Assignment 5

Probability and Random Processes

MA6.102 - Monsoon 2022

Date: October 10, 2022 **Deadline: 17 Oct 2022**

Topics: **Functions of Random Variables** **Marks: 100**

Instructions:

- Answer all the questions.
- Clearly state the assumptions made (*if any*) that are not specified in the question.
- For analytical problems, write your answers on A4 sheets and scan them in pdf format.
- For the simulation problems, please write a single script for each of the two questions in MATLAB. In addition to codes, please provide a report (in pdf format) including the analytical description and clearly depicting the generated plots with appropriate labels. Submit a zipped folder (named as 'Roll-number_A5') containing the analytical solution pdf, report pdf and the two scripts.
- Do not copy from your peers or online. Copied assignments will fetch **zero** marks.

Problems

1. Five men and seven women go to prom. They start dancing and get jumbled up. After the prom, they all go to their hostels in a line. How many men are immediately followed by a woman on average? (Assume that all the orders obtained after jumbling are equally likely.)
2. Let X and Y be two independent Gaussian random variables with mean μ and variance σ^2.
   (i) Find the pdf of $U = \sqrt{X^2 + Y^2}$.
   (ii) For $\mu = 0$, find the joint pdf of $U = \sqrt{X^2 + Y^2}$ and $V = \frac{Y}{X}$
3. Let X_1 and X_2 be bi-variate normal random variables with correlation ρ, given by $X_1 \sim Normal(\mu_1, \sigma_1^2)$ and $X_2 \sim Normal(\mu_2, \sigma_2^2)$, with corresponding log-normal distributions $Z_1 = exp(X_1)$ and $Z_2 = exp(X_2)$.
   (i) Show that $Z_1 Z_2$ is also log-normal.
   (ii) Find the parameters corresponding to $Z_1 Z_2$.
   (iii) How would the parameters change if X_1 and X_2 are independent?
4. Severus Snape, the Potions master loathes Harry. Knowing that Harry does not know about probability distributions, he asks the following questions.
   (i) If you have a uniform random variable U[0, 1], does e^U give you an exponential distribution? If yes, prove it. If not, think of a way to obtain exponential distribution from uniform distribution.
   (ii) If you have a log-normal random variable X, does e^X give you a normal distribution? If yes, prove it. If not, think of a way to obtain normal distribution from log-normal distribution.

(iii) Is there a way of obtaining Bernoulli distribution from Poisson distribution? If it is directly not possible, suggest a good approximation.

(iv) Is there a way of obtaining Poisson distribution from Bernoulli distribution? If it is directly not possible, suggest a good approximation.

(v) What would be the resultant distribution if you have a sum of exponential random variables? What would be the resultant distribution if you take the minimum of two exponential random variables?

(vi) What would be the resultant distribution if you have a sum of chi-squared random variables?

(vii) How would you obtain chi-squared distribution from gamma distribution?

(viii) Is there a way of obtaining gamma distribution from chi-squared distribution? Why or why not?

Snape is looking for an opportunity to give Harry a detention. He expects proper proof for every claim and would not accept a simple yes or no answer. Help Harry out.

5. Let X and Y be $i.i.d$ Gaussian random variables with mean μ and variance σ^2. If $Z = X + Y$ and $W = X - Y$, then find joint pdf of Z and W and also determine whether they are independent or not.

6. (i) Let X be an exponential random variable with parameter λ. Find the pdf and cdf of $Y = lnX$.

(ii) Let X be a Rayleigh distributed variable with parameter σ. Find the pdf of $Y = X^3$.

Simulation Problems

(i) Consider X and Y to be two exponential random variables. Consider $Z = X + Y$. Verify that the pdf of Z is the convolution of the pdfs of X and Y.

(ii) Let X be a complex Gaussian random variable. Find the pdf and cdf of $|X|$ through simulation and verify them using analytically found pdf and cdf.