Tutorial 1

1. Discuss the applicability of Rolle's theorem for the function $f(x) = 2 + (x-1)^{\frac{2}{3}}$
2. Discuss applicability of Rolle's theorem to the function $|x|$ in $[-1, 1]$
3. For what values of a, m and b does the function.

$$f(x) = \left\{ \begin{array}{ll} 3, & \text{if } x = 0 \\ -x^2 + 3x + a, & \text{if } 0 < x < 1 \\ mx + b, & \text{if } 1 \leq x \leq 2 \end{array} \right\}$$

satisfy the hypothesis of Mean Value Theorem on the interval $[0, 2]$?
Find the value of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

In the conclusion of Mean Value Theorem for the functions and intervals in exercise 4-5

4. $f(x) = x + \frac{1}{x}$ in $[\frac{1}{2}, 2]$.
5. $f(x) = \left\{ \begin{array}{ll} x^3, & \text{if } -2 \leq x \leq 2 \\ x^2, & \text{if } 0 < x \leq 2 \end{array} \right\}$
6. Find the value of θ Lagrange's Mean Value Theorem $f(a+h) = f(a) + hf'(a+\theta h)$ for the function $f(x) = x^2 + 2px + q$, p and q being constants.
7. Use Lagrange's Mean Value Theorem to show that

$$\frac{x}{1+x} < log(1+x) < x \; \forall \; x$$

8. Discuss the applicability of LMVT of$f(x) = x^{\frac{1}{3}}$in $[-1, 1]$
9. Find the Taylor's polynomial $P_6(x)$ for the following functions
   $(i)\ e^x$ $(ii)\ sinx$ $(iii)\ cosx$ $(iv)a^x$
10. Find the critical points and identify the intervals on which the function is increasing or decreasing
    $(i)\ f(x) = x^3 - 12x - 5$ $(ii)\ f(x) = x^{\frac{1}{3}}(x-4)$.
11. Discuss the maxima and minima for the following functions
    $(i)\ f(x) = (x-1)^3(x-2)(x-4)$ $(ii)\ f(x) = x^5$ $(iii)\ f(x) = c(x-1)^3 - 2$
12. Find the following limits.

    $(i)\ \lim\limits_{x\to 0} \frac{tanx - x}{x^2 tanx}$ $(ii)\ \lim\limits_{x\to 0} \frac{log_e(1+x^3)}{sin^3 x}$ $(iii)\ \lim\limits_{x\to 0} \frac{logx^2}{cotx^2}$ $(iv)\ \lim\limits_{x\to\infty} \frac{log_e x}{a^x},\ a > 1$

    $(v)\ \lim\limits_{x\to 0} sinx.logx$ $(vi)\ \lim\limits_{x\to 0} x.log_e sinx$ $(vii)\ \lim\limits_{\to 1}\{\frac{2}{x^2-1} - \frac{1}{x-1}\}$ $(viii)\ \lim\limits_{x\to 0}(cosx)^{cotx}$

    $(ix)\ \lim\limits_{x\to 0}(cosecx)^{\frac{1}{log_e x}}$ $(x)\ \lim\limits_{x\to 0}(\frac{a^x + b^x}{2})^{\frac{1}{x}}$ $(xi)\ \lim\limits_{x\to \frac{\pi}{2}}(sinx)^{tanx}$ $(xii)\ \lim\limits_{x\to 0}(\frac{1}{x})^{2sinx}$
13. Find the value of a,b and c so that

$$\lim_{x\to 0} \frac{ae^x - bsinx + ce^{-x}}{xsinx} = 2$$