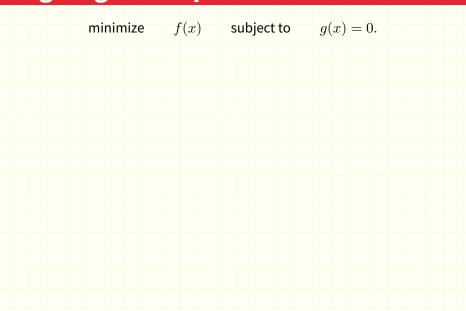
About the Lagrangian

Optimization and Computational Linear Algebra for Data Science

Léo Miolane

Lagrange multipliers



Encoding the constraint in L

We have $L(x, \lambda) = f(x) + \lambda g(x)$, hence

$$\max_{\lambda \in \mathbb{R}} L(x,\lambda) = \begin{cases} f(x) & \text{if} \quad g(x) = 0 \\ +\infty & \text{otherwise}. \end{cases}$$

Primal and dual problems

Primal optimization problem:

Dual optimization problem:

$$\mbox{maximize} \quad \min_{x \in \mathbb{R}^n} \ L(x, \lambda) \quad \mbox{subject to} \quad \lambda \in \mathbb{R}.$$

Saddle point interpretation

Theorem

Assume that:

- $\star x^*$ is a solution to the primal problem.
- λ^* is a solution to the dual problem.
- Strong duality holds: $\min_x \max_{\lambda} L(x, \lambda) = \max_{\lambda} \min_x L(x, \lambda)$.

Then (x^*, λ^*) is a saddle-point of the Lagrangian L.