EE100: Basic Electrical Engineering

Module-II

Alternating Voltages and Currents

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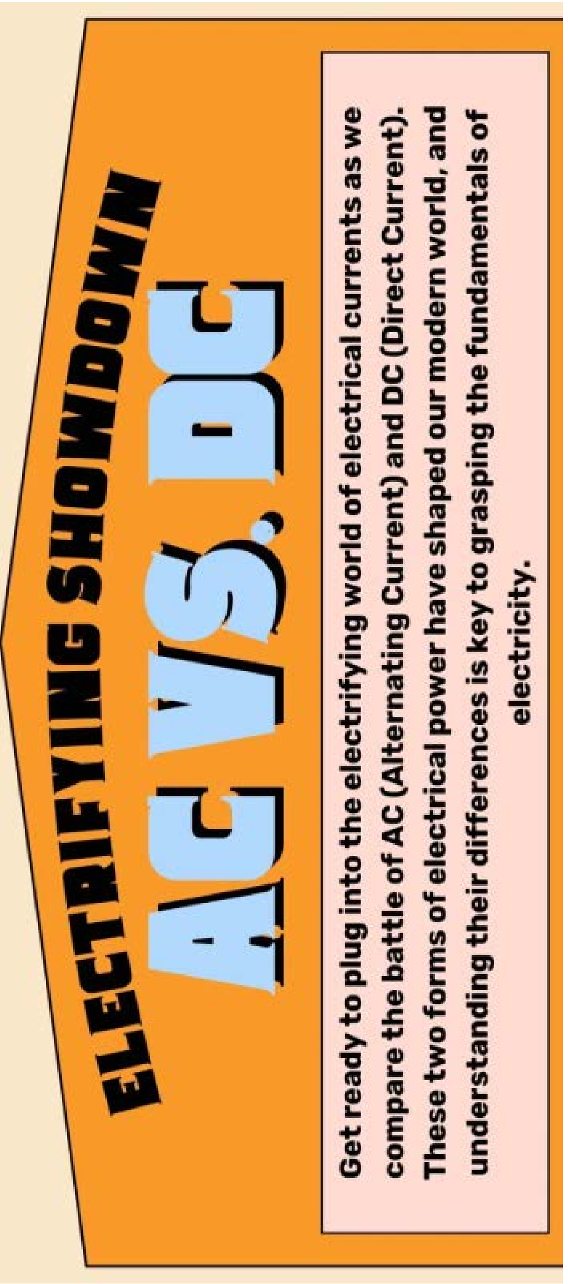
M 2 AC circuits:

- ❖ Alternating voltages and currents, AC values,
- ❖ Single Phase RL, RC, RLC Series circuits,
- ❖ Power in AC circuits and Power Factor



DC (DIRECT CURRENT)

- Current that flows in a single direction
- Commonly used in batteries and electronic devices
- Voltage level remains constant over time
- Well-suited for short-distance transmission and storage

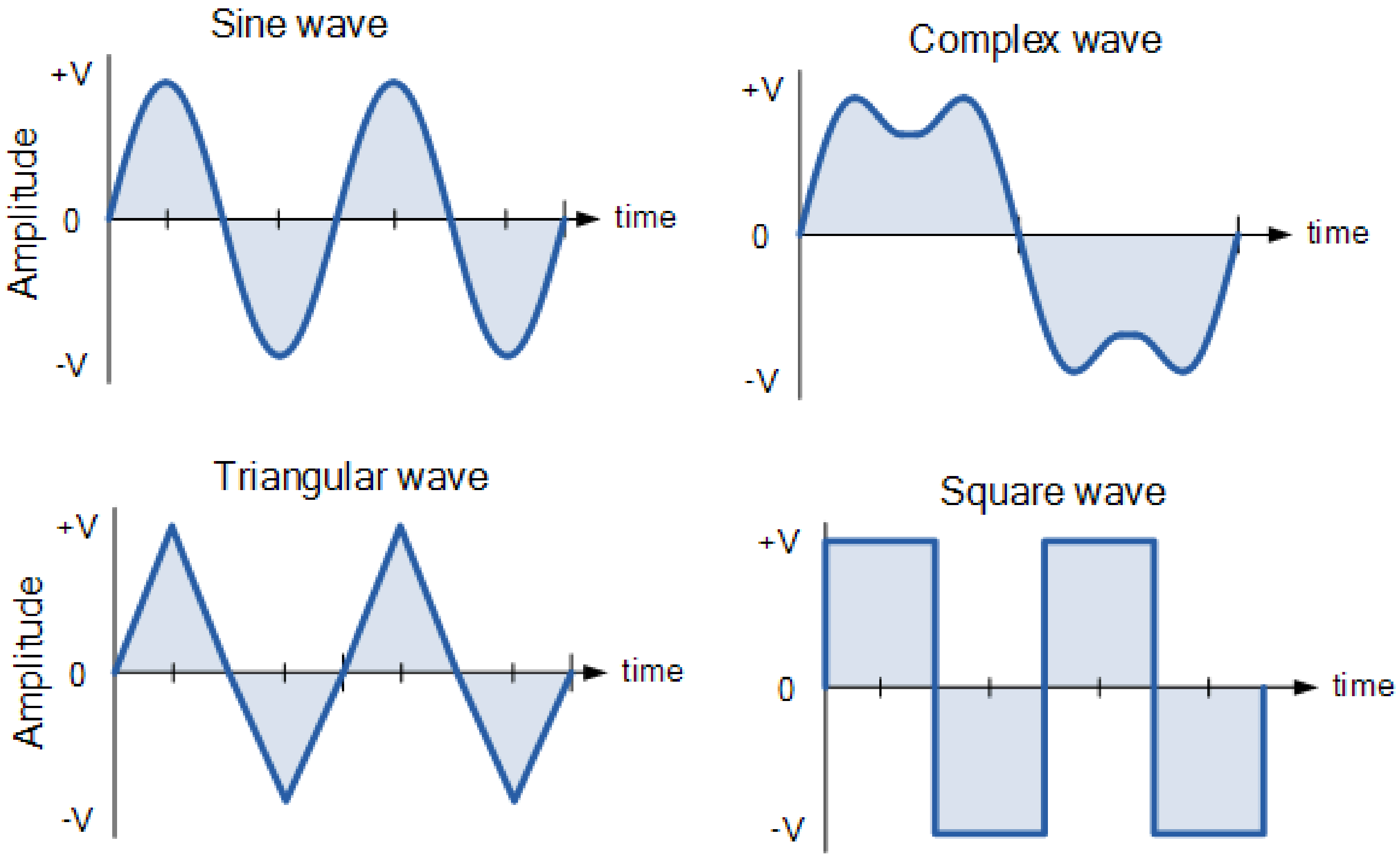
AC (ALTERNATING CURRENT)

- Current that periodically reverses direction
- Widely used in homes and businesses for powering appliances
- Can be easily converted to different voltage levels using transformers
- Efficient for long-distance transmission of electricity

Introduction to AC circuits

Alternating Voltage

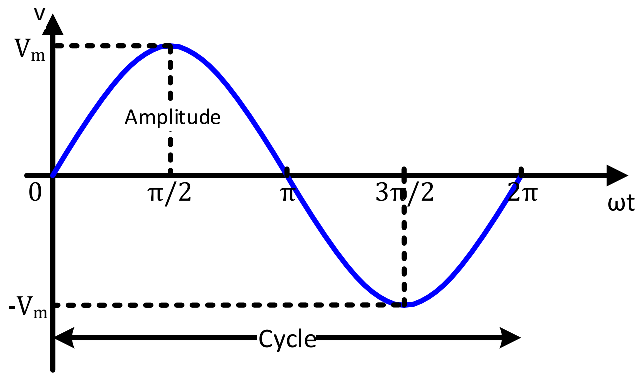
A voltage which periodically changes its polarity at regular intervals of time is called an alternating voltage.



Why Sine Waveform?

- The sine waveform produces the least disturbance in the electrical circuit and is the smoothest and efficient waveform.
- The use of sinusoidal voltages applied to appropriately designed coils results in a revolving magnetic field which has the capacity to do work.
- The mathematical computations are much simpler with this waveform.
- By means of Fourier series analysis, it is possible to represent any periodic function of whatever waveform in terms of sinusoids.

A.C. Terminology



Instantaneous value

The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by *v* and *i* respectively.

Cycle

One complete set of positive and negative values of an alternating quantity is known as a cycle.

Time period

The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.

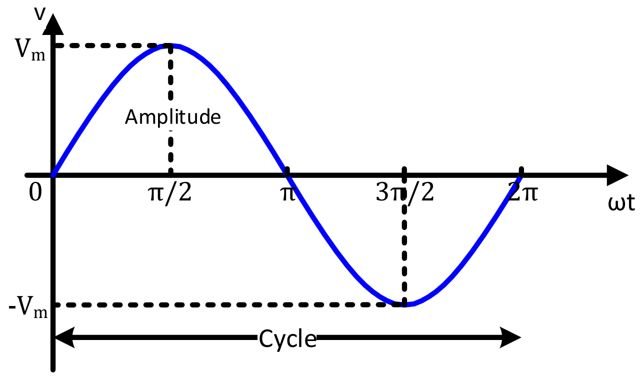
Frequency

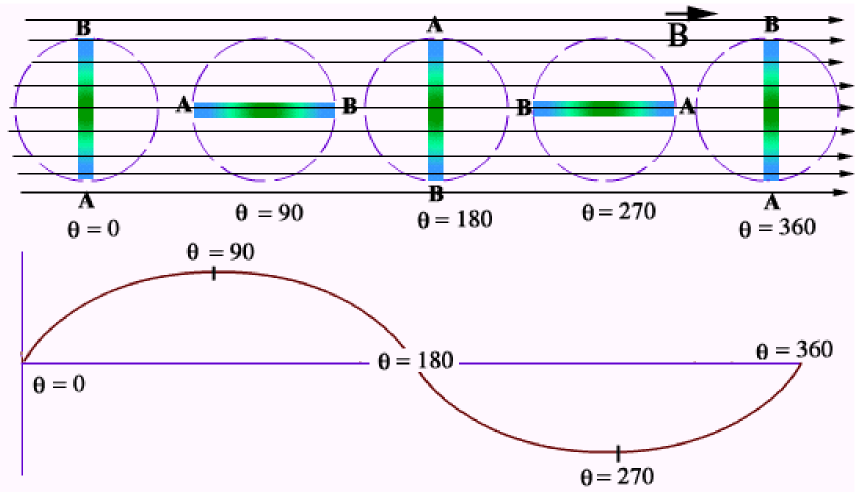
The number of cycles that occur in one second is called the frequency (f) of the alternating quantity. It is measured in cycles/sec (C/s) or Hertz (Hz). One Hertz is equal to 1C/s.

Amplitude

The maximum value (positive or negative) attained by an alternating quantity is called its amplitude or peak value.

A.C. Terminology





Time period and frequency.

Consider an alternating quantity having a frequency of *f* C/s and time period *T* second

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

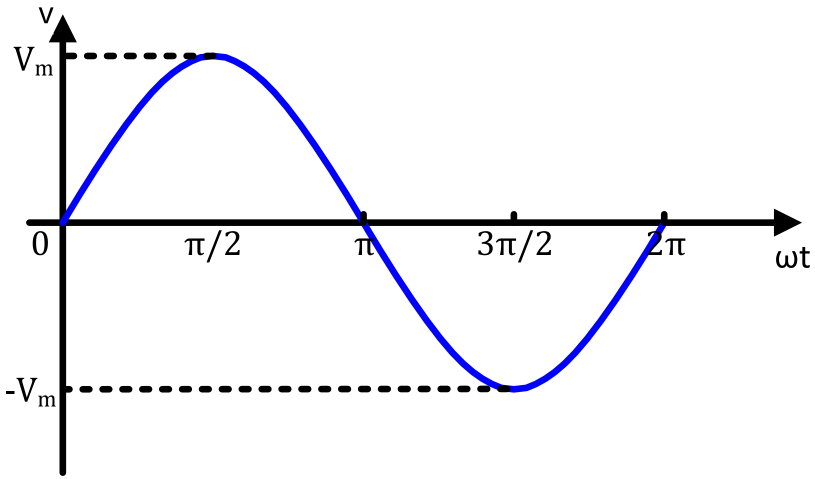
Angular velocity and frequency

In one revolution of the coil, the angle turned is 2π radians and the voltage wave completes 1 cycle. The time taken to complete one cycle is the time period T of the alternating voltage

Angular velocity $$\omega = \frac{Angle\ turned}{Time\ taken} = \frac{2\pi}{T} = 2\pi f$$

Introduction to AC circuits

Representation

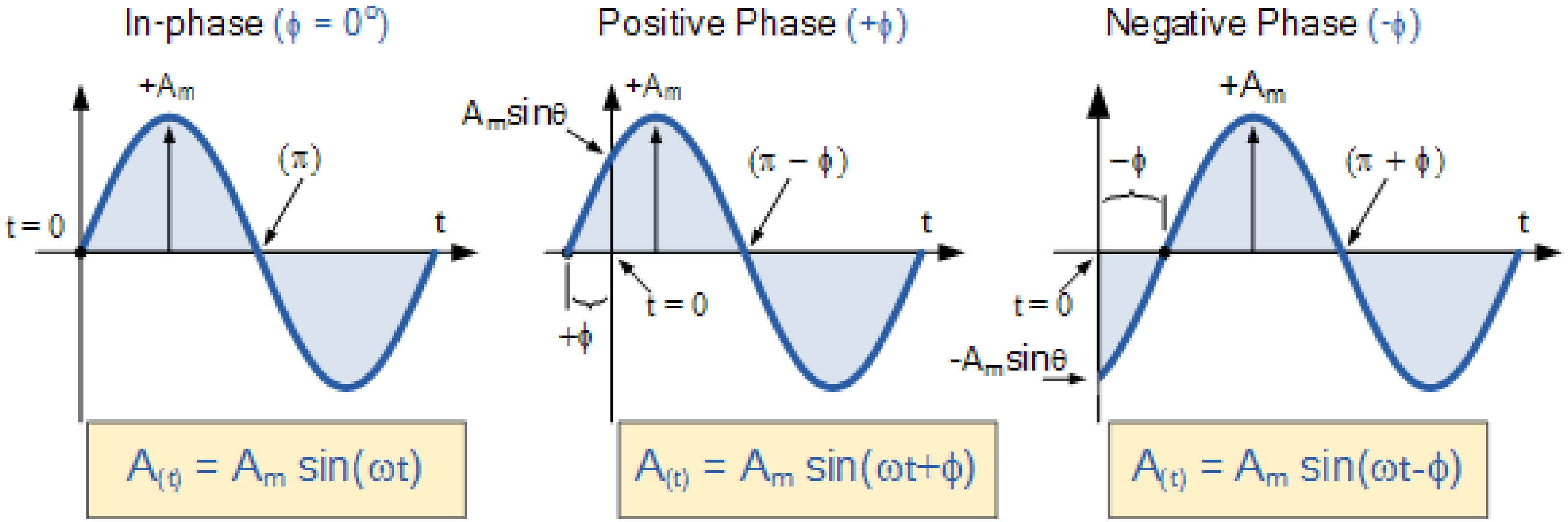


$$v(t) = V_m \sin \omega t$$

V_m = the amplitude of the sinusoid
ω = the angular velocity in radians/s
ωt = the argument of the sinusoid

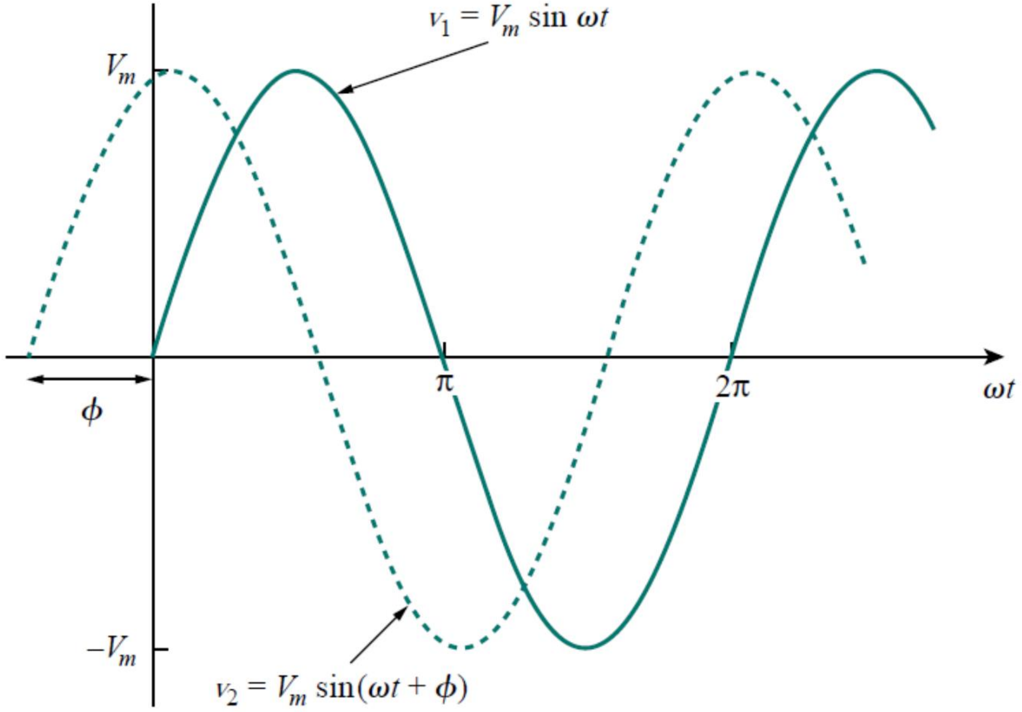
In general $v(t) = V_m \sin(\omega t + \emptyset)$

$(\omega t + \varphi)$ is the argument and $\emptyset$ is the phase. Both argument and phase can be in radians or degrees



Representation

Consider two sinusoids



$$v_1(t) = V_m \sin \omega t$$

$$v_2(t) = V_m \sin(\omega t + \emptyset)$$

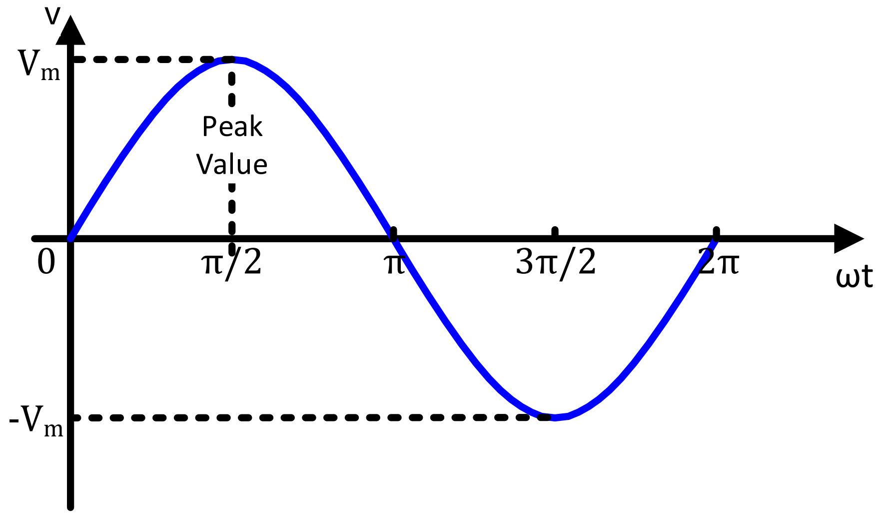
- The starting point of $v_2(t)$ occurs first in time. Hence, $v_2(t)$ *leads* $v_1(t)$ by ∅ or that $v_1(t)$ *lags* $v_2(t)$ by ∅.
- If, $\emptyset \neq 0^0$ then $v_1(t)$ and $v_2(t)$ are *out of phase.*
- If $\emptyset = 0^0$, then $v_1(t)$ and $v_2(t)$ are said to be *in phase* i.e., they reach their minima and maxima at exactly the same time.
- $v_1(t)$ and $v_2(t)$ is compared in this manner because they operate at the same frequency and they do not need to have the same amplitude.

Representation

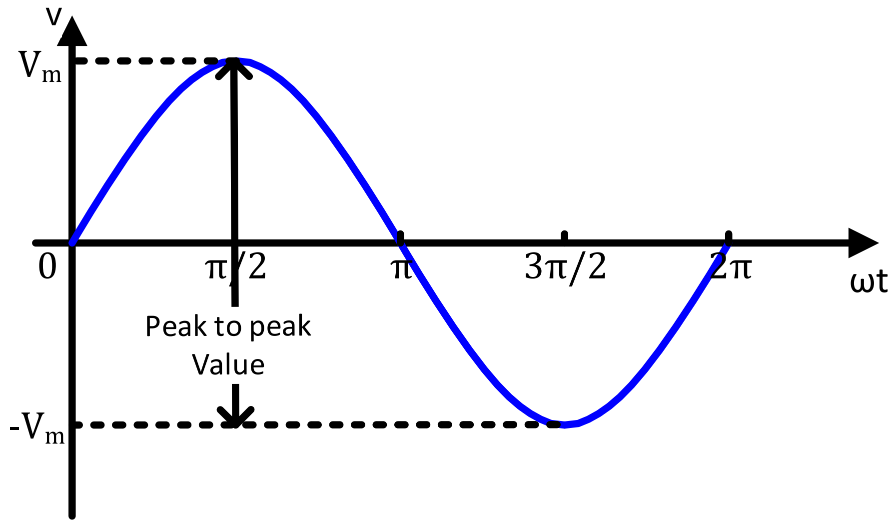
There are four ways to express the magnitude of an alternating voltage or current.

- ❖ Peak value
- ❖ Peak to peak value
- ❖ Average value
- ❖ RMS value

Peak value



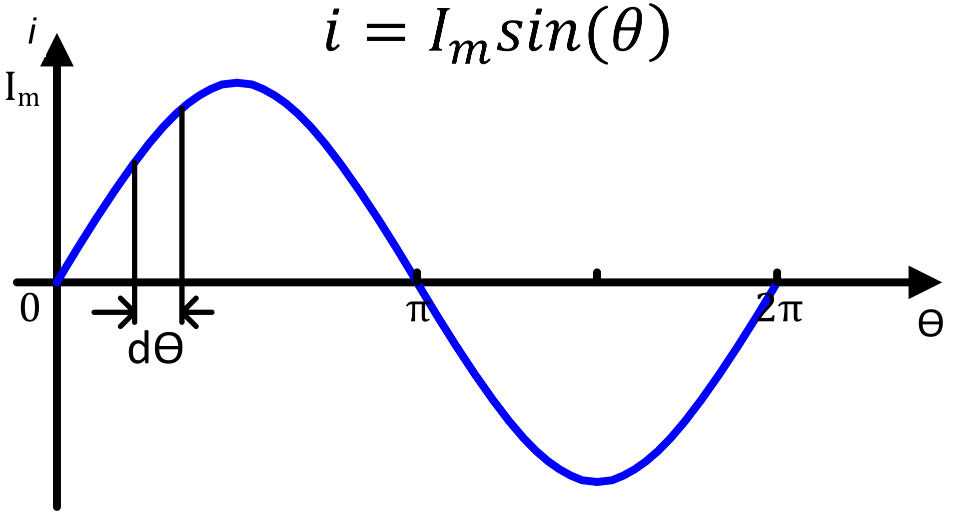
Peak to peak value



Average value

- The average value of alternating current (or voltage) over one cycle is zero. It is because the waveform is symmetrical about time axis and positive area exactly cancels the negative area. However, the average value over a half-cycle (positive or negative) is not zero. *Therefore, average value of alternating current (or voltage) means half-cycle average value unless stated otherwise*.
- The half-cycle average value of a.c. is that value of steady current (d.c.) which would send the same amount of charge through a circuit for half the time period of a.c. as is sent by the a.c. through the same circuit in the same time.

$$i = I_m sin(\theta)$$



Area of the strip $= id\theta$

Area of half−cycle $= \int_0^\pi id\theta = \int_0^\pi I_m \sin\theta\, d\theta = I_m[-\cos\theta]_o^\pi = 2I_m$

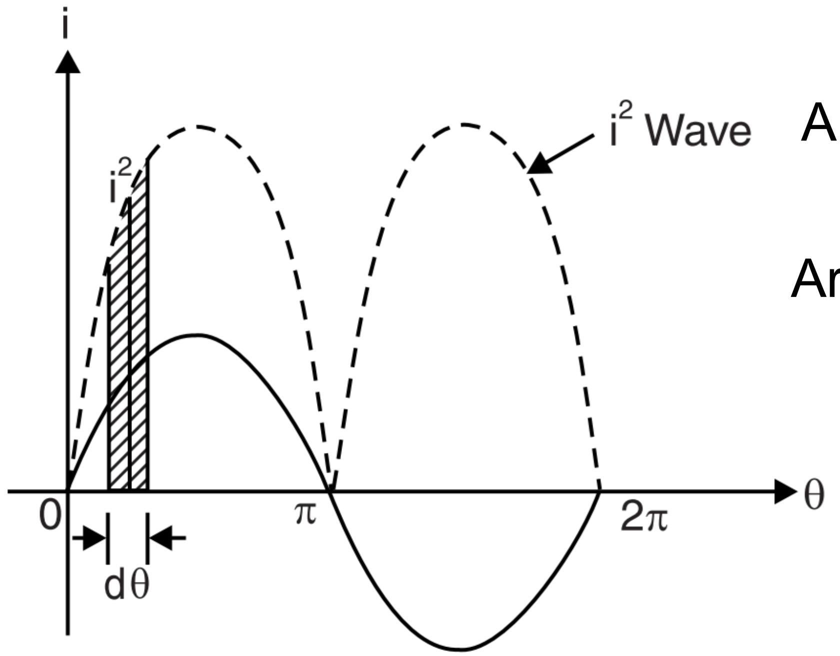
Average Value $I_{av} = \dfrac{\text{Area of half−cycle}}{\text{Base length of half−cycle}} = \dfrac{2I_m}{\pi}$

$$\boldsymbol{I_{av} = 0.637 I_m}$$

RMS value

The effective or r.m.s. value of an alternating current is that steady current (d.c.) which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.

$$\boldsymbol{i = I_m sin(\theta)} \qquad \boldsymbol{i^2 = I_m^2 sin^2(\theta)}$$



Area of the strip $= i^2 d\theta$

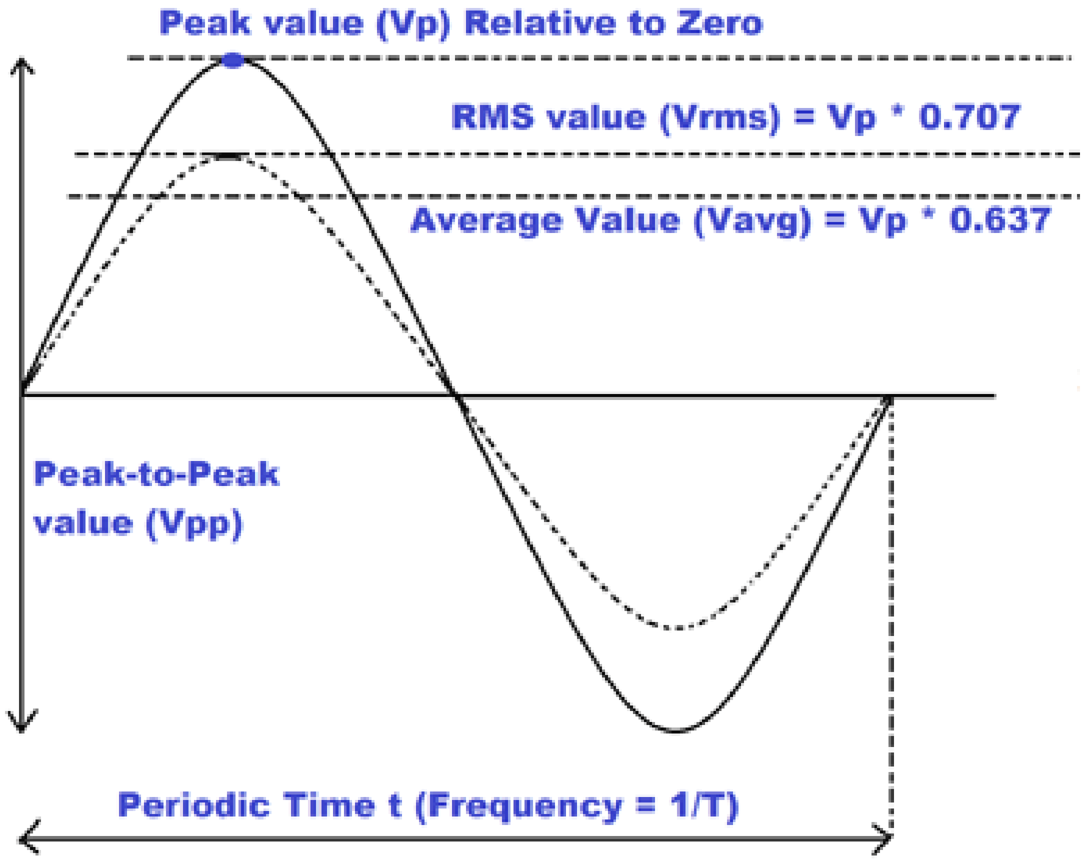
Area of half−cycle $= \int_0^{\pi} i^2 d\theta = \int_0^{\pi} I_m^2 sin^2(\theta) d\theta = I_m^2 \int_0^{\pi} sin^2(\theta) d\theta = \frac{\pi I_m^2}{2}$

$$I_{av}^2 = \frac{\text{Area of half−cycle squared wave}}{\text{Base length o fhalf−cycle}} = \frac{\pi\, I_m^2/2}{\pi}$$

$$\boldsymbol{I_{rms} = 0.707 I_m}$$

$$I_{rms} = \sqrt{I_{av}^2} = \sqrt{\frac{I_m^2}{2}} = 0.707 I_m$$

Introduction to AC circuits



$$V_{P-P} = 2V_m$$

$$V_{av} = 0.637V_m$$

$$V_{rms} = 0.707V_m$$

$$V_m = 1.414V_{rms}$$

Note:

- ✓ The domestic a.c. supply is 230 V, 50 Hz. It is the r.m.s. or effective value.

$$v = V_m sin(\omega t)$$

$$v = 230 \times \sqrt{2}\, sin(2\pi f \times t)$$

$$v = 230 \times \sqrt{2}\, sin(314t)$$

Introduction to AC circuits

Form factor

The ratio of r.m.s. value to the average value of an alternating quantity is known as form factor.

$$\text{Form factor} = \frac{\text{R. M. S value}}{\text{Average value}}$$

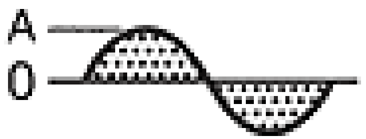
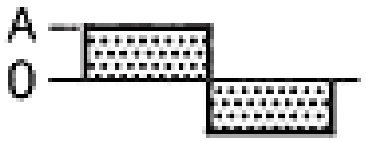
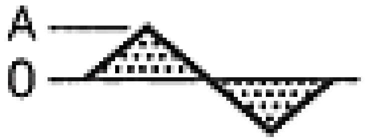
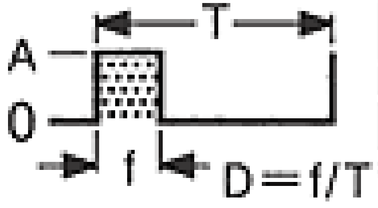
$$\text{Form factor} = \frac{0.707 \times \text{Max.value}}{0.637 \times \text{Max.value}} = 1.11$$

Peak factor

The ratio of maximum value to the r.m.s. value of an alternating quantity is known as peak factor.

$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R. M. S. value}}$$

$$\text{Peak factor} = \frac{\text{Max.value}}{0.707 \times \text{Max.value}} = 1.414$$

Waveform	Effective value V rms	Average value V avg	Conversion factor V rms/V avg	Reading errors for average sensing Instruments	Crest factor CF
	$\frac{1}{\sqrt{2}}A$ $\fallingdotseq 0.707$	$\frac{2}{\pi}A$ $\fallingdotseq 0.637$	$\frac{\pi}{2\sqrt{2}}$ $\fallingdotseq 1.111$	0%	$\sqrt{2}$ $\fallingdotseq 1.414$
	A	A	1	$\frac{A\times1.111-A}{A}\times100$ $=11.1\%$	1
	$\frac{1}{\sqrt{3}}A$	0.5 A	$\frac{2}{\sqrt{3}}$ $\fallingdotseq 1.155$	$\frac{0.5A\times1.111-\frac{A}{\sqrt{3}}}{\frac{A}{\sqrt{3}}}$ $\times100=-3.8\%$	$\sqrt{3}$ $\fallingdotseq 1.732$
	$A\sqrt{D}$	$A\frac{f}{T}$ $=A\cdot D$	$\frac{A\sqrt{D}}{AD}=\frac{1}{\sqrt{D}}$	$(1.111\sqrt{D}-1)$ $\times100\%$	$\frac{A}{\sqrt{AD}}=\frac{1}{\sqrt{D}}$

Example

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12sin(50t + 10^o)$$

Solution:

The amplitude is $v_m = 12\ V$

The phase is $\emptyset = 10^o$

The angular frequency is $\omega = 50\ rad/s$

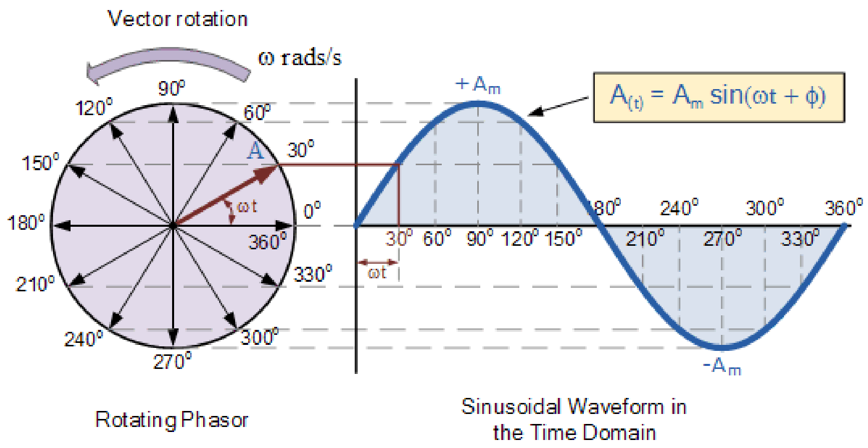
The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257s$

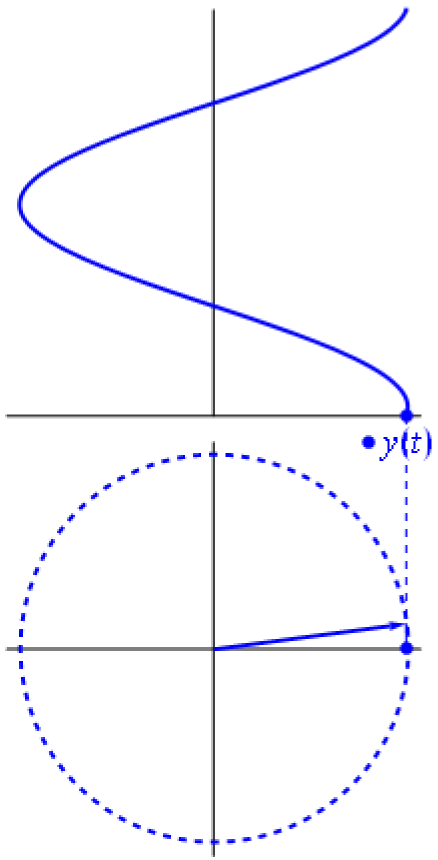
The frequency is $f = \frac{1}{T} = 7.958Hz$

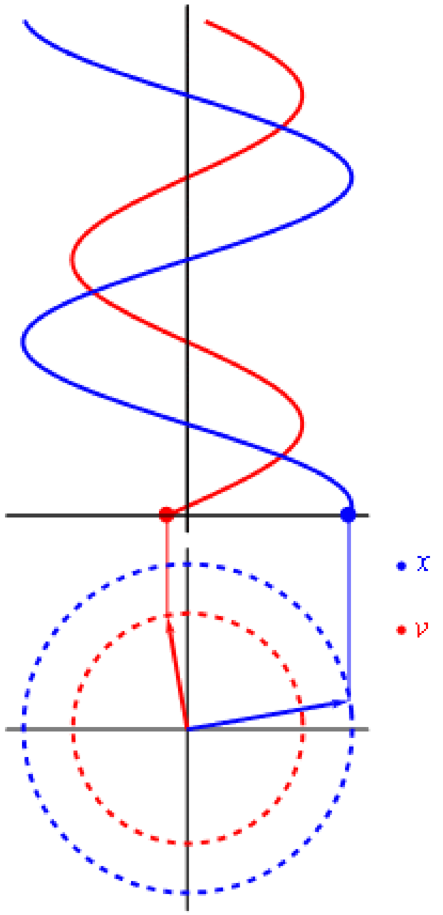
Concept of Phasors

Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions.

- The sinusoidal alternating voltage or current is represented by a line of definite length rotating in anticlockwise direction at a constant angular velocity (ω). Such a rotating line is called a **phasor**.
- The length of the phasor is taken equal to the maximum value (on a suitable scale) of the alternating quantity and angular velocity equal to the angular velocity of the alternating quantity.







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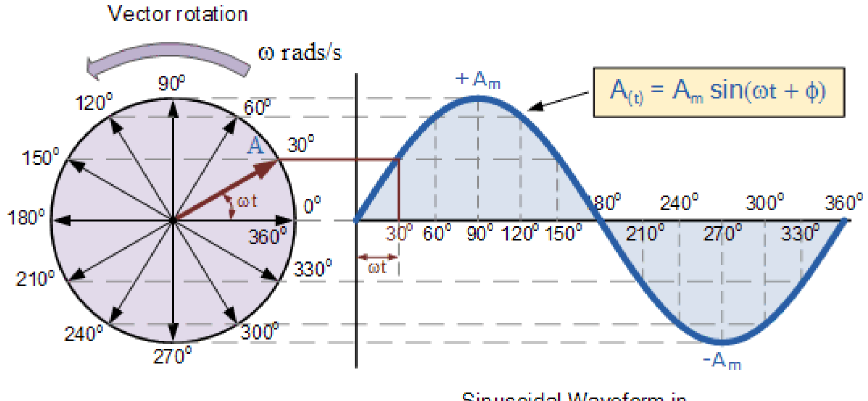
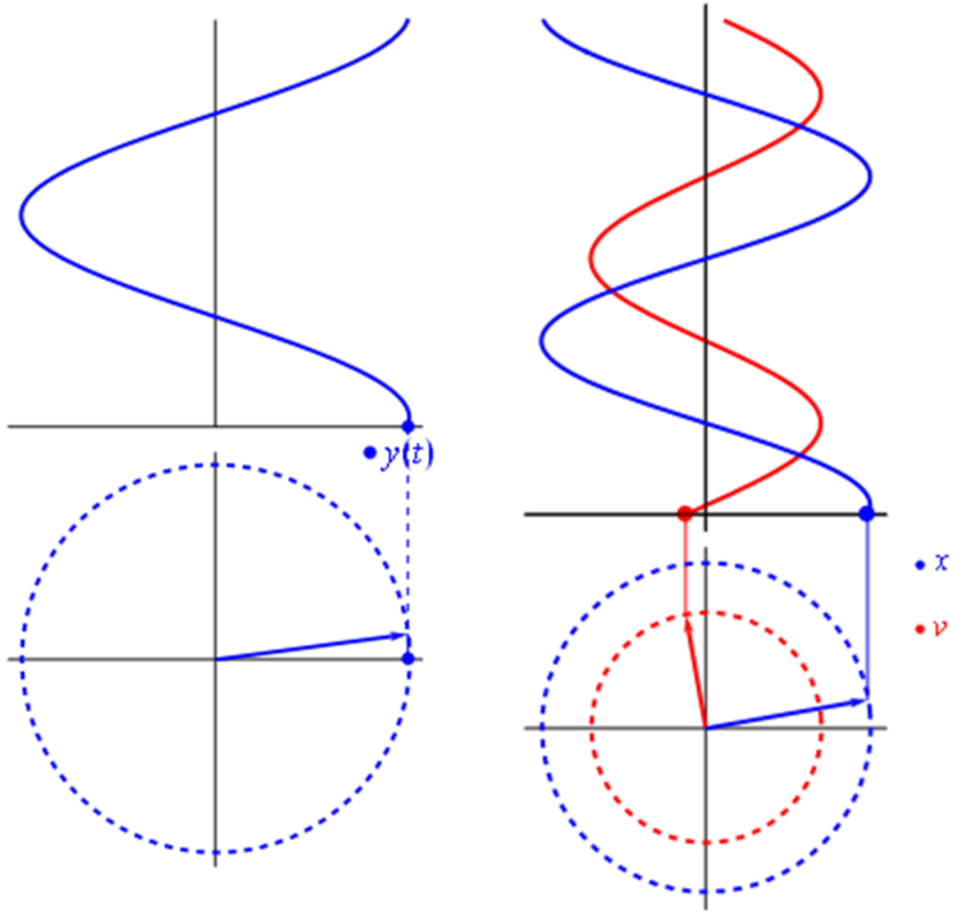


Rotating Phasor

Sinusoidal Waveform in the Time Domain



Concept of Phasors

Representation

There are four ways of representing a phasor in the mathematical form *viz*.

(i) Rectangular form

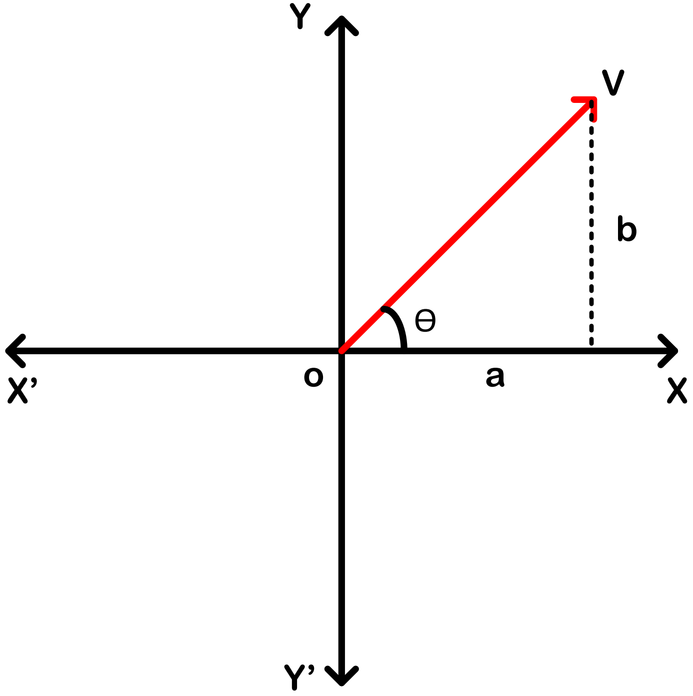
(ii) Trigonometrical form

(iii) Polar form

(iv) Exponential form.

Rectangular form

In this representation, the phasor is resolved into horizontal and vertical components and is expressed in the complex form



$$\mathbf{V} = a + jb$$

$$V = \sqrt{a^2 + b^2}$$

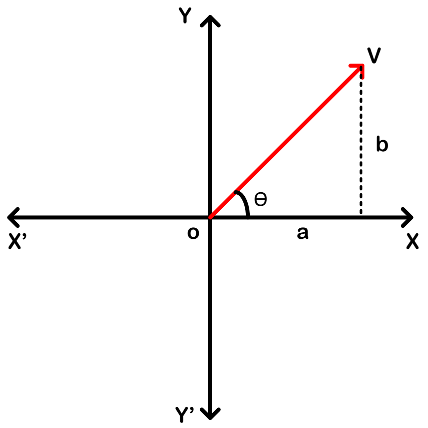
$$\theta = tan^{-1}\frac{b}{a}$$

where $\mathbf{j} = \sqrt{-1}$; a is the real part of $\mathbf{V}$; b is the imaginary part of $\mathbf{V}$.

Concept of Phasors

Trigonometrical form

It is similar to the rectangular form except that in-phase and quadrature components of the phasor are expressed in the trigonometrical form.



$$a = V\cos\theta$$

$$b = V\sin\theta$$

$$\mathbf{V} = V\,(\cos\theta + j\sin\theta)$$

In general

$$\mathbf{V} = V\,(\cos\theta \pm j\sin\theta)$$

Polar form

$$\mathbf{V} = V\angle\theta$$

V is the magnitude of the phasor and q is its phase angle measured *CCW* from the reference axis

In general $$\mathbf{V} = V\angle \pm \theta$$

Exponential form

According to Euler's equation :

$$e^{j\theta} = cos(\theta) + jsin(\theta)$$
$$e^{-j\theta} = cos(\theta) - jsin(\theta)$$

Trigonometrical form of $\mathbf{V}$

$$\mathbf{V} = V(cos(\theta) \pm jsin(\theta))$$

$$\mathbf{V} = Ve^{\pm j\theta}$$

Concept of Phasors

Phasor Representation

Given a sinusoid

$$v(t) = V_m cos(\omega t + \phi)$$

$$v(t) = Re(V_m e^{j(\omega t+\phi)})$$

$$v(t) = Re(V_m e^{j\phi} e^{j\omega t})$$

$$v(t) = Re(\mathbf{V} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi$$

V is thus the *phasor representation* of the sinusoid *v(t)*
i.e., a phasor is a complex representation of the magnitude and phase of a sinusoid.

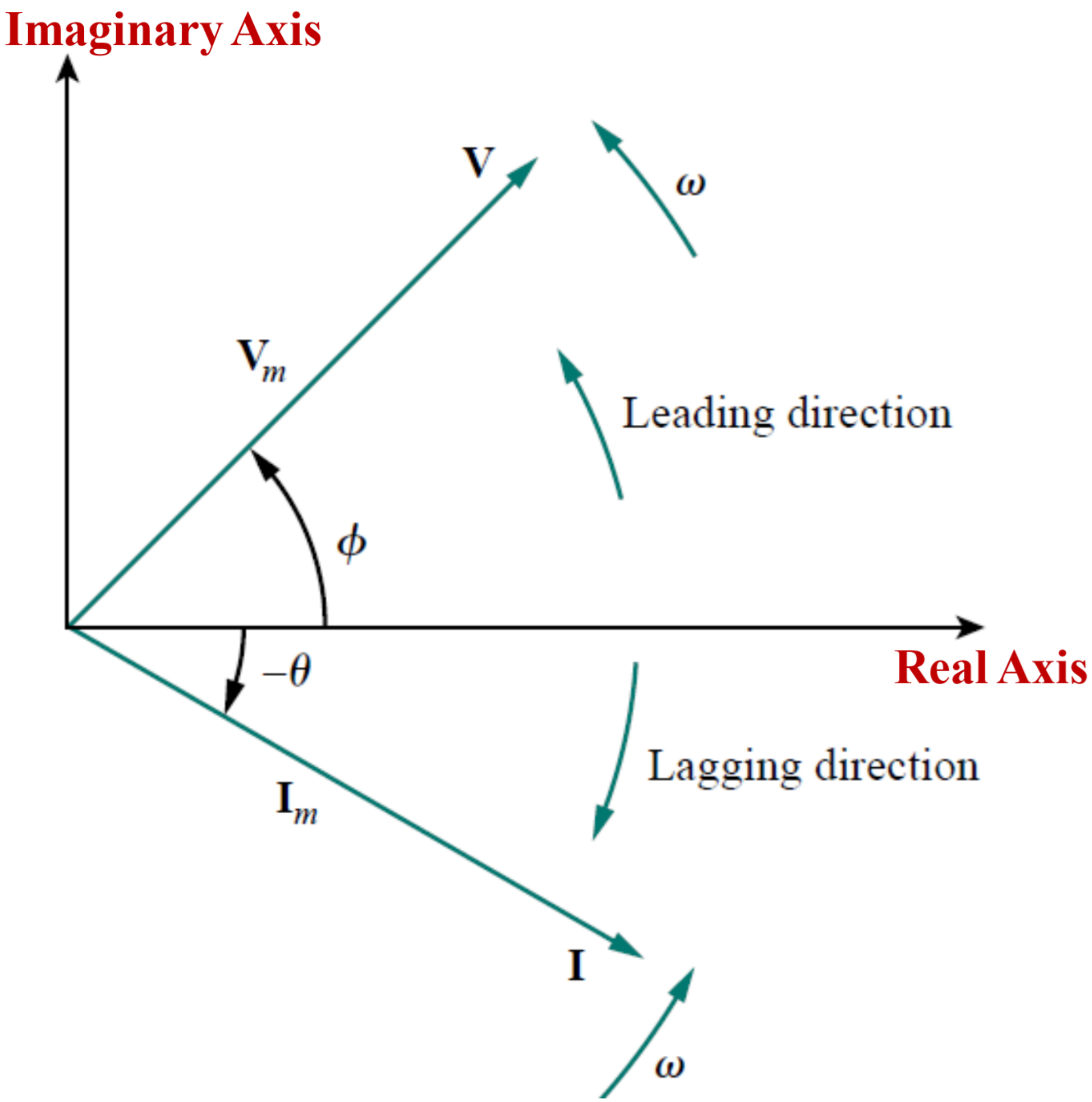
$$v(t) = V_m cos(\omega t + \phi) \Leftrightarrow \mathbf{V} = V_m \angle \phi$$

Concept of Phasors

Phasor Diagram

$$\mathbf{V} = V_m\angle\phi$$

$$\mathbf{I} = I_m\angle -\theta$$



Concept of Phasors

Basic Operations

$$\mathbf{V} = a + jb = V\angle\phi$$

Given the Complex number $\mathbf{V}_1 = a_1 + jb_1 = V_1\angle\theta_1$ $\mathbf{V}_2 = a_2 + jb_2 = V_2\angle\theta_2$

Addition

$$\mathbf{V}_1 + \mathbf{V}_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Subtraction

$$\mathbf{V}_1 - \mathbf{V}_2 = (a_1 - a_2) + j(b_1 - b_2)$$

Multiplication

$$\mathbf{V}_1\mathbf{V}_2 = V_1V_2\angle(\theta_1 + \theta_2)$$

Division

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{V_1}{V_2}\angle(\theta_1 - \theta_2)$$

Reciprocal

$$\frac{1}{\mathbf{V}} = \frac{1}{V}\angle(-\theta)$$

Square Root

$$\sqrt{\mathbf{V}} = \sqrt{V}\angle(\theta/2)$$


Thank you