Class 10
Trigonometry

Answer the questions

(1) Simplify $\frac{\sin x - \sin y}{\cos x + \cos y} + \frac{\cos x - \cos y}{\sin x + \sin y}$

(2) Simplify 1 + $\frac{\tan^2\beta}{1 + \sec\beta}$.

(3) Simplify $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\mathrm{cosec}^3\theta}$

(4) If cosec θ = a/b and 0° > θ > 90°, find value of cot θ.

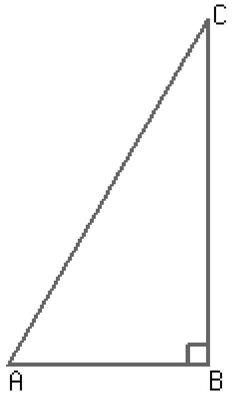
(5) Simplify $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

(6) Simplify $3(\sin^4\theta + \cos^4\theta) - 2(\sin^6\theta + \cos^6\theta)$

(7) A rope is tightly stretched and attached from top of a vertical tower to the ground. The angle made by rope with ground is 30°. If height of the tower is 2 m, find length of the rope.

(8) If 7 tanθ = 4, find the value of $\frac{7 \sin\theta - \cos\theta}{7 \sin\theta + \cos\theta}$.

(9) If AB = 6 and AC = 12, find value of (sin A × tan A).



(10) If 2sinθ – cosθ = 2, find the value of sinθ + 2cosθ.

(11) If tan θ + cot θ = 5, find the value of $\tan^2\theta + \cot^2\theta$.

(12) Simplify $\tan^2\theta\left(\frac{\mathrm{cosec}\theta-1}{1+\cos\theta}\right)-\mathrm{cosec}^2\theta\left(\frac{\cos\theta-1}{1+\mathrm{cosec}\theta}\right)$

(13) Simplify (1 + cotθ + cosecθ) (1 + tanθ - secθ)

Choose correct answer(s) from the given choices

(14) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} = ?$

a. $\frac{2}{2\sin^2\theta - 1}$

b. $\frac{2}{1 - \sin^2\theta}$

c. $\frac{2}{1 - \cos^2\theta}$

d. $\frac{2}{1 - 2\sin^2\theta}$

Check True/False

(15) $\tan 8° - \cot 8° < 0$

- True
- False

Answers

(1) 0

Step 1

On combining two fractions

$= \frac{\sin x - \sin y}{\cos x + \cos y} + \frac{\cos x - \cos y}{\sin x + \sin y}$

$= \frac{(\sin x - \sin y)(\sin x + \sin y) + (\cos x + \cos y)(\cos x - \cos y)}{(\cos x + \cos y)(\sin x + \sin y)}$

$= \frac{\sin^2 x - \sin^2 y + \cos^2 x - \cos^2 y}{(\cos x + \cos y)(\sin x + \sin y)}$

Step 2

Since $\sin^2\theta + \cos^2\theta + = 1$

$= \frac{1-1}{(\cos x + \cos y)(\sin x + \sin y)} = 0$

(2) sec β

Step 1

Let, $S = 1 + \frac{\tan^2\beta}{1 + \sec\beta}$

Step 2

Using identity $\tan^2\theta = \sec^2\theta - 1$,

$\Rightarrow S = 1 + \frac{\sec^2\beta - 1}{1 + \sec\beta}$

Step 3

Using $a^2 - b^2 = (a+b)(a-b)$,

$\Rightarrow S = 1 + \frac{(\sec\beta + 1)(\sec\beta - 1)}{1 + \sec\beta}$

$\Rightarrow S = 1 + (\sec\beta - 1)$

$\Rightarrow S = \sec\beta$

(3) $\sin^2\theta \cos^2\theta$

Step 1

We have been asked to simplify the $\dfrac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{\sec^3\theta - \csc^3\theta}$.

Step 2

$$\frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{\sec^3\theta - \csc^3\theta} = \frac{(1 + \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta})(\sin\theta - \cos\theta)}{(\sec\theta - \csc\theta)(\sec^2\theta + \sec\theta \times \csc\theta + \csc^2\theta)}$$) [Since, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$= \frac{(\dfrac{\sin\theta \cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta \cos\theta})(\sin\theta - \cos\theta)}{(\dfrac{1}{\cos\theta} - \dfrac{1}{\sin\theta})(\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin\theta \cos\theta} + \dfrac{1}{\cos^2\theta})}$$ [Since, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\csc\theta = \dfrac{1}{\sin\theta}$]

$$= \frac{(\dfrac{1 + \sin\theta \cos\theta}{\sin\theta \cos\theta})(\sin\theta - \cos\theta)}{\dfrac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta \cos\theta + \cos^2\theta)}{(\sin\theta \cos\theta)(\sin^2\theta \cos^2\theta)}}$$

$$= \frac{(1 + \sin\theta \cos\theta)(\sin\theta - \cos\theta)(\sin^3\theta \cos^3\theta)}{(\sin\theta \cos\theta)(\sin\theta - \cos\theta)(1 + \sin\theta \cos\theta)}$$

$= \sin^2\theta \cos^2\theta$

Step 3

Therefore, $\dfrac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{\sec^3\theta - \csc^3\theta}$ is equal to the $\mathbf{\sin^2\theta \cos^2\theta}$.

(4) $\sqrt{\frac{a^2-b^2}{b^2}}$

Step 1

We know that,

cot θ = √(cosec^{2}θ - 1)

Step 2

Now replace value of cosec(θ) in above equation

⇒ cot(θ) = $\sqrt{\left(\frac{a}{b}\right)^2-1}$

Step 3

Simplify RHS of above equation

⇒ cot(θ) = $\sqrt{\frac{a^2-b^2}{b^2}}$

(5) 2 tanθ

Step 1

$=\sqrt{\frac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}}+\sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$

Step 2

$=\sqrt{\frac{(\sec\theta-1)^2}{\cot^2\theta}}+\sqrt{\frac{(\sec\theta+1)^2}{\cot^2\theta}}$

Step 3

$=\frac{(1+\sec\theta+1-\sec\theta)}{\cot\theta}=2\tan\theta$

(6) 1

Step 1

= 3 ($\sin^4\theta + \cos^4\theta$) - 2 [$(\sin^2\theta)^3 + (\cos^2\theta)^3$ }]

Step 2

= 3 ($\sin^4\theta + \cos^4\theta$) - 2 [($\sin^2\theta + \cos^2\theta$) {$(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta \cos^2\theta$ }]

Step 3

Since $\sin^2\theta + \cos^2\theta = 1$

=3 ($\sin^4\theta + \cos^4\theta$) - 2 {$\sin^4\theta + \cos^4\theta - \sin^2\theta \cos^2\theta$ }

Step 4

= $\sin^4\theta + \cos^4\theta + 2 \sin^2\theta \cos^2\theta$

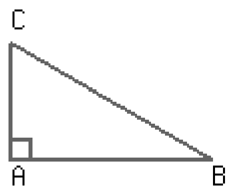
Step 5

= $(\sin^2\theta + \cos^2\theta)^2 = 1^2 = 1$

(7) 4 m

Step 1

In following figure, AC shows the towerm BC is the rope connected from ground to tower, and angle ∠B is the angle angle made by rope with ground.



Step 2

AC/BC = sin(∠B)

⇒ AC/BC = sin(30°)

⇒ $\frac{2}{BC} = \frac{1}{2}$

⇒ $BC = \frac{2}{\frac{1}{2}}$

⇒ $BC = 4$

(8) 3/5

Step 1

Lets

$$S = \frac{7 \sin\theta - \cos\theta}{7 \sin\theta + \cos\theta}$$

Step 2

Now divide numerator and denominator of this fraction by cosθ

$$\Rightarrow S = \frac{(7 \sin\theta - \cos\theta)/\cos\theta}{(7 \sin\theta + \cos\theta)/\cos\theta}$$

$$\Rightarrow S = \frac{(7 \sin\theta/\cos\theta - 1)}{(7 \sin\theta/\cos\theta + 1)}$$

$$\Rightarrow S = \frac{(7 \tan\theta - 1)}{(7 \text{tab}\theta + 1)}$$

Step 3

Now replace 7 tanθ with is value (7 tanθ = 4)

$$\Rightarrow S = \frac{(4 - 1)}{(4 + 1)}$$

⇒ S = 3/5

(9) $\frac{3}{2}$

Step 1

$$BC = \sqrt{AC^2 - AB^2}$$

Step 2

BC = $6\sqrt{3}$

Step 3

sin A × tan A = (BC/AC) × (BC/AB) = $\frac{3}{2}$

(10) 1

Step 1

It is given that

$2\sin\theta - \cos\theta = 2$

$\Rightarrow (2\sin\theta - \cos\theta)^2 = 2^2$ .... [On squaring both sides]

Step 2

Now add $(\sin\theta + 2\cos\theta)^2$ to both sides of equation

$\Rightarrow (2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2 = 2^2 + (\sin\theta + 2\cos\theta)^2$

$\Rightarrow 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta + \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 4 + (\sin\theta + 2\cos\theta)^2$

$\Rightarrow 5\sin^2\theta + 5\cos^2\theta + \cancel{4\sin\theta\cos\theta} - \cancel{4\sin\theta\cos\theta} = 4 + (\sin\theta + 2\cos\theta)^2$

$\Rightarrow 5(\sin^2\theta + \cos^2\theta) = 4 + (\sin\theta + 2\cos\theta)^2$

$\Rightarrow 5 = 4 + (\sin\theta + 2\cos\theta)^2$

$\Rightarrow (\sin\theta + 2\cos\theta)^2 = 5 - 4$

$\Rightarrow (\sin\theta + 2\cos\theta)^2 = 1$

$\Rightarrow \sin\theta + 2\cos\theta = \pm 1$

Step 3

Since $2\sin\theta - \cos\theta = 2$, ignore the negative value.

Therefore, $\sin\theta + 2\cos\theta = 1$

(11) 23

Step 1

$\tan\theta + \cot\theta = 5$

Step 2

On squaring both sides,

$(\tan\theta + \cot\theta)^2 = 5^2$

Step 3

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

Step 4

$\tan^2\theta + \cot^2\theta + 2 = 25$

Step 5

$\tan^2\theta + \cot^2\theta = 25 - 2 = 23$

(12) 0

Step 1

On adding two fractions

$$= \frac{\tan^2(\text{cosec}\theta - 1)(\text{cosec}\theta + 1) - \text{cosec}^2(\cos\theta - 1)(\cos\theta + 1)}{(1+\cos\theta)(1+\text{cosec}\theta)}$$

Step 2

$$= \frac{\tan^2(\text{cosec}^2\theta - 1) - \text{cosec}^2(\cos^2\theta - 1)}{(1+\cos\theta)(1+\text{cosec}\theta)}$$

Step 3

$$= \frac{-\tan^2\cot^2\theta + \text{cosec}^2\sin^2\theta}{(1+\cos\theta)(1+\text{cosec}\theta)}$$

Step 4

$$= \frac{-1+1}{(1+\cos\theta)(1+\text{cosec}\theta)} = 0$$

(13) 2

Step 1

We need to find following product

$S = (1 + \cot\theta + \text{cosec}\theta)(1 + \tan\theta - \sec\theta)$

Step 2

On multiplying each terms

$\Rightarrow S = 1(1 + \tan\theta - \sec\theta) + \cot\theta(1 + \tan\theta - \sec\theta) + \text{cosec}\theta(1 + \tan\theta - \sec\theta)$

$\Rightarrow S = (1 + \tan\theta - \sec\theta) + (\cot\theta + \cot\theta\tan\theta - \cot\theta\sec\theta) + (\text{cosec}\theta + \text{cosec}\theta\tan\theta - \text{cosec}\theta\sec\theta)$

Step 3

Using identities $\cot\theta\tan\theta = 1$, $\cot\theta\sec\theta = \text{cosec}\theta$ and $\text{cosec}\theta\tan\theta = \sec\theta$

$\Rightarrow S = (1 + \tan\theta - \sec\theta) + (\cot\theta + 1 - \text{cosec}\theta) + (\text{cosec}\theta + \sec\theta - \text{cosec}\theta\sec\theta)$

Step 4

Now positive $\text{cosec}\theta$ and $\sec\theta$ will cancel each other

$\Rightarrow S = (1 + \tan\theta - \cancel{\sec\theta}) + (\cot\theta + 1 - \cancel{\text{cosec}\theta}) + (\cancel{\text{cosec}\theta} + \cancel{\sec\theta} - \text{cosec}\theta\sec\theta)$

$\Rightarrow S = 2 + \tan\theta + \cot\theta - \text{cosec}\theta\sec\theta$

Step 5

Using identities $\tan\theta = \sin\theta/\cos\theta$, and $\cot\theta = \cos\theta/\sin\theta$

$$\Rightarrow S = 2 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} - \text{cosec}\theta\sec\theta$$

$$\Rightarrow S = 2 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} - \text{cosec}\theta\sec\theta$$

$$\Rightarrow S = 2 + \frac{1}{\sin\theta\cos\theta} - \text{cosec}\theta\sec\theta$$

$\Rightarrow S = 2 + \text{cosec}\theta\sec\theta - \text{cosec}\theta\sec\theta$

$\Rightarrow S = 2$

(14) **a.** $\dfrac{2}{2\sin^2\theta - 1}$

Step 1

On adding two fractions,

S = $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

Step 2

Let,

$\Rightarrow$ S = $\dfrac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)+(\sin\theta-\cos\theta)(\sin\theta-\cos\theta)}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$\Rightarrow$ S = $\dfrac{(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta)+(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}{(\sin^2\theta-\cos^2\theta)}$

Step 3

Using identity $\sin^2\theta + \cos^2\theta = 1$

, $\Rightarrow$ S = $\dfrac{(1+2\sin\theta\cos\theta)+(1-2\sin\theta\cos\theta)}{(\sin^2\theta-\cos^2\theta)}$

$\Rightarrow$ S = $\dfrac{2}{(\sin^2\theta-\cos^2\theta)}$

$\Rightarrow$ S = $\dfrac{2}{\sin^2\theta-(1-\sin^2\theta)}$

$\Rightarrow$ S = $\dfrac{2}{2\sin^2\theta-1}$

(15) True

We know that $\cot\theta$ is greater than $\tan\theta$, if $\theta < 45°$.

Therefore $\tan 8°$ - $\cot 8°$ will be negative, and statement "$\tan 8° - \cot 8° < 0$" is **True**.