Assignment 8

Probability and Random Processes

MA6.102 - Monsoon 2022

Date: November 16, 2022 **Deadline: 30 Nov 2022**

Topics: **Random Processes** **Marks: 100**

Instructions:

- Answer all the questions.
- Clearly state the assumptions made (*if any*) that are not specified in the question.
- For analytical problems, write your answers on A4 sheets and scan them in pdf format.
- For the simulation problems, please write a **single script** for each of the three questions in MATLAB. In addition to codes, please provide a report (in pdf format) including the analytical description and clearly depicting the generated plots with appropriate labels. Submit a zipped folder (named as 'Rollnumber_A8') containing the analytical solution pdf, report pdf and the two scripts.
- Do not copy from your peers or online. Copied assignments will fetch **zero** marks.

Problems

1. Let $\{X(t), t \in \mathbb{R}\}$ be a WSS process. The time average mean is defined as

$$\hat{\mu}_X = \frac{1}{2T}\int_{-T}^{T} X(t)dt$$

The process $X(t)$ is said to be mean ergodic if $\hat{\mu}_X$ converges to μ_X as $T \to \infty$.
Let $A_0, A_1, A_{-1}, A_2, A_{-2}, ...$ be a sequence of *i.i.d* random variables with mean $\mu < \infty$. Let $X(t)$ be defined as

$$X(t) = \sum_{k=-\infty}^{\infty} A_k g(t-k)$$

where $g(t)$ is given by

$$g(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$$

Show that $X(t)$ is mean ergodic.

2. a) Let $X(t)$ be a Gaussian process with $\mu_X(t) = 2t$ and $R_X(t_1, t_2) = 1 + 3t_1t_2$ for all $t, t_1, t_3 \in \mathbb{R}$. Find $P(X(1) + X(2) < 3)$

   b) Let $X(t)$ be a WSS Gaussian process with $\mu_X(t) = 1$ and $R_X(\tau) = 1 + 4sinc(\tau)$. Find $P(1 < X(1) < 2, X(2) < 3)$

Simulation Problems

3. After finding out that an assignment is going to be released in the last week of the course, a student got tensed. To ease his tension he started walking. Let this process be modelled as $X[n] = \Sigma_{i=0}^{n} R[i]$. Here, $R[n]$ is an independent and identically distributed random process with pmf as $p(-1) = p(1) = \frac{1}{2}$.

   - Write a MATLAB script to generate a realization of this random process. Plot the graph for 100 samples i.e. for $n = 1, 2, \ldots, 100$.
   - Is the process stationary? Argue by looking at the graph.
   - Find the mean function $E[X[n]]$ and covariance function $c_X[n, n+k]$ in the same script. Is the random process wide-sense stationary? Justify your answer.

4. Let $X[n] = 2W[n] - 4W[n-1]$ be a random process, where $W[n] \sim \mathcal{N}(0, 1)$.

   - Write a MATLAB script to generate a realization of this process. Plot the graph for 1000 samples i.e. for $n = 1, 2, \ldots, 1000$.
   - Then, whiten/decorrelate $X[n]$ to obtain $d[n]$.
   - Verify that $X[n]$ is a WSS process by calculating the mean and auto-correlation function.
   - The generated $X[n]$ is passed through an LTI system with impulse response $h[n] = sin(\frac{\pi}{4}n)$ to obtain $y[n]$. Plot $y[n]$. Find the mean and auto-correlation function of $y[n]$. What do you infer?

5. Let $\{X[n], n \in \mathbb{Z}\}$ be a discrete time random process defined as

$$X[n] = \sum_{k=0}^{N} A_k n^k$$

   where $A_0, A_1, ..., A_N$ are standard normal variables and N is a fixed positive integer.

   1. write a MATLAB script to generate realization of this process and plot the process for $n = 0, 1, ..., 2017$ and $N = 3$.
   2. repeat 1 for $N = 2, 4, 5, 6, 7$ and for $n = 0, 1, ..., 1000$.
   3. plot the mean function of $X[n]$ for $n = 0, 1, ..., 1000$ and $N = 3$.
   4. plot the auto-correlation function of $X[n]$ using a surface-plot. Use $n_1 = 0, 1, ..., 1000$ and $n_2 = 0, 1, ..., 1000$ and $N = 3$.
   5. generate the mean and auto-correlation function for different values of N.
   6. is this a WSS process. How can you determine this by looking at the graphs you generated?

6. Generate a zero-mean unit variance white GAussian random process $X(t)$. Analytically find the auto-correlation of the output $Y(t)$.