0144	0.0134	0.0145	0.0162	0.0197	0.0222	0.0184
V6	-0.170	0.0050	0.0089	0.0111	0.0122	0.0144	0.0153	0.0141	0.0158	0.0199	0.0226	0.0271	0.0246
V7	-0.176	0.0049	0.0085	0.0101	0.0112	0.0134	0.0141	0.0145	0.0152	0.0180	0.0217	0.0250	0.0234
V8	-0.176	0.0048	0.0087	0.0108	0.0122	0.0145	0.0158	0.0152	0.0169	0.0216	0.0245	0.0295	0.0269
V9	-0.192	0.0039	0.0085	0.0114	0.0139	0.0162	0.0199	0.0180	0.0216	0.0317	0.0327	0.0428	0.0377
V10	-0.256	0.0059	0.0114	0.0143	0.0168	0.0197	0.0226	0.0217	0.0245	0.0327	0.0360	0.0446	0.0406
V11	-0.266	0.0055	0.0118	0.0156	0.0191	0.0222	0.0271	0.0250	0.0295	0.0428	0.0446	0.0579	0.0511
V12	-0.352	0.0043	0.0085	0.0118	0.0165	0.0184	0.0246	0.0234	0.0269	0.0377	0.0406	0.0511	0.0518

Showing 1 to 13 of 13 entries, 13 total columns

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Proof: R code

```
x = as.matrix(read.csv("rachmaninov_pc2_onset.csv"))

## Covariance matrix proof
c <- ncol(x) #number of variables
r <- nrow(x) #number of elements

#means for each column
x_mean <-matrix(data=1, nrow=r) %*%
cbind(mean(x[,1]),mean(x[,2]),mean(x[,3]),mean(x[,4]),mean(x[,5]),mean(x[,6]),mean(x[,
7]),mean(x[,8]),mean(x[,9]),mean(x[,10]),mean(x[,11]),mean(x[,12]))

#difference matrix

v1 <- c(x[,1])
v2 <- c(x[,2])
v3 <- c(x[,3])
v4 <- c(x[,4])
v5 <- c(x[,5])
```

```
v6 <- c(x[,6])
v7 <- c(x[,7])
v8 <- c(x[,8])
v9 <- c(x[,9])
v10 <- c(x[,10])
v11 <- c(x[,11])
v12 <- c(x[,12])

new_x <- cbind(v1,v2,v3,v4,v5,v6,v7,v8,v9,v10,v11,v12)
D <- new_x - x_mean

#covariance matrix creation
cova_x <- (r-1)^-1*t(D) %*% D
```

Solution:

	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12
v1	4.66666	-0.04800	-0.09066	-0.10133	-0.13866	-0.1440	-0.17066	-0.17600	-0.17600	-0.19200	-0.25600	-0.26666
v2	-0.0480	0.003218	0.005022	0.005656	0.005120	0.00638	0.005022	0.004900	0.004827	0.003925	0.005924	0.005510
v3	-0.0906	0.005022	0.008825	0.009825	0.009045	0.01104	0.008996	0.008508	0.008728	0.008533	0.011410	0.011800
v4	-0.1013	0.005656	0.009825	0.011410	0.010752	0.01306	0.011190	0.010118	0.010873	0.011434	0.014384	0.015652
v5	-0.1386	0.005120	0.009045	0.010752	0.010922	0.01297	0.012288	0.011264	0.012288	0.013994	0.016896	0.019114
v6	-0.1440	0.006387	0.011044	0.013068	0.012970	0.01565	0.014457	0.013458	0.014531	0.016213	0.019772	0.022211
v7	-0.1706	0.005022	0.008996	0.011190	0.012288	0.01445	0.015311	0.014140	0.015896	0.019968	0.022674	0.027160
v8	-0.1760	0.004900	0.008508	0.010118	0.011264	0.01345	0.014140	0.014579	0.015286	0.018090	0.021747	0.025014
v9	-0.1760	0.004827	0.008728	0.010873	0.012288	0.01453	0.015896	0.015286	0.016969	0.021674	0.024502	0.029598
v10	-0.1920	0.003925	0.008533	0.011434	0.013994	0.01621	0.019968	0.018090	0.021674	0.031744	0.032768	0.042837
v11	-0.2560	0.005924	0.011410	0.014384	0.016896	0.01977	0.022674	0.021747	0.024502	0.032768	0.036083	0.044641
v12	-0.2666	0.005510	0.011800	0.015652	0.019114	0.02221	0.027160	0.025014	0.029598	0.042837	0.044641	0.057929

Showing 1 to 12 of 12 entries, 12 total columns

The answer is the same as the one we find with the automated R function of cov(x). Hence we have proved that this is correct.

c)

Multivariate Normal Distribution. A random vector $X = (X1, X2,..., Xn)$ T is said to follow a multivariate normal distribution with mean μ and covariance matrix Σ if X can be expressed as $X = AZ + \mu$,

where $\Sigma = AAT$ and $Z = (Z1,Z2,...,Zn)$ with Zi, $i = 1, 2,...,n$ iid $N(0, 1)$ variables.

Multivariate normal distribution uses the equation given in the problem to generate its representative samples. In the R code below we will be generating and synthesizing the data points using multivariate normal distribution.

```
#Part c
install.packages("MASS")                    # Install MASS package
library("MASS")                             # Load MASS package
set.seed(98989)                             # Set seed for reproducibility
my_n <- 1000                                # Specify sample size
my_mu <- colMeans(x, dims = 1)                      # Specify the means of the variables
my_Sigma <- cov(x)  # Specify the covariance matrix of the variables
mynormal <- mvrnorm(n = my_n, mu = my_mu, Sigma = my_Sigma)  # Random sample from
multivariate normal distribution

library("dplyr")

# creating a data frame
data_frame <- data.frame(mynormal)

# eliminating NA values
data_without_na <- data_frame %>%
  replace(is.na(.), 0)
print("Row Wise Sum")

data_mod <- data_without_na%>%
  mutate(sum_of_rows = rowSums(.))
```

```
> print(data_mod)
           X       V1       V2       V3       V4       V5       V6       V7       V8       V9
1   0.13974656 0.4754233 0.8237397 1.1815687 1.575730 1.942866 2.360101 2.727466
3.048831 3.443871
2   2.40180829 0.5244056 0.8677106 1.2560097 1.614226 2.011423 2.374877 2.726034
3.049753 3.387027
3  -1.26048540 0.5862061 1.0404989 1.4169783 1.781059 2.199917 2.547172 2.925823
3.238442 3.617651
4   3.47796722 0.4739580 0.8209390 1.1550996 1.506564 1.901814 2.245926 2.727476
2.976393 3.271039
5  -1.18879738 0.5050951 0.8639027 1.2590850 1.665396 2.034999 2.468407 2.765587
3.130614 3.560538
6   5.69660701 0.4577721 0.7610785 1.1094615 1.446312 1.824796 2.182627 2.560873
2.866054 3.196266
7   6.33437642 0.4192513 0.6972131 0.9962903 1.323408 1.678712 2.029138 2.441960
2.712926 2.999984
8   0.77322105 0.4579428 0.8222900 1.1739852 1.549938 1.904962 2.293731 2.638668
2.961339 3.339316
```

9 4.46981247 0.4441171 0.7608055 1.1128927 1.463766 1.832681 2.201062 2.576345 2.883959 3.226129
10 -0.53676394 0.5141705 0.8642553 1.2272481 1.610729 1.982854 2.375636 2.724064 3.042324 3.387009
11 5.22753500 0.4289834 0.7767944 1.1153932 1.452344 1.809681 2.181708 2.499555 2.843142 3.261941
12 5.13230975 0.4189434 0.7377929 1.0826056 1.429195 1.791086 2.154810 2.537275 2.840501 3.186131
13 4.89421557 0.4710437 0.8242786 1.1836816 1.517221 1.895667 2.250169 2.577329 2.914788 3.300809
14 3.63113329 0.5054322 0.8243942 1.1866356 1.529202 1.917744 2.262561 2.638943 2.936785 3.225017
15 1.92333956 0.4399786 0.7966603 1.1496306 1.518374 1.863334 2.261310 2.558305 2.910552 3.328901
16 5.83904553 0.4210174 0.7518844 1.0687224 1.391365 1.746383 2.085562 2.454948 2.753875 3.087973
17 3.27693823 0.4785547 0.8562628 1.1979400 1.523568 1.894079 2.205957 2.567662 2.862713 3.159835
18 2.05796821 0.5654540 0.9365148 1.3197215 1.658551 2.069911 2.393589 2.755675 3.062135 3.354546
19 3.04460853 0.4200928 0.7274519 1.0974518 1.492386 1.861638 2.289041 2.693707 3.008138 3.411934
20 4.41559247 0.4540975 0.7638942 1.1184358 1.460446 1.833093 2.169386 2.572228 2.850972 3.115285
21 3.17896078 0.4797818 0.8908593 1.3083380 1.684695 2.080051 2.505839 2.772428 3.188435 3.764181
22 5.01270411 0.4227128 0.7595167 1.1599610 1.534074 1.909812 2.330081 2.646126 3.021634 3.509370
23 1.15102535 0.4915442 0.9019985 1.2787664 1.661994 2.056323 2.464815 2.827318 3.173052 3.655183
24 6.14665581 0.4686858 0.7908731 1.1438454 1.466187 1.847115 2.187479 2.535910 2.854950 3.193388
25 4.77716082 0.4318794 0.7751609 1.1385405 1.478439 1.834457 2.193984 2.498816 2.843341 3.224181
26 3.18776884 0.4597993 0.8188045 1.1850530 1.529124 1.886360 2.241616 2.534259 2.879516 3.246236
27 5.56334288 0.5151662 0.8802021 1.2306765 1.512817 1.884546 2.161492 2.417957 2.757442 3.045158
28 6.89754636 0.3788808 0.6847352 1.0227821 1.363123 1.711705 2.089698 2.442824 2.768812 3.163898
29 5.07118127 0.4883727 0.8998931 1.2841121 1.603876 1.992420 2.327337 2.601506 2.974916 3.416058
30 4.68662238 0.3448255 0.6202673 0.9313995 1.301565 1.624697 2.027294 2.434595 2.720117 3.063200
31 8.99270992 0.4199137 0.7499798 1.1046403 1.412695 1.791959 2.132326 2.472079 2.810212 3.213535

32 0.48255386 0.5010945 0.9499758 1.3581551 1.755816 2.154989 2.598886 2.889392 3.297267 3.900473
33 5.54551818 0.4386052 0.7476250 1.0746396 1.424572 1.806942 2.185305 2.625651 2.908146 3.267432
34 5.13529172 0.2561379 0.4541258 0.7740516 1.180196 1.458359 1.934757 2.289739 2.612860 3.007334
35 2.88846023 0.4838136 0.8091668 1.1829381 1.546064 1.922248 2.306749 2.641451 2.975263 3.340044
36 5.21864139 0.5258733 0.8317238 1.2331028 1.567072 1.973715 2.315015 2.652762 2.981170 3.285797
37 3.53066863 0.4097578 0.7242765 1.0773185 1.448567 1.793615 2.199785 2.533066 2.869552 3.268853
38 3.56160587 0.5188822 0.8604666 1.2340589 1.559329 1.936433 2.263711 2.548704 2.887293 3.192295
39 4.16272681 0.4420147 0.7802244 1.1342693 1.481815 1.842215 2.211478 2.542109 2.873978 3.250207
40 3.17514200 0.4668744 0.8361861 1.2048425 1.558243 1.928160 2.304528 2.612476 2.964777 3.382604
41 6.61939036 0.4094133 0.6798024 1.0564137 1.402486 1.755576 2.139017 2.441468 2.791931 3.157257
42 6.32742286 0.4656462 0.7812104 1.1448205 1.470611 1.844675 2.205210 2.500232 2.854220 3.238029
43 3.58895970 0.4452893 0.7769361 1.1621630 1.540962 1.917043 2.332632 2.675744 3.026030 3.466263
44 5.66061710 0.4593857 0.8007672 1.1812418 1.506882 1.882289 2.213860 2.529876 2.865739 3.206999
45 0.16032705 0.4324112 0.7790812 1.1567936 1.575638 1.932042 2.394038 2.737204 3.091641 3.570769
46 4.82609488 0.2689564 0.4344759 0.7552650 1.173772 1.456732 1.954899 2.323503 2.642328 3.027836
47 3.95324032 0.3923713 0.7107229 1.0591008 1.441430 1.785899 2.229508 2.552915 2.913380 3.399512
48 8.53152182 0.3979195 0.6963115 1.0750628 1.427990 1.806164 2.234956 2.551883 2.935868 3.459871
49 4.20859280 0.4891426 0.8122398 1.1418250 1.484377 1.871432 2.235297 2.638099 2.930334 3.267242
50 5.28608552 0.4185889 0.7412643 1.1249587 1.508071 1.885403 2.338542 2.665659 3.046796 3.583287
51 3.67826568 0.4995888 0.8619702 1.2256528 1.576787 1.978217 2.344488 2.734361 3.047790 3.431803
52 -1.67326391 0.5625923 0.9962314 1.3735277 1.735888 2.127430 2.465163 2.802989 3.119934 3.452068
53 4.27437713 0.4136964 0.7009086 1.0558892 1.438577 1.793333 2.238374 2.573175 2.926125 3.379566
54 5.96321226 0.4766974 0.7653977 1.1175844 1.429139 1.801930 2.106595 2.465353 2.753195 2.973195

55 -0.61123740 0.4878910 0.8398622 1.2044330 1.621065 2.003588 2.457209 2.851897 3.177689 3.626928
56 0.88044891 0.4293545 0.7995718 1.1687614 1.566675 1.918586 2.354088 2.673844 3.034017 3.513607
57 1.22258582 0.4400619 0.8088925 1.1543364 1.535285 1.890610 2.295252 2.652962 2.978903 3.398140
58 6.02150261 0.4407976 0.7662785 1.1596969 1.503809 1.874114 2.256497 2.531191 2.909603 3.342099
59 2.84012601 0.5940063 1.0204602 1.4237534 1.758227 2.192680 2.541539 2.840692 3.212200 3.658753
60 4.34236955 0.4385226 0.7699652 1.1515591 1.510559 1.872529 2.266309 2.558818 2.924136 3.348842
61 0.01147987 0.3608615 0.7033373 1.0470682 1.461202 1.762721 2.229661 2.508604 2.880267 3.370518
62 1.89806743 0.4825040 0.8193389 1.1735274 1.551006 1.934523 2.331193 2.734682 3.036986 3.405443
63 5.58612882 0.4534495 0.8310812 1.1968387 1.538241 1.923591 2.305685 2.627178 2.990798 3.475314
64 3.59689502 0.4527500 0.8072818 1.1884190 1.557157 1.934351 2.335637 2.667289 3.021287 3.468131
65 6.04413878 0.4168118 0.7457412 1.1165234 1.462981 1.826016 2.212349 2.520117 2.880511 3.317037
66 3.05727740 0.6087439 1.0728553 1.4828642 1.799064 2.245582 2.550536 2.866096 3.221953 3.634962
67 2.97958488 0.4566882 0.8186722 1.2075190 1.583060 1.956264 2.371847 2.670550 3.046354 3.525708
68 4.04811073 0.4800686 0.8204727 1.2038246 1.570040 1.961198 2.369576 2.700669 3.059924 3.513986
69 1.88229620 0.4243357 0.7565974 1.1298185 1.532942 1.893454 2.340190 2.693972 3.041085 3.503725
70 5.10510345 0.4777040 0.8225210 1.2162701 1.563844 1.959960 2.327593 2.670375 3.013542 3.412307
71 5.14518407 0.3774390 0.6886247 1.0392808 1.385342 1.707825 2.087007 2.359965 2.715828 3.102370

	V10	V11	V12	sum_of_rows
1	3.859440	4.220095	4.767510	30.56639
2	3.817428	4.144800	4.627682	32.80318
3	4.108345	4.469090	4.880284	31.55098
4	3.738392	4.019272	4.443614	32.75845
5	3.966396	4.363611	4.937894	30.33273
6	3.571040	3.884071	4.329006	33.88596
7	3.361010	3.619096	4.115845	32.72921
8	3.734576	4.073295	4.586887	30.31015
9	3.607053	3.926095	4.384603	32.88932
10	3.824173	4.140035	4.734985	29.89072
11	3.573250	3.953203	4.339992	33.46352

12	3.554206	3.873318	4.301350	33.03952
13	3.652651	4.013900	4.399007	33.89476
14	3.650511	3.927989	4.430334	32.66668
15	3.669451	4.042474	4.534014	30.99632
16	3.431451	3.731485	4.144090	32.90780
17	3.566071	3.836809	4.263403	31.68979
18	3.817825	4.106241	4.571008	32.66914
19	3.808968	4.187987	4.664867	32.70827
20	3.537520	3.788101	4.271595	32.35065
21	4.085001	4.631305	4.901604	35.47148
22	3.834238	4.297272	4.642019	35.07952
23	4.057118	4.508758	4.871736	33.09963
24	3.551760	3.872824	4.277090	34.33676
25	3.556213	3.905491	4.315588	32.97325
26	3.599768	3.933909	4.385794	31.88801
27	3.383588	3.648664	4.064988	33.06604
28	3.469549	3.830040	4.214650	34.03824
29	3.744990	4.161003	4.426688	34.99235
30	3.407762	3.706412	4.232426	31.10118
31	3.512873	3.896072	4.158130	36.66712
32	4.256992	4.822964	5.114939	34.08350
33	3.656409	3.994133	4.410392	34.08537
34	3.275550	3.610941	4.247023	30.23637
35	3.728099	4.071417	4.572996	32.46871
36	3.695069	4.003892	4.457279	34.74111
37	3.609034	3.970066	4.462671	31.89723
38	3.571937	3.859271	4.354263	32.34825
39	3.601131	3.945974	4.385716	32.65386
40	3.735749	4.121524	4.531865	32.82297
41	3.468456	3.811694	4.276854	34.00976
42	3.554928	3.916559	4.328150	34.63171
43	3.829673	4.245310	4.668365	33.67537
44	3.564959	3.889901	4.269004	34.03152
45	3.948617	4.386267	4.899307	31.06414
46	3.309228	3.640109	4.343007	30.15621
47	3.699649	4.139133	4.588436	32.86530
48	3.720696	4.219859	4.506596	37.56470
49	3.668046	3.983924	4.453646	33.18420
50	3.886823	4.393019	4.745301	35.62380
51	3.843206	4.211276	4.596799	34.03020
52	3.926996	4.234706	4.726989	29.85125
53	3.697943	4.114960	4.610894	33.21782
54	3.369381	3.580808	4.084876	32.88736
55	4.058029	4.474808	5.010319	31.20248
56	3.868243	4.301842	4.763597	31.27264
57	3.778147	4.154135	4.626949	30.93626

58 3.647787 4.057545 4.430114 34.94103
59 4.060526 4.498691 4.806699 35.44835
60 3.675856 4.071091 4.497161 33.42772
61 3.670666 4.090328 4.658403 28.75512
62 3.832422 4.177541 4.676709 32.05394
63 3.792584 4.247220 4.528409 35.49652
64 3.825224 4.244831 4.634395 33.73365
65 3.622747 4.029597 4.392643 34.58721
66 4.065804 4.477449 4.710099 35.79329
67 3.866029 4.313564 4.712203 33.50804
68 3.871239 4.304186 4.702919 34.60621
69 3.870661 4.296956 4.775418 32.14145
70 3.787924 4.175515 4.531019 35.06368
71 3.385680 3.727135 4.195839 31.91752
[reached 'max' / getOption("max.print") -- omitted 929 rows]

sum_of_rows is the synthesized 1000 data points from Rachmaninov distribution using the equation given in the problem.

D)

i. The measures are generally centered around μ =0. When we set the true value of μ a nonzero value, it shifts the distribution of u and has no effect on the variance necessarily and also on the orthonormal basis. The variability of the sampling distribution of the measures would decrease as the maximum variance increases but increases as the sample size also increases.

ii. Same reasoning applies as (i) but one more thing to keep in mind is that just because both of the points are normal random variables - it does not mean that their sum will be random.

Q3.

```
> #######Q3#######
mystery <- as.matrix(read.csv("mystery.csv"))
cov_mystery <- cov(mystery) #covariance matrix
mystery.svd <- svd(mystery) #singular value decomposition

> mystery.svd

$d
 [1] 2.073163e+02 1.710831e+02 1.338359e+02 1.230630e+02 7.400490e+01 1.568536e-13
1.185302e-13
 [8] 1.107252e-13 1.051540e-13 9.987181e-14 8.942903e-14 8.085713e-14 7.327570e-14
7.025569e-14
[15] 6.882758e-14 6.770368e-14 6.585686e-14 6.459208e-14 5.667311e-14 4.512825e-14

$u
             [,1]         [,2]         [,3]         [,4]         [,5]         [,6]
```

```
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 [2,]  2.880919e-02  2.905414e-02 -4.906980e-02  3.443017e-02  3.684605e-03  4.601277e-01
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  [2,] -2.994259e-02  2.336855e-02
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 [8,]  2.442729e-02 -3.695948e-03
 [9,]  2.352466e-05 -8.702273e-02
[10,]  1.478967e-02  8.270919e-02
[11,]  2.499965e-02  9.178661e-03
[12,] -1.632441e-02  1.242142e-02
[13,]  8.628951e-03 -8.264359e-03
[14,]  1.467613e-02  3.372798e-02
[15,] -3.393636e-02  1.161006e-02
[16,] -1.914772e-02  9.156804e-03
[17,]  6.286895e-03  7.333088e-02
[18,]  2.569315e-02  5.397072e-03
[19,] -2.266160e-02 -1.155852e-03
[20,] -2.354484e-02 -6.291715e-02
[21,]  3.190729e-02 -9.599630e-03
[22,]  2.518774e-02 -1.217862e-02
[23,] -1.244608e-02  8.990568e-03
[24,] -3.575628e-03  1.174705e-01
[25,] -7.061673e-02 -1.157246e-02
[26,] -1.091375e-02  2.527511e-03
[27,] -1.592327e-02 -1.297408e-02
[28,]  3.480802e-03 -5.889869e-02
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[32,]  3.601092e-02 -5.103282e-03
[33,] -1.534099e-02  8.802388e-03
[34,]  6.321563e-02  4.452798e-03
[35,] -6.381714e-02 -3.352234e-02
[36,]  1.908115e-03  2.369938e-02
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[38,]  1.434929e-02  8.902880e-03
[39,] -2.590269e-04 -2.686510e-02
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[44,] -1.664118e-02 -4.006352e-02
[45,] -1.417900e-04  7.570012e-02
[46,] -3.385124e-03  1.189914e-02
[47,]  2.570969e-02  1.000973e-01
[48,] -2.619192e-02  5.811793e-03
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  [50,]  4.732477e-02 -1.119653e-03
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-0.177430285
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[19,] -0.338814267  0.038259548 -0.179100946 -0.074582108 -0.044303443 -0.0744445345
[20,] -0.009425493  0.005182024  0.007884365  0.012871138 -0.023241929  0.0308795501
```

b)
The dimension of the hyperplane is n-1 where n is the number of columns of the matrix mystery (mystery <- as.matrix(read.csv("mystery.csv"))).

c)

```
####Q3 - part c #####

# Installing required package
install.packages("dplyr")

# Loading the package
library(dplyr)

mystery <- as.matrix(read.csv("mystery.csv"))

# Importing file
>str(mystery)

num [1:1000, 1:20] -2.1064 -1.2097 0.0423 -0.0203 -1.1918 ...
 - attr(*, "dimnames")=List of 2
  ..$ : NULL
  ..$ : chr [1:20] "V1" "V2" "V3" "V4" ...


data <- as.matrix(read.csv("mystery.csv"))

# Apply PCA using prcomp function
# Need to scale / Normalize as
# PCA depends on distance measure
```

```
my_pca <- prcomp(mystery, scale = TRUE,
                 center = TRUE, retx = T)
>names(my_pca)

[1] "sdev"     "rotation" "center"   "scale"    "x"

# Summary
>summary(my_pca)

Importance of components:
                          PC1    PC2    PC3    PC4     PC5       PC6       PC7       PC8       PC9
Standard deviation     2.5843 2.2996 2.0009 1.5649 1.25733 2.631e-15 2.508e-15 2.013e-15
1.868e-15
Proportion of Variance 0.3339 0.2644 0.2002 0.1225 0.07904 0.000e+00 0.000e+00 0.000e+00
0.000e+00
Cumulative Proportion  0.3339 0.5983 0.7985 0.9210 1.00000 1.000e+00 1.000e+00 1.000e+00
1.000e+00
                          PC10     PC11      PC12      PC13      PC14      PC15      PC16
Standard deviation     1.578e-15 1.47e-15 1.411e-15 1.364e-15 1.349e-15 1.237e-15 1.164e-15
Proportion of Variance 0.000e+00 0.00e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
0.000e+00
Cumulative Proportion  1.000e+00 1.00e+00 1.000e+00 1.000e+00 1.000e+00 1.000e+00
1.000e+00
                          PC17      PC18      PC19      PC20
Standard deviation     1.121e-15 1.111e-15 1.033e-15 9.318e-16
Proportion of Variance 0.000e+00 0.000e+00 0.000e+00 0.000e+00
Cumulative Proportion  1.000e+00 1.000e+00 1.000e+00 1.000e+00




# View the principal component loading
# my_pca$rotation[1:5, 1:4]
> my_pca$rotation

            PC1         PC2         PC3         PC4         PC5         PC6         PC7
V1   0.290508762  0.22329525 -0.12938533 -0.201034764 -0.065076152  0.10896268
-0.106226706
V2  -0.225297704 -0.31993866  0.01234387 -0.046634848  0.268269051 -0.20703159
0.074087510
V3  -0.038485533  0.25595574  0.22676669 -0.292237321 -0.380308469 -0.70188486
-0.297899918
V4   0.246920407 -0.17756842 -0.11979190  0.383634297 -0.072005002 -0.20288827
0.049982803
V5  -0.067400745  0.24317876  0.26451632  0.325642833  0.272197347 -0.11076819
0.075894064
```

V6 -0.050176459 0.33450844 0.30523322 0.086810125 0.003477128 -0.03792438
0.353948555
V7 -0.015159819 0.13193938 -0.46145792 -0.145953535 -0.033004546 -0.10341297
0.515350217
V8   0.007393602 0.22368684 0.17954312 -0.413826721 0.343560602 -0.02105214
0.118835235
V9 -0.181701202 0.31811943 -0.08548886 0.291690684 -0.065224870 0.06002652
0.059449873
V10 -0.244335763 -0.05469524 -0.32704610 -0.188327764 -0.211026950 -0.08307279
0.303749948
V11 -0.367569257 -0.11705628 -0.07100227 0.041994991 0.021262422 -0.15623507
0.008721104
V12 0.049603413 0.39101878 -0.15507378 0.179292092 0.005179062 0.12982813
-0.061430260
V13 -0.332136640 0.08453845 -0.11461951 0.165337413 -0.258843789 0.21737701
-0.422646538
V14 -0.265393354 0.31406970 0.04274998 0.005182458 -0.019552619 0.23801840
-0.028470905
V15 -0.293929211 -0.13672448 0.15696586 -0.122643824 -0.345472721 0.20202323
0.084326616
V16 0.093774659 -0.06825756 0.28407152 0.340367855 -0.442720199 -0.11318497
0.390291474
V17 -0.278304994 -0.22513092 0.06403231 0.207750581 0.242004283 -0.15509905
-0.096919312
V18 -0.072285026 0.23520808 -0.33636579 0.225932559 0.244596596 -0.38081151
-0.094646616
V19 0.285965260 -0.04062692 -0.30834860 0.096576396 -0.162281983 -0.07643521
-0.155609815
V20 0.352669018 -0.06835983 0.18177116 0.049498323 0.063413853 0.04898515
-0.012009775
          PC8   PC9   PC10   PC11   PC12   PC13   PC14
V1   0.030138937 -0.10836544 0.16838778 -0.0245997556 0.528966132 -0.060822736
0.05245827
V2   0.447852375 -0.26697849 0.20183589 -0.2494285551 0.102124476 0.076047792
0.32518698
V3   0.075060940 0.04347068 -0.13874068 -0.0320520772 0.081658913 0.038065718
-0.01934756
V4 -0.017910846 0.04125827 0.04735443 -0.0031903911 0.082846774 0.074832509
0.06425956
V5 -0.325394987 -0.33353652 -0.06702414 -0.0607640586 0.199793633 0.513632079
-0.30837800
V6 -0.070056741 0.25873099 -0.02176184 0.3500428591 -0.086986312 -0.193817131
0.21390637
V7   0.015697520 -0.24509418 -0.58390312 0.0035577843 0.175632425 -0.006329479
0.10852189

V8 -0.156680950 -0.26605765 0.25397695 0.0131680498 -0.037593149 -0.061686977
0.29380616
V9   0.029504104 -0.03010182 0.08180639 -0.2536644886 -0.031652776 -0.344574546
-0.11808720
V10 -0.108288473 0.24336580 0.44473660 0.3215866744 -0.010917885 0.434531988
-0.06029959
V11 -0.136372453 -0.05714867 0.27202840 0.0497390431 0.450718396 -0.290835364
-0.28858981
V12 -0.006840676 0.21940886 0.22555893 -0.1584021159 0.287556824 -0.052815296
0.29472982
V13 -0.075877589 -0.44992959 -0.06667697 0.4140716345 -0.054263031 0.087059516
0.37114148
V14 0.623338786 0.18489946 -0.09999721 -0.0921350467 0.067817586 0.350716003
-0.14463431
V15 -0.414335399 0.14478768 -0.09697531 -0.5856139941 0.044511239 0.153303864
0.27008117
V16 0.217802679 -0.26481094 0.20278925 0.0007992019 0.006014589 -0.065453944
0.14521159
V17 -0.037099354 0.35768631 -0.29909905 0.1952848100 0.356772532 -0.087718086
0.24673456
V18 -0.085999726 0.10076917 0.11576967 -0.1976859111 -0.378651289 0.087598477
0.23363478
V19 -0.058989727 -0.03244073 0.03830169 -0.0878618634 0.054342455 0.086563354
0.01106986
V20 0.008128043 0.16106824 -0.01847708 0.0811257163 0.215301611 0.319510904
0.30337197
         PC15   PC16   PC17   PC18   PC19   PC20
V1   0.59523775 -0.26510687 -0.07196672 0.05061919 0.113017711 0.090691056
V2   0.01376335 0.11661783 -0.22985120 0.32506504 0.220915017 -0.079883660
V3 -0.06897918 0.13834190 -0.07039234 0.02850346 -0.034457690 0.054912273
V4   0.05394926 -0.03421758 0.02758539 0.42802174 -0.661293284 0.225622613
V5   0.06861367 0.10787468 0.02018437 0.08144891 0.140417146 -0.030654353
V6   0.02995886 -0.22884957 -0.13503358 0.46025625 0.144780374 -0.260981590
V7 -0.14189119 -0.01166968 -0.02732492 -0.04476616 -0.014262240 0.059888934
V8 -0.02978869 0.10136273 0.22959822 -0.16357820 -0.491940351 -0.172128522
V9   0.17214906 0.33691798 -0.54675374 -0.18976851 -0.228561360 -0.158642420
V10 0.13972527 0.22556713 -0.10979291 -0.06175495 -0.005453431 0.012152005
V11 -0.39076654 -0.42802148 -0.01269986 -0.05746317 -0.078105000 -0.068047444
V12 -0.39656359 0.34396846 0.27411224 0.10718513 0.208208742 0.256400567
V13 -0.03060435 -0.05085568 -0.08284045 0.05317697 -0.054419213 0.025332222
V14 -0.01916495 -0.23096379 0.16674875 -0.01760765 -0.281765564 -0.142339573
V15 0.07194685 -0.15462308 0.00170061 0.09863247 -0.062224985 -0.043083783
V16 0.08215102 -0.05261694 0.29612368 -0.37227128 0.094486729 0.006847458
V17 0.32359121 0.20658454 0.20998726 -0.25961723 -0.050722888 -0.135493553
V18 0.12841550 -0.44734779 0.04145146 -0.22230067 0.107084871 0.081116629
V19 -0.09075112 0.06131047 0.16195683 0.08136426 0.045906662 -0.827366411

V20 -0.33078973 -0.16221930 -0.53242283 -0.36196191 -0.063393033 -0.035228714

See the principal components
> dim(my_pca$x)

[1] 1000   20

> my_pca$x

	PC1	PC2	PC3	PC4	PC5	PC6
[1,]	-0.200869475	-4.564292913	2.800919e+00	-0.331382945	-1.1054428269	2.563975e-15
[2,]	-0.360568324	-1.493571535	4.144423e+00	-1.173362131	0.2156264778	1.319890e-15
[3,]	0.485395356	-0.985149316	-4.482148e+00	3.134803767	-2.1938137678	-4.524848e-15
[4,]	2.649199168	-1.231285484	1.519769e+00	1.210785185	1.2209119239	-3.505519e-15
[5,]	0.384850663	-2.795266188	-2.023589e-01	0.903504508	1.1350933626	-4.200674e-16
[6,]	-3.974544794	-0.915434592	-8.671602e-01	-0.709032978	-0.0481395938	3.376680e-16
[7,]	2.130849225	-1.336365484	1.342359e-01	-0.881688480	0.2360216243	-5.656581e-17
[8,]	1.062783892	0.454888169	-1.868122e+00	2.056430802	-2.0539740964	-1.279506e-15
[9,]	-1.509306412	-1.921821815	2.966359e+00	-1.524589108	0.9641005532	3.616221e-15
[10,]	2.223413737	3.478674719	-2.247785e+00	0.825253795	-0.0760847258	-2.806903e-15
[11,]	-3.627341103	-3.159692326	8.154630e-01	-0.831474952	-3.7572841116	6.974303e-15
[12,]	0.030553019	2.985182282	2.020039e+00	-0.533100391	-1.4675692582	4.474009e-15
[13,]	-4.626039212	-1.964375924	-1.586032e+00	1.022382229	-1.3312871236	-7.575394e-16
[14,]	2.497377952	0.630143729	-4.882796e+00	3.364600733	-0.0233990490	-7.133884e-15
[15,]	1.325195576	-1.495957626	-2.179614e+00	-2.077635453	1.0884737389	8.672738e-16
[16,]	1.574590403	-1.183135414	1.279944e+00	-1.423418840	-0.6522035734	3.703670e-15
[17,]	1.318032299	-0.323460659	5.161668e+00	0.623361359	-0.8654757606	1.128568e-15
[18,]	3.167923029	1.133784753	1.473452e+00	0.224379661	-0.5116410189	1.645334e-15
[19,]	0.693293596	0.611491653	8.062201e-01	-0.882609144	-0.0928441320	3.724038e-15
[20,]	-0.700941620	2.301340445	-2.394873e+00	-0.562685958	-0.6527471195	1.485747e-15
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[31,]	-4.271946869	-2.312220080	-2.332133e+00	-1.333988576	-1.8596771360	3.905774e-15
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[34,]	2.752218638	2.707501446	-5.267348e-01	-2.089148393	0.7206109519	3.389947e-17
[35,]	1.789680218	-2.327874066	2.965441e+00	-1.143116793	0.6809660417	3.800228e-15
[36,]	-1.307587117	-2.429602447	6.976278e-01	-3.994429942	-1.6330532224	5.490306e-15

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[38,] 3.338620686 4.145552677 -3.469976e+00 -1.130069691 0.1328282449 3.347190e-16
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 [2,] 3.042507e-15 1.876831e-15 1.330367e-15 1.554601e-15 4.442859e-15 -1.952000e-15
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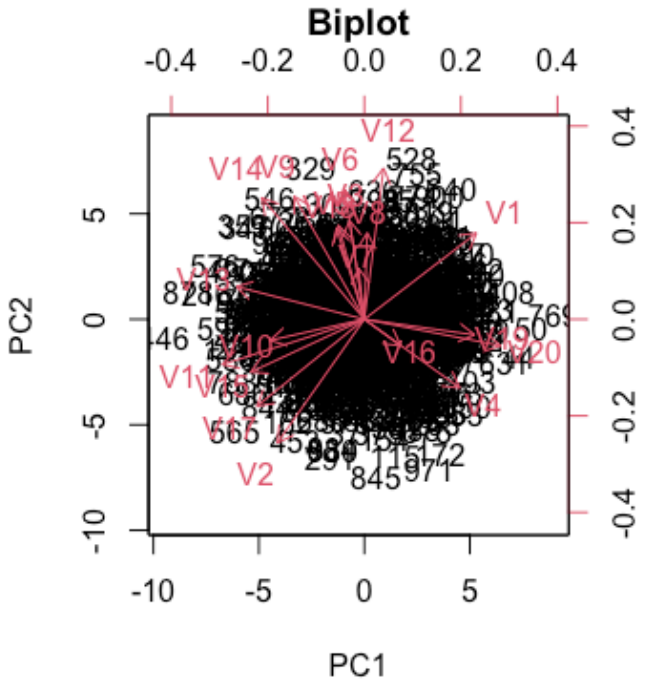
 [25,] -1.044062e-15 -8.151925e-17 -3.688936e-17 -8.520447e-16 -7.424396e-16
-4.008062e-16
 [26,] -7.846161e-16 -1.251916e-15 -1.232053e-15 -4.373614e-16  1.407085e-15  2.606447e-16
 [27,] -9.743601e-16 -7.083291e-16 -1.053282e-15 -2.830463e-15 -4.124675e-16  9.152661e-16
 [28,]  1.748428e-15 -6.548769e-16 -2.127778e-15  3.081814e-16  2.823306e-15 -7.875897e-17
 [29,]  1.431852e-15  2.240617e-16  2.019120e-15 -9.941774e-16 -1.809271e-16  3.051371e-16
 [30,] -3.028208e-15  5.868714e-16 -1.260510e-15  2.454316e-15  2.156270e-16  9.239710e-16
 [31,] -5.944181e-16  1.226420e-15  3.276241e-16 -1.891050e-16 -1.420782e-15  2.983998e-15
 [32,] -1.030008e-15 -2.285487e-15 -1.126382e-15 -2.103511e-15 -5.919926e-16
-5.162659e-16
 [33,] -2.130983e-15  2.798809e-15 -9.553231e-16 -1.062229e-15  8.742901e-16 -4.096381e-16
 [34,] -8.630121e-16 -5.172303e-16  1.849380e-15  4.801605e-16  3.957365e-16 -1.313805e-15
 [35,]  1.546564e-15  8.239157e-16  1.175459e-16  1.360257e-15 -1.291302e-15  1.770898e-15
 [36,] -8.869655e-16  2.020290e-15 -3.641280e-17 -9.446396e-16 -8.840649e-18  1.745164e-15
 [37,]  5.235927e-16 -1.119809e-15  3.403457e-17  1.407171e-15 -8.374419e-16 -2.025914e-15
 [38,] -1.020610e-15 -2.191401e-15  1.630219e-15  5.798564e-16 -2.799126e-17 -8.609978e-16
 [39,]  1.033927e-15  9.686498e-16  1.579687e-16  2.162281e-16  4.594200e-16  3.348513e-16
 [40,]  5.301158e-16 -7.241353e-16 -7.412775e-17  1.212620e-16  4.887285e-16 -5.808477e-16
 [41,]  2.781333e-16  5.133150e-16 -9.710759e-16  1.393168e-15 -4.831243e-16 -2.150437e-15
 [42,]  5.615704e-16  4.516483e-16  4.839350e-16 -6.443626e-16 -1.266554e-15 -9.724335e-16
 [43,] -1.720207e-15 -3.520809e-15  9.270924e-16  1.701774e-16 -9.836593e-16 -7.551666e-17
 [44,]  1.010672e-15 -9.457961e-16  2.100994e-15  1.911869e-15  3.828413e-15  5.373998e-16
 [45,] -1.999176e-15  2.550975e-15 -3.261241e-15 -3.761908e-16  1.170143e-15 -3.932378e-16
 [46,] -8.222672e-16  4.155377e-16 -6.781952e-16  1.068269e-16 -1.776936e-15 -4.143249e-16
 [47,] -7.992741e-16  2.236369e-16 -1.181600e-15 -5.371741e-16  5.665683e-16  8.739592e-16
 [48,]  4.578388e-17 -8.251813e-16 -2.777452e-16  6.128582e-16 -3.339795e-16  7.873280e-16
 [49,] -1.927309e-15  6.240149e-17 -6.831640e-16 -6.423289e-16  1.752287e-16  6.606138e-16
 [50,]  3.246322e-15 -7.358822e-16 -9.471025e-16  5.710351e-16  6.067718e-16 -8.353504e-16
              PC19          PC20
 [1,]  9.353081e-16 -1.313415e-15
 [2,]  4.730353e-16  1.364412e-15
 [3,]  1.412791e-15 -5.702224e-16
 [4,]  4.721166e-16  4.819068e-16
 [5,]  1.630459e-16  1.898847e-16
 [6,]  4.896785e-16 -1.409695e-15
 [7,]  5.154145e-16  1.990479e-16
 [8,]  2.523907e-16  7.950532e-16
 [9,]  1.208817e-16 -5.341843e-16
[10,] -1.198252e-16 -7.502775e-16
[11,]  2.358203e-15  1.250666e-16
[12,]  2.630599e-16  5.296191e-16
[13,] -1.126181e-15 -4.754404e-16
[14,]  3.222255e-15 -1.701317e-16
[15,]  6.808361e-17  3.487832e-16
[16,]  1.034128e-15  6.130206e-16
[17,]  2.213715e-16  4.913176e-16
```

```
 [18,]  1.052102e-16  1.183019e-15
 [19,] -2.957593e-16 -3.124037e-16
 [20,]  9.269535e-18 -9.939854e-16
 [21,] -9.507107e-16 -1.893611e-15
 [22,]  1.153317e-15  8.543142e-16
 [23,]  1.381805e-15  7.041817e-16
 [24,] -3.772687e-16 -1.083654e-15
 [25,] -1.914636e-15  1.495190e-15
 [26,]  9.316218e-16 -2.041927e-16
 [27,] -1.968365e-16 -1.530502e-15
 [28,] -2.120991e-15 -2.607671e-16
 [29,]  9.419822e-16 -2.000054e-17
 [30,]  1.689340e-15  8.860171e-16
 [31,]  7.819751e-16  4.572462e-16
 [32,] -3.945653e-16 -9.421317e-16
 [33,] -1.384568e-15  1.673122e-16
 [34,]  1.094616e-15 -5.321174e-16
 [35,]  6.426231e-16  1.534550e-15
 [36,] -6.284577e-16  3.398279e-16
 [37,]  8.543891e-16 -5.974886e-16
 [38,] -3.450517e-16  1.629056e-15
 [39,]  1.772702e-15 -1.129137e-15
 [40,] -3.821182e-16  1.601797e-15
 [41,]  4.638255e-18 -8.944584e-16
 [42,]  1.598229e-15  1.022190e-15
 [43,]  1.925989e-15 -2.519634e-15
 [44,]  2.934647e-15  7.538194e-16
 [45,] -5.553776e-16  5.115036e-16
 [46,]  2.000152e-15  3.164946e-17
 [47,]  2.854230e-16  4.216045e-16
 [48,]  4.949197e-16  8.273350e-17
 [49,] -1.403037e-15  5.121364e-16
 [50,]  1.439087e-15  6.004299e-16
 [ reached getOption("max.print") -- omitted 950 rows ]


# Plotting the resultant principal components
# The parameter scale = 0 ensures that arrows
# are scaled to represent the loadings
> biplot(my_pca, main = "Biplot", scale = 0)
```



Compute standard deviation
> my_pca$sdev

[1] 2.584297e+00 2.299585e+00 2.000875e+00 1.564913e+00 1.257326e+00 2.630646e-15 2.508323e-15
[8] 2.012537e-15 1.867531e-15 1.578043e-15 1.470052e-15 1.411042e-15 1.363986e-15 1.348657e-15
[15] 1.236797e-15 1.163937e-15 1.121026e-15 1.110799e-15 1.033126e-15 9.317638e-16

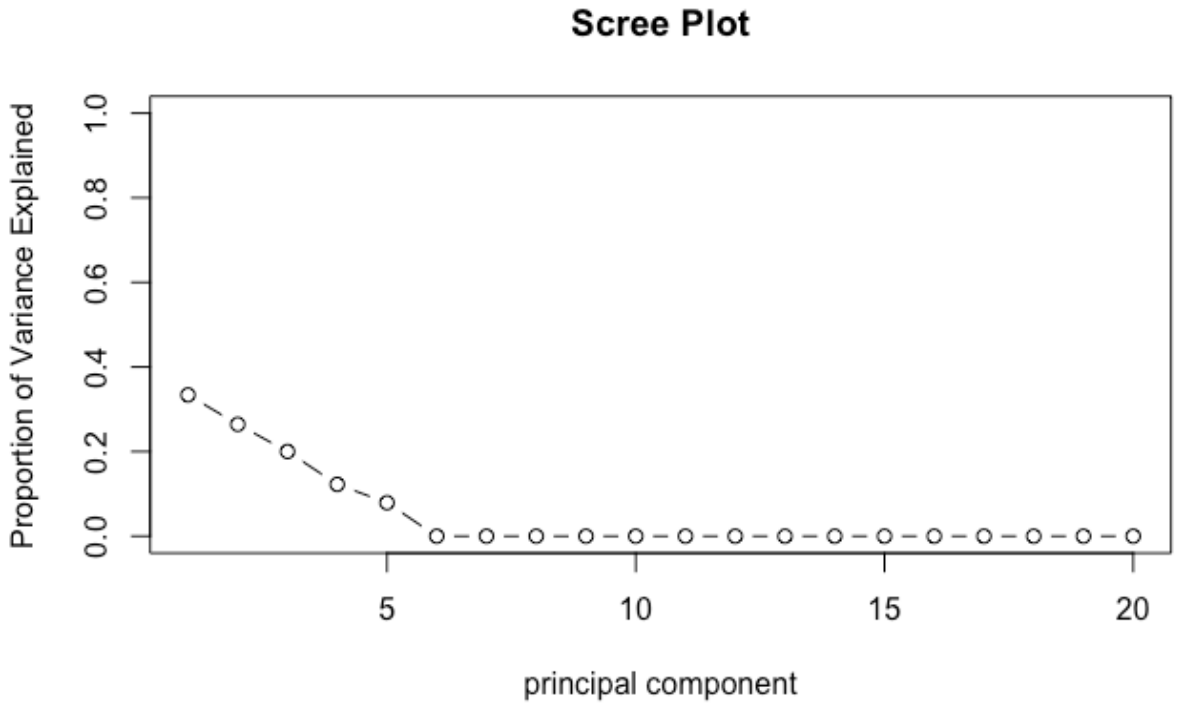
# Compute variance
my_pca.var <- my_pca$sdev ^ 2
> my_pca.var

[1] 6.678589e+00 5.288089e+00 4.003502e+00 2.448952e+00 1.580868e+00 6.920299e-30 6.291686e-30
[8] 4.050304e-30 3.487673e-30 2.490220e-30 2.161054e-30 1.991038e-30 1.860457e-30 1.818876e-30
[15] 1.529667e-30 1.354750e-30 1.256700e-30 1.233875e-30 1.067350e-30 8.681838e-31

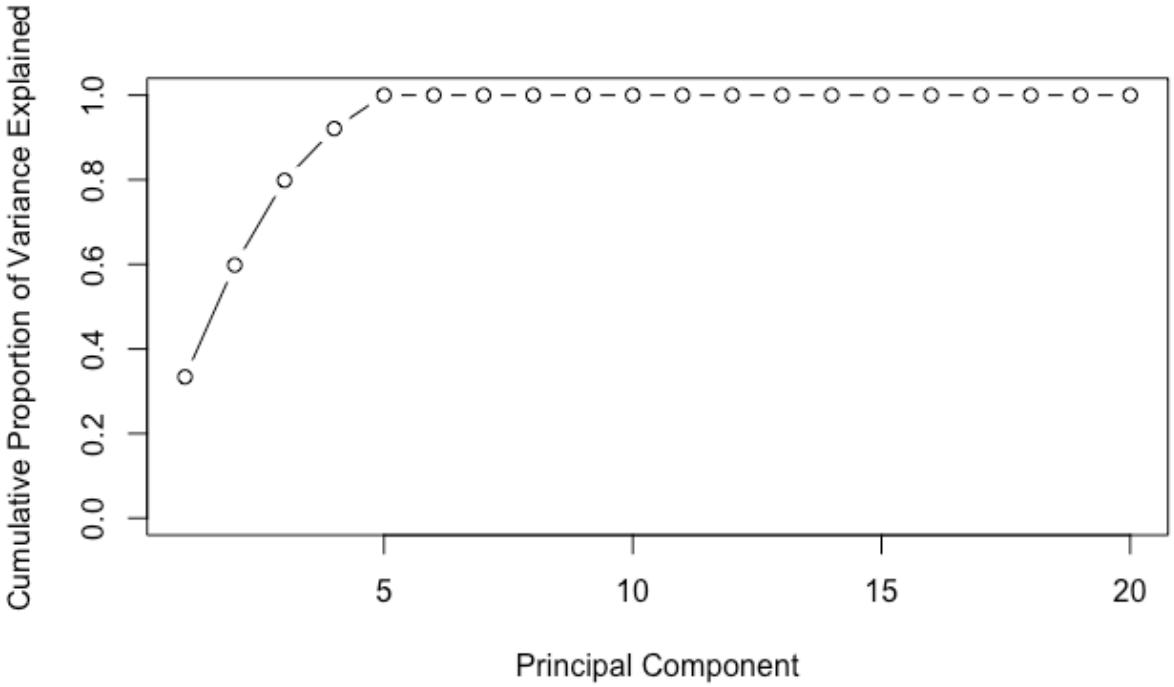
# Proportion of variance for a scree plot
propve <- my_pca.var / sum(my_pca.var)

```
> propve
[1] 3.339294e-01 2.644045e-01 2.001751e-01 1.224476e-01 7.904340e-02 3.460149e-31
3.145843e-31
 [8] 2.025152e-31 1.743837e-31 1.245110e-31 1.080527e-31 9.955192e-32 9.302287e-32
9.094380e-32
[15] 7.648333e-32 6.773751e-32 6.283500e-32 6.169375e-32 5.336751e-32 4.340919e-32

# Plot variance explained for each principal component
plot(propve, xlab = "principal component",
     ylab = "Proportion of Variance Explained",
     ylim = c(0, 1), type = "b",
     main = "Scree Plot")
```




```
# Plot the cumulative proportion of variance explained
plot(cumsum(propve),
     xlab = "Principal Component",
     ylab = "Cumulative Proportion of Variance Explained",
     ylim = c(0, 1), type = "b")
```



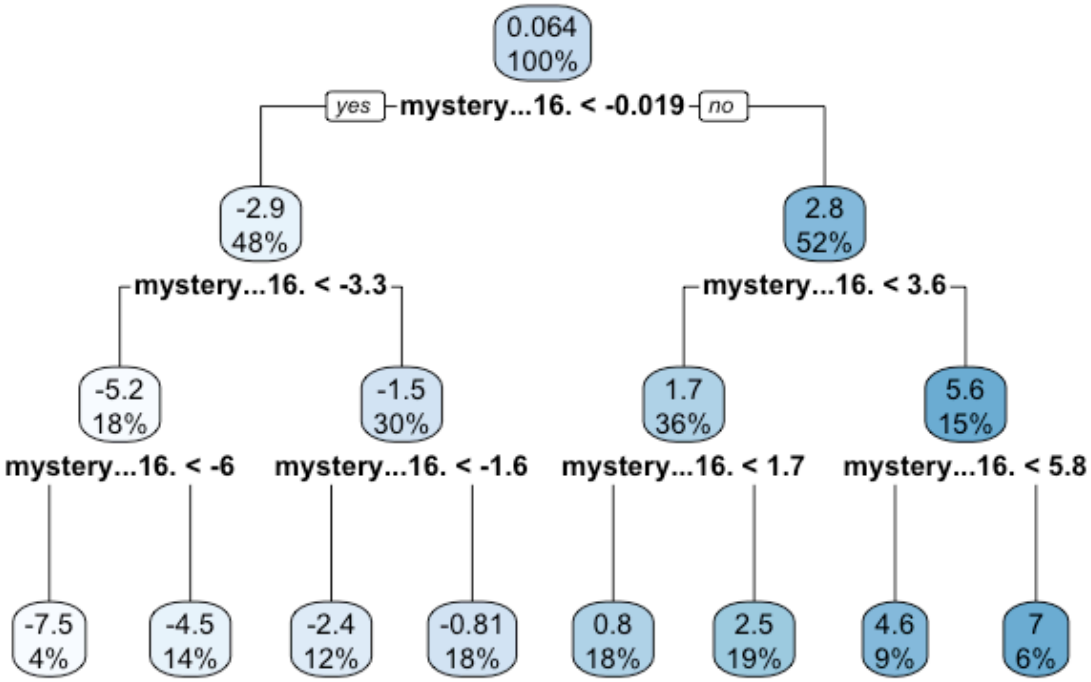

```
# Find Top n principal component
# which will atleast cover 90 % variance of dimension
> which(cumsum(propve) >= 0.9)[1]
[1] 4


# Predict mpg using first 4 new Principal Components
# Add a training set with principal components
train.data <- data.frame(mystery[,16], my_pca$x[, 1:4])

# Running a Decision tree algporithm
## Installing and loading packages
install.packages("rpart")
install.packages("rpart.plot")
library(rpart)
library(rpart.plot)

rpart.model <- rpart( mystery[,16] ~ .,
                      data = train.data, method = "anova")

rpart.plot(rpart.model)
```



It can be observed that the proportion of variance decreases for the remaining principal components, and the first two principal components account for (52 + 48 = 100) of the total variance. Using PCA to convert the original features to lesser new principal components greatly simplifies the computation and analysis without losing fidelity. We have used principal components to reduce dimensions by transforming numerical data into smaller set of weighted averages of the original data that contain fewer variables.

Q4.

R Code is below. Outputs were very large - so R Code file is attached.

```
######## Q4 ########

install.packages("mlbench")

library("mlbench")
```

```
data("Ionosphere")
ion=as.matrix(Ionosphere)
library(dplyr)

good <- ion[which(ion[,35] == "good"), ]
bad <- ion[which(ion[,35] == "bad"), ]

goodmat <- data.matrix(good)
badmat <- data.matrix(bad)

goodmat<- goodmat[,-c(35)]
goodmat<- goodmat[,-c(1,2)]
badmat<- badmat[,-c(35)]
badmat<- badmat[,-c(1,2)]
goodmat<- matrix(as.numeric(goodmat),    # Convert to numeric matrix
                 ncol = ncol(goodmat))
badmat<- matrix(as.numeric(badmat),    # Convert to numeric matrix
                ncol = ncol(badmat))

covGood <- cov(goodmat)
```

```
covBad <- cov(badmat)




mahGood <- data.matrix(mahalanobis(goodmat, colMeans(goodmat),
covGood))
mahBad <- data.matrix(mahalanobis(badmat, colMeans(badmat), covBad))

##Confusion matrix
install.packages("caret")

library(caret)
library(InformationValue)
library(ISLR)

set.seed(1)
mydata <- ion
sample <- sample(c(TRUE, FALSE), nrow(mydata), replace=TRUE,
prob=c(0.7,0.3))
train <- mydata[sample, ]
```

```
test <- mydata[!sample, ]


test <- ifelse(mahGood < mahBad[,1], 1, 0) #1 if bad, 0 if good

predicted <- (mydata=train)

optimal <- optimalCutoff(test[,36], predicted)[1]

confusionMatrix(test$default, predicted)
```