Session 13: Stochastic gradient descent

Optimization and Computational Linear Algebra for Data Science

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Stochastic gradient descent

Setting

In machine learning, one often has to minimize functions of the form

$$f(x) = \frac{1}{N} \sum_{i=1}^{N} f_i(x).$$

where $f_i : \mathbb{R}^n \to \mathbb{R}$.

Setting

Stochastic gradient descent

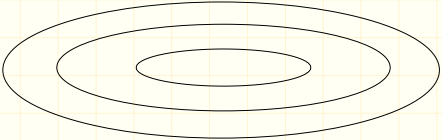
$$f(x) = \frac{1}{N}\sum_{i=1}^{N} f_i(x).$$

Starting at some $x_0 \in \mathbb{R}^n$, perform the updates:

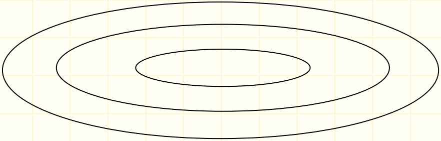
$$\begin{array}{lll} \text{Pick} & i & \text{uniformly at random in} \quad \{1, \ldots, N\}, \\ \text{Update} & \multicolumn{2}{l}{x_{t+1} = x_t - \alpha_t \nabla f_i(x_t),} \end{array}$$

Tradeoffs in SGD

Rapidly decaying step sizes



Slowly decaying step sizes



Convergence analysis

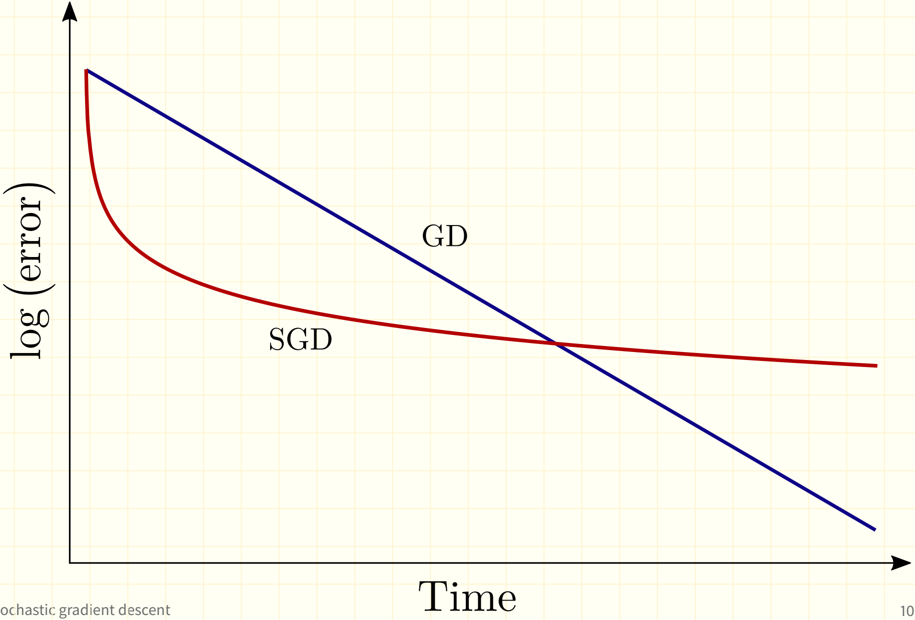
Convergence analysis

GD vs SGD

Gradient descent

Stochastic gradient descent

GD vs SGD



Questions?

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