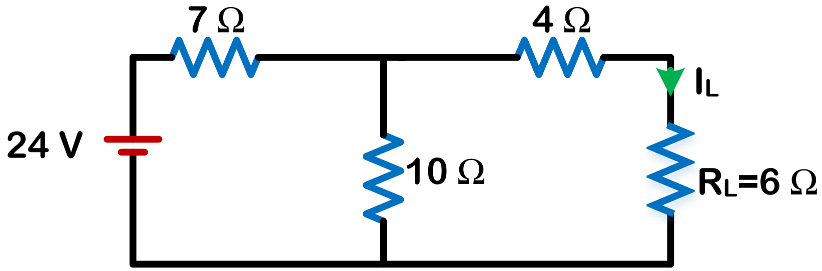
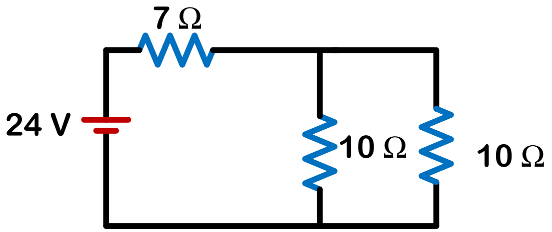
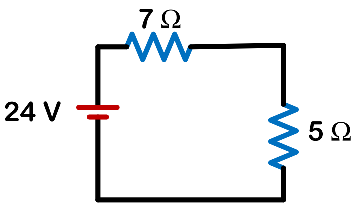
Thevenin's and Maximum Power Transfer Theorem

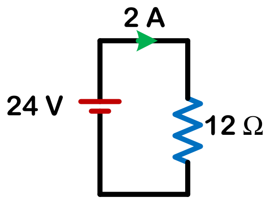
Thevenin's Theorem

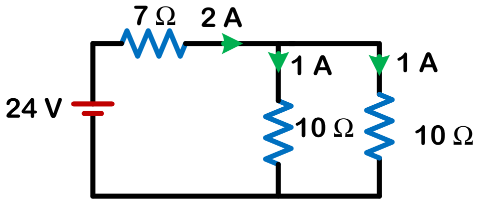
Find I_L for the given circuit

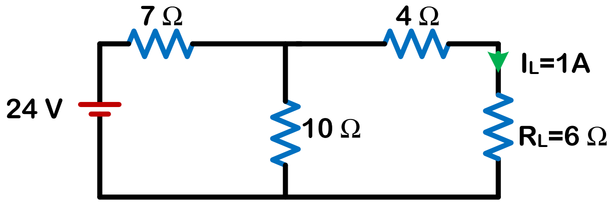


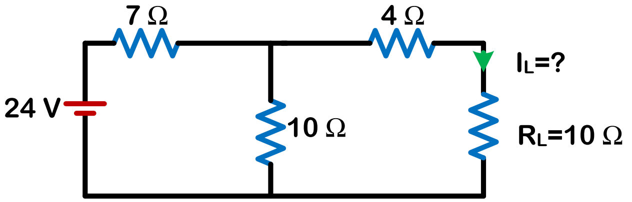


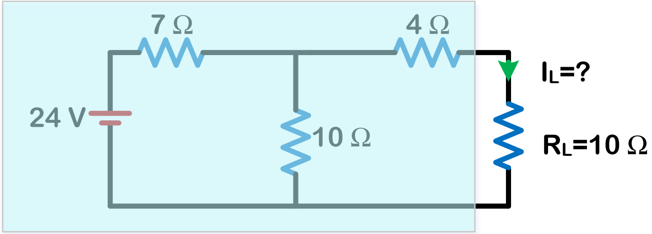


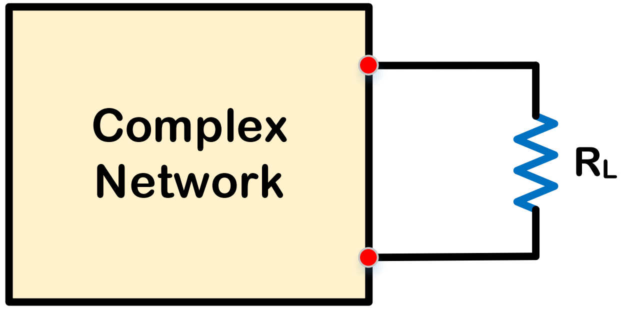


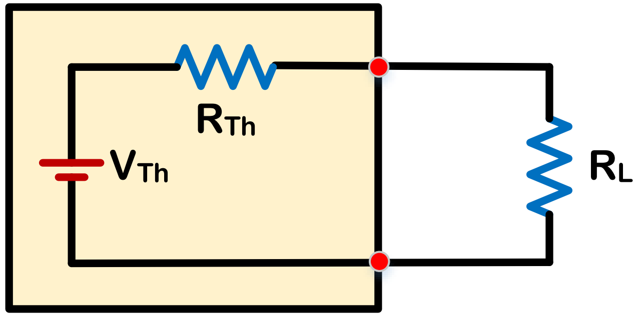












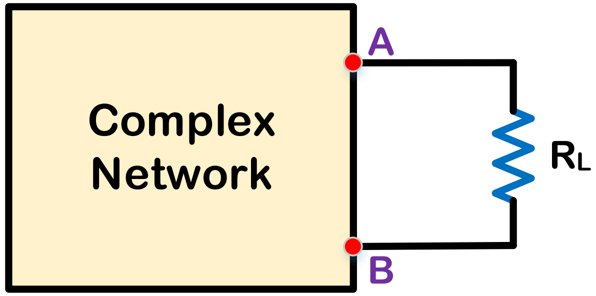
Thevenin's Equivalent Circuit

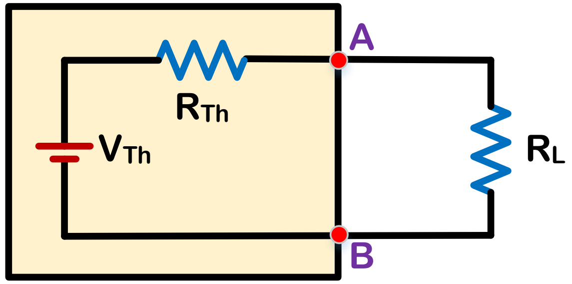
Thevenin's Theorem

Any linear, bilateral network having terminals A and B can be replaced by a single source of *e.m.f.* V_{Th} in series with a single resistance R_{Th}.

(i) The e.m.f. V_{Th} is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.

(ii) The resistance R_{Th} is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Voltage sources are replaced with short circuits and current sources are replaced with open circuits.

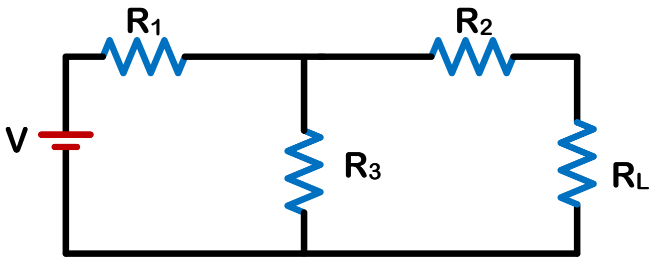


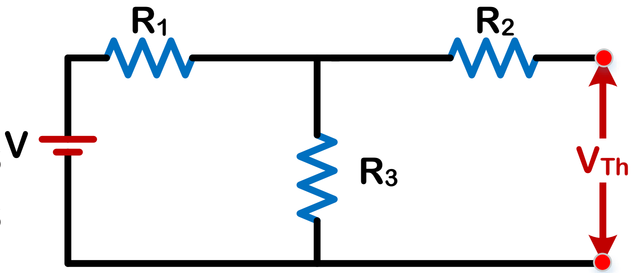


Thevenin's Equivalent Circuit

Procedure for Finding Thevenin Equivalent Circuit

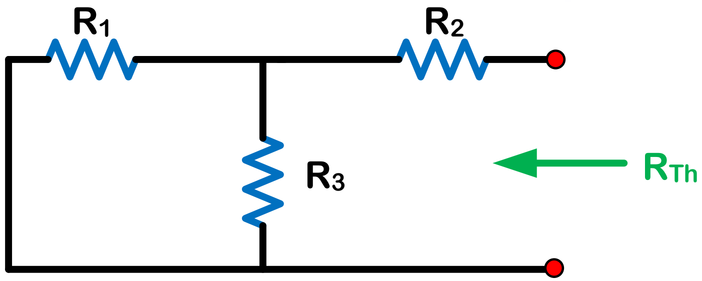
1. Open the two terminals (*i.e.*, remove any load) between which you want to find Thevenin equivalent circuit.
2. Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage V_{Th}.
3. Determine the resistance between the two open terminals with all voltage sources shorted and all current sources opened. It is called Thevenin resistance R_{Th}.
4. Connect V_{Th} and R_{Th} in series to produce Thevenin equivalent circuit between the two terminals under consideration.
5. Place the load resistor removed in step (1) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.



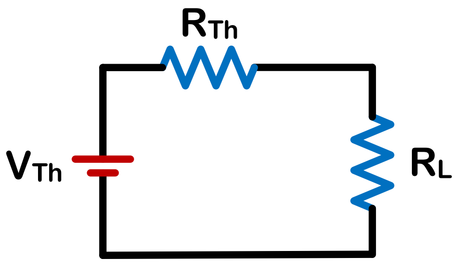


$$V_{Th} = Volatge\ across\ R_3$$

$$V_{Th} = \frac{V}{R_1 + R_3} R_3$$



$$R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

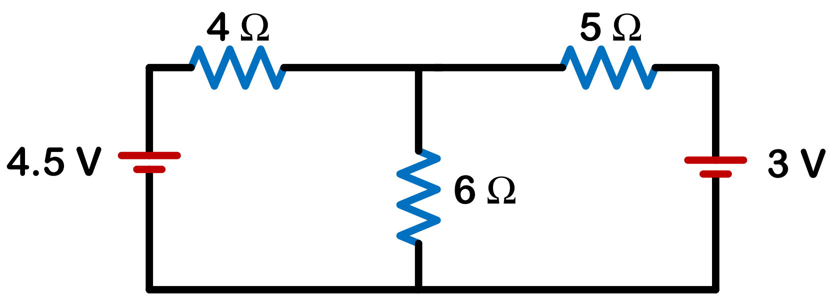


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Thevenin's Theorem

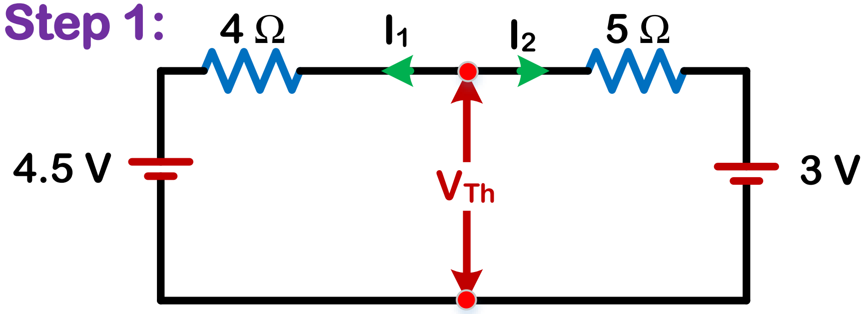
Example

Find the current flowing through 6Ω resistor using Thevenin's theorem.



Solution:

Step 1:

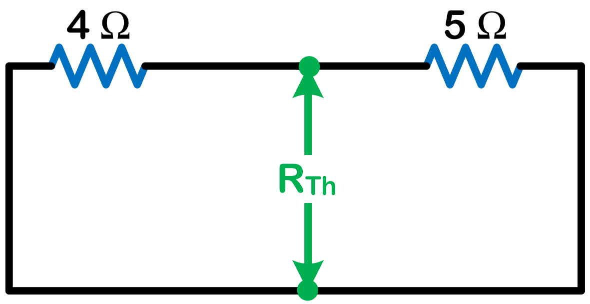


$$I_1 + I_2 = 0$$

$$\frac{V_{Th} - 4.5}{4} + \frac{V_{Th} - 3}{5} = 0$$

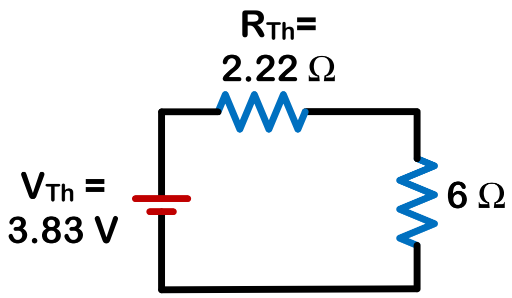
$$V_{Th} = 3.83V$$

Step 2:



$$R_{Th} = \frac{4 \times 5}{4 + 5} = 2.22\Omega$$

Step 3:

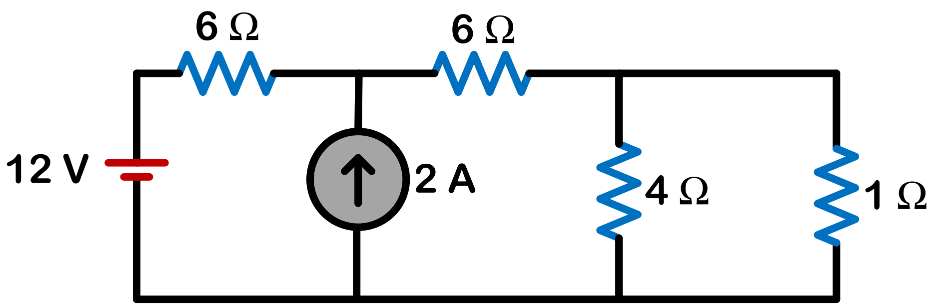


$$I_{6\Omega} = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_{6\Omega} = \frac{3.83}{2.22 + 6} = 0.47A$$

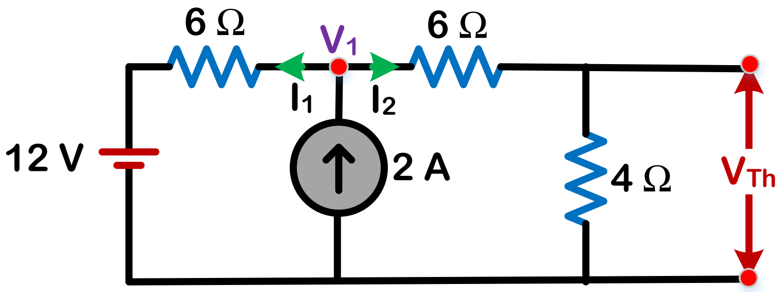
Thevenin's Theorem

Find the current flowing through 1Ω resistor using Thevenin's theorem.



Solution:

Step 1:



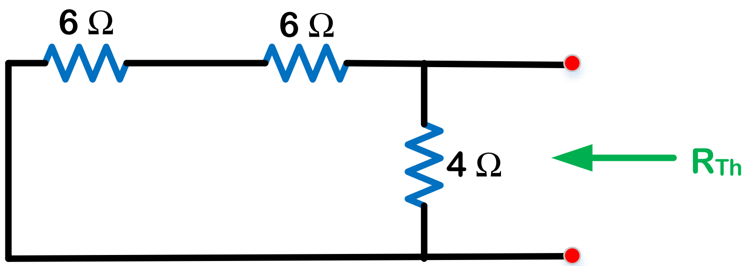
$$I_1 + I_2 = 2$$

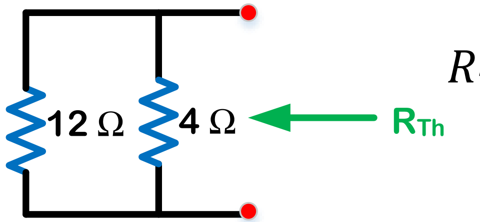
$$\frac{V_1 - 12}{6} + \frac{V_1}{10} = 2; \quad V_1 = 15V$$

$$I_2 = \frac{V_1}{10} = 1.5A$$

$$V_{Th} = I_2 \times 4 = 6V$$

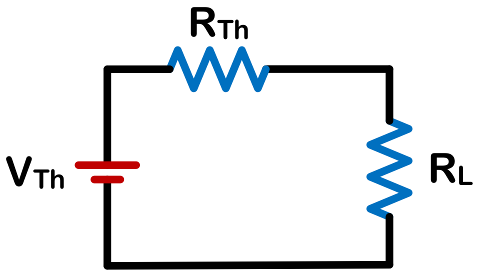
Step 2:





$$R_{Th} = \frac{12 \times 4}{12 + 4} = 3\Omega$$

Step 3:

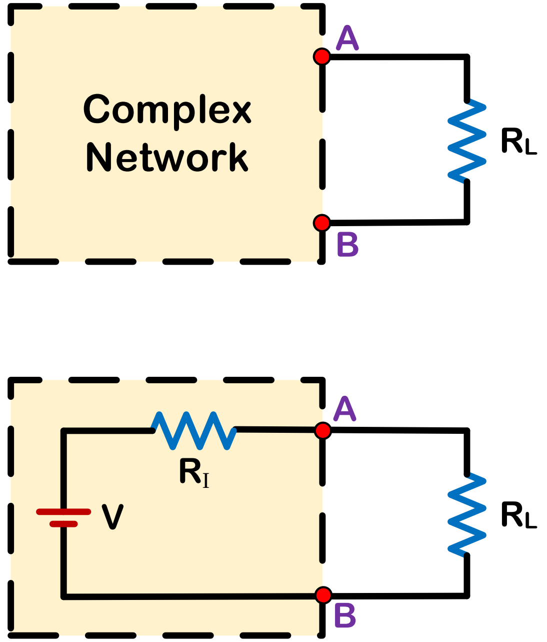


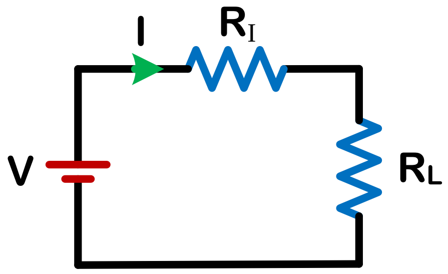
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{6}{3 + 1} = 1.5A$$

Maximum Power Transfer Theorem

In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.

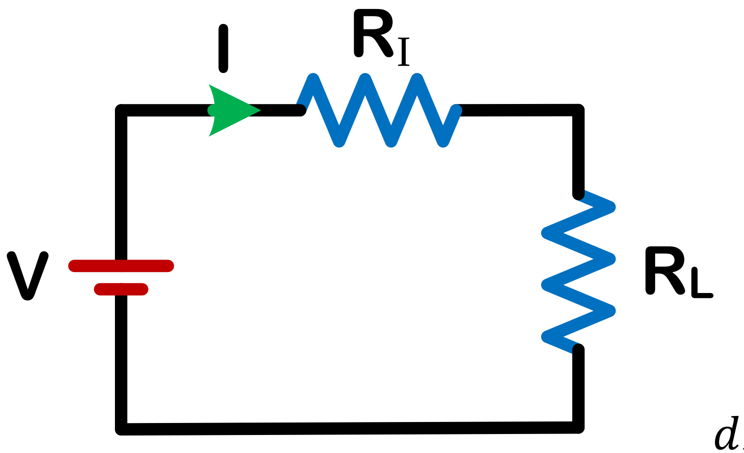




V = 12V $R_I = 3\ \Omega$

R_L	$I = \dfrac{V}{R_I + R_L}$	Power Delivered to Load $P_L = I^2 * R_L$
1Ω	I=3 A	$P_L = 9$ W
2Ω	I=2.4 A	$P_L = 11.52$ W
3Ω	I=2 A	$P_L = 12$ W
4Ω	I=1.71 A	$P_L = 11.69$ W
5Ω	I=1.5 A	$P_L = 11.25$ W
6Ω	I=1.33 A	$P_L = 10.61$ W

Proof of Maximum Power Transfer Theorem



$$I = \frac{V}{R_L + R_I}$$

$$P_L = I^2 R_L = \left(\frac{V}{R_L + R_I}\right)^2 R_L$$

$$\frac{dP_L}{dR_L} = V^2 \left[\frac{(R_L + R_I)^2 - 2R_L(R_L + R_I)}{(R_L + R_I)^4}\right] = 0$$

$$(R_L + R_I)^2 - 2R_L(R_L + R_I) = 0$$

$$(R_L + R_I)(R_L + R_I - 2R_L) = 0$$

$$(R_L + R_I)(R_I - R_L) = 0$$

$$(R_L + R_I) \neq 0$$

$$R_I - R_L = 0$$

$$R_L = R_I$$

Load resistance = Internal resistance

Maximum Power Transfer Theorem

Important Points

- ✓ The circuit efficiency at maximum power transfer is only 50% as one-half of the total power generated is dissipated in the internal resistance R_i of the source.

$$\eta = \frac{Output\ Power}{Input\ Power} = \frac{I^2 R_L}{I^2(R_L + R_I)} = \frac{R_L}{2R_L} = \frac{1}{2} = 50\%$$

- ✓ Under the conditions of maximum power transfer, the load voltage is one-half of the open circuited voltage at the load terminals.

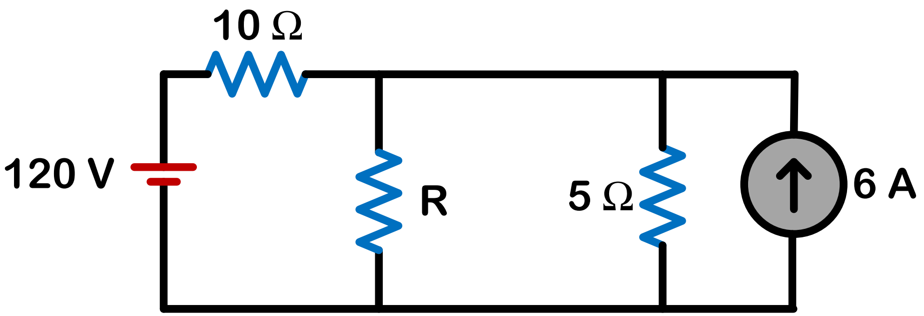
$$V_L = IR_L = \frac{V}{R_L + R_I} R_L = \frac{V}{2}$$

- ✓ Value of maximum power transferred

$$P_{L,max} = \left(\frac{V}{R_L + R_I}\right)^2 R_L = \frac{V^2}{4R_L}$$

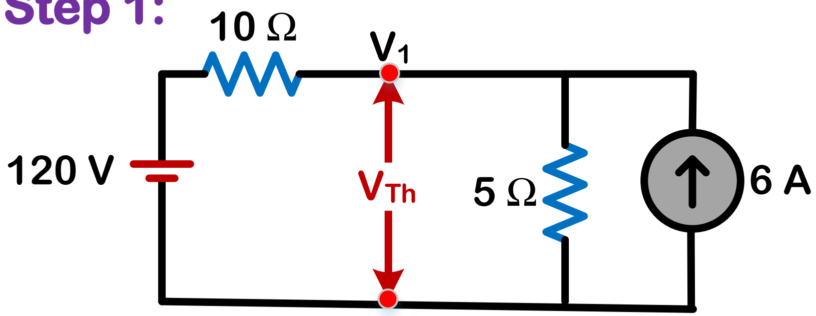
Example

Find the value of resistance R to have maximum power transfer in the circuit shown in figure. Also obtain the amount of maximum power



Solution:

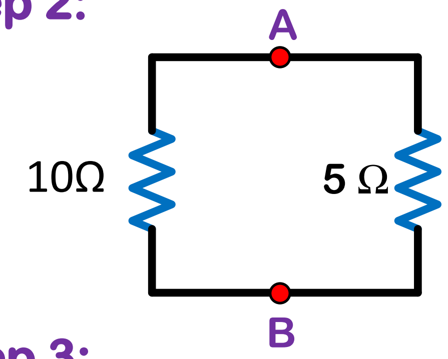
Step 1:



$$\frac{V_1 - 120}{10} + \frac{V_1}{5} = 6$$

$$V_{Th} = V_1 = 60V$$

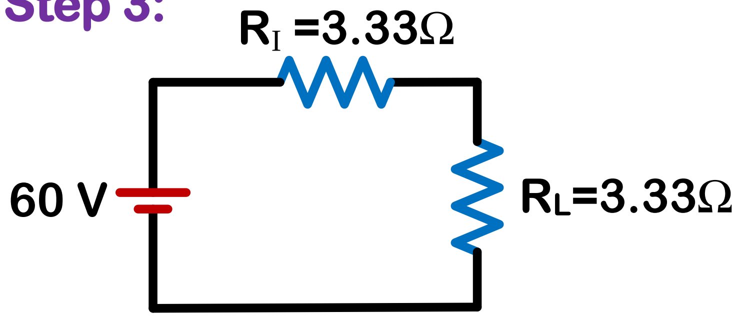
Step 2:



$$R_{Th} = 10\Omega||5\Omega = \frac{10 \times 5}{10 + 5} = 3.33\Omega$$

$$R = R_{Th} = 3.33\Omega$$

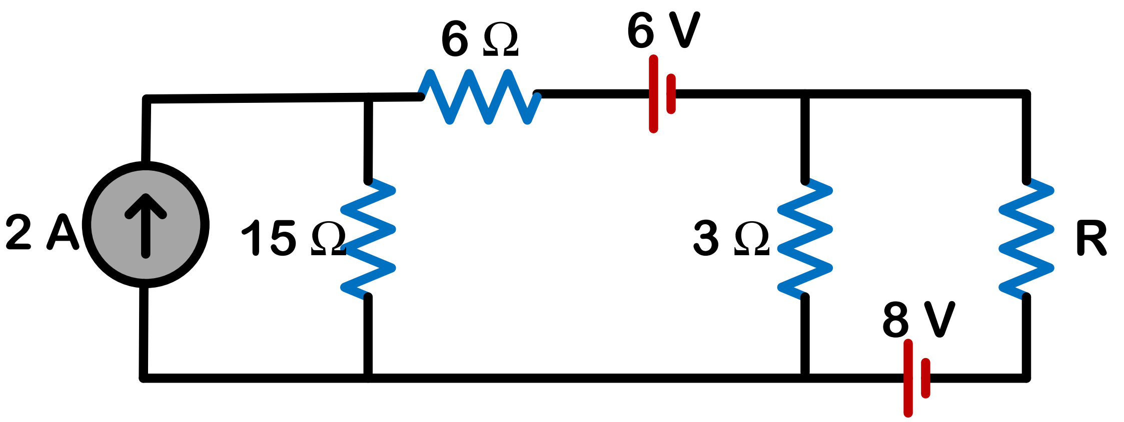
Step 3:



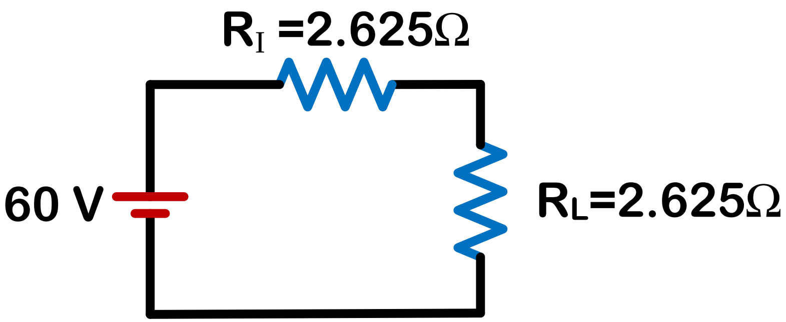
$$P_{max} = \frac{V_T^2}{4R_L} = \frac{60^2}{4 \times 3.33} = 270W$$

Tutorial

Calculate the value of R which will absorb maximum power from the circuit shown in figure. Also find the value of maximum power.



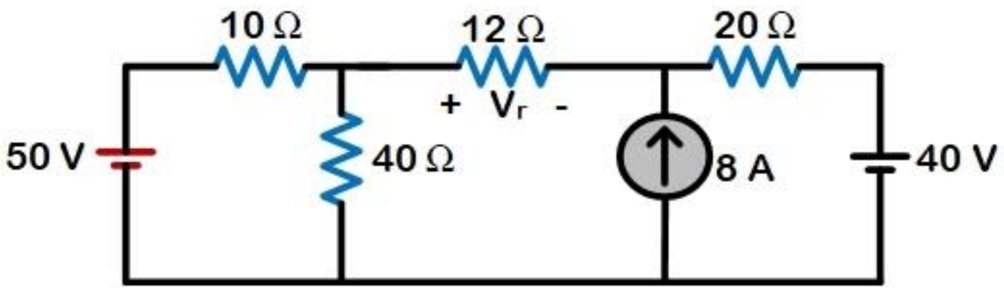
Solution:



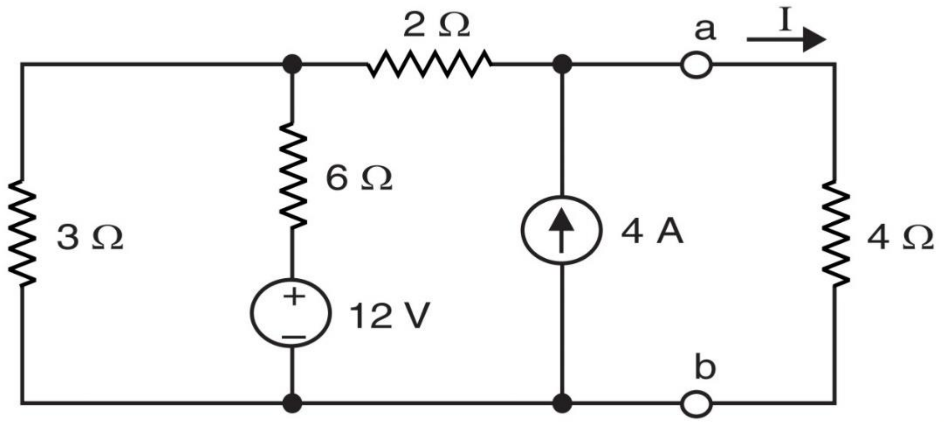
maximum power transferred =11.52W

Tutorial Problems

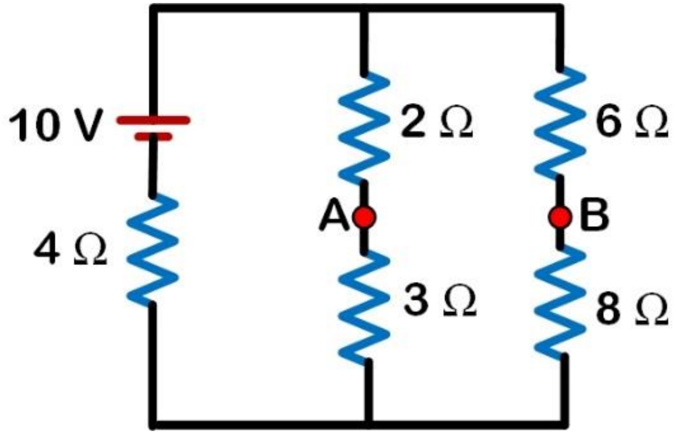
Use nodal analysis to find the value of V_r



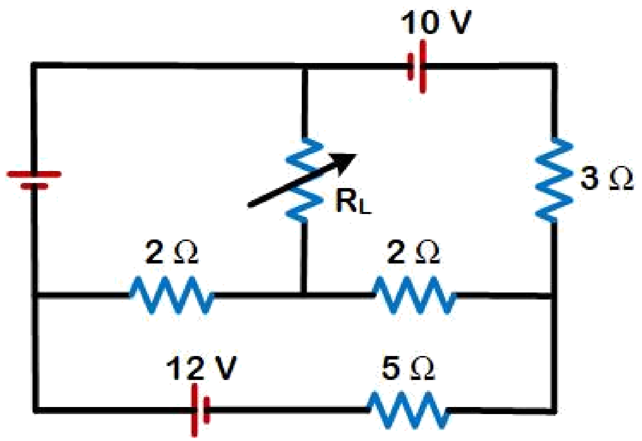
By using Thevenin's theorem, find current I



Using mesh analysis find the voltage across A and B V_{AB}



Calculate the value of R_L which will absorb maximum power from the circuit. Also find the value of maximum power.



Tutorial Problems

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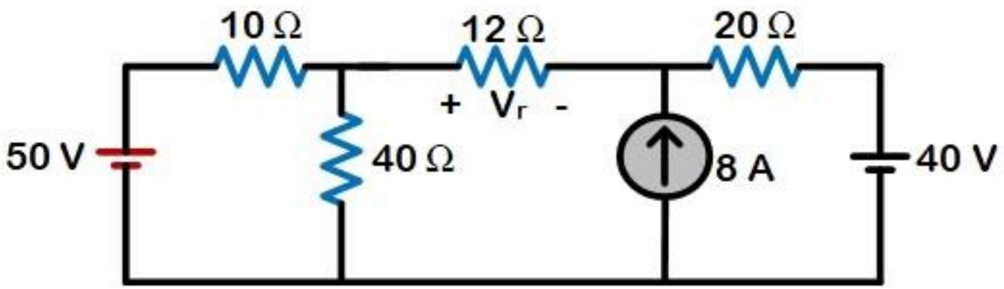

Tutorial Problems

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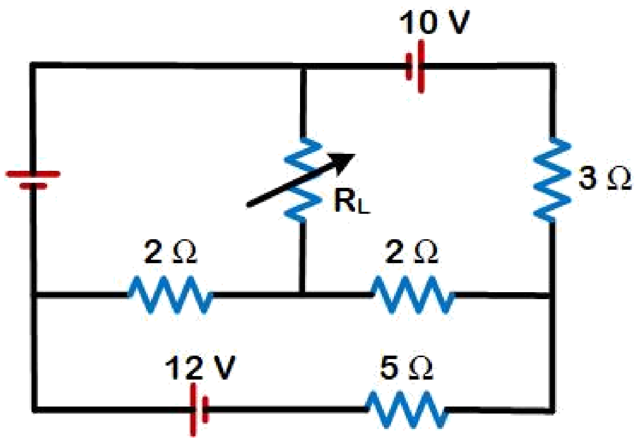


Thank you