DEPARTMENT OF MATHEMATICs

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MATHEMATICS - I

TEXT BOOK: DIFFERENTIAL CALCULUS BY SHANTI NARAYAN & P.K.MITTAL

LECTURE - 2

Meaning of the Sign of Derivatives of Lagrange's mean value theorem(Chapter-8.3 & 8.4)

DEPARTMENT OF MATHEMATICS, CVRCE

Ex-1: Show that $f(x) = x^3 - 3x^2 + 3x + 2$ is strictly increasing in every interval.

Solution

We have $f(x) = x^3 - 3x^2 + 3x + 2$.

Then $f'(x) = 3x^2 - 6x + 3 = 3(x-1)^2$

Thus f '(x) > 0 for every value of x except one point i.e 1, where it is zero.

Hence the given function is strictly increasing in every interval.

Ex-3: Show that

$$x/1+x<\log(1+x)<x \qquad \forall x>0$$

Solution

Now

$$f(x)=\log(1+x)-\frac{x}{(1+x)}, \qquad x>0$$

$$\Rightarrow f'(x)=\frac{1}{1+x}-\frac{1}{(1+x)^2}=\frac{x}{(1+x)^2}$$

$\Rightarrow f'(x)>0 \quad \forall x>0$ and is 0 for x = 0.

$\Rightarrow$ f is strictly increasing in $[0, \infty)$,

Also f(0)=0.

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It follows that $f(x) > f(0) = 0 \quad \forall x > 0$

$$\Rightarrow \log(1+x) > \frac{x}{1+x} \qquad \ldots(i)$$

Again $F(x) = x - \log(1+x), \quad x > 0$

$$\Rightarrow F'(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x}.$$

$\Rightarrow F'(x) > 0 \quad \forall x > 0$ and is 0 for x=0.

$\Rightarrow F(x)$ is strictly increasing in the interval $[0, \infty)$

Also F(0)=0

Thus $F(x)>F(0)=0 \Rightarrow x > \log(1+x) \quad \forall x>0 \qquad \ldots(ii)$

Thus from (i) and (ii) we get the required result.

Ex-5: Show that for any $x > 0$

$$1+x < e^x < 1+xe^x .$$

Solution

Take $f(x) = e^x - (1+x)$. Then $f'(x) = e^x - 1 > 0$ for any $x>0$,

So f is an increasing function and $f(0)=0$.

Therefore $f > 0$ for any $x > 0$

i.e. $e^x - (1+x) > 0$ or $1 + x < e^x$ ………(1)

Similarly choose $g(x) = 1 + xe^x - e^x$

Then $g'(x) = xe^x \geq 0$ for any $x > 0$ and $g(0)=0$

Thus g is an increasing function and therefore $g(0) > 0$

i.e. $1 + xe^x - e^x > 0$ or $e^x < 1 + xe^x$……. (2)

Results (1) and (2) form the required inequality.

Question: Without solving the equation prove that the equation $x^5 + 10x + 3 = 0$ has exactly one root.

- Assume it has two solution a and b. so f(a)=f(b)=0 then by Rolle's theorem for some c ε [a,b], f'(c)=0. but f'(c)=$5x^4 + 10 > 0$. hence contradiction.
- But f(-1)=-8 and f(0)=3. thus there is a c ε[-1,0]:f(c)=0.
- From the two statement we prove that the equation has one real root.

Thank You