Problem Set #2

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Question 1

In this question you are required to build a classification model for CIFAR-10 visual dataset. I have uploaded to the Moodle two resources:

- get_cifar.sh - a shall script to download the data.
- read_cifar.ipynb - a notebook that reads the data.

You are required to present two models: (i) using fully connected layers only and (ii) combining convolution layers and fully connected layers. Analyze the performance of your models.
Notes: (i) For both models you may use Batchnorm and Dropout layers. (ii) The models should be written in PyTorch.

Question 2

A data follows the following model for a time series:

- Use the notebook get_GNP_data.ipynb to get the data
- Use AR model on rgnp to predict the last 10 observations. Adjust the number of lags to consider using any model selection criteria you choose.
- Use VAR model (vector auto-regressive) to predict the last 10 observations of the data
- Replace the prediction models in the last two sections with corresponding LSTM models.

The LSTM models should be implemeted in PyTorch. You should optimize the hidden state size using validation set.
Hint: For the regression model you may prefer to work on derivatives of the series. You can check the order of derivation using Augmented Dickey-Fuller test.

Question 3

A function is called *equivariant* to transformation T if $f(T(x)) = T(f(x))$. A function is called *invariant* to transformation T if $f(T(x)) = f(x)$. In the above definitions x may be a vector.

3.a

Prove that two-dimensional convolution is equivariant to the shift transformation, i.e., $T : x \to x + c \ \ \forall c$.

3.b

Given an image I of size W, H, and a filter F of size d_x, d_y. Give a necessary and sufficient condition for the I to be equivariant to convolution with F

3.c

Is MAX-POOLING 3×3 shift invariant to one pixel shift? Assume you apply the MAX-POOLING with stride 3 so the windows are disjoint.

3.d

Given inputs $x_1, x_2, \cdots, x_d$ to a fully connected layer give a sufficient condition for the layer to be permutation invariant.
Bonus: Find a necessary and sufficient condition for permutation invariance.

Question 4

In this question you will generate data using Kalman filter and do the prediction with an LSTM.
A Kalman filter is a model for which the following holds:

$$X_{k+1} = AX_k + \epsilon_{1,k+1}$$
$$Y_{k+1} = BX_{k+1} + \epsilon_{2,k+1}$$

Where X_k is the unobserved state vector, Y_k is the observation, A is the state dynamic model, and B is the observation model. The variable ϵ_1 is called the process noise and ϵ_2 is called observations noise, where $\forall i\ \epsilon_{1,i} \sim \mathcal{N}(0, \sigma_1^2)$ and $\epsilon_{2,i} \sim \mathcal{N}(0, \sigma_2^2)$.

4.a

Generate data from the Kalman filter with the following parameters:

$$A = \begin{bmatrix} \frac{\sqrt{99}}{10} & -\frac{1}{10} \\ \frac{1}{10} & \frac{\sqrt{99}}{10} \end{bmatrix} B = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{2}{2} \\ \frac{\sqrt{(2)}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$
$$\sigma_1 = 0.01, \sigma_2 = 0.2$$

Assume that the initial state is the vector $[1, 0]$, and the length of the process is 2000 steps. Use the first 1000 steps as a training data and the last 1000 steps as the test data. Note that for the train you see both X, Y, whereas for the test you only sees Y.

4.b

Use LSTM to predict the next observation.

4.c

Use LSTM to infer the value of the hidden variable X.

4.d

Bonus: Use a Kalman filter package to infer X and compare your result.