BEEE100

Basic Electrical Engineering

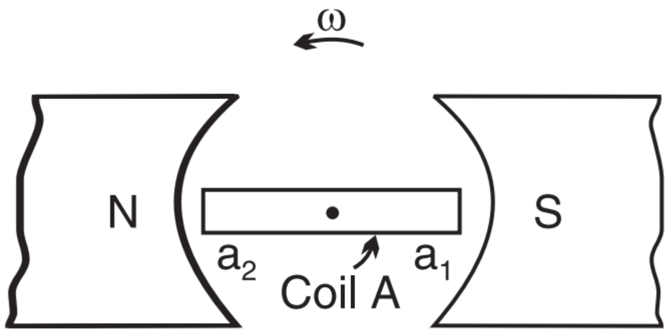
Module – 02: AC Circuits

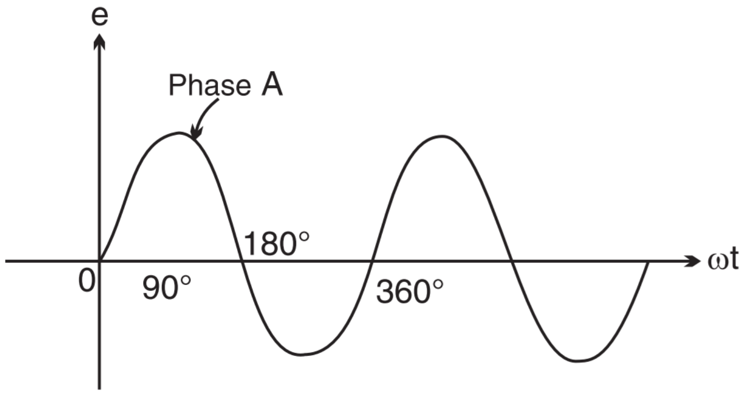
Three phase balanced

Three Phase Systems

Single Phase System

$$e_{a_1 a_2} = E_m \sin \omega t$$



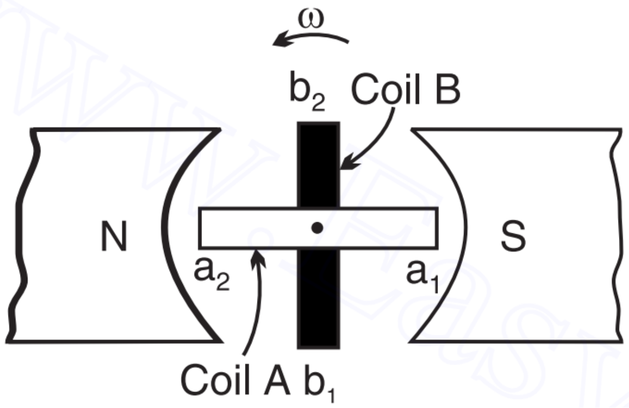


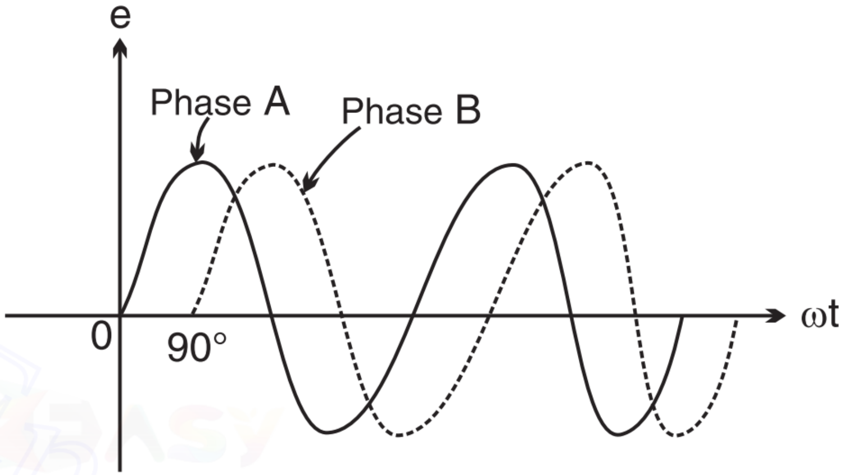
Circuits or systems in which the ac sources operate at the same frequency but different phases are known as *polyphase*

Two Phase System

$$e_{a_1 a_2} = E_m \sin \omega t$$

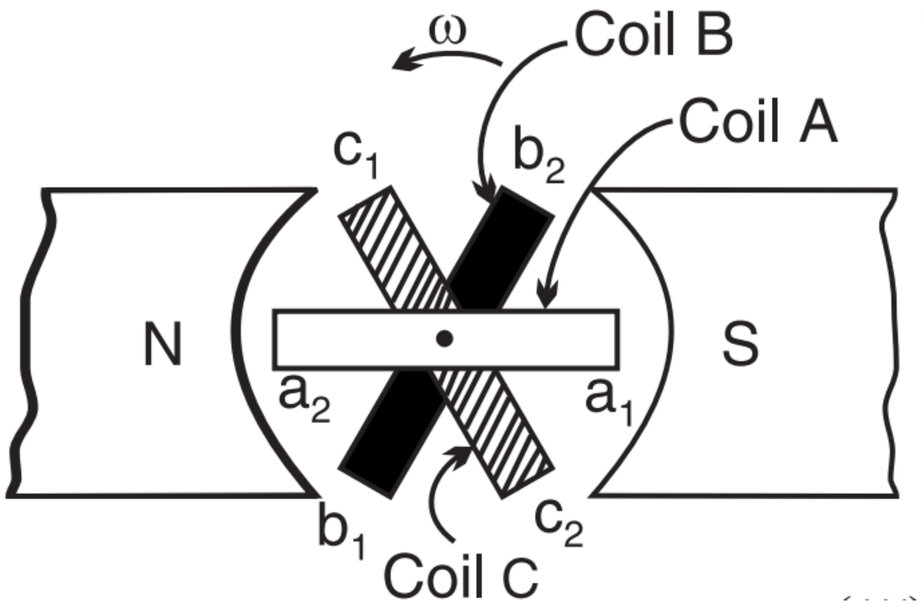
$$e_{b_1 b_2} = E_m \sin(\omega t - 90^\circ)$$

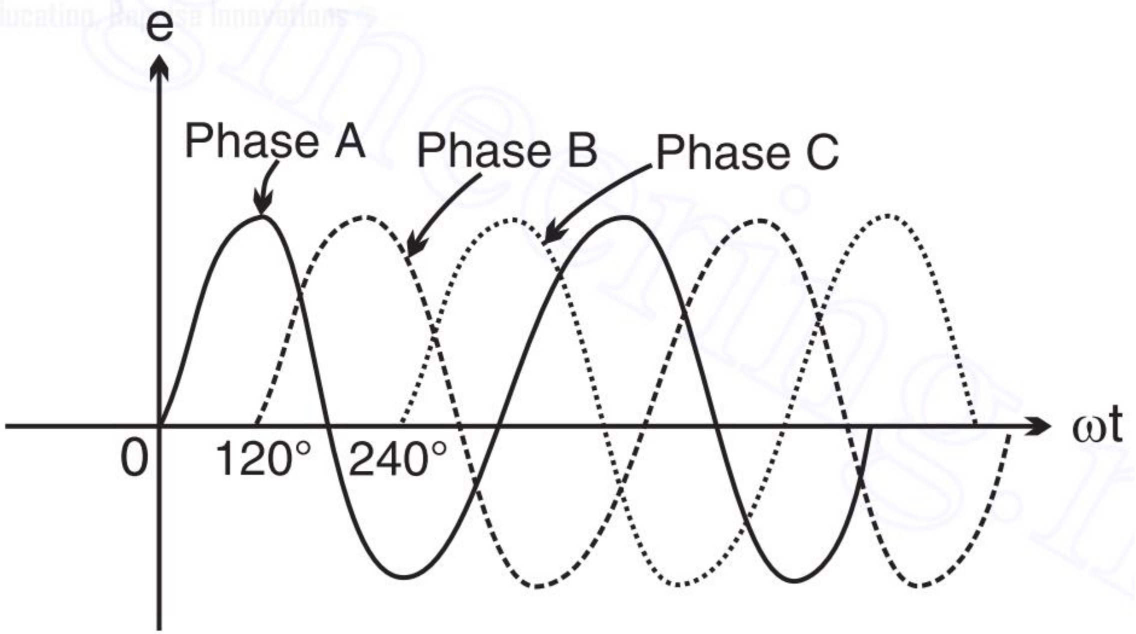




Three phase systems

Three Phase System





$$e_{a_1a_2} = E_m \sin \omega t$$

$$e_{b_1b_2} = E_m \sin(\omega t - 120^\circ)$$

$$e_{c_1c_2} = E_m \sin(\omega t - 240^\circ)$$

Three phase systems

Why Three Phase?

- **Constant power.** In a single-phase circuit, the instantaneous power varies sinusoidally from zero to a peak value at twice the supply frequency. However, in a balanced 3-phase system, the power supplied at all instants of time is constant.

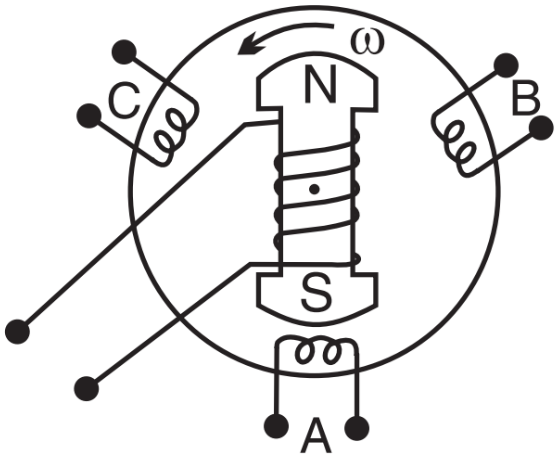
- **Greater output.** The output of a 3-phase machine is greater than that of a single-phase machine for a given volume and weight of the machine.

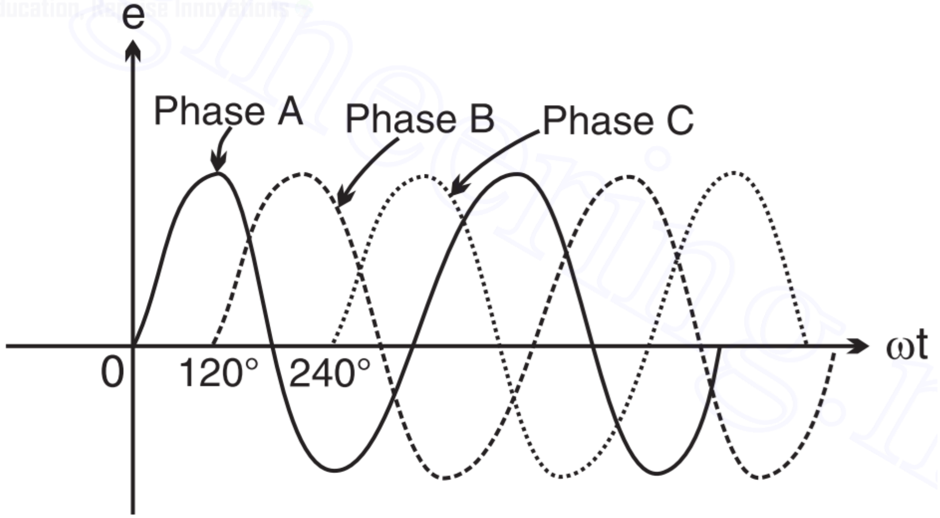
- **Cheaper.** The three-phase motors are much smaller and less expensive than single-phase motors because less material (copper, iron, insulation) is required.

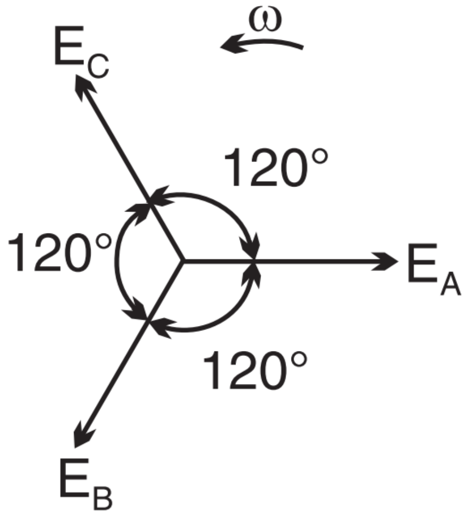
- **Power transmission economics.** Transmission of electric power by 3-phase system is cheaper than that of single-phase system, even though three conductors are required instead of two.

- **Rotating Magnetic Field** A 3-phase system can set-up a rotating uniform magnetic field in stationary windings. This cannot be done with a single-phase current.

Three phase systems







$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin(\omega t - 120°)$$

$$e_C = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

$$\text{Resultant} = e_A + e_B + e_C$$

$$= E_m[\sin \omega t + \sin(\omega t - 120°) + \sin(\omega t - 240°)]$$

$$= E_m[\sin \omega t + 2\sin(\omega t - 180°)\cos 60°]$$

$$= E_m\ [\sin \omega t - 2 sin\omega t \cos 60°] = 0$$

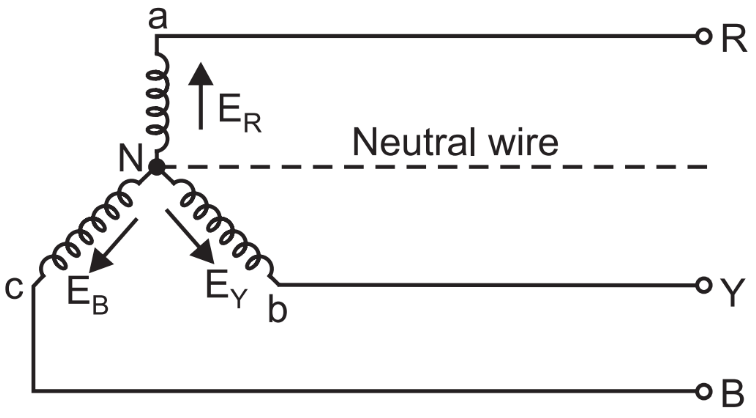
It is a usual practice to name the three phases or windings after the three natural colors viz. Red (R), yellow (Y) and blue (B). In that case, the phase sequence is RYB i.e. voltage in phase R attains maximum positive value first, next phase Y and then phase B. It may be noted that there are only two possible phase sequences viz RYB and RBY

Three phase systems

Three Phase Configurations

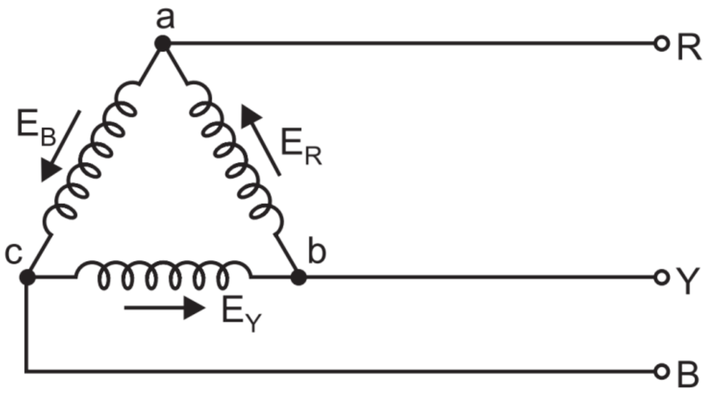
(i) Star or Wye (Y) connection
(ii) Delta (D) connection

Y-connection



If a neutral conductor exists, the system is called *3-phase, 4 wire system*. If there is no neutral conductor, it is called *3-phase, 3-wire system*.

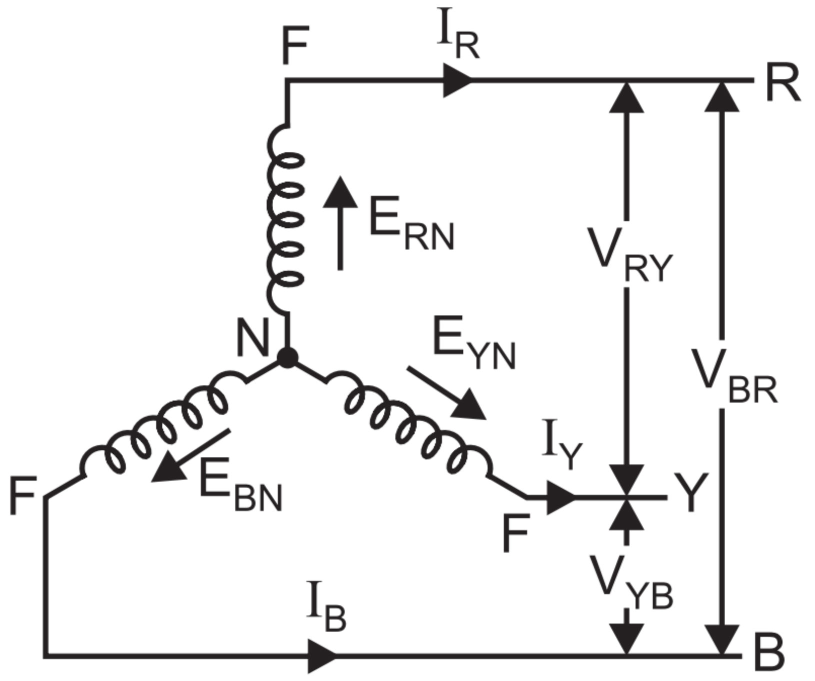
D -connection



In a D-connection, no neutral point exists and only 3-phase, 3-wire system can be formed.

Three phase systems

Star or Wye Connected System



The voltages E_{RN}, E_{YN}, and E_{BN} are respectively between lines *R*, *Y*, and *B*, and the neutral line *N*. These voltages are called *phase voltages*. If the voltage sources have the same amplitude and frequency *ω* and are out of phase with each other by 120º, the voltages are said to be *balanced*.

Phase Voltage

$$\boldsymbol{E_{RN}} = V_{Ph}\angle 0^\circ \qquad \boldsymbol{E_{YN}} = V_{Ph}\angle -120^\circ$$

$$\boldsymbol{E_{BN}} = V_{Ph}\angle - 240^\circ$$

When voltages are balanced $E_{RN} + E_{YN} + E_{BN} = 0$

$$|E_{RN}| = |E_{YN}| = |E_{BN}| = V_{Ph}$$

Line Voltage

$$\boldsymbol{V_{RY}} = V_L\angle 0^\circ \qquad \boldsymbol{V_{YB}} = V_L\angle -120^\circ$$

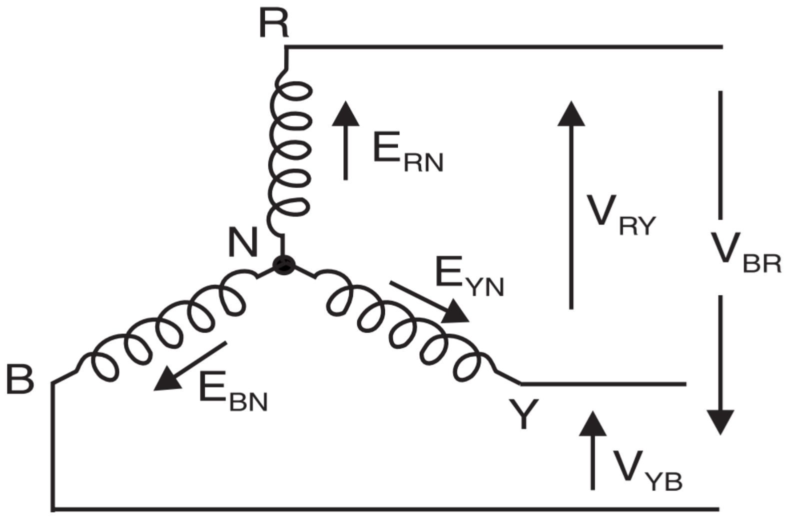
$$\boldsymbol{V_{BR}} = V_L\angle -240^\circ$$

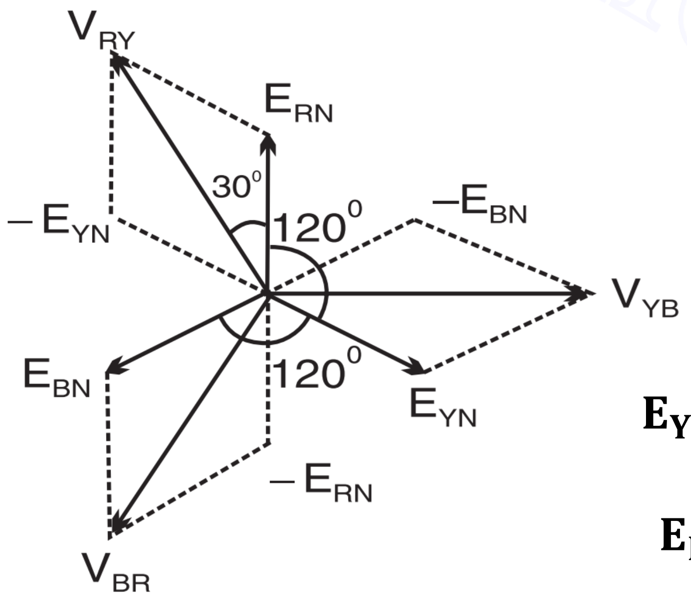
Line voltage = $\sqrt{3}\times$ Phase voltage $\qquad V_L = \sqrt{3}V_{Ph}$

Line current = Phase current $\qquad I_L = I_{Ph}$

Three phase systems

Star or Wye Connected System





$$V_{RY} = E_{RN} + E_{NY} = E_{RN} - E_{YN}$$

$$V_{RY} = 2V_{Ph}\cos(60°/2) = 2V_{Ph}\cos 30° = \sqrt{3}V_{Ph}$$

$$V_{YB} = E_{YN} - E_{BN} = \sqrt{3}V_{Ph}$$

$$V_{BR} = E_{BN} - E_{RN} = \sqrt{3}V_{Ph}$$

$$\mathbf{E_{RN}} = V_{Ph}\angle 0° = V_{Ph}(1 + j0)$$

$$\mathbf{E_{YN}} = V_{Ph}\angle -120° = V_{Ph}(-0.5 - j0.866)$$

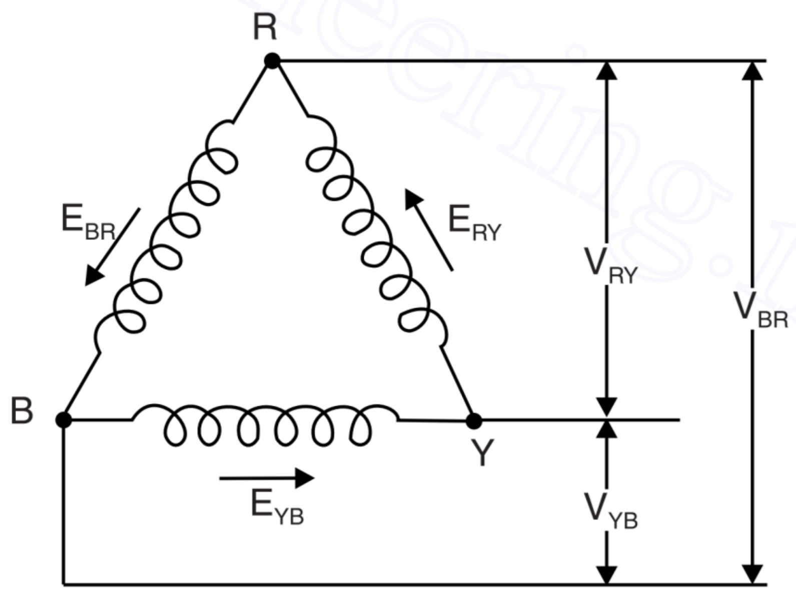
$$\mathbf{E_{BN}} = V_{Ph}\angle -240° = V_{Ph}(-0.5 + j0.866)$$

$$\mathbf{V_{RY}} = \mathbf{E_{RN}} - \mathbf{E_{YN}} = V_{Ph}(1 + j0) - V_{Ph}(-0.5 - j0.866)$$

$$\mathbf{V_{RY}} = V_{Ph}(1.5 + j0.866) = \sqrt{3}V_{Ph}\angle 30°$$

Three phase systems

Delta Connected System



When voltages are balanced

$$\mathbf{E_{RY}} = V_{Ph}\angle 0^\circ = V_{Ph}(1 + j0)$$

$$\mathbf{E_{YB}} = V_{Ph}\angle -120^\circ = V_{Ph}(-0.5 - j0.866)$$

$$\mathbf{E_{BR}} = V_{Ph}\angle -240^\circ = V_{Ph}(-0.5 + j0.866)$$

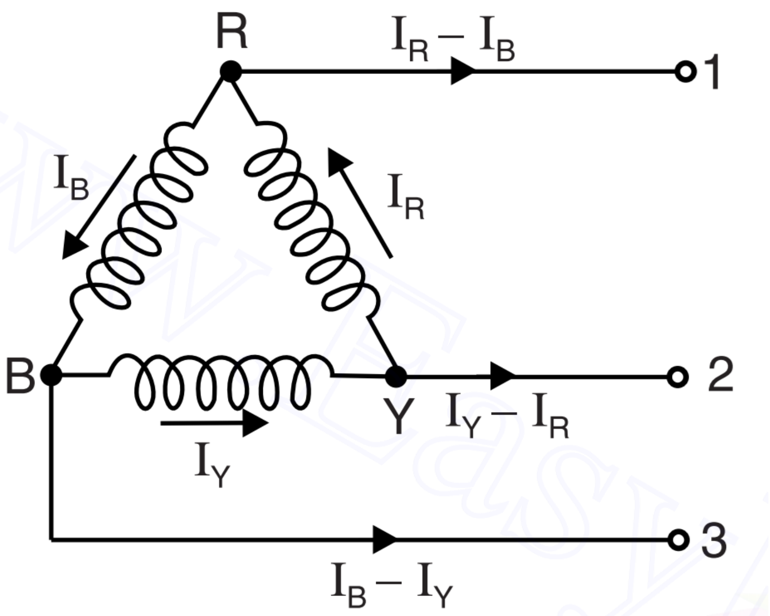
$$\mathbf{E_{RY}} + \mathbf{E_{YB}} + \mathbf{E_{BR}} = V_{Ph}(1 + j0) + V_{Ph}(-0.5 - j0.866) + V_{Ph}(-0.5 + j0.866) = 0$$

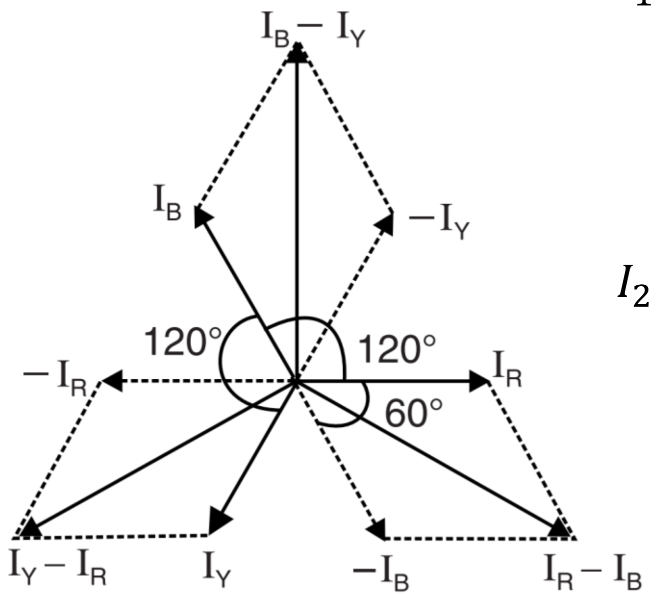
Line voltage = Phase voltage $\qquad V_L = V_{Ph}$

Line current = $\sqrt{3}\times$ Phase current $\qquad I_L = \sqrt{3}\, I_{Ph}$

Three phase systems

Delta Connected System





$$I_1 = I_R - I_B \qquad I_2 = I_Y - I_R \qquad I_3 = I_B - I_Y$$

$$I_1 = 2I_{Ph}\cos(60°/2) = 2I_{Ph}\cos 30° = \sqrt{3}I_{Ph}$$

$$I_2 = I_Y - I_R = \sqrt{3}I_{Ph} \qquad I_3 = I_B - I_Y = \sqrt{3}I_{Ph}$$

$$\mathbf{I_R} = I_{Ph}\angle 0° = I_{Ph}(1 + j0)$$

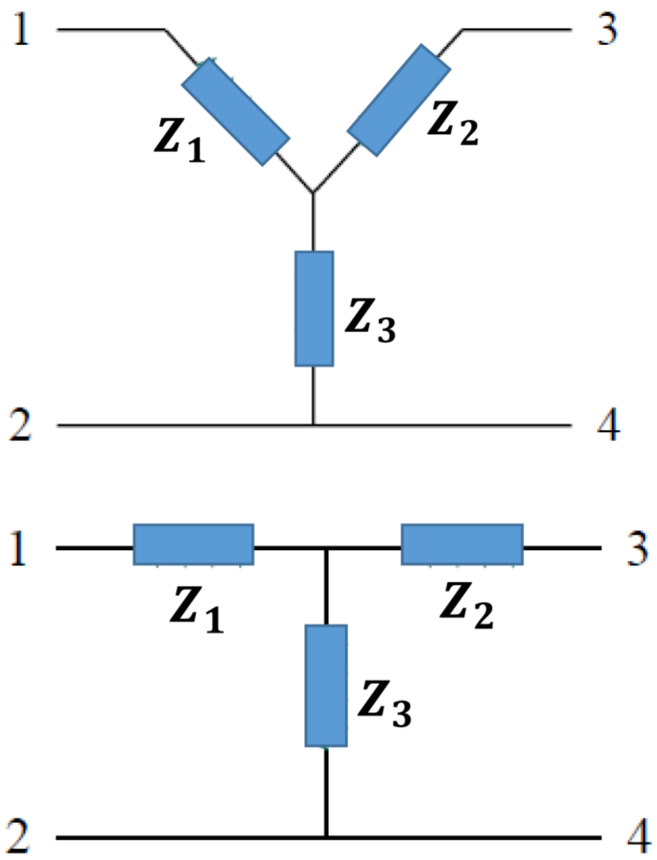
$$\mathbf{I_Y} = I_{Ph}\angle -120° = I_{Ph}(-0.5 - j0.866)$$

$$\mathbf{I_B} = I_{Ph}\angle -240° = I_{Ph}(-0.5 + j0.866)$$

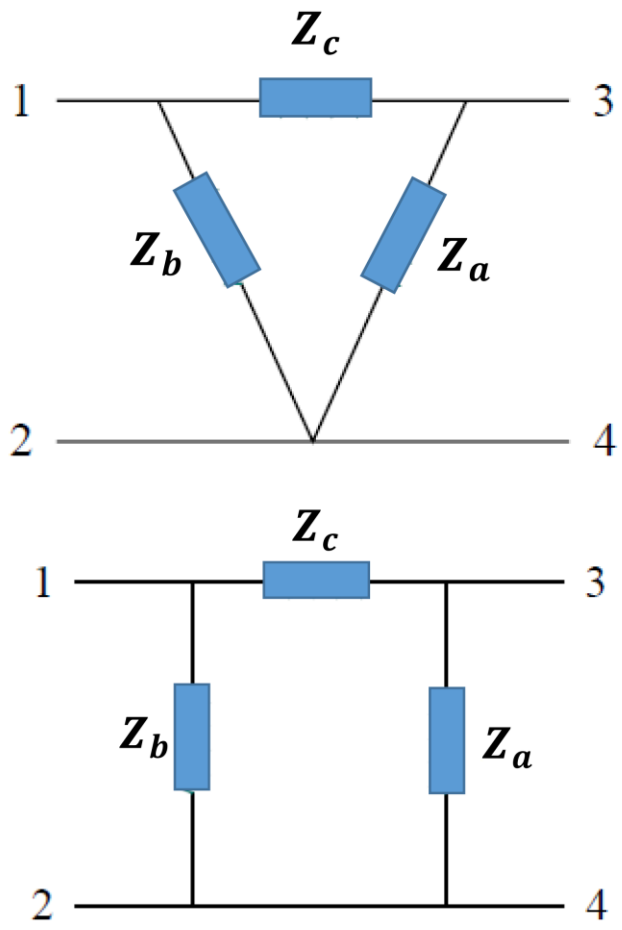
$$\mathbf{I_1} = \mathbf{I_R} - \mathbf{I_B} = I_{Ph}(1 + j0) - I_{Ph}(-0.5 + j0.866)$$

Three Phase Load

Star or Y-connected Load

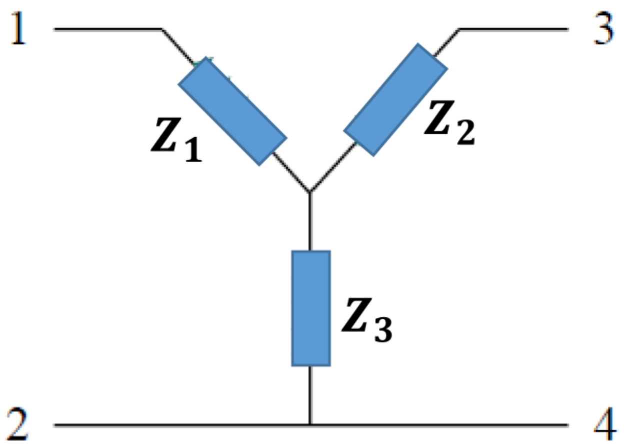


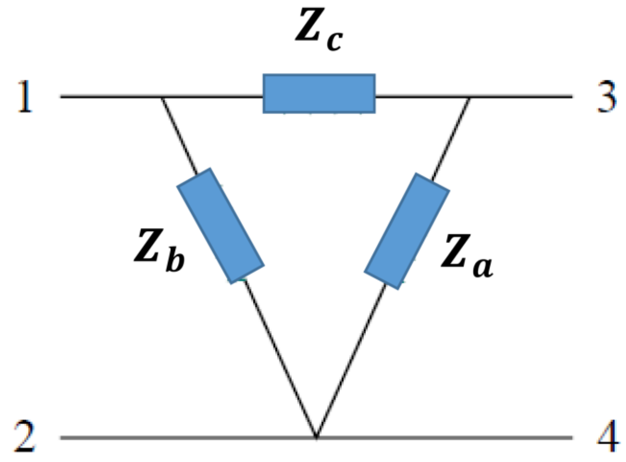
Δ-connected Load



Three phase systems

Delta to Wye Conversion





$$Z_{12}(Y) = Z_1 + Z_3$$

$$Z_{12}(\Delta) = Z_b||(Z_a + Z_c)$$

$$Z_{12}(Y) = Z_{12}(\Delta)$$

$$Z_{12} = Z_1 + Z_3 = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c}$$

$$Z_{13} = Z_1 + Z_2 = \frac{Z_c(Z_a + Z_b)}{Z_a + Z_b + Z_c}$$

$$Z_{34} = Z_2 + Z_3 = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c}$$

Subtracting Z_{34} from Z_{12}

$$Z_1 - Z_2 = \frac{Z_c(Z_b - Z_a)}{Z_a + Z_b + Z_c}$$

Adding Z_{13} and (Z_1- Z_2) gives

$$\boxed{Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}} \qquad \boxed{Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}}$$

$$\boxed{Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}}$$

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \frac{Z_aZ_bZ_c(Z_a + Z_b + Z_c)}{(Z_a + Z_b + Z_c)^2}$$

$$= \frac{Z_aZ_bZ_c}{(Z_a + Z_b + Z_c)}$$

$$\boxed{Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}} \qquad \boxed{Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}}$$

$$\boxed{Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}}$$

Three Phase Load

A balanced load is one in which the phase impedances are equal in magnitude and in phase.

For a *balanced* wye-connected load

$$Z_1 = Z_2 = Z_3 = Z_Y$$

where Z_Y is the load impedance per phase in this case

For a *balanced* delta-connected load

$$Z_a = Z_b = Z_c = Z_\Delta$$

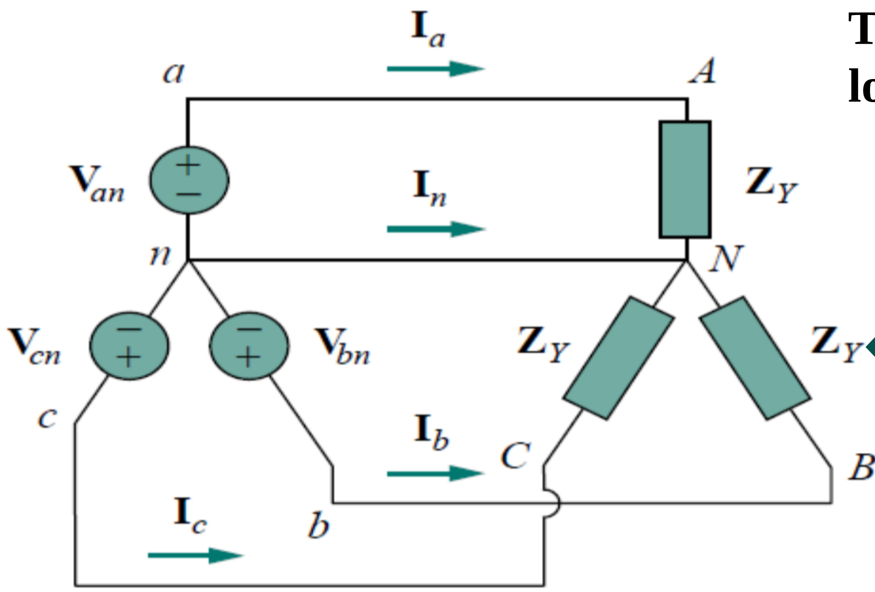
where Z_Δ is the load impedance per phase in this case

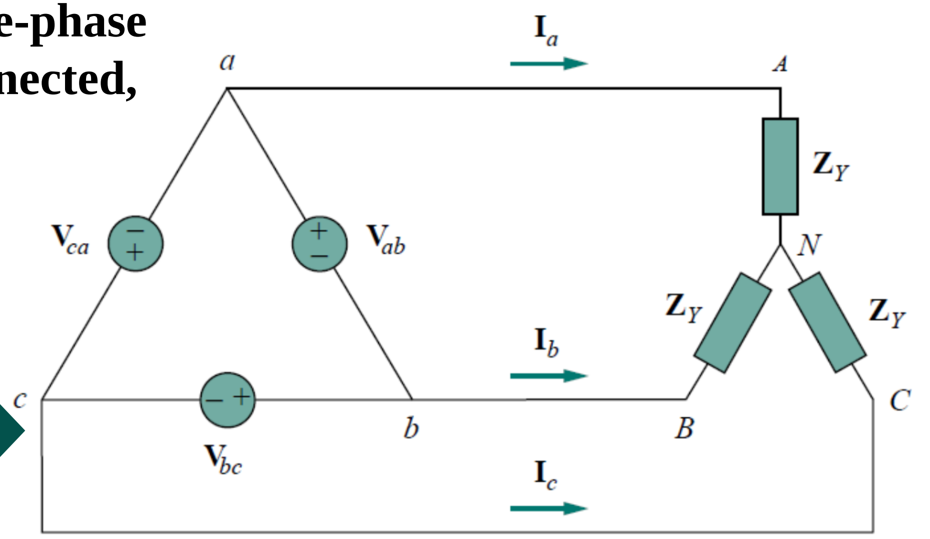
$$\boxed{\boldsymbol{Z_\Delta = 3Z_Y}}$$

$$\boxed{\boldsymbol{Z_Y = \frac{Z_\Delta}{3}}}$$

Three phase systems

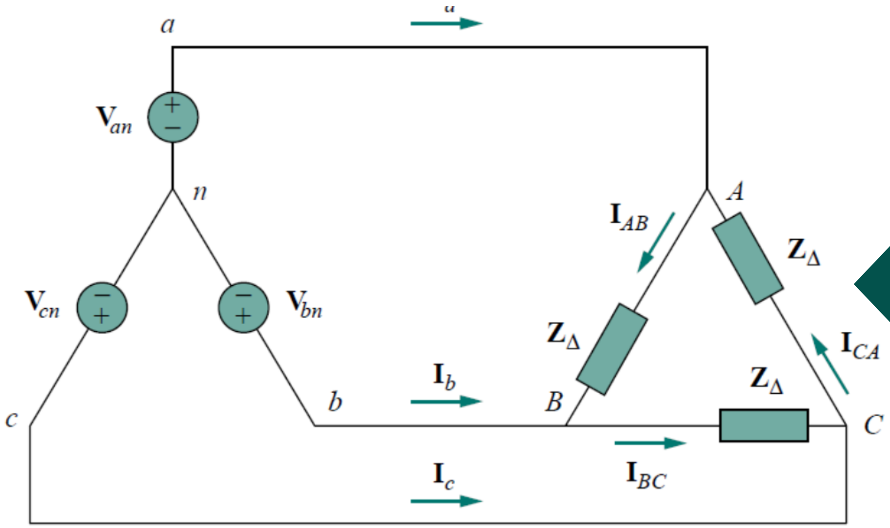
Three-phase source and the three-phase load can be either wye- or delta-connected,

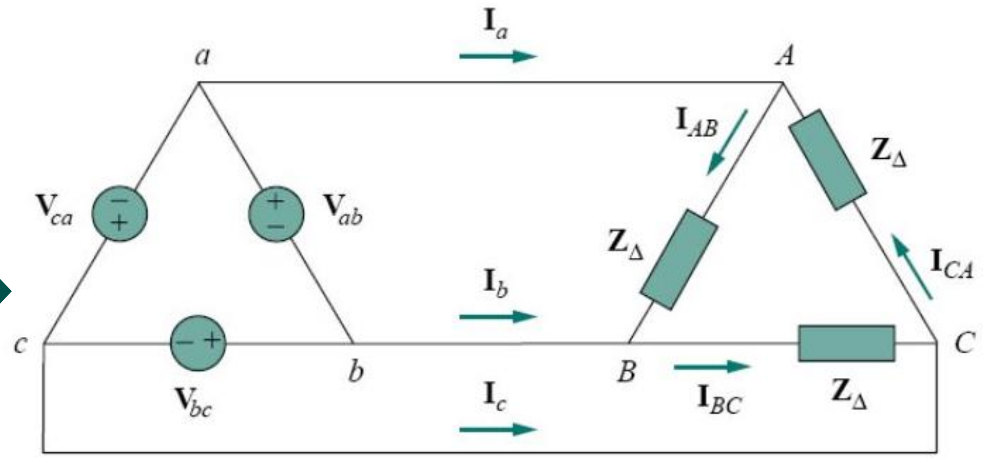




Y - Y Connection

Δ - Y Connection





Y - Δ Connection

Δ - Δ Connection

Three phase systems

Three Phase Power

Total Power, $P = 3 \times \text{Power in each phase}$

Y-System

$$P = 3 \times V_{Ph} I_{Ph} \cos\phi$$

$$V_{Ph} = \frac{V_L}{\sqrt{3}} \qquad I_{Ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$$

Real Power $P = \sqrt{3} V_L I_L \cos\phi$

Reactive Power, $Q = \sqrt{3} V_L I_L \sin\phi$

Apparent Power, $S = \sqrt{3} V_L I_L = \sqrt{P^2 + Q^2}$

Power Factor $= \cos\phi = \frac{P}{S}$

Δ-System

$$P = 3 \times V_{Ph} I_{Ph} \cos\phi$$

$$V_{Ph} = V_L \qquad I_{Ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

Real Power $P = \sqrt{3} V_L I_L \cos\phi$

Reactive Power, $Q = \sqrt{3} V_L I_L \sin\phi$

Apparent Power, $S = \sqrt{3} V_L I_L = \sqrt{P^2 + Q^2}$

Power Factor $= \cos\phi = \frac{P}{S}$

For balanced condition $\mathbf{I_N} = \mathbf{I_R} + \mathbf{I_Y} + \mathbf{I_B} = 0$

Thank You !