Assignment 4

Probability and Random Processes

MA6.102 - Monsoon 2022

Date: September 12, 2022 **Deadline: 21 Sep 2022**

Topics: **Random Variables** **Marks: 100**

Instructions:

- Answer all the questions.
- Clearly state the assumptions made (*if any*) that are not specified in the question.
- For analytical problems, write your answers on A4 sheets and scan them in pdf format.
- For the simulation problems, please write a single script for each of the two questions in MATLAB. In addition to codes, please provide a report (in pdf format) including the analytical description and clearly depicting the generated plots with appropriate labels. Submit a zipped folder (named as 'Roll-number_A4') containing the analytical solution pdf, report pdf and the two scripts.
- Do not copy from your peers or online. Copied assignments will fetch **zero** marks.

Problems

1. It's Arjo's birthday and he invites N friends to Tantra canteen for a treat. His friends sit around a table and each person has ordered either fried-rice or noodles with 40% and 60% probability respectively. (Assume Arjo is popular and has more than 5 friends)

   a) Since they are all friends, each person can also eat some of the food ordered by their neighbors. Taking X to be the number of people who can eat both types of food, find $E[X]$ and $Var[X]$

   b) A person can access a good quantity of a food item if either i) *they ordered the food item* or ii) *both neighbors have ordered the food item.* Taking Y to be the number of people who can eat a good quantity of both food types, find $E[Y]$ and $Var[Y]$.

2. There are N students in UG2k21 batch of IIIT-H. We assume the birth month of each person in the batch to be distributed uniformly. Let X and Y be the number of people with their birth month as November and December respectively. Find $Cov(X, Y)$ and $\rho(X, Y)$.

3. Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables, where each X_i is a Bernoulli r.v. with parameter p.

   a) Let $Y_1 = X_1X_2, Y_2 = X_2X_3, \cdots, Y_n = X_nX_1$ and $Y = \sum_{i=1}^{n} Y_i$. Find $E[Y]$ and $Var(Y)$.

   b) Let $Z_1 = max(X_1, X_2), Z_2 = max(X_2, X_3), \cdots, Z_n = max(X_n, X_i)$ and $Z = \sum_{i=1}^{n} Z_i$. Find $E[Z]$ and $Var(Z)$.

   c) Find $Cov(Y, Z)$ and $\rho(Y, Z)$.

4. Consider the following set of points P:

$$P = \{(x, y) | x, y \in \mathbb{Z}, x^4 + y^{\frac{1}{4}} \leq 2\}$$

   Let (X,Y) denote an element picked from this set. Suppose that an element of this set is selected at random with uniform probability then answer the following questions:

a) find joint PMF of X,Y.
b) find marginal PMF of X and marginal PMF of Y.
c) find conditional PMF of X given Y=8.
d) Verify whether X,Y are independent.
e) find E[XY].

5. Consider i.i.d random variables $X_1, X_2, ..., X_n$. Find the distribution of:

   a) $min(X_1, X_2, ..., X_n)$
   b) $max(X_1, X_2, ..., X_n)$
   c) if $X_1, X_2, ..., X_n$ are independent Gaussian random variables with parameters $\mu_1, \mu_2, ..., \mu_n$ and $\sigma_1, \sigma_2, ..., \sigma_n$ then what is the distribution of $X = \sum_{i=1}^{n} X_i$?

6. You are given the following information about different parts of a computer:

   - The lifetime of hard-disk is an exponential random variable with parameter 1/2.
   - The lifetime of RAM is an exponential random variable with parameter 1/3.
   - The lifetime of motherboard is an exponential random variable with parameter 1/5.

   Assuming that each part fails independently of the other what is the expected time before the computer fails?

Simulation Problems

7. Write a Matlab code for generating a random variable for the following distributions

   1. Exponential
   2. Rayleigh
   3. Gaussian
   4. Laplace