Department of Physics, C. V. Raman Global University

Lecture Notes of Module: 1

Subject: Physics (PH101)

Topics covered: Oscillation, Interference, Diffraction, Polarization

Lecture: 1

Simple harmonic motion (SHM)

Simple harmonic motion can be defined as the simplest kind of oscillatory motion in which displacement varies sinusoidally with time i.e. displacement is sine or cosine function of time.

Let us consider a particle Pat a point P(x,y) rotating on a circumference of a circle of radius A with uniform angular velocity ω as shown in figure 1. O is the origin (centre of circle).

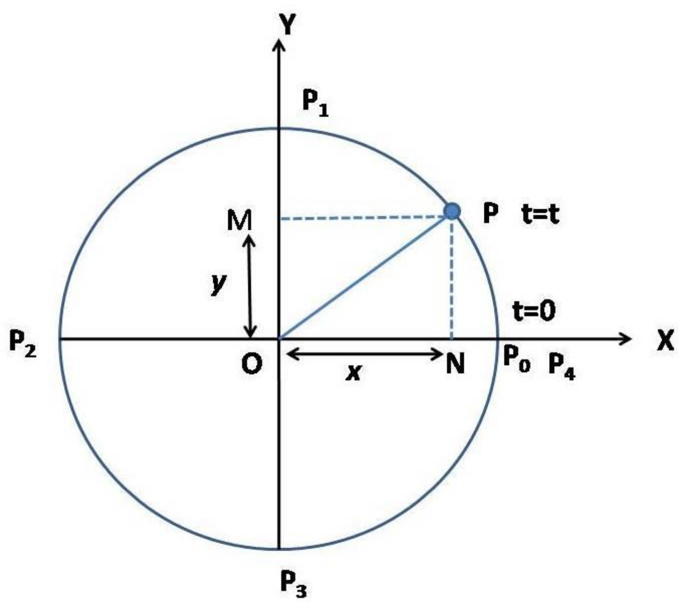


Figure 1: Particle rotating on a circumference of a circle

Let a time t=0, P lies on x-axis i.e. at P_0. At an arbitrary time t, the point will be at P where angle POX = ωt. OP is the position vector of the particle. Let, N be the foot of the perpendicular (projection on point P on x-axis).

i.e., ON=A cos ωt & OM =A sin ωt

Let, ON = x: so x= A cosωt

As particle (P) rotates on the circumference of the circle N moves to and fro about the origin on the diameter. The position on the particle P and that of N at different times:

Serial No.	**Position of particle (P)**	**Position of projection on X-axis (N)**	**Time as a fraction of time period (T)**	**Angle described by radius vector (θ)**
1.	P_0	P_0 (extreme position)	0	0
2.	P_1	O (mean position)	T/4	π/2
3.	P_2	P_2(Extreme position)	T/2	π
4.	P_3	O (mean position)	3T/4	3π/2
5.	P_4	P_0 (extreme position)	T	2 π

Thus, it is evident that as P moves on the circumference of the circle, N moves to and fro about the centre O along the diameter POP_2. Hence, when a particle rotates on the circumference of a circle with an angular velocity the foot of the perpendicular on its diameter executes simple harmonic motion.

Characteristics of simple harmonic motion

- Amplitude: The maximum displacement of N on either side of the mean position O is called amplitude of simple harmonic motion (S.H.M) i.e. $OP_0 = A$, $OP_2 = -A$.
- Time period (t): Time required to complete one revolution i.e. $\omega t = 2\pi$.
  $t = \frac{2\pi}{\omega}$. This t is called time period of S.H.M. and denoted as t.
- Frequency (υ): Inverse of time period i.e. $\upsilon = \frac{1}{t} = \frac{2\pi}{\omega}$. Hence, $\omega = 2\pi\upsilon$.
- Phase (ϕ): The choice of time t= 0 is arbitrary and one could have chosen t=0 to be instant when P was at P' (not a P_0).

  Now if angle P'OX= θ and POX = (ωt+θ). Hence, ON= A cos (ωt+θ). The quantity (ωt+θ) is known as the phase of the motion. Θ represents initial phase. The value of θ is arbitrary and depends on the instants from which one start measuring time.

Equation of motion of a particle executing SHM:

The displacement of a particle executing SHM:

$$x = A\cos(\omega t + \theta)$$

$$v = \frac{dx}{dt}$$

$$= -\omega\sqrt{A^2 - x^2}$$

$$a = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \theta)$$

$$a = -\omega^2 x$$

Now, we know that, $F=ma$ or, $F=-m\omega^2 x$

Again, $F=-kx$ [if we write $k=m\omega^2$=force constant]

$$\therefore m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0\left[k = m\omega^2\right]$$

The solution of this equation:

$$x = \alpha e^{i\omega t} + \beta e^{-i\omega t}$$

$$= \alpha\left(\cos\omega t + i\sin\omega t\right) + \beta\left(\cos\omega t - i\sin\omega t\right)$$

$$= \left(\alpha + \beta\right)\cos\omega t + i\left(\alpha - \beta\right)\sin\omega t$$

$$C\cos\omega t + D\sin\omega t\begin{bmatrix}\text{where c=}\alpha+\beta \\ D = i\left(\alpha\text{-}\beta\right)\text{are const.}\end{bmatrix}$$

Again, let

C=A$\cos\theta$

D=-A$\sin\theta$

$$x = A\cos\left(\omega t + \theta\right)$$

Graphical representation of SHM

$$x = A\cos\omega t$$

$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

Time (t)	Angle (ωt)	Position (x)	Velocity (v)	Acceleration (a)
0	0	A	0	$-A\omega^2$
T/4	π/2	0	-Aω	0
T/2	π	-A	0	$A\omega^2$
3T/4	3π/2	0	Aω	0
T	2π	A	0	$-A\omega^2$

❖ **Displacement vs. time plot:**

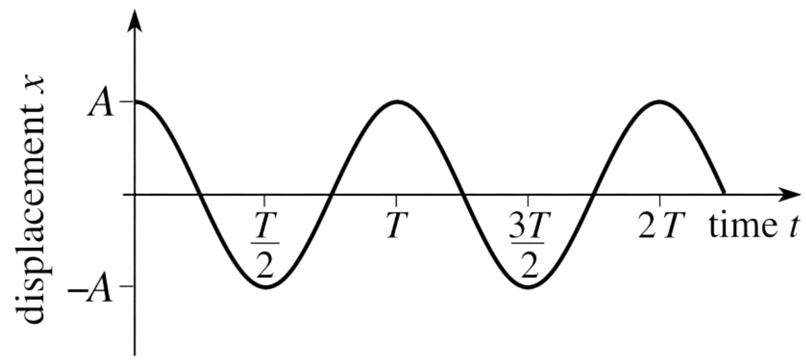


Figure 2: Displacement vs. time plot

Energy in SHM:

When a particle executes SHM, its total energy consists of potential energy and kinetic energy. Let us consider, a particle of mass m executing SHM along X axis OX. If the particle is displaced through a distance OL= S from the mean position O, the restoring forced acting on the particle is

$$F = -kS \; ; K = \text{force constant}$$

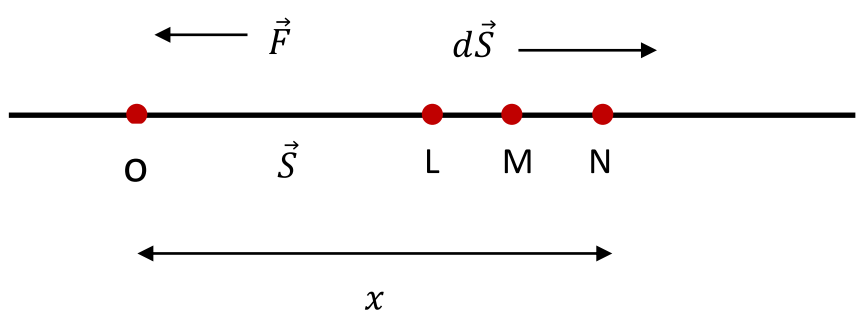


Figure 3: Energy of a particle executing SHM

Let the particle be further displaced by distance dS; then work done on the particle against force F is:

$$dW = \vec{F}.d\vec{S} = FdS\cos 180°$$

$$dW = -FdS = -(-kS)dS = kSdS$$

Let at any instant t the displacement of the particle from the mean position be x.

Total work done on the particle for displacement x is given by:

$$W = \int_0^W dW = k\int_0^x SdS = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

This work done stored as potential energy, E_p which is given by:

$$E_p = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2cos^2(\omega t + \theta)$$

And kinetic energy, E_k is:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2sin^2(\omega t + \theta) = \frac{1}{2}m\omega^2(A^2 - x^2)$$

So, the total energy E is given by:

$$\boldsymbol{E} = E_p + E_k = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2) = \boldsymbol{\frac{1}{2}m\omega^2A^2}$$

Hence, the total energy is independent of time.

Variation of total energy ($\boldsymbol{E}$), kinetic energy ($\boldsymbol{E_k}$) and potential energy ($\boldsymbol{E_p}$) as function of position:

Sr. No.	Physical Quantity	At mean position (x=0)	At extreme position (x=A)	At x=A/√2
1	Potential Energy	0 (min)	$\frac{1}{2}m\omega^2A^2$(max)	$\frac{1}{4}m\omega^2A^2$
2	Kinetic Energy	$\frac{1}{2}m\omega^2A^2$(max)	0 (min)	$\frac{1}{4}m\omega^2A^2$
3	Total Energy	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$

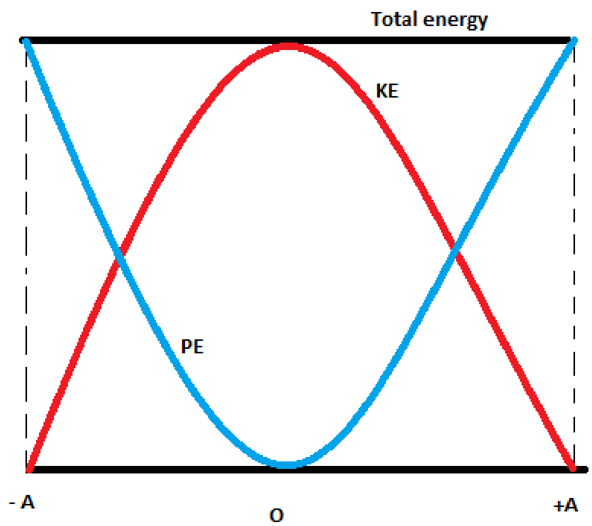


Figure 4: Kinetic and potential energies of a particle executing SHM

Lecture: 2

Damped Harmonic Oscillation

Damping force acts in direction opposite to the motion or displacement for small velocity, proportional to velocity.

$F_{restoring} = -k_0 x$

$F_{damping} = -r\frac{dx}{dt}$; where r is damping constant

Equation of motion of the damped oscillatory particle-

$$F_{net} = F_{restoring} + F_{damping}$$

$$\Rightarrow m\frac{d^2x}{dt^2} = -k_o x - r\frac{dx}{dt}$$

$$\Rightarrow m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + k_o x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k_o}{m}x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_o{}^2 x = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\begin{bmatrix} 2b = \dfrac{r}{m} = \text{damping coefficient.} \\ \omega_{\text{o}} = \text{natural frequency of undamped oscillation} \end{bmatrix}$$

Solution of this equation:

$$x(t) = \varepsilon(t)e^{-bt}$$

$$\frac{dx}{dt} = \frac{d\varepsilon}{dt}.e^{-bt} - \varepsilon b e^{-bt}$$

$$= \left(\frac{d\varepsilon}{dt} - \varepsilon b\right)e^{-bt}$$

$$\frac{d^2x}{dt^2} = \left(\frac{d^2\varepsilon}{dt^2} - b\frac{d\varepsilon}{dt}\right)e^{-bt} + \left(\frac{d\varepsilon}{dt} - \varepsilon b\right)(-b)e^{-bt}$$

$$\frac{d^2x}{dt^2} = \left(\frac{d^2\varepsilon}{dt^2} - 2b\frac{d\varepsilon}{dt} + b^2\varepsilon\right)e^{-bt}$$

From equation (1);

$$\left[\frac{d^2x}{dt^2} - 2b\frac{d\varepsilon}{dt} + b^2\varepsilon\right]e^{-bt} + 2b\left[\frac{d\varepsilon}{dt} - \varepsilon b\right]e^{-bt} + \omega_0{}^2\varepsilon(t)e^{-bt} = 0$$

$$\therefore \frac{d^2\varepsilon}{dt^2} + \left(\omega_0{}^2 - b^2\right)\varepsilon = 0$$

Now, $\left(\omega_0{}^2 - b^2\right)$ can be positive, negative or zero.

Case 1:

$\omega_o^2 > b^2$; damping is small (Underdamping)

$$\varepsilon(\mathrm{t}) = A\cos\left(\sqrt{\omega_o^2 - b^2}\,\mathrm{t}+\theta\right)$$

$$x(t) = Ae^{-bt}\cos\left(\sqrt{\omega_o^2 - b^2}\,\mathrm{t}+\theta\right)$$

- ❖ **Logarithmic decrement:**

The rate at which the amplitude dies away:

$$x(t) = Ae^{-bt}\cos\left(\sqrt{\omega_o^2 - b^2}\,\mathrm{t}+\theta\right)$$

Let θ=0;

$$\therefore x(t) = Ae^{-bt}\cos\left(\sqrt{\omega_o^2 - b^2}\,\mathrm{t}+\theta\right)$$

At t=0; x=A

Suppose $\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3, ...$are the amplitudes at t=T,2T,3T,..................

Then, $\mathrm{A}_1 = Ae^{-bt} = Ae^{-bT}$

$\mathrm{A}_2 = \mathrm{A}_1 e^{-2bT}$

$\mathrm{A}_3 = \mathrm{A}e^{-3bT}$

Hence, the decrement in amplitudes; $d = \dfrac{A}{A_1} = \dfrac{A_2}{A_3} = \dfrac{A_3}{A_4} .... = e^{bt}$

Natural log of decrement; $\lambda\text{=bT=}\dfrac{2\pi \mathrm{b}}{\sqrt{\omega_\mathrm{o}^2 - b^2}}$

- ❖ **Mean life time (τ_m):**

Time interval in which the amplitudes of the under-damped oscillator falls to $\frac{1}{e}$of its initial value is known as mean life time.

$$\frac{1}{e}A = Ae^{-b\tau_m}$$

$$\Rightarrow e^{-b\tau_m} = \frac{1}{e}$$

$$\Rightarrow b\tau_m = -1; \therefore \tau_m = \frac{1}{b}$$

- **Quality Factor (Q):**

Half of the mean life time is called relaxation time (τ). Now, quality factor (Q)= $\tau\omega_0$. (ω_0- natural frequency)

Case 2:

$$b^2 > \omega_o^{\ 2} \text{ (Overdamping)}$$

$$\frac{d^2\varepsilon}{dt^2} - \left(b^2 - \omega_o^{\ 2}\right)\varepsilon = 0$$

$$\varepsilon(t) = A_1 e^{\left(\sqrt{b^2-\omega_o^{\ 2}}\right)t} + A_2 e^{\left(\sqrt{b^2-\omega_o^{\ 2}}\right)t}$$

$$\therefore x(t) = A_1 e^{\left(-b+\sqrt{b^2-\omega_o^{\ 2}}\right)t} + A_2 e^{\left(-b+\sqrt{b^2-\omega_o^{\ 2}}\right)t}$$

These two powers are negative, the displacement decreases exponentially to zero when time increases without performing any oscillation.

Case 3:

$$b^2 = \omega_0^{\ 2} \text{ critical damping}$$

$$\frac{d^2\varepsilon}{dt^2} = 0$$

$$\varepsilon(t) = ct + D$$

$$\therefore x(t) = (ct + D)e^{-bt}; \text{C and D are const. of Integration}$$

displacement approaches zero asymptotically.

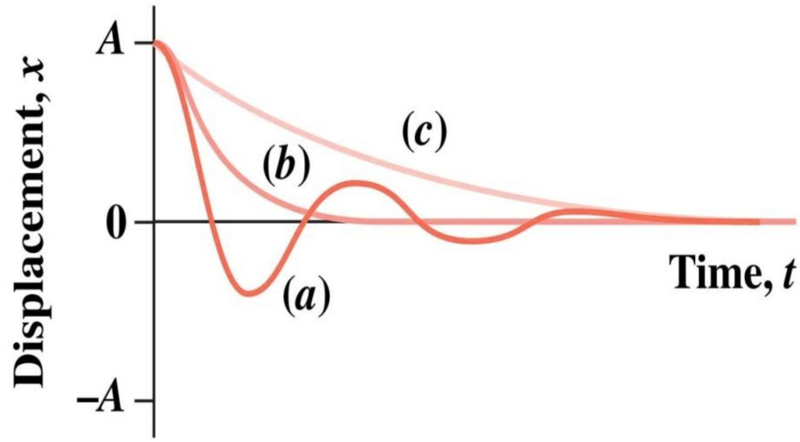


Figure 5: (a) underdamping, (b) over damping, (c) critical damping

Lecture: 3

Forced Oscillation

The Phenomenon of setting body into vibrations with the help of periodic force having frequency different from the natural frequency of the body is called forced vibrations.

Example: vibration of bridge under the influence of marching soldiers. Let us consider external periodic force $FCos\omega t$, where ω= frequency of applied force.

The equation of motion,

$$F_{net} = F_{rest} + F_{damp} + F_{ext.}$$

and

$$m\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} + F\cos\omega t$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F}{m}\cos\omega t$$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0{}^2 x = G\cos\omega t; \text{where}\left[\mathrm{G}=\frac{\mathrm{F}}{\mathrm{m}} = const.; 2b = \frac{r}{m}; \omega_0{}^2 = \frac{k}{m}\right] \text{..............}(1)$$

Equation (1) is an inhomogeneous linear second order differntial equation.

Solution is given by:

x(t)=x_c(t)+x_p(t); x_c(t):complimentary solution; x_p(t)- Particular integral
The complimentary solution;

Assuming $b^2 < {\omega_0}^2$; (weak damping or under damping).

$$x_c(t) = Ae^{-bt}\cos\left(\sqrt{{\omega_0}^2 - b^2}\,t + \theta\right)$$

The particular integral;
Let's try

$$x_p(t) = a\cos(\omega t - \phi)$$

$$\frac{dx_p}{dt} = -a\omega\sin(\omega t - \phi)$$

$$\frac{d^2x_p}{dt^2} = -a\omega^2\cos(\omega t - \phi)$$

From equation(1):

$$-a\omega^2\cos(\omega t - \phi) - 2ba\omega\sin(\omega t - \phi) + a{\omega_0}^2\cos(\omega t - \phi) = G\cos((\omega t - \phi) + \phi)$$

$$-a\omega^2\cos(\omega t - \phi) - 2ba\omega\sin(\omega t - \phi) + a{\omega_0}^2\cos(\omega t - \phi) = G\cos(\omega t - \phi)\cos\phi - G\sin(\omega t - \phi)\sin\phi ....(2)$$

Equating coefficients of $Cos(\omega t - \emptyset)$and $Sin(\omega t - \emptyset)$

$$\begin{bmatrix} a\left(\omega_0^2 - \omega^2\right) = G\cos\phi \\ 2ab\omega = G\sin\phi \end{bmatrix} ....(3)$$

using equation (3);

$$a^2\left(\omega_0^2 - \omega^2\right)^2 + 4a^2b^2\omega^2 = G^2$$

$$\Rightarrow a^2\left[\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2\right] = G^2$$

$$\Rightarrow a = \frac{G}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2}} ....(4)$$

$$x_p(t) = a\cos(\omega t - \phi)$$

$$= \frac{G}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2}}\cos(\omega t - \phi) ....(5)$$

Thus general solution x(t)=x_c(t)+x_p(t);

$$x_c(t) + x_p(t)$$

$$= Ae^{-bt}\cos\left(\sqrt{\omega_0^{\,2} - b^2}\,t + \theta\right) + a\cos(\omega t - \phi)$$

$$= Ae^{-bt}\cos\left(\sqrt{\omega_0^{\,2} - b^2}\,t + \theta\right) + \frac{G}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2}}\cos(\omega t - \phi)............(6)$$

First term of RHS:

Transient solution corresponds to natural vibrations which die away exponentially with time.

Second term of RHS:

Steady state solutions (corresponds to forced vibrations by external force. Now from equation (3):

$$\tan\phi = \frac{2b\omega}{\omega_0^2 - \omega^2},$$

$\phi =$ phase difference between oscillator and external driving force

$$= \tan^{-1}\frac{2b\omega}{\omega_0^2 - \omega^2}..................(7)$$

Amplitude (a):

$$a = \frac{G}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2}}..............(7a)$$

$'a'$ will be maximum when $\left(\omega_0^2 - \omega^2\right)^2 + 4b^2\omega^2$ is minimum.

Resonance:

When a periodic force of frequency equal to the natural frequency of a body is applied, the body slowly gains its amplitude and finally begins to vibrate with a very large amplitude.

Frequency of driving force = Natural frequency of body.

So, the condition of resonance; $\left(\omega_0^2-\omega^2\right)^2+4b^2\omega^2$ is minimum

$$i.e$$

$$\frac{d}{d\omega}\left[\left(\omega_0^2-\omega^2\right)^2+4b^2\omega^2\right]=0$$

$$\Rightarrow 2\left(\omega_0^2-\omega^2\right)\left(-2\omega\right)+8b^2\omega=0$$

$$\Rightarrow -\left(\omega_0^2-\omega^2\right)+2b^2=0$$

$$\Rightarrow \omega^2=\omega_0^2-2b^2$$

$$\therefore \omega=\sqrt{\omega_0^2-2b^2}=\omega_0\sqrt{1-\frac{2b^2}{\omega_0^2}}\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

When damping is extremely small (b tends to zero), $\omega=\omega_0$[from equation 8].

(I) Resonant frequency: $\omega_R=\omega_0\sqrt{1-\frac{2b^2}{\omega_0^2}}\ldots\ldots\ldots\ldots(8a)$

(II) Resonant Amplitude: From equation 7(a) taking $\omega=\omega_R$

$$a_R=\frac{G}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+4b^2\omega^2}}$$

$$=\frac{G}{\sqrt{\left(2b^2\right)^2+4b^2\left(\omega_0^2-2b^2\right)}}$$

$$=\frac{G}{\sqrt{4b^2+4b^2\omega_0^2-8b^4}}=\frac{G}{\sqrt{4b^2\omega_0^2-4b^4}}=\frac{F}{2bm\sqrt{\omega_0^2-b^2}},\left[\text{as G=}\frac{\text{F}}{\text{m}}\right]\ldots\ldots\ldots\ldots(9)$$

For very weak damping $b^2<<\omega_0^2$;

$$a_R=\frac{F}{2bm\omega_0}=\frac{F}{r\omega_0},\left[\text{as 2b=}\frac{\text{r}}{\text{m}}\right]\ldots\ldots\ldots\ldots\ldots(10)$$

$$Thus, a_R\alpha\frac{1}{r}\ldots\ldots\ldots\ldots$$

(III) Sharpness of Resonance (SOR): Indicates how fast the amplitude falls on either side of resonant frequency. SOR depends upon 'b' [equation (8)a]

$$\omega_R = \omega_0 \pm b$$
$$\omega_1 = \omega_0 + b$$
$$\omega_2 = \omega_0 - b$$
$\therefore \omega_1 - \omega_2 = 2b =$ width of resonance curve.

If 'b' is very small $\omega_R \approx \omega_0$

If 'b' is small : $\omega_1 - \omega_2 =$ small and resonance is sharp. i.e

$$\text{SOR}\,\alpha\,\frac{1}{\text{b}} \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

(IV) Quality Factor (Q):

$$Q = 2\pi.\frac{\text{average energy stored per unit cycle}}{\text{average energy dissipated per unit cycle}} = \frac{\omega_0}{2b}$$

Lecture: 4

Interference of light waves

Wave interference is the phenomenon that occurs when two waves meet while traveling along the same medium. The interference of waves causes the medium to take on a shape that results from the net effect of the two individual waves upon the particles of the medium. The phenomenon of the interference of light has proved the validity of the wave theory of light. In physics, interference is a phenomenon in which two waves superpose to form a resultant wave of greater, lower, or the same amplitude. According to it, when two light waves of the same frequency and having a constant phase difference traverse simultaneously in the same region of a medium and cross each other, then there is a modification in the intensity of light in the region of the superposition, which is in general different from the sum of intensities due to individual waves at that point. This modification in the intensity of light resulting from the superposition of two (or more) waves of light is called interference.

At certain points the waves superimpose in such a way that the resultant intensity is greater than the sum of the intensities due to individual waves. The interference produced at these points is called constructive interference or reinforcement, while at certain other points the resultant intensity is less than the sum of the intensities due to individual waves. The interference produced at these points is called destructive interference. Beyond the region of superposition the waves come through completely uninfluenced by each other.

Types of Interference:

The phenomenon of interference may be grouped into two categories namely: (a) division of wavefront, (b) division of amplitude.

(a) Division of Wavefront:

Under this category, the coherent sources are obtained by dividing the wave front, originating from a common source, by employing mirrors, biprism or lenses. This class of interference requires essentially a point source or a narrow slit source. The instruments used to obtain interference by division of wavefront are the Fresnel biprism, Fresnel mirrors, Lloyd's mirror, lasers, Young’s double slit experiment, etc.

(b) Division of Amplitude:

In this method, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus we have coherent beams produced by division of amplitude. These beams travel different paths and are finally brought together to produce interference. The effects resulting from the superposition of two beams are referred to as two beam interference and those resulting from superposition of more than two beams are referred to as multiple beam interference. The interference in thin films, Newton's rings, and Michelson's interferometer are examples of two beam interference and Fabry-Perot's interferometer is an example of multiple beam interference.

Newton's Ring Experiment

Newton's rings, in optics, a series of concentric light- and dark-coloured bands observed between two pieces of glass when one is convex and rests on its convex side on another piece having a flat surface. Thus, a layer of air exists between them.

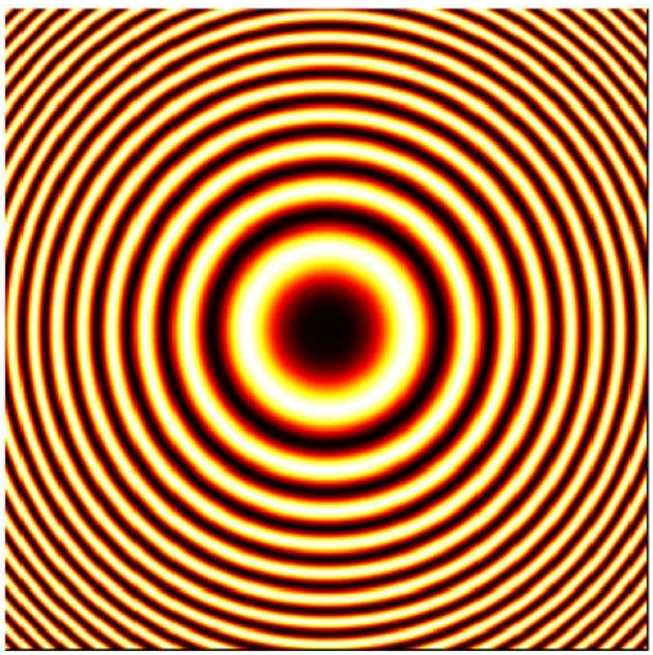


Figure 6: Formation of Newton's rings

Basically, it is a phenomenon in where an interference pattern is created by the reflection of light between two surfaces: a spherical surface and an adjacent touching flat surface. The phenomenon is named after Isaac Newton, who investigated the effect in his 1704 treatise optics. When viewed with monochromatic light, Newton's rings appear as a series of concentric, alternating bright and dark rings centered at the point of contact between the two surfaces. However, when the same phenomena were viewed with white light, it forms a concentric ring pattern of rainbow colors, because the different wavelengths of light interfere at different thicknesses of the air layer between the surfaces.

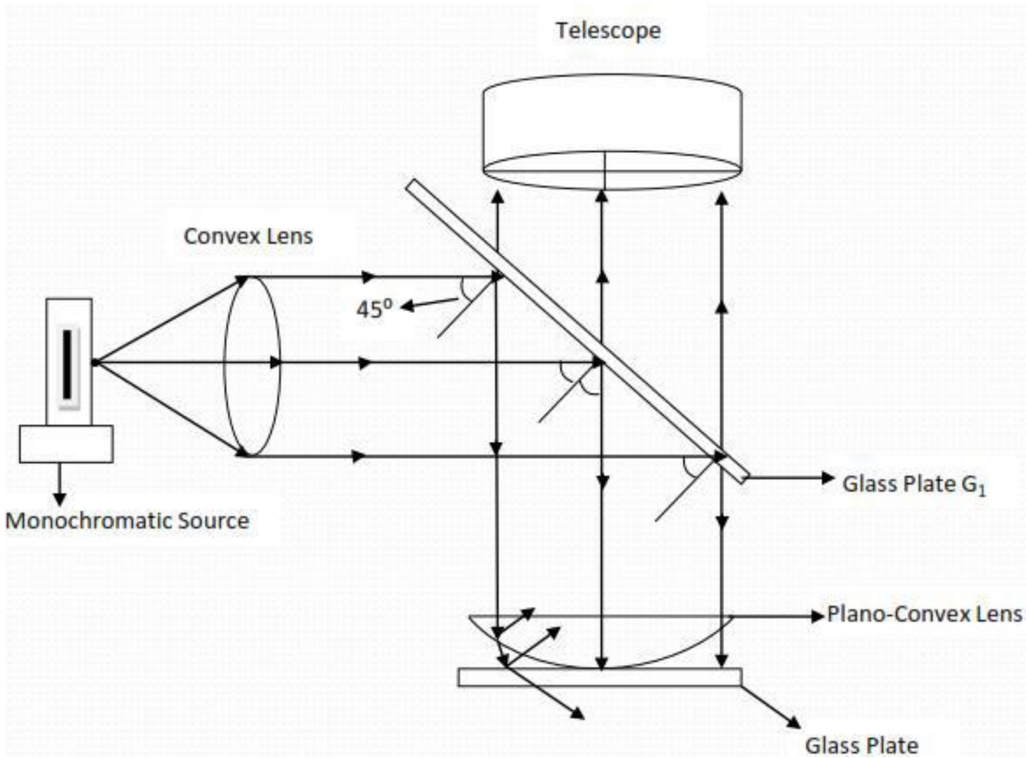


Figure 7: Schematic of Newton’s ring experimental setup

When light from a monochromatic source (e.g., sodium lamp) is allowed to fall on a convex lens, then it renders a parallel beam of light. This parallel beam of light is allowed to fall on plane glass plate. That is placed at an angle to the direction of incident beam of light. Then the glass plate reflects the incident beam of light normally towards the air film enclosed between the plano-convex lens and the glass plate. First of all this light is allowed to fall on a plane surface of Plano convex lens. Then a part of this light is reflected and a part of light is transmitted, then this transmitted light is allowed to fall on a curve surface of Plano convex lens. Then a part of this transmitted light is reflected and comes out in the form of ray no. 1 and a part of light is transmitted, after that this transmitted light is allowed to fall on a plane glass late then a part of light is reflected and comes out in the form of ray no. 2 and a part of light is transmittedand comes out in the form of ray no. 3. Thus at a particular constant thickness interference take place due to the reflected ray no. 1 and 2. Due to convexity of the plano-convex lens and at the particular constant thickness the radii or foci are constant so that the interference pattern is take place in the form of concentric ring. Thus interference pattern is either dark or bright depend upon path difference between the two reflected rays. Thus the path difference between two reflected ray will be: $\boldsymbol{2\mu t\, cos(r+\alpha)+\frac{\lambda}{2}}$**.**

However, in the experiment rays are incident normally, thus for normal incidence, angle of refraction is zero ($r = 0$). For air film $\mu = 1$ and for small wedge angle 'α' is very small. Thus, the value of $(r + \alpha)$ tends to zero, in turn $cos(r + \alpha) = 1$. Hence, the path difference between both the reflected ray will be: $2t + \frac{\lambda}{2}$. But, at the point of contact thickness of air film is zero so that when t=0 then path difference between the two reflected ray is zero, which is condition of the minima so that at the point of contact in case of reflected light interference pattern will be dark.

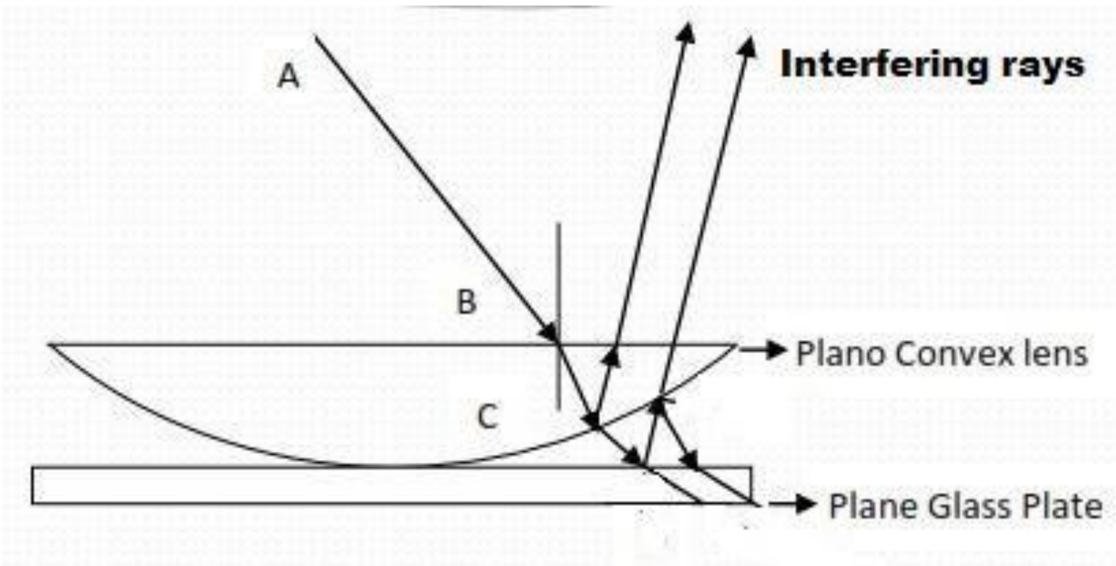


Figure 8: Interference in Newton's ring experiment

(a) Condition for bright fringe

$$2t + \frac{\lambda}{2} = 2n.\frac{\lambda}{2} = n\lambda; n = 0,1,2,3,.......$$

$$2t = (2n-1)\frac{\lambda}{2}$$

(b) Condition for dark fringe

$$2t + \frac{\lambda}{2} = (2n+1).\frac{\lambda}{2} = n\lambda; n = 0,1,2,3,.......$$

$$2t = n\lambda$$

N.B.: Here it is seen that for t=0, n=0. Hence, the central fringe is dark.

(c) Thickness of the film

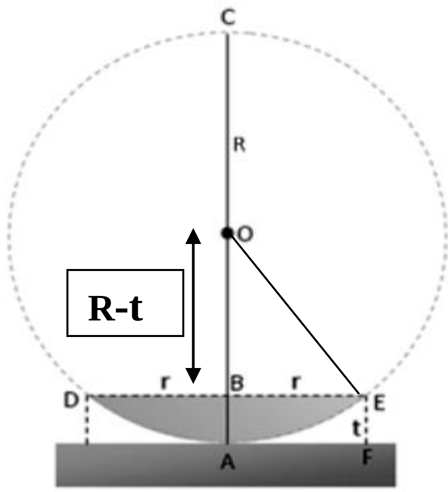


Figure 9: Thickness of the film

According to the geometrical theorem (i.e. property of the circle), the product of intercepts of the intersecting chord is equal to the product of sections of the diameter.

$DB \times BE = AB \times BC$

$\Rightarrow r \times r = t(2R - t)$

$\Rightarrow r^2 = 2Rt - t^2$

Since t is very small, t^2 would be negligible. Hence,

$\Rightarrow r^2 = 2Rt$

Or, $t = \frac{r^2}{2R}$

Considering the radius represents that of the N'th ring

$t = \frac{r_N^2}{2R}$

(d) Diameter of the bright fringe

$$2t = (2n-1)\frac{\lambda}{2}$$
$$\Rightarrow 2.\frac{r_N^{\ 2}}{2R} = (2n-1)\frac{\lambda}{2}$$
$$\Rightarrow r_N^{\ 2} = \left(\frac{2n-1}{2}\right).\lambda R$$
$$\Rightarrow \frac{D_N^{\ 2}}{4} = \frac{2n-1}{2}.\lambda R$$
$$\Rightarrow D_N^{\ 2} = 2(2n-1).\lambda R$$
$$D_N = \sqrt{2\lambda R(2n-1)} = K\sqrt{2n-1}\left[K = \sqrt{2\lambda R} = const.\right]$$
$$\therefore D_N \alpha \sqrt{2n-1}$$

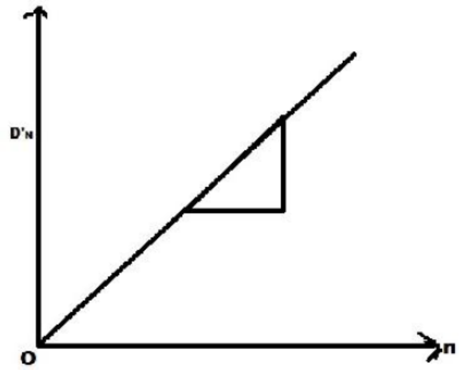
(e) Diameter of the dark fringe

$$2t = n\lambda$$
$$\Rightarrow 2.\frac{r'_N{}^{2}}{2R} = n\lambda$$
$$\Rightarrow r'_N{}^{2} = n\lambda R$$

$$\Rightarrow \frac{D'_N{}^{2}}{4} = n\lambda R$$
$$\Rightarrow D'_N{}^{2} = 4n\lambda R$$
$$D'_N = \sqrt{4\lambda R}\sqrt{n} = K'\sqrt{n}\left[K' = \sqrt{4\lambda R} = const.\right]$$
$$\therefore D'_N \alpha \sqrt{n}$$

Determination of wavelength of light:

$$\lambda = \frac{D'^2_N}{n} \cdot \frac{1}{4R} = \frac{slope}{R}$$



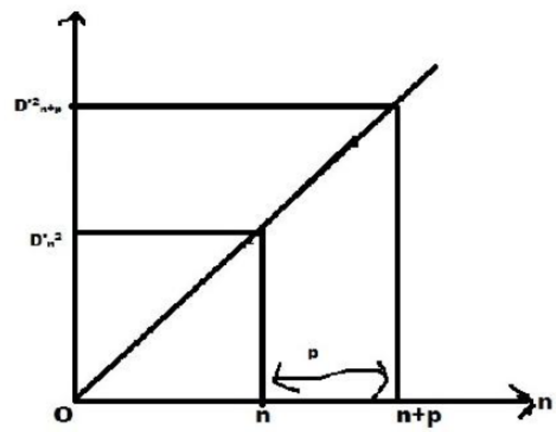
If the contact of lens and the glass plate is not perfect the order of nth ring cannot be known precisely, due to uncertainty in the thickness of the film. To overcome this; diameter of 'n' and (n+p)of dark ring is measured.

$$D'^2_n = 4n\lambda R$$

$$D'^2_{n+p} = 4(n+p)\lambda R$$

$$D'^2_{n+p} - D'^2_n = 4p\lambda R$$

$$\therefore \lambda = \frac{D'^2_{n+p} - D'^2_n}{4PR}$$



Lecture: 5

Diffraction

In his 1704 treatise on the theory of optical phenomena (Optics), Sir Isaac Newton wrote that "*light is never known to follow crooked passages nor to bend into the shadow*". He explained this observation by describing how particles of light always travel in straight lines, and how objects positioned within the path of light particles would cast a shadow because the particles could not spread out behind the object. Thus, the phenomenon of bending of light around edges of obstacles or narrow slits, and hence its encroachment into the region of geometrical shadow is known as diffraction. Diffraction is often explained in terms of the Huygens principle, which states that each point on a wave-front can be considered as a source of a new wave.

Definition:

If the size of an obstacle or aperture is comparable with the wavelength of light, light deviates from its rectilinear propagation and bends near the edges of the obstacles and enters into the region of the geometrical shadow.

As a result of diffraction, the edges of shadow do not remain well defined and sharp but become blurred and fringed. The phenomenon could be explained on the basis of wave theory.

Wave Fronts:

A wave front is the locus of the points or particles which are in the same phase. For example if we drop a small stone in a calm pool of water, circular ripples spread out from the point of impact. Each point on the circumference of the circle oscillates with the same amplitude and same phase and thus we have a circular wave front.

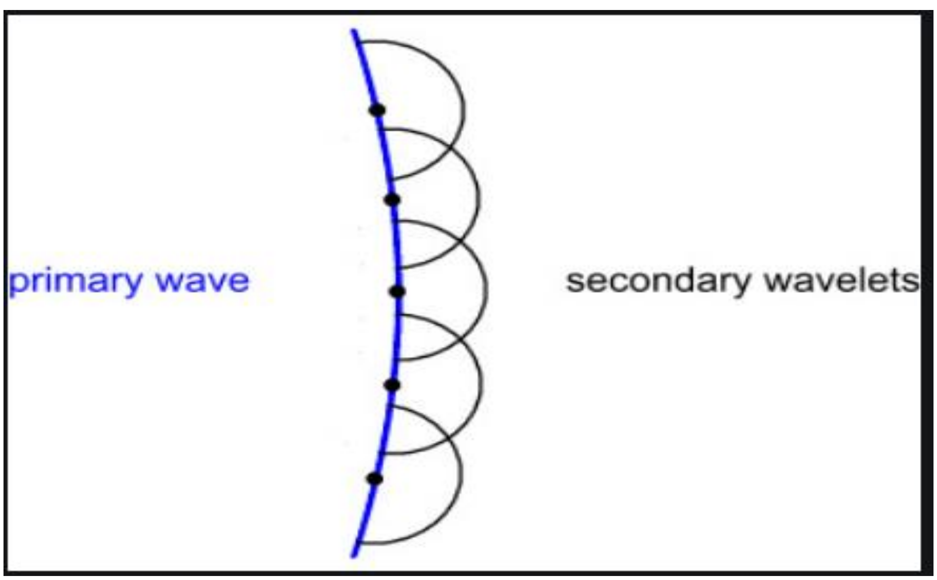


Figure 10: Wavefronts expanding from a point source

Huygen's Principle

- *Each point of a wave front acts as a centre (or source) of new disturbances called secondary wavelets which travel in all direction with speed of light.*

- *The tangent envelope of these wavelets gives the shape and position of the new wave front at any subsequent time.*

As the straight waves passed through a narrow hole, they spread out in a circular pattern. Giving proof to the fact that every point on a wave front is a new source for a new set of wavelets.

Classification of Diffraction

We can define two distinct types of diffraction:

- *Fresnel diffraction is produced when light from a point source meets an obstacle, the waves are spherical and the pattern observed is a fringed image of the object.*
- *Fraunhoffer diffraction occurs with plane wave-fronts with the object effectively at infinity.*

Fraunhoffer diffraction:

Fraunhoffer diffraction is the type of diffraction that occurs in the limit of small Fresnel number. In Fraunhoffer, diffraction, the diffraction pattern is independent of the distance to the screen, depending only on the angles to the screen from the aperture.

- ✓ Source and the screen are far away from each other.
- ✓ Incident wave fronts on the diffracting obstacle are plane.
- ✓ Diffracting obstacle give rise to wave fronts which are also plane.

- ✓ Plane diffracting wave fronts are converged by means of a convex lens to produce

Derivation of Fraunhoffer diffraction due to single slit:

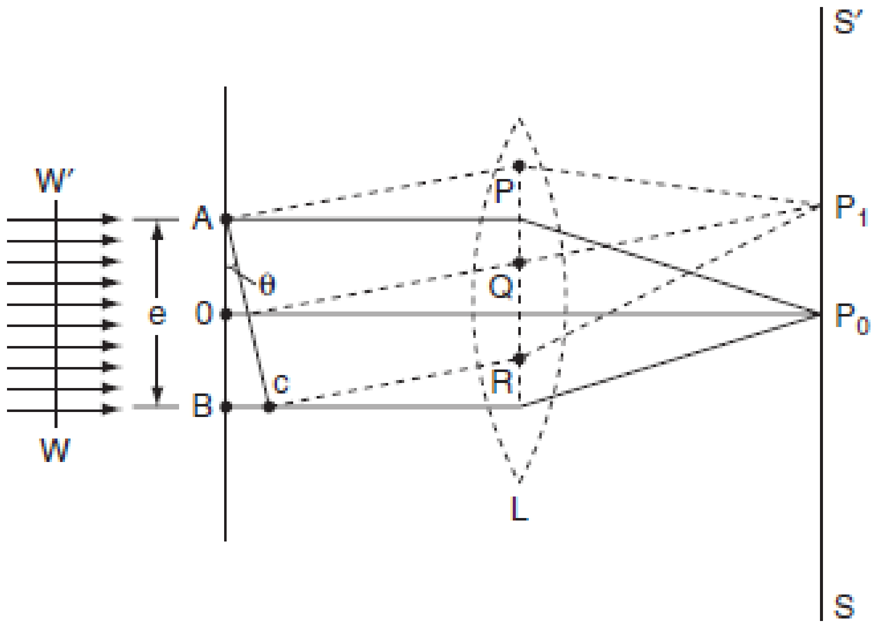


Figure 11: Fraunhoffer diffraction from single slit

The adjacent figure represents a narrow slit AB of width ‘e’. Let a plane wavefront of monochromatic light of wavelength λ is incident on the slit. Let the diffracted light be focused by means of a convex lens on a screen. According to Huygen Fresnel, every point of the wavefront in the plane of the slit is a source of secondary wavelets. The secondary wavelets traveling normally to the slit i.e., along OP_o are brought to focus at P_o by the lens. Thus Po is a bright central image. The secondary wavelets traveling at an angle 'This is the rendered form of the equation are focused at a point P_1 on the screen. The intensity at the point P_1 is either minimum or maximum and depends upon the path difference between the secondary waves originating from the corresponding points of the wavefront. In order to find out the intensity at P_1, draw a perpendicular AC on BR.

The path difference between secondary wavelets from A and B in direction θ is BC i.e. ;

$$\Delta = BC = AB\sin\theta = e\sin\theta$$

So, the phase difference,

$$= \frac{2\pi}{\lambda} \times \Delta = \frac{2\pi}{\lambda}(e\sin\theta)$$

Let us consider that the width of the slit is divided into ‘n’ equal parts and the amplitude of the wave from each part is ‘a’.

So, the phase difference between two consecutive points will be:

$$\delta = \frac{1}{n}\left\{\frac{2\pi}{\lambda}(e\sin\theta)\right\}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Then the resultant amplitude R is calculated by using the method of vector addition of amplitudes.

The resultant amplitude of n number of waves having same amplitude 'a' and having common phase difference of 'δ' is:

$$R = a\frac{\sin(n\delta/2)}{\sin(\delta/2)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting the value of in above equation:

$$R = a\frac{\sin\left(\frac{\pi}{\lambda}.e\sin\theta\right)}{\sin\left\{n\left(\frac{\pi}{\lambda}.e\sin\theta\right)\right\}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substituting $\alpha = \frac{\pi}{\lambda}.e\sin\theta$ in the above equation, we have:

$R = a\frac{\sin\alpha}{\sin(\alpha/n)}$, As α/n is small value; $\sin(\alpha/n) \to \alpha/n$

$R = na\dfrac{\sin\alpha}{\alpha}$ and **na=A.**

Therefore,

$$R = A\frac{\sin\alpha}{\alpha}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

So, the intensity is given by:

$$\mathrm{I}^2 = \mathrm{R}^2 = \mathrm{A}^2\frac{\sin^2\alpha}{\alpha^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Lecture: 6

Contd…..

$$\mathrm{I}^2 = \mathrm{R}^2 = \mathrm{A}^2\frac{\sin^2\alpha}{\alpha^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Case (a): Principal Maximum:

Eqn (4) takes maximum value for: α=0;

Thus, $\alpha=\frac{\pi}{\lambda}.e\sin\theta=0;\Rightarrow\sin\theta=0$ or $\theta=0$

The condition θ=0means that this maximum is formed by the secondary wavelets which travel normally to the slit along OP_o and focus at P_o. This maximum is known as ***"Principal maximum".***

❖ **Intensity of Principal maxima**

$$R_{\max} = \mathrm{Lim}_{\alpha\to0}\frac{\mathrm{A}\sin\alpha}{\alpha} = A\mathrm{Lim}_{\alpha\to0}\frac{\sin\alpha}{\alpha}$$

$$R_{\max} = A.1 = A$$

Therefore, $I_{\max}^2 = R_{\max}^2 = A^2$

Case (b): Minimum Intensity positions:

Equation (3) takes minimum values for sinα=0. The values of 'α' which satisfies sinα=0 are:

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi .... \pm n\pi$$

$$\frac{\pi}{\lambda}.e\sin\theta = \pm n\pi$$

$$e\sin\theta = \pm n\lambda, where n = 1, 2, 3..........(6)$$

In the above Eq. (6) n = 0 is not applicable because corresponds to principal maximum. Therefore, the positions according to Eq. (6) are on either side of the principal maximum.

Case (c): Secondary maximum:

In addition to principal maximum at α= 0, there are weak secondary maxima between minima positions. The positions of these weak secondary maxima can be obtained with the rule of finding maxima and minima of a given function in calculus. So, differentiating Eq.(4) and equating to zero, we have:

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(A^2 \frac{\sin^2\alpha}{\alpha^2} \right) = 0$$

$$\Rightarrow \frac{dI}{d\alpha} = 2A^2 \left(\frac{\sin\alpha}{\alpha} \right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} \right) = 0$$

$$\because A^2 \neq 0; \sin\alpha \neq 0$$

Because, $\sin\alpha$ =0 correspond to minimum positions,

$$\therefore \alpha \cos\alpha - \sin\alpha = 0$$

$$\Rightarrow \alpha = \tan\alpha = 0 \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The values of 'α' satisfying the equation (7) are obtained graphically, by plotting the curves y=α and y= *tan α*, on the same graph. The points of intersection of the two curves give the values 'α' which satisfy equation (7). The points of intersection are:

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \pm\frac{(2n-1)\pi}{2}$$

But α=0, gives the principal maximum, substituting the values of 'α' in equation (5), we have:

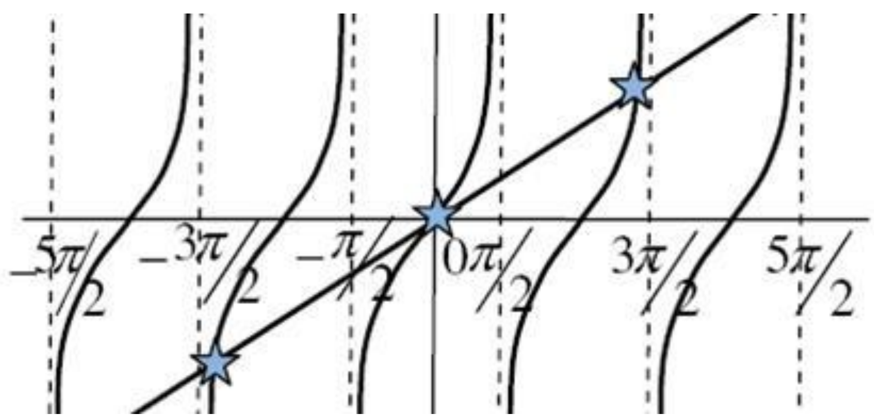


Figure 12: Fraunhoffer's diffraction from single slit

$$I_1 = A^2 \left[\frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right]^2 = \frac{A^2}{22}$$

$$I_2 = A^2 \left[\frac{\sin \frac{5\pi}{2}}{\frac{5\pi}{2}} \right]^2 = \frac{A^2}{62}$$

$$I_3 = A^2 \left[\frac{\sin \frac{7\pi}{2}}{\frac{7\pi}{2}} \right]^2 = \frac{A^2}{125}$$

and so on. From the above expression, Imax, I_1, I_2, I_3,..... it is evident that most of the incident light is concentrated at the principal maximum.

❖ **Intensity distribution graph:**

A graph showing the variation of intensity with α

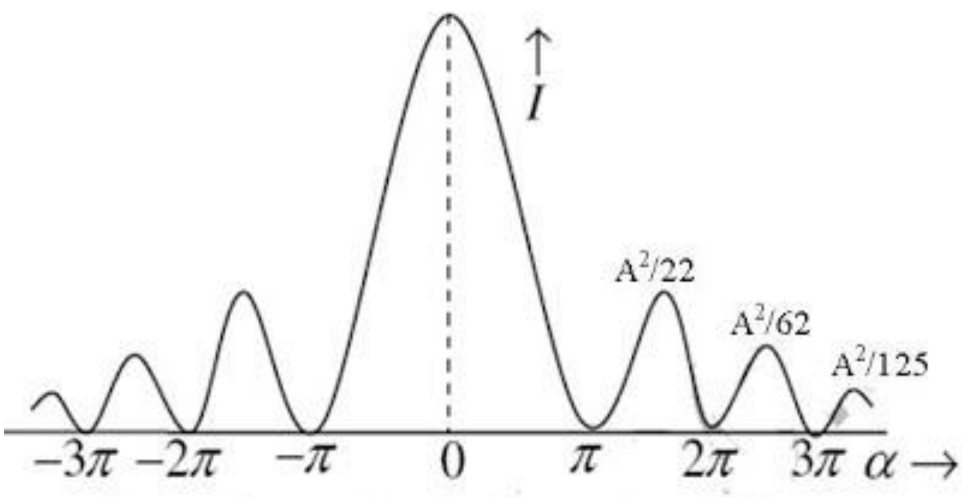


Figure 13: Intensity distribution graph

Lecture: 7

Derivation of Fraunhoffer Diffraction due to N-slits (Grating):

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as Diffraction grating.

Gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass, with a fine diamond point. *The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as plane transmission grating.* When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.

Explanation:

A section of a plane transmission grating AB placed perpendicular to the plane of the paper is as shown in the figure.2.6

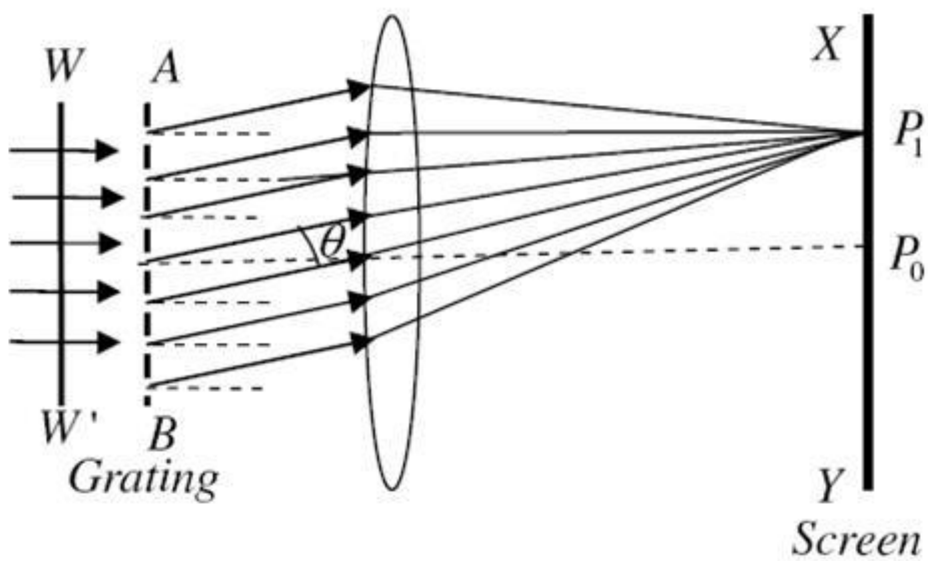


Figure: 14 Fraunhoffer's diffraction from multiple slits

Let 'e' be the width of each slit and 'd' the width of each opaque space. Then (e+d) is known as grating element and XY is the screen. Suppose a parallel beam of monochromatic light of wavelength 'λ' be incident normally on the grating. By Huygen's principle, each of the slit sends secondary wavelets in all directions. Now, the secondary wavelets travelling in the direction of incident light will focus at a point P_o on the screen. This point P_o will be a central maximum.

Now consider the secondary waves travelling in a direction inclined at an angle 'θ' with the incident light will reach point P_1 in different phases. As a result dark and bright bands on both sides of central maximum are obtained.

The intensity at point P_1 may be considered by applying the theory of Fraunhoffer diffraction at a single slit. The wavelets proceeding from all points in a slit along their direction are equivalent to a single wave of amplitude $\mathrm{A}\frac{\sin\alpha}{\alpha}$; starting from the middle point of the slit where $\alpha = \frac{\pi}{\lambda}.e\sin\theta$.

If there are N slits, then we have N diffracted waves. The path difference between two consecutive slits is $(e+d)\sin\theta$. Therefore, the phase difference:

$$\delta = \frac{2\pi}{\lambda}.(e+d)\sin\theta = 2\beta ....(8)$$

Since in the previous case:

$$\alpha = \mathrm{A}\frac{\sin\alpha}{\alpha}; n = N; \delta = 2\beta$$

$$R = a\frac{\sin(n\delta/2)}{\sin(\delta/2)}$$

Substituting these in equation:

The resultant amplitude on screen at P_1 becomes:

$$R=\left(A\frac{\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta}\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

The intensity at P_1 will be:

$$I^2=R^2=\left(A\frac{\sin\alpha}{\alpha}\right)^2\frac{\sin^2 N\beta}{\sin^2\beta}\ldots\ldots\ldots\ldots(10)$$

- The factor: $\left(A\frac{\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to single slit
- While the factor: $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the distribution of intensity as a combined effect of all the slits.

Intensity Distribution:

Case (a): Principal maxima:

The Eq. (9) will take a maximum value if:

$$\sin\beta=0$$
$$\beta=\pm n\pi; n=0,1,2,3\ldots\ldots\ldots$$
$$\frac{\pi}{\lambda}(e+d)\sin\theta=\pm n\pi$$
$$(e+d)\sin\theta=\pm n\lambda\ldots\ldots\ldots\ldots(11)$$

n=0, corresponds to zero order maximum. For n = 1,2,3,… we obtain first, second, third,… principal maxima respectively. The ± sign indicates that there are two principal maxima of the same order lying on either side of zero order maximum.

Case (b): Minima Positions:

The eq. (9) takes minimum value if $\sin N\beta=0$ but $\sin\beta\neq 0$,

$$\therefore N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\lambda ....(12)$$

where m has all integral values except m = 0, N, 2N, …, nN, because for these values $\sin\beta$ becomes zero and we get principal maxima. Thus, m=1,2,3,............,(N-1).

Hence, $N(e+d)\sin\theta = \pm m\lambda; m = 1,2,3..........(N-1),(N+1),.............(2N-1)....$ gives the minima positions which are adjacent to the principal maxima.

Case (c): Secondary maxima:

As there are (N-1) minima between two adjacent principal maxima there must be (N-2) other maxima between two principal maxima. These are known as secondary maxima. To find their positions

$$\frac{dI}{d\beta} = 0$$

$$\frac{dI}{d\beta} = \left(A\frac{\sin\alpha}{\alpha}\right)^2 2\left(\frac{\sin N\beta}{\sin\beta}\right)\left[\frac{N\cos N\beta\text{-}\sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

$$\because \frac{\sin\alpha}{\alpha} \neq 0; \sin N\beta \neq 0;$$

only

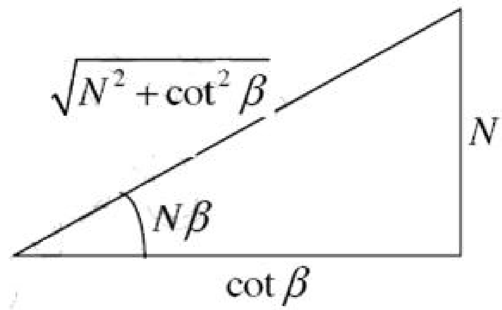
$$\left[N\cos N\beta\sin\beta\text{-}\sin N\beta\cos\beta\right] = 0$$

$$N\tan\beta = \tan N\beta ....(13)$$

The roots of the above equation other than those for which $\beta = \pm n\pi$ gives the position of secondary maxima.

The equation (13) can be written as:

$$\tan N\beta = \frac{N}{\cot\beta}$$



From the triangle we have:

$$\sin N\beta = \frac{N}{\sqrt{N^2+\cot^2\beta}}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\left(N^2+\cot^2\beta\right)\sin^2\beta}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{N^2\sin^2\beta+\left(1-\sin^2\beta\right)}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1+\left(N^2-1\right)\sin^2\beta}$$

$$I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{N^2}{1+\left(N^2-1\right)\sin^2\beta}$$

Since intensity of principal maxima is proportional to N^2,

$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{\dfrac{N^2}{1+\left(N^2-1\right)\sin^2\beta}}{N^2} = \frac{1}{1+\left(N^2-1\right)\sin^2\beta}$$

Hence if the value of N is larger, then the secondary maxima will be weaker and becomes negligible when N becomes infinity.

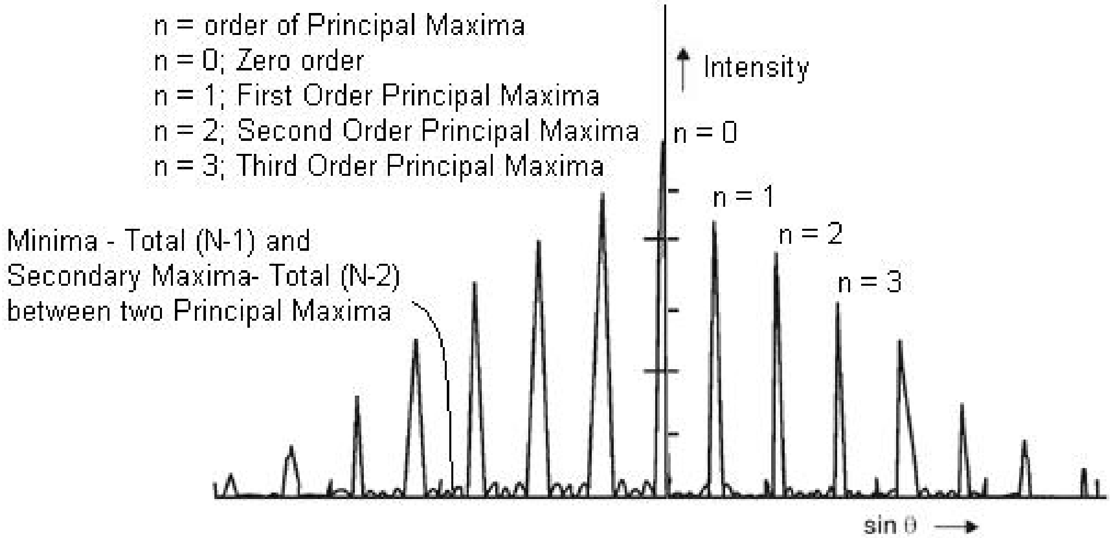


Figure: 15 Fraunhoffer's diffraction from multiple slits

N.B. Diffraction Grating:

A series of closely spaced parallel slits used to separate colors of light. Different colors have different wavelengths and diffract at different rates. So they constructively interfere at different places.

Lecture: 8

Polarization of Light waves

The transverse nature of light waves was established by an optical phenomenon called polarization. So in a transverse wave, the vibrations can be restricted to one particular direction. It can be made asymmetrical about the direction of propagation. A wave having such a characteristic of asymmetry is said to be polarized and the phenomenon is called polarization.

Unpolarized light

According to Maxwell's electromagnetic theory, light is an electromagnetic wave which consists of mutually perpendicular vibrating electric and magnetic vectors, both being perpendicular to the direction of propagation of light. The planes of vibration of light vectors have random orientation due to random orientations at excited atoms or molecules of the source.

Polarized light

Light having vibrations of electric or magnetic vectors only in selected directions or planes is said to be polarized.

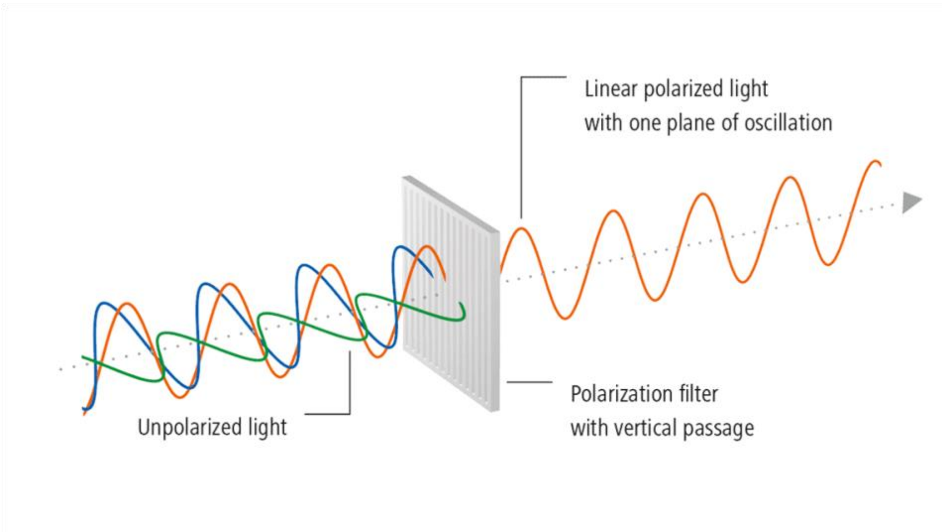
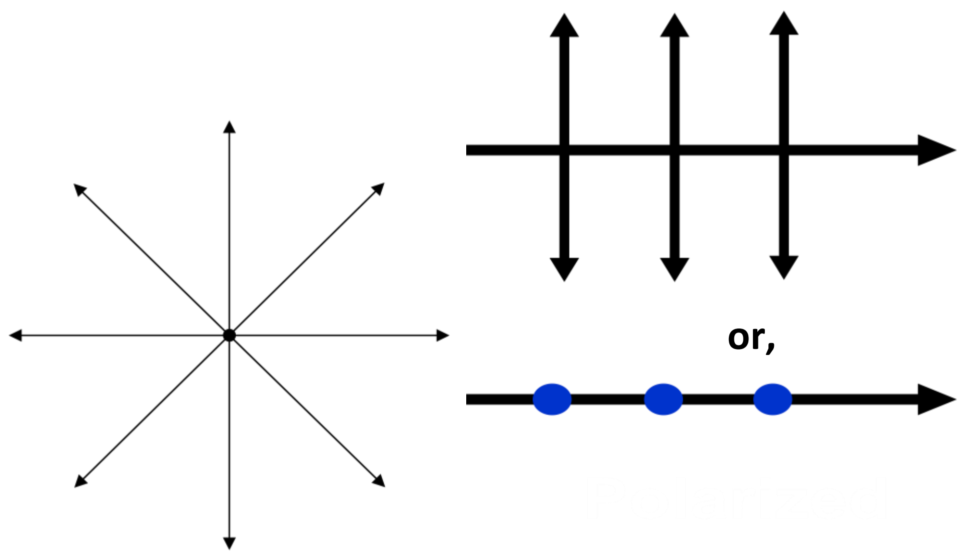


Figure: 16 Unpolarized and polarized light

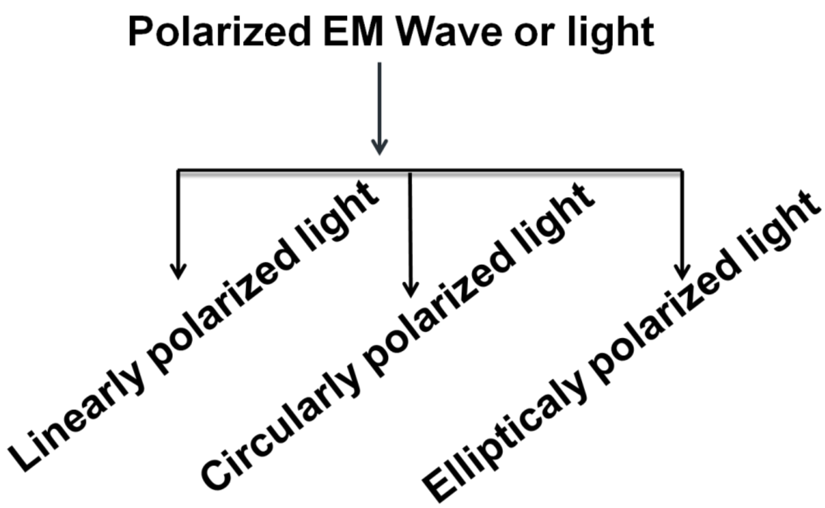
Pictorial representation:



(a) **(b)**

Figure: 17 (a) unpolarized light and (b) polarized light

Categories of polarized light:



- **Plane polarized light:-**If the vibrating electric or light vectors of the light wave are confined to a single linear direction perpendicular to the direction of propagation, light is said to be linearly polarized or plane polarized.
- **Circularly polarized Light:** If the tips of the vibrating electric vector moves in a circular path over a plane perpendicular to the direction of propagation of light is said to be circularly polarized.
- **Elliptically polarized Light:** If the tips of the vibrating electric vector moves in a elliptical path over a plane perpendicular to the direction of propagation of light is said to be circularly polarized.

Plane of vibration:

The plane containing the direction of propagation of light and plane of vibration is called the plane of polarization.

Plane of polarization

The plane containing the direction of propagation of light and perpendicular to the plane of vibration is called the plane of polarization.

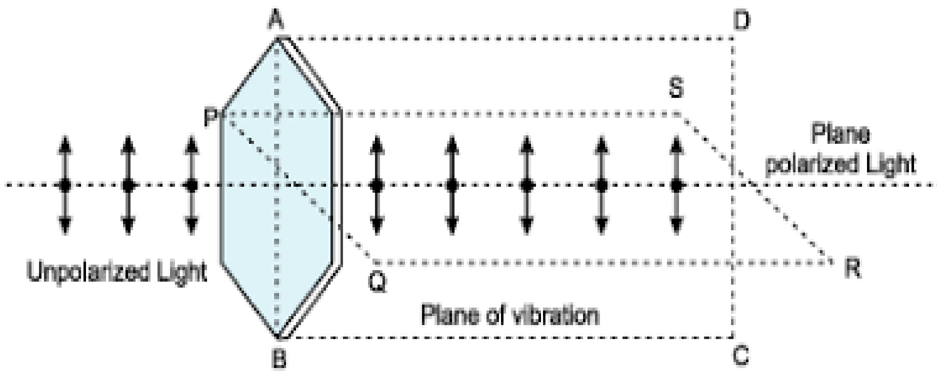


Figure: 18 Plane of vibration and plane of polarization

PQRS= Plane of polarization, ABCD= Plane of vibration.
For Example:-If the direction of propagation is along X-axis and the plane of vibration is the XY-plane, then the plane of polarization lies in the XZ-plane.
In b-The plane of polarization is perpendicular to the plane of the paper.
In c-The plane of polarization is on the plane of paper.

- **Production of plane polarized light**

  1. by reflection
  2. by refraction
  3. by double refraction
  4. by scattering
  5. by selective absorption in crystals

Brewster's Law

In 1811, Sir David Brewster found that there is a relation between the polarizing angle I_p and the refractive index μ of the refracting medium.
This is called Brewster's Law.

Statement: *It states that the refractive index of the refracting medium is equal to the target of angle of polarization.*

$$\mu = \tan i_p$$

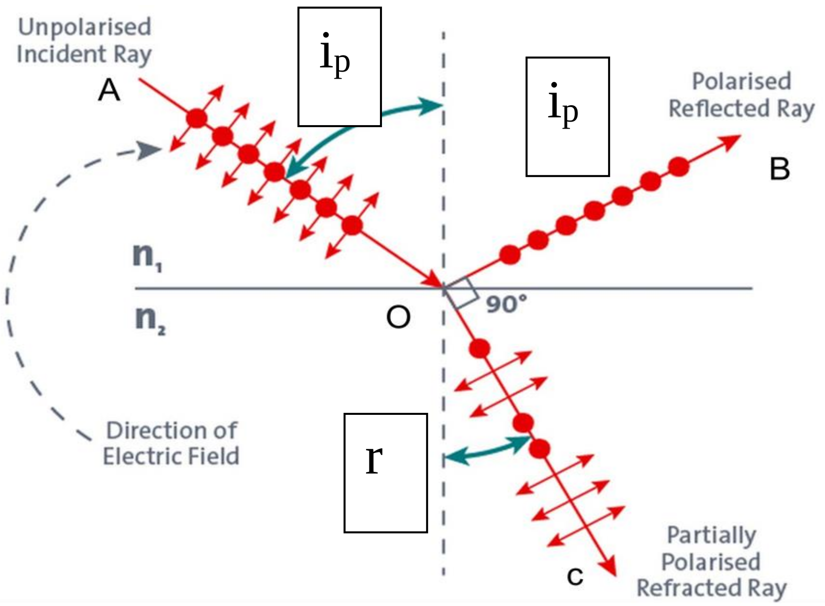


Figure: 19 Brewster's law

If light is incident at the polarizing angle, the reflected beam is right angles to the refracted beam.

AO= Unpolarized light

I_p=57.5°

OB=Plane polarized with the plane of the vibration perpendicular to the plane of incidence

OC=Refracted ray is partially polarized

r= angle of reflection=angle of incidence=angle of refraction

According to Brewster's, <BOC=90°, I_p+r=90°, r=90°- I_p

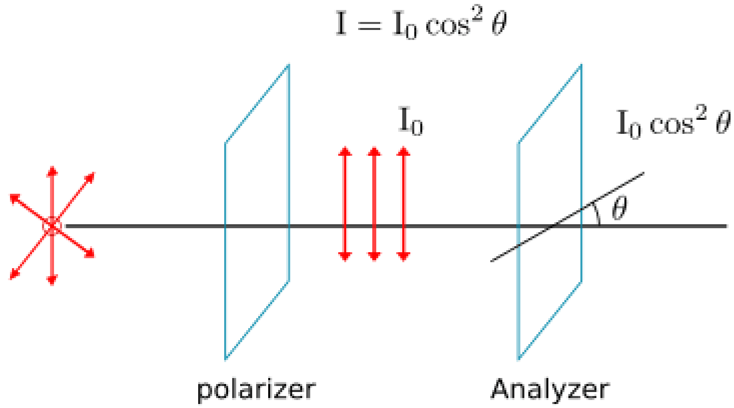
Snell's Law: $\mu = \frac{\sin i}{\sin r} = \frac{siniP}{\sin(90°-iP)} = \frac{\sin iP}{\cos\ iP} = tan\ i_p$

Brewster's law; $\mu = tan\ i_p$

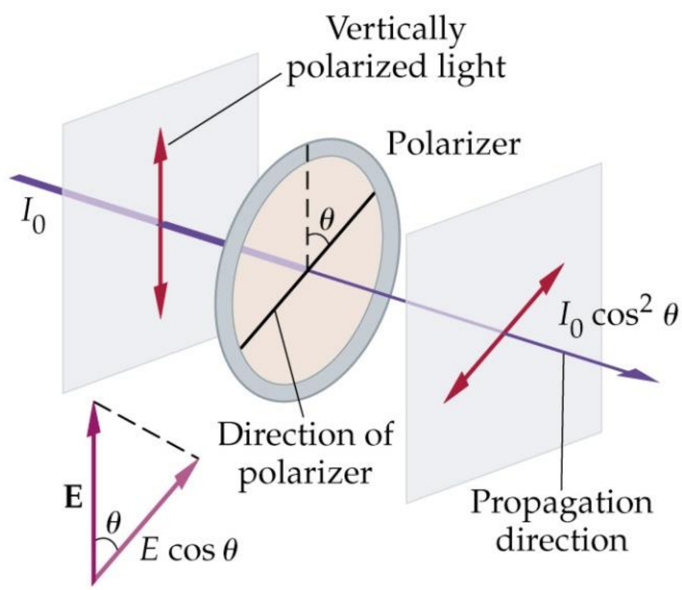
Malus' law

It states that the intensity I of polarized light transmitted through the analyzer varies as the square of the cosine of the angle θbetween the plane of transmission of the analyzer and that of polarizer.

$$I \; \alpha \; COS^2\theta$$



(a)



(b)

Figure: 20 (a) and (b) Schematic of Malus' Law

Proof: Any polarized light can be split into two rectangular components.

(i) Parallel to the plane of transmission.
(ii) Perpendicular to the plane of analyzer.

The first component will only be transmitted through the analyzer.

Let E is the amplitude of vibrations (electric vector) transmitted by the polarizer and θ is the angle between the planes of polarizer and analyzer.

The components of E are

$E_1 = E\cos\theta$ along OY

$E_2 = E\sin\theta$ along OX

The component E_1 is transmitted through the analyzer
Intensity $I_1 = kE_1^2 = kE_1^2\cos^2\theta$

k = Constant , $I_1 = kE^2 = I_0$=maximum intensity
for $\theta = 0°$ or $180°$, $I_1 = I_0\cos^2\theta$
(two planes will be parallel)
$I \,\alpha\, \cos^2\theta$, $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $I = I_0 (\cos\frac{\pi}{2})^2 = 0$
Therefore no light is travelled when the plane of the analyzer becomes perpendicular to the plane of polarizer.

Lecture: 9

Double refraction

When a beam of ordinary light is allowed to pass through a calcite or quartz crystal, we get two refracted beams instead of one in case of glass. This phenomenon is called double refraction or birefringence.

Calcite crystal, (Iceland space & crystallized $caco_3$)

Hexagonal system, two corners -102°

6 corners- two acute & one obtuse

Uniaxial crystal:- one optic axis, calcite , tourmaline & quartz

Biaxial crystal:- two optic axis, topaz & aragonite.

Optic axis: It is a direction inside a double refracting crystal along which both the refracted behave like in all respect.
Principal section: A plane passing through the optic axis and normal to a crystal surface is called a principal section.
Principal plane:
The plane in the crystal drawn through the optic axis and ordinary ray or drawn through the optic axis and the extraordinary ray is called as principal plane. These are two principal planes corresponding to refracted ray.

Difference between O-Ray and E-ray:

O-ray	**E-ray**
It obeys law of refraction.	It does not obey laws of refraction.
Image does not rotate.	Image undergoes rotation.
O-ray lies in the plane of incidence.	E-ray may not lie in the plane of incidence.
Speed is same in all direction	Speed is different in different direction

Nicol prism

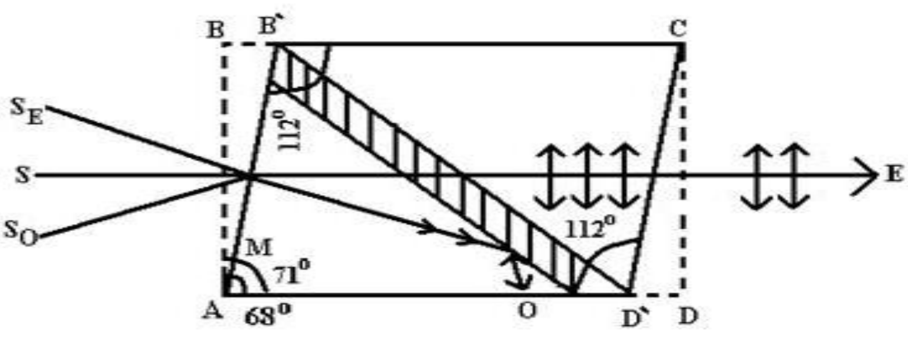


Figure: 21 Schematic of Nicol prism

When an ordinary light is passed through a calcite crystal, it splits into o-ray and E-ray which one plane polarized with plane at polarization perpendicular to each other. A calcite crystal whose length is 3 times of its breadth. The end faces of the crystal are at so as to make angles of 68° and 112° in the principal section instead of 71° & 109°. The crystal is then cut into two pieces from blunt corner to the other along a plane perpendicular to the principal section as well as AB' & CD'. The two cut surfaces are polished optically flat & then centered together with a thin layer of Canada Balsam. The end faces are left transparent while remaining faces are painted black. Canada balsam is a transparent liquid having refractive index $\mu_B$between the refractive indices of calcite for the O-ray (μ_o) & E-ray (μ_E). For sodium light of wavelength 5893 $\dot{A}$, $\mu_E(1.486)$ <$\mu_B(1.55)$ <$\mu_o(1.658)$. The Canada balsam is optically rarer than calcite for O-ray and denser than calcite for E-ray.

Action:- SM⟶unpolarised light is incident as the face AB', it splits up into o-ray & E-ray; the O-ray is incident at an angel more than the critical angel at rarer medium (Canada Balsam). The critical angle θ= $\sin^{-1}$ ($\frac{\mu_B}{\mu_o}$)=69.2^0. The O-ray is totally reflected and absorbed by side AD' which is blackened. The E-ray travels from rarer to denser medium (Canada Balsam) & therefore refracted to emerge from CD' parallel to incident ray. Thus finally Nicol prism transmits only E-ray plane polarized with vibration continued in principal section.

Nicol prism as polarizer and as analyzer:

The Nicol prism can be used both as a polariser and also an analyser. When a ray of unpolarised light is incident on a Nicol prism, then the ray emerging from the Nicol prism is plane polarised with vibration in principal section. As this, ray falls on a second Nicol which is parallel to that of 1st, its vibration will be in the principal section of 2nd and will be completely transmitted and the intensity of emergent light is the maximum, thus the Nicol prism behaves as a polariser.

If the second Nicol is rotated such that its principal section is perpendicular to that of 1st then the vibration in the plane polarisation may incident on 2nd will be perpendicular to the principal section of 2nd. Hence, the ray will behave as a ray inside the 2nd and will lose by total reflection at the balsam surface. If the second Nicol is further rotated to hold its principal section again parallel to that of 1st the intensity will be again the maximum then the 1st prism acts as a polariser and the 2nd prism acts as an analyser.

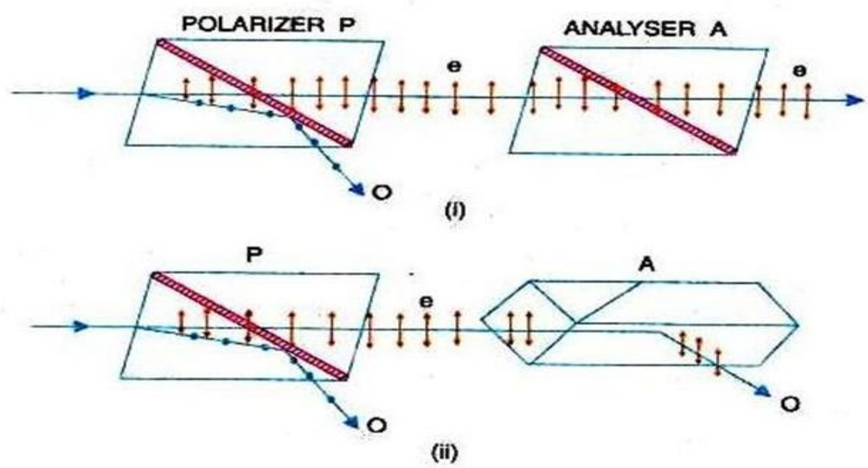


Figure: 22 Nicol prism as polarizer and as analyzer

Retardation plates:

Retardation plates can be made of calcite or quartz using the phenomenon of double diffraction because such a plate retards the motion of one of two refracted beams. There are two practically important phase retardation plates: aquarter wave plate and a half wave plate.

Quarter wave plate (QWP):

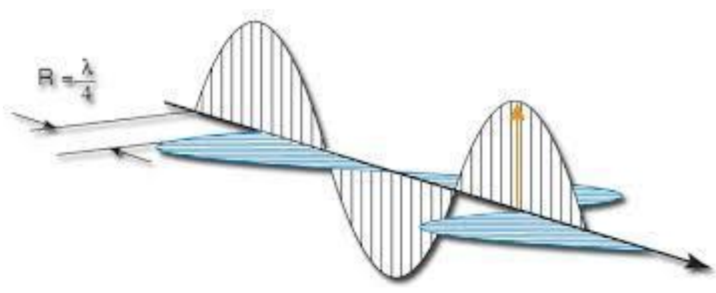


Figure: 23 Quarter wave plate

It is a plate of a double refracting uniaxial crystal (e.g. quartz or calcite) whose refracting faces are cut parallel to the direction of optic axis and whose thickness is such as to produce a phase difference of $\frac{\pi}{2}$ or a path difference of $\frac{\lambda}{4}$ between the O and E rays.

If t is the thickness of the quarter wave plate, μ_O and μ_E are the refractive indices for the O ray and E ray respectively, then for normal incidence in case of a negative crystal like calcite ($\mu_O > \mu_E$) the path difference introduced between the O and E rays = $(\mu_O - \mu_E)t$

But for a quarter wave plate, path difference = $\frac{\lambda}{4}$

For negative crystals, $(\mu_O - \mu_E)t = \frac{\lambda}{4}$

$$t = \frac{\lambda}{4(\mu_O - \mu_E)}$$

For a positive crystal like quartz, $\mu_E > \mu_O$

Hence, thickness of plate,

$$\boldsymbol{t = \frac{\lambda}{4(\mu_E - \mu_O)}}$$

A quarter wave plate is used to produce circularly and elliptically polarized light. When a plane polarized light is incident on a quarter wave plate with its vibrations making an angle of 45^{o} with the optic axis, the emergent light is circularly polarized. But if the vibration of the incident plane polarized light do not make an angle of 45^{o} with the optic axis, the emerging light is elliptically polarized.

Half wave plate (HWP):

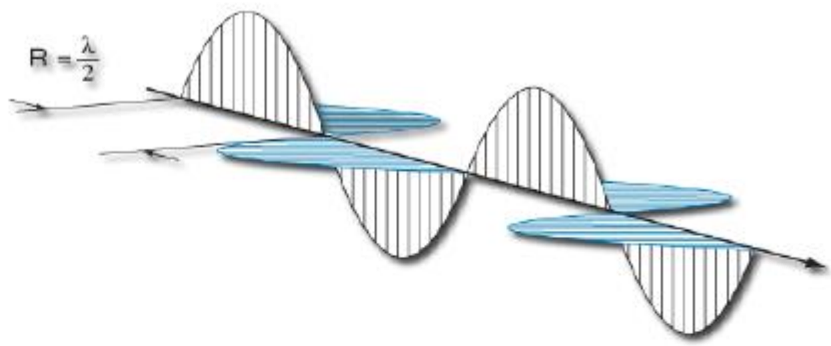


Figure: 24 Half wave plate

It is a plate of doubly refracting uniaxial crystal (e.g. quartz or calcite), whose refracting faces are cut parallel to the direction of optic axis and whose thickness is such as to produce a phase difference of π or a path difference of $\frac{\lambda}{2}$between the O and E waves.

If t is the thickness of half wave plate, μ_O and μ_E are the refractive indices for the O and E rays, then for normal incidence in case of a negative crystal like calcite $(\mu_O > \mu_E)$ the path difference introduced between O and E rays = $(\mu_O - \mu_E)t$

But for a half wave plate, path difference = $\frac{\lambda}{2}$

For negative crystals,$(\mu_O - \mu_E)t = \frac{\lambda}{2}$

$$t = \frac{\lambda}{2(\mu_O - \mu_E)}$$

For a positive crystal like quartz, $\mu_E > \mu_O$

Hence, thickness of plate,

$$\boldsymbol{t = \frac{\lambda}{2(\mu_E - \mu_O)}}$$

When a plane polarized light is passed through a half wave plate, the emergent light is also plane polarized for all orientations of the plate with respect to the plane of vibration of the incident light but if the direction of vibration in the incident light is inclined at an angle θ with the optic axis then the direction of vibration of the emergent light makes an angle 2θ with that of the incident light.

OPEN SOURCE NPTEL

- **https://nptel.ac.in/courses/122/107/122107035/**
- **https://www.youtube.com/watch?v=rZATW8dpiQA**

Question Bank:

Sl. No.	Short Questions
1	Define Simple Harmonic Motion (SHM) and write down the differential equation of motion for SHM.
2	Define amplitude, time period and frequency of a simple pendulum moving under simple harmonic motion.
3	Find the position at which the potential energy is equal to kinetic energy in case of SHM.
4	What is the physical significance of damping coefficient? What is its unit?
5	What is damping coefficient and how it is related to quality factor?
6	Plot the displacement ~ time graph for three types of motion possible for a damped harmonic oscillator.
7	What is resonance and sharpness of resonance? How the sharpness of resonance does depends on damping.
8	Why diffraction cannot occur if slit width is less than the wavelength of the light?
9	Differentiate between transverse wave and longitudinal wave.
10	Define diffraction grating. In a diffraction grating of 2.5 cm have 12500 rulings.
11	What are the differences between the Interference and diffraction patterns?
12	Differentiate between Fresnel and Fraunhofer classes of diffraction.
13	Differentiate between the ordinary and the extra-ordinary rays.
14	Explain briefly the role of Canada balsam layer in a Nicol prism?
15	What is Brewster's angle?
16	Write down the classification of polarization.

17 Define plane of vibration and plane of polarization.

18 Define angle of polarization. Express refractive index in terms of angle of polarization.

19 Discuss the methods (logarithmic decrement, relaxation time and quality factor) for quantitative measurement of damping effect in a damped simple harmonic oscillator.

20 What do you understand by double refraction?

21 Explain damped vibrations, forced vibrations and resonance, giving one example of each.

22 Discuss which source is preferred in Newton's ring's experiments, point source or extended source?

23 What do you mean by grating element? Explain the effect of increasing the number of lines in a grating on the diffraction pattern.

24 Define optic axis and what is an uniaxial crystal

25 What are the different methods of production of polarising light?

Sl. No.	Broad Questions
1	Write the expression for displacement, velocity, acceleration and find the positions of maxima and minima for displacement, velocity, acceleration.
2	Set equation of motion for a particle in SHM. Deduce the expressions for its displacement, velocity and acceleration and plot all these in a graph versus time.
3	Derive the differential equation for a damped harmonic oscillator and discuss under damped, critically damped, over damped oscillations conditions. Represent these oscillations on displacement vs time graph.
4	Set up the differential equation of damped harmonic oscillator. Discuss the critical-damped oscillation. Give a graphical (displacement ~ time) of this oscillation.

5 Obtain the equation of motion of a forced harmonic oscillator starting the different force acting on it. Identify the steady state and transient parts in the general solution to this equation.

6 Obtain the equation of motion of a forced harmonic oscillator starting the different force acting on it. Define the Resonance and find the condition for it in forced harmonic oscillator.

7 With a suitable labelled diagram explain the formation of Newton's ring as interference of light. Derive an expression for the diameter of dark and bright rings in Newton's ring experiment.

8 State and prove Brewster law.

9 State and prove Malus' law of polarization.

10 Write the principle and construction of Nicol prism. With a proper diagram explain Nicol prism work as polarizer and analyser.

11 Obtain the condition for maxima and minima for Fraunhoffer diffraction due to single slit with a labelled diagram.

12 Obtain the condition for principle maxima of Fraunhoffer diffraction due to single slit with a labelled diagram.

Sl. No. **Numericals**

1 The differential equation of motion of a freely oscillating body is given by

$$2\frac{d^2x}{d^2t} + 18\pi^2 x = 0$$

Calculate the natural frequency of the body.

2 In a Newton's arrangement a source of light having two wavelengths 6000 Å and 4500 Å is used. It is found that nth dark ring due to 6000 Å coincides with (n+1) th dark ring due to 4500 Å. Calculate the radii of nth dark rings 6000 Å and 4500 Å, if radius of curvature of plano-convex lens is 100 cm.

3 Find the angle between the axis of an analyzer with incident beam so that the intensity of the polarized light is **25%** less than that of initial intensity.

4 A glass plate having refractive index **1.66** is to be used as a polarizer. Calculate polarizing angle and angle of refraction.

5 The total energy of one dimensional simple harmonic oscillator is 0.8 erg. What is its kinetic energy when it is midway between mean position and extreme position?

6 Two simple harmonic motions are represented by $\boldsymbol{y = a \sin(\omega t - kx)}$ and $\boldsymbol{y = a \cos(\omega t - kx)}$ respectively. Find the phase difference between them.

7 A plane diffraction grating of width 4 cm has slits each of width 0.0001 cm separated by a gap of 0.0002 cm. Find the grating element of diffraction grating.

8 The equation of a particle executing simple harmonic motion is $x = 3\sin\left(2\pi t + \frac{\pi}{3}\right)$, where x is in meter and t is in seconds. Find amplitude, time period.

9 A plane diffraction grating of width 2.5 cm has 15000 ruling on it. Monochromatic light of wavelength 5893 Å is incident normally in it. Find the angle at which second order principal maximum occur.

10 The displacement of particle performing simple harmonic motion is given by,

y = 3sin2t + 4cos2t. Find the time period and amplitude.

11 Calculate Brewster's angle for a glass slab (n=1.5) immersed in water (n=1.33).

12 A plane diffraction grating of width 2.5 cm has 15000 ruling on it. Monochromatic light of wavelength 5893 Å is incident normally in it. Find the angle at which second order principal maximum occur.

13 Considering quality factor of sonometer wire of frequency 260 Hz as 2000, calculate the time in which the amplitude decreases to $1/e^2$ of its initial value.

14 A grating having 15000 lines per inch produces spectra of a mercury arc. The green line of the mercury spectrum has a wavelength of 5461 Å. What is the angular separation between the first order and second order green line?

15 The diameter of 10th dark ring for a light of wavelength 540 nm is found to be 0.54 cm. Find the radius of curvature of the plano-convex lens used.

16 Monochromatic light from He-Ne laser (λ= 5893Å) is incident normally on a diffraction grating 6000 lines per cm. Find the angle at which one would observe the first order and secondordermaxima.

17 Two Nicols are oriented with their principal planes making an angle of 30°. What percentage of light will pass through the system?

18 Two Nicols have parallel polarising directions so that the intensity of transmitted light is maximum. Through what angle must either Nicol be turned if intensity is to drop by one-fourth of its maximum value?

19 In a Newton's arrangement, the diameter of a bright ring is 0.5 cm. What would be the diameter of the ring if the lens placed on the plane glass plate is replaced by another one having double the radius of curvature?

20 In Newton's rings with reflected light are observed between a planoconvex lens of radius of curvature of 120 cm and a plane glass plate. The diameter of the 15th bright ring is 0.58 cm. Calculate the diameter of the 25th bright ring and the wavelength of the light used.

21 In a Newton's arrangement, the diameter of a bright ring is 0.5 cm. What would be the diameter of the ring if the lens placed on the plane glass plate is replaced by another one having double the radius of curvature?

22 A transmission grating has 8000 rulings per cm. The first order principal maximum due to a monochromatic source of light occurs at an angle of 30°. Determine the wavelength of light.

23 The diameter of 10th dark ring for a light of wavelength 540 nm is found to be 0.54 cm. Find the radius of curvature of the plano-convex lens used.

24 Find the angle between polarizer and analyzer so that intensity become half of its initial value.