VAR 99-Alpha Analysis under Merton Model, CRE Loan Portfolio (hotel)

For many private capital funded CRE lenders, loan Value At Risk (VAR) analysis is seldom studied due to the limit of the size N of the number of loans held in the portfolio P of the firm. Traditional VAR analysis is conducted over a pool of thousands of loans where traditional statistical distributions with parameters can be measured with sufficient statistical accuracy and which can be applied to estimate the probability of a debtor default over a defined time period T and where the number of loans is sufficient enough to achieve debtor diversification and minimization of debtor default correlation.

The purpose of this study is to estimate the VAR of a loan portfolio of CRE assets, specifically hotel properties under traditional loan underwriting assumptions prevalent in the private lender subspace of commercial real estate finance. Assumptions will be explicitly stated and which can be relaxed as part of ongoing investigation in default risk of the loan portfolio in subsequent studies.

Portfolio Assumptions

We make the following assumptions explicitly, noting these are on average consistent with private equity funded CRE loans.

$N = 6$, number of loans in portfolio
ltv is 80% is the loan to value of each loan
P_n is the value of the asset at time t=0 (time of loan n issue) [$90MM,$150MM]
L_n is the value of the loan size at par assuming no OID
$timeperiod$ or T is the amount of time until maturity or refinance, 2 years

Merton Model for Asset movement

The Merton model makes use of the well known equation that the movement of an asset exhibits a brownian motion with drift and diffusion such that the diffusion is a wiener process. Otherwise, we restate the well known logarithmic change in the asset as $dA_t = \mu A dt + \sigma A dwt$ We note that dwt is $N(0, \Delta t)$. If we assume that $f(x)$ is C^2 on the domain of x then the Ito integral solution to $f(A_T)$

$A_T = A_t e^{(\mu - \sigma^2/2)(T-t) + \sigma(w_T - w_t)}$

The reader is encouraged to consult any textbook on stochastic integrals as the proof of this analytical closed form solution is beyond the scope of this paper.

Thus the basis of the Merton approach to the VAR estimation is to simulate the progression of the asset A_i over a time period T and define the following stochastic indicator function as a default event such as:

$I_D = 1$ with $P(A_t <= K)$ otherwise 0 with $(1 - P(A_t <= K))$

The indicator function thus will yield the value of 1 per simulation run if the asset value of the property falls under a specific value K. Under this definition, we define the value of K to be the principal book value of the loan that is collateralized by the asset A. In principle, this simply states that if the value of the asset falls below the value of the loan during time period T, the loan will be in default and a loss will occur. This is a very usual and customary definition of a loan default and thus satisfies the basis of this simulation study.

We define the loss function $L_D = \sum_i L_i * I_{D_i} \ \forall i \in N$

One critical assumption we make in this simulation is that the wiener processes that serves as the basis of the diffusion of the asset process are unique to each asset. In this case, the diffusions of the assets are not correlated and thus in keeping with our original assumption that the assets are not correlated as part of this study and the default of one property shall not trigger the default of another property. We can simply enforce this assumption by restricting the set of assets in this study to reside in different geographical MSAs.

Thus the $E(I_D) = P(A_t <= K) \ \forall t \in T$.

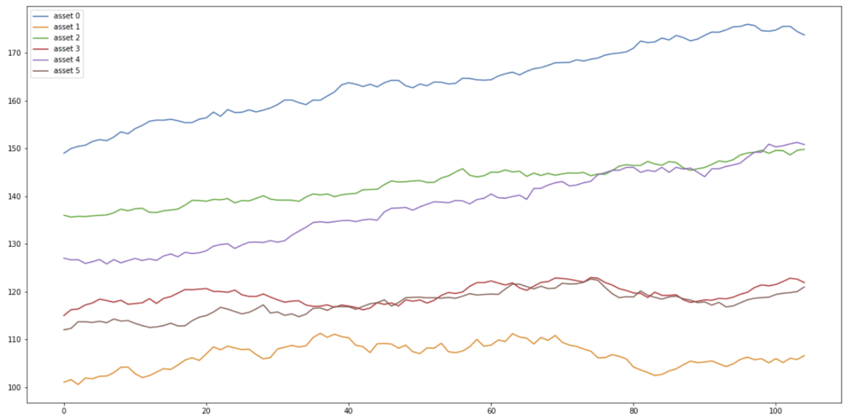
The final component prior to implementation of the Merton asset simulation for the VAR analysis is estimation of the Merton input parameters for Asset movement, namely μ and σ. A proxy for estimation of the asset prices of a hotel asset is the movement of its equity price, which claim is satisfied if the firm has no leverage and thus the entirety of the equity value is a claim on the value of the assets. Under this assumption and for the purposes of this study, we select a publicly traded firm most closely related to the assets held in the portfolio, and in this case this would be a hotel REIT. Upon cursory inspection we note there are several publicly traded REITs, and thus selection of the proper reit for parameter estimation is an exercise in review of the assets held by the reit most closely resembling the portfolio. We elect to choose Apple Hospitality REIT (ticker: APLE) and select the time period of study from 1/1/2019 - 1/1/2020 to negate the effects of COVID-19 on asset volatility.

In order to measure the asset volatility, we compute the log 1 day change in the asset price movement over the 2019 daily closing values of the APLE reit and compute the sample period standard deviation of the delta log values. The estimate of σ is then:

$\sigma = \frac{samplestd}{\sqrt{(T)}}$ where $T = \frac{1}{252}$ since the price changes are daily.

Thus the estimate of σ is found to be 15.25% and we note this is an annual figure.

In order to estimate asset drift, we utilize the expected cap rates of hotel assets within the MSAs selected, and set the value of the supremum of the cap rate of hotel assets to 9%, consistent with cap rates for hotel assets that the industry has averaged over the 2019 period.



Simulation of the asset price movements via geometric brownian motion

Simulation and Results

The Merton simulation of the hotel assets then proceeds in a straight forward manner. First for each asset price a starting value is computed P_i, and from which a principal loan value is calculated using an 80% ltv. The μ and σ are randomly sourced from [.06,.09] and [.15,.40] respectively and are deterministic for purposes of the Merton model. The model then proceeds to compute the asset price over the time period of 2 years and over 1,000 simulations. Loss is computed via the loss function L_D if the value of the asset A_i falls below K_i and the loss amount is the notional principal amount of the loan. For purposes of this simulation we note the loans are non-amortizing and are repaid in full upon maturity, consistent with construction loans for hotel properties.

Using the stated assumptions with an 80% ltv and given the domains of σ and μ we note that the 90% VAR is \$0.0, and thus at current underwriting terms an 80% ltv exhibits extremely low risk assuming the value of the hotel properties follows a geometric brownian motion with constant drift and difussion over the time period the loan is outstanding.

Naturally an extension of this analysis is then to stress various ltv and diffusion values to determine the impact on these variables on the VAR. This analysis was completed and the VAR 99.5% results indicate that at ltv 85% with a σ of 0.30 the 99.5% VAR is still \$0, whereas at 80% ltv the VAR 99.5% remains \$0 until a σ of 0.35.

The results of this analysis indicate that lenders can increase leverage up to 85% without significant VAR loss at the 99.5% confidence level in order to be more competitive with respect to construction loan financing. An extension of this risk could be to require loan documents to covenant that if volatility of the asset, which is a measurable value, increases over a certain threshold, the debtor will be required to provide additional collateral securing the loan to bring back the ltv to 80%. In this way the debtor realizes an increased leverage and thus maximizes her return to equity while the lender realizes incrementally negligible risk given low VAR 99.5% rates at the 85% ltv.

One notes that a natural extension of this analysis would be re-compute the VAR analysis assuming a non-zero asset movement correlation among the property assets A_i and subsequent default correlations among the assets. In this case the expectation is the tail risk of default would be increased given the default dependence and as a result would yield a more conservative analysis under the VAR model.