Assignment 7

Probability and Random Processes

MA6.102 - Monsoon 2022

Date: November 9, 2022 **Deadline: Nov 2022**

Topics: **Law of Large Numbers, Central Limit Theorem** **Marks: 100**

Instructions:

- Answer all the questions.
- Clearly state the assumptions made (*if any*) that are not specified in the question.
- For analytical problems, write your answers on A4 sheets and scan them in pdf format.
- For the simulation problems, please write a single script for each of the two questions in MATLAB. In addition to codes, please provide a report (in pdf format) including the analytical description and clearly depicting the generated plots with appropriate labels. Submit a zipped folder (named as 'Rollnumber_A5') containing the analytical solution pdf, report pdf and the two scripts.
- Do not copy from your peers or online. Copied assignments will fetch **zero** marks.

Problems

1. A bank models the number of loans defaulted by borrowers as a Poisson random variable. The average number of loans defaulted is 200 per month. Consider the number of loans defaulted in each month to be independent. Find the probability that the number of loans defaulted exceeds 24000 in a decade.

2. There are 256 students in UG2k19 batch. The Advanced Computer Systems course has 40 seats exclusively for UG2k19 students. If the probability of selecting the course for any student is independent and equal to $\frac{1}{8}$, what is the probability of students that more students apply for the course than its capacity? (Use CLT with continuity correction.)

3. Consider n rolls of a die with X_i denoting the outcome of the i^{th} roll. Let $S_n = \sum_{i=0}^{n} X_i$. Show that, $\forall \epsilon > 0$:

$$\lim_{n \to +\infty} P(|S_n - \mu| \geq \epsilon n) \to 0$$

   Also, find μ.

4. You pick one card from a deck of cards n times. Let X be the ratio of times black suit cards appear. How large does n have to be so that you are 98% sure that $0.45 \leq X \leq 0.55$?

Simulation Problems

5. a) Let $X_1, X_2, ..., X_n$ be $i.i.d$ Bernoulli random variables with parameter 0.4. Perform the following steps in MATLAB and note down your observations.

    1. Generate a sample of 1000000 Bernoulli variables. This will be our population for the experiment.
    2. Let the sample size be $n = 100$, collect samples from the population 61 times and compute the mean each time. This will give a distribution on the sample-mean. Store all the means generated.
    3. Repeat step-2 by increasing the sample size by 400 each time until the sample size reaches 10000 and store the distribution of mean each time.
    4. Plot the sample-mean using a box-plot for every sample size.

   b) Repeat the same for γ-distribution with scale 1 and shape 3 and note down your observations.

6. 1. Let $X1, X2, ...Xn$ be $iid\ Uniform(0,1)$ random variables. Plot the distribution of $\frac{X1+X2+...Xn-n\mu}{\sqrt{n\sigma^2}}$ for $n = 2, 3, 10, 30$ in a single plot using subplot in MATLAB. Where μ is the mean and σ^2 is the variance of a Uniform(0,1) random variable.
   2. Repeat the same experiment for a binomial random variable.

7. Plot the multivariate Gaussian distribution for $\mu = [0, 0]$ and K. Compare your result with the distribution obtained by the using the in-built function mvnpdf. Do this for the following values of K

   1. $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   2. $K = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
   3. $K = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$
   4. $K = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 4 \end{bmatrix}$